Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

7-Inverse-hyperbolic-functions/7.3-Inverse-hyperbolic-tangent/192-7.3.2-d-x-^m-a+b-arctanh-c-x^n-^p

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [243]. This is test number [192].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (243)	0.00 (0)
Mathematica	95.06 (231)	4.94 (12)
Maple	84.77 (206)	15.23 (37)
Maxima	63.37 (154)	36.63 (89)
Fricas	60.49 (147)	39.51 (96)
Mupad	52.67 (128)	47.33 (115)
Giac	52.26 (127)	47.74 (116)
Sympy	34.57 (84)	65.43 (159)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

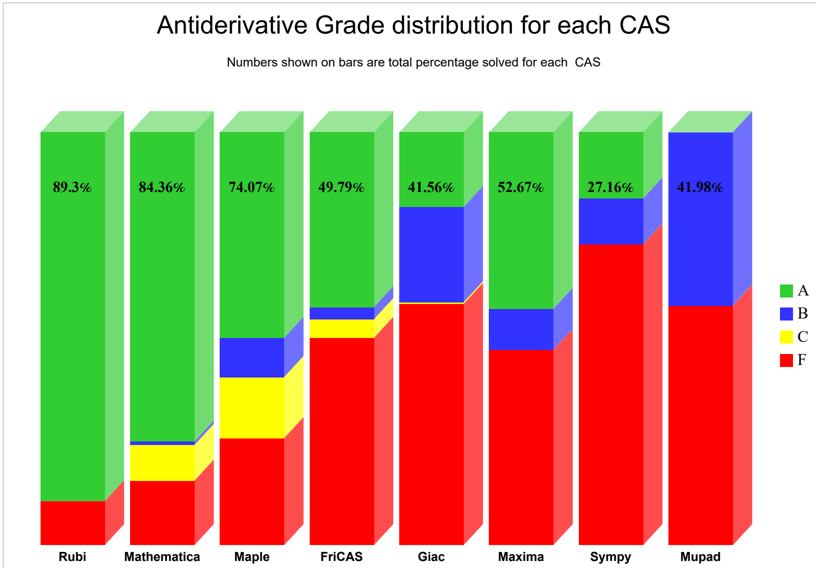
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

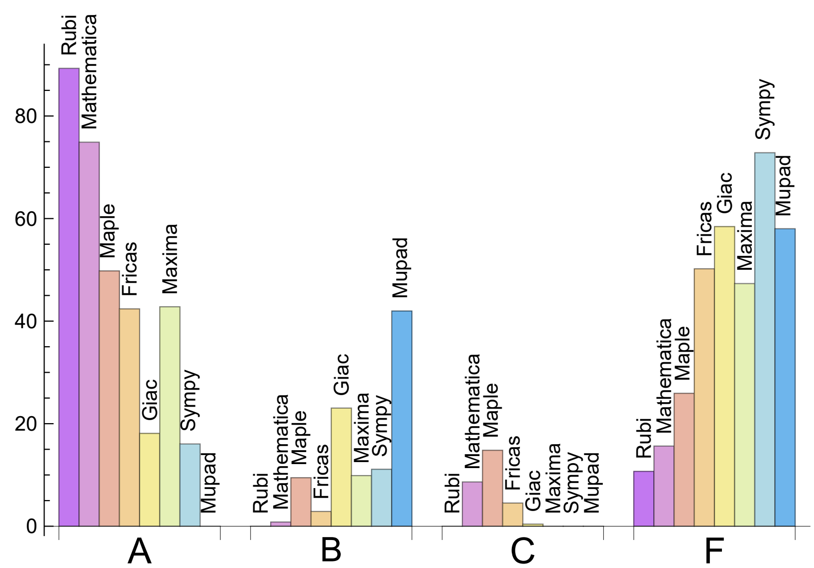
System	% A grade	% B grade	% C grade	% F grade
Rubi	88.066	1.235	0.000	10.700
Mathematica	74.897	0.823	8.642	15.638
Maple	49.794	9.465	14.815	25.926
Maxima	42.798	9.877	0.000	47.325
Fricas	42.387	2.881	4.527	50.206
Giac	18.107	23.045	0.412	58.436
Sympy	16.049	11.111	0.000	72.840
Mupad	0.000	41.975	0.000	58.025

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	12	100.00	0.00	0.00
Maple	37	100.00	0.00	0.00
Maxima	89	100.00	0.00	0.00
Fricas	96	100.00	0.00	0.00
Mupad	115	0.00	100.00	0.00
Giac	116	100.00	0.00	0.00
Sympy	159	69.18	30.82	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.28
Giac	0.38
Maxima	0.38
Mathematica	0.51
Rubi	0.81
Mupad	3.96
Sympy	10.94
Maple	11.26

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	91.72	1.32	59.00	1.12
Mathematica	129.12	1.23	99.00	1.15
Maxima	157.56	2.93	97.00	1.13
Fricas	161.63	1.88	69.00	1.40
Giac	163.75	2.47	109.00	1.41
Sympy	236.46	4.13	69.00	1.26
Rubi	247.66	1.07	90.00	1.01
Maple	406.94	3.32	93.00	1.07

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

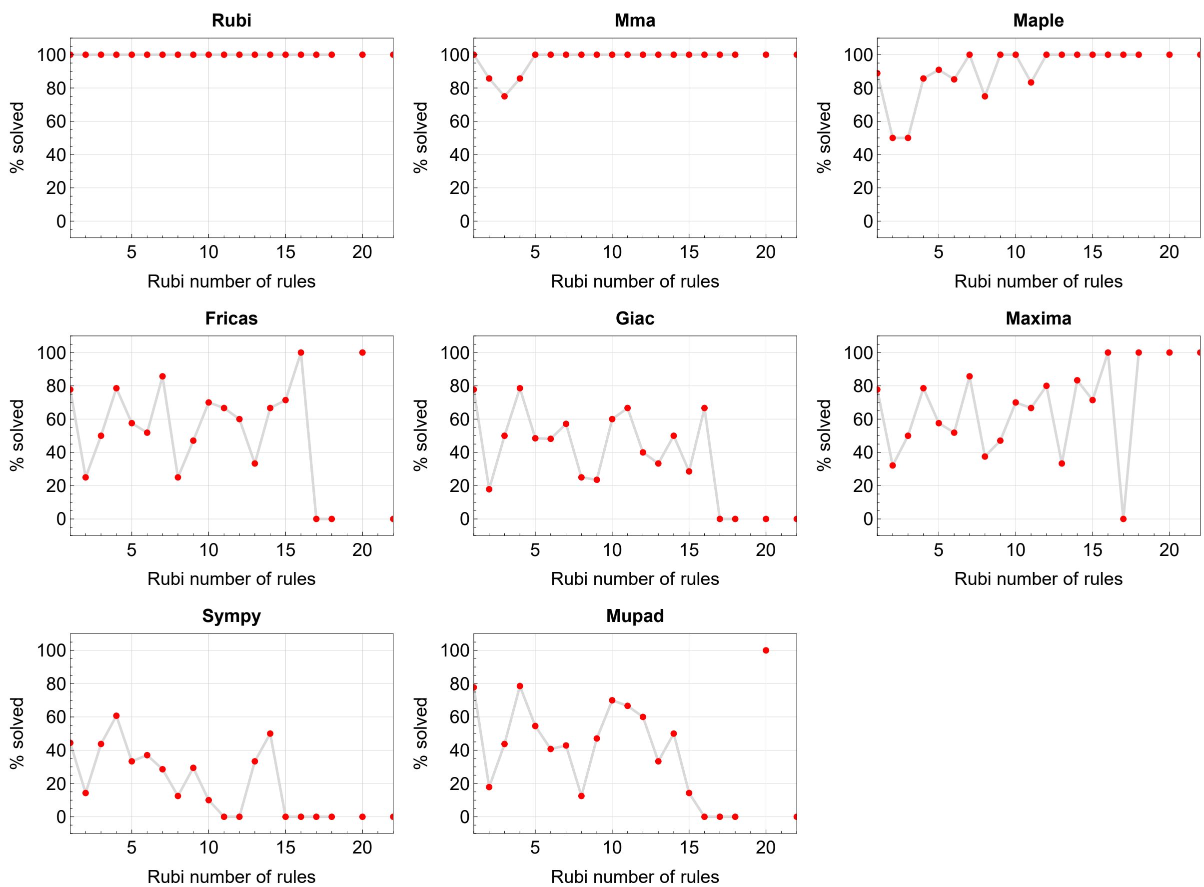


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

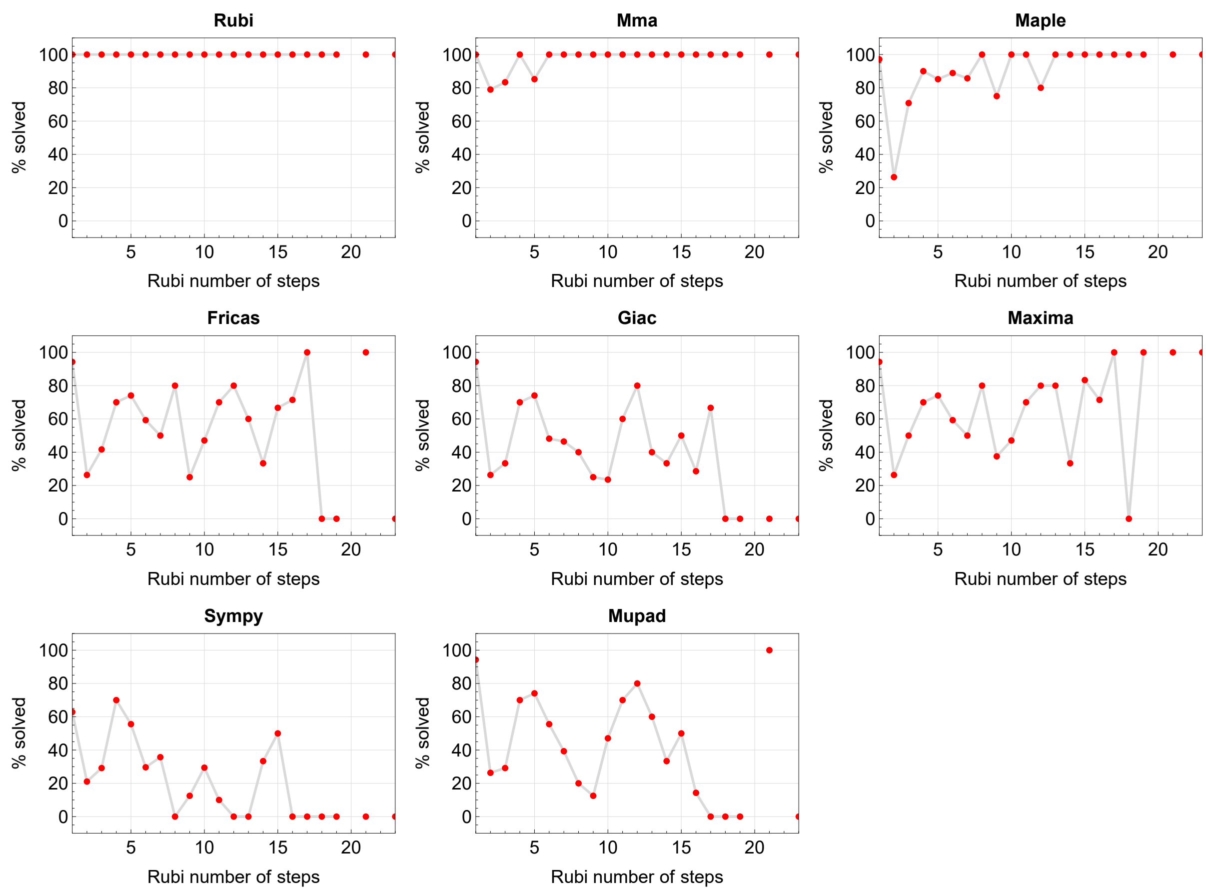


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

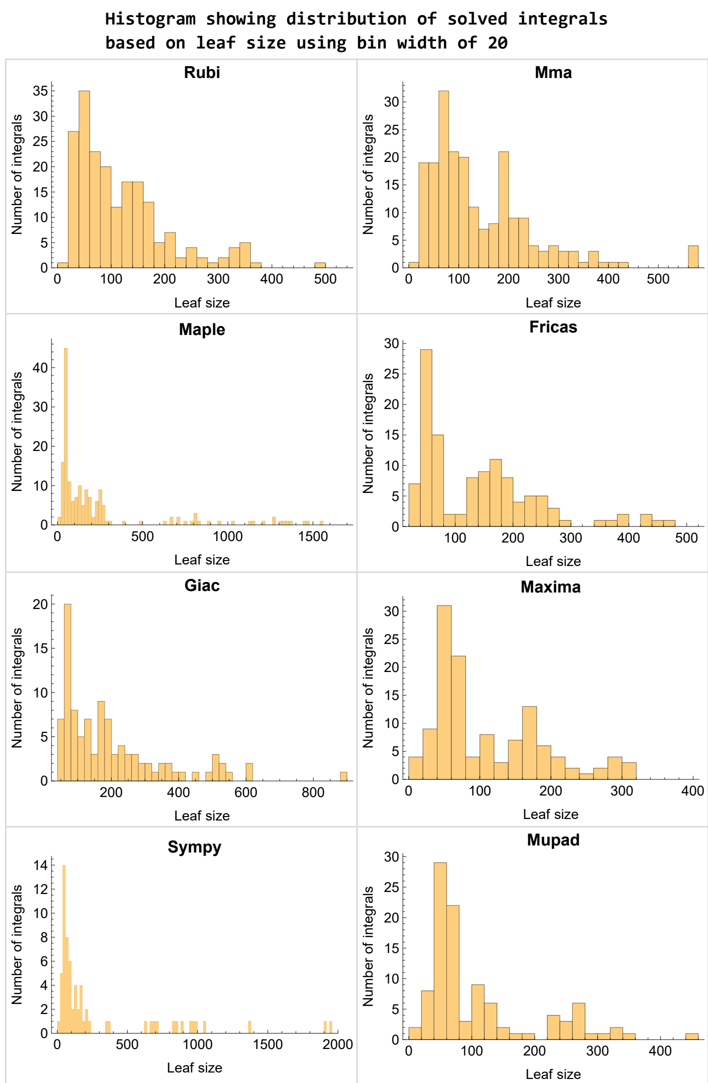


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

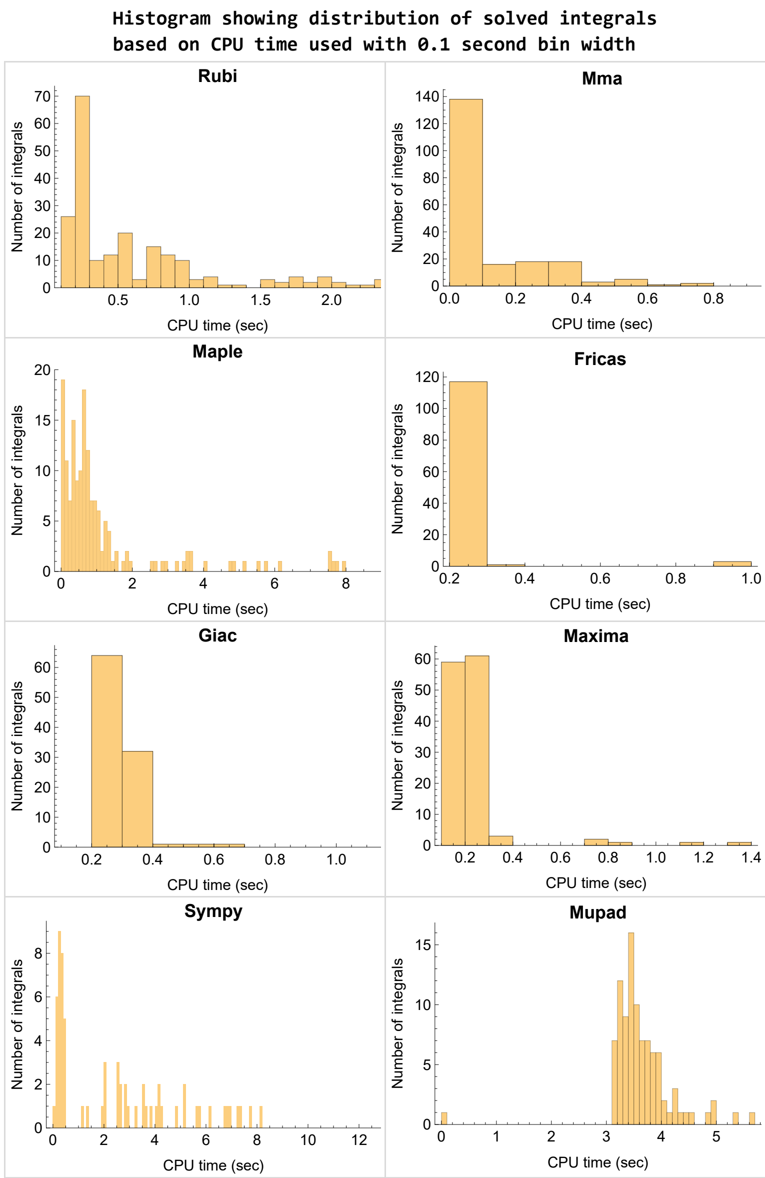


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

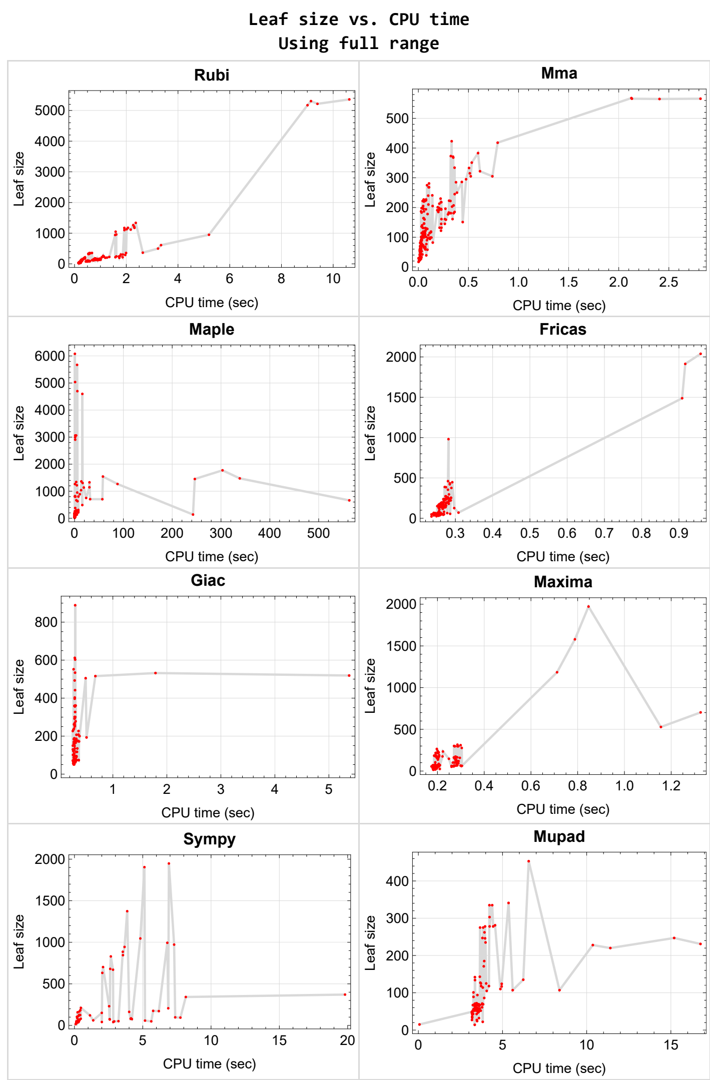


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{43, 44, 46, 47, 48, 49, 94, 95, 97, 98, 130, 131, 133, 134, 182, 183, 185, 186, 231, 232, 234, 235, 239, 240, 242, 243}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {90, 93, 143, 195, 196, 201}

Mathematica {}

Maple {19, 25, 27, 30, 31, 32, 33, 34, 69, 77, 117, 121, 123, 125, 147, 150, 151, 152, 153, 154, 156, 171, 172, 199, 202, 203, 204, 205, 206, 207, 208, 222, 223, 233}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

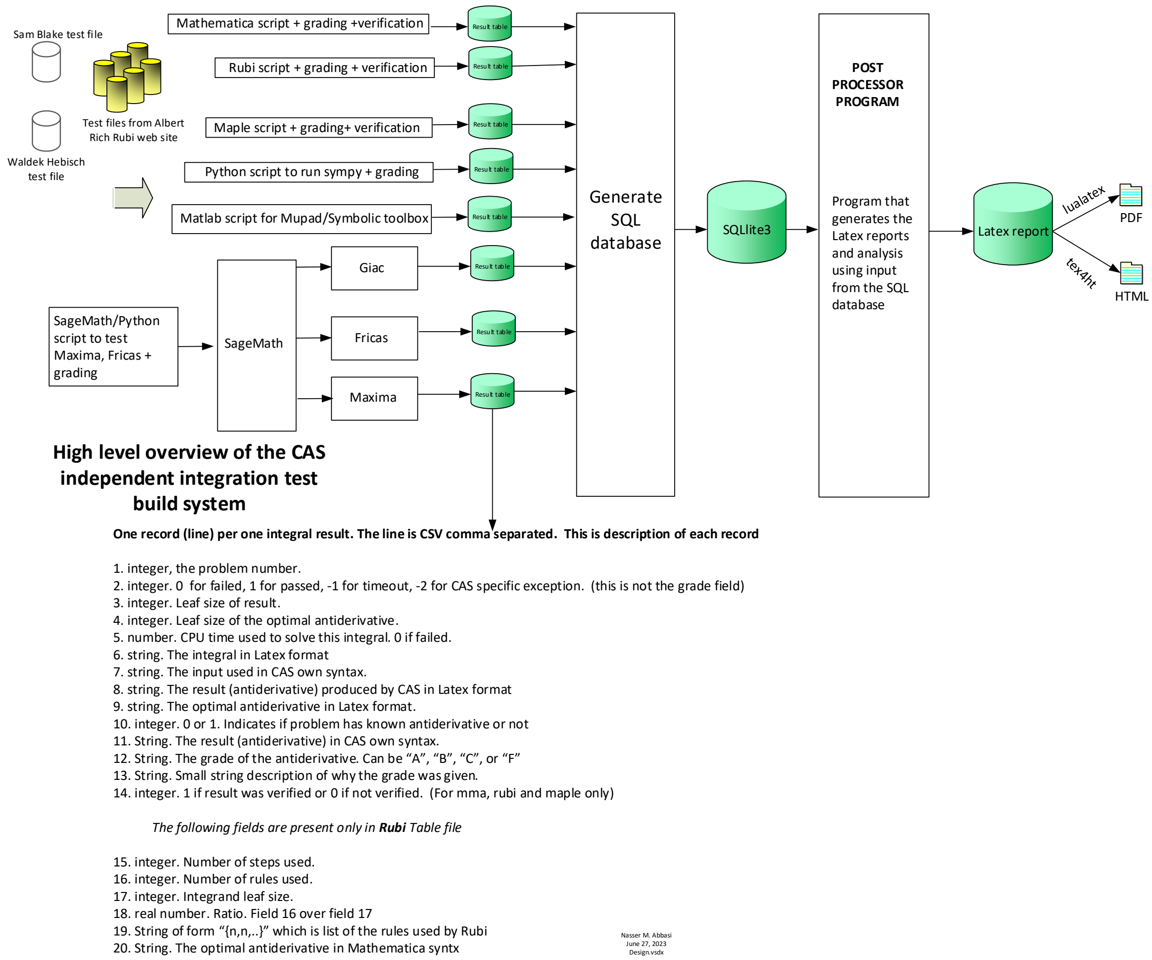
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 238, 241 }

B grade { 24, 202, 203 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 32, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 73, 74, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 96, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 129, 132, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 152, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 178, 179, 184, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 224, 225, 226, 228, 229, 230, 237, 238, 241 }

B grade { 220, 223 }

C grade { 19, 30, 31, 33, 68, 79, 80, 120, 127, 128, 147, 151, 153, 154, 173, 199, 206, 222, 227, 233, 236 }

F normal fail { 71, 72, 75, 76, 90, 91, 92, 93, 176, 177, 180, 181 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 70, 82, 83, 84, 85, 86, 87, 88, 89, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 119, 122, 135, 136, 137, 138, 140, 141, 142, 143, 145, 148, 149, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 187, 188, 189, 190, 192, 193, 194, 195, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 227, 236, 238 }

B grade { 20, 22, 24, 26, 28, 29, 54, 65, 78, 126, 139, 144, 146, 155, 161, 191, 196, 197, 198, 200, 201, 217, 221 }

C grade { 19, 25, 27, 30, 31, 32, 33, 34, 69, 77, 103, 117, 121, 123, 125, 147, 150, 151, 152, 153, 154, 156, 171, 172, 199, 202, 203, 204, 205, 206, 207, 208, 222, 223, 233, 237 }

F normal fail { 45, 68, 71, 72, 73, 74, 75, 76, 79, 80, 81, 90, 91, 92, 93, 96, 120, 124, 127, 128, 129, 132, 173, 176, 177, 178, 179, 180, 181, 184, 224, 225, 226, 228, 229, 230, 241 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 52, 53, 55, 56, 57, 58, 59, 61, 62, 63, 64, 66, 70, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 122, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 201, 209, 210, 211, 212, 214, 218, 219, 220, 238 }

B grade { 60, 198, 200, 221, 223, 227, 236 }

C grade { 82, 83, 84, 85, 86, 87, 88, 89, 213, 215, 216 }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 191, 199, 202, 203, 204, 205, 206, 207, 208, 217, 222, 224, 225, 226, 228, 229, 230, 233, 237, 241 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 35, 36, 37, 38, 39, 40, 41, 42, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 82, 83, 84, 85, 86, 87, 88, 89, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 135, 136, 137, 138, 140, 141, 142, 143, 145, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 187, 188, 189, 190, 192, 193, 194, 195, 196, 209, 210, 211, 212, 213, 214, 215, 216, 218, 219, 220, 238 }

B grade { 17, 21, 23, 66, 70, 118, 122, 149, 175, 191, 197, 198, 200, 201, 202, 203, 204, 207, 208, 217, 221, 223, 236, 237 }

C grade { }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 199, 205, 206, 222, 224, 225, 226, 227, 228, 229, 230, 233, 241 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.6 Giac

A grade { 38, 39, 40, 41, 42, 50, 51, 55, 56, 57, 58, 59, 63, 64, 99, 100, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 157, 158, 162, 163, 164, 165, 166, 167, 168, 169, 170, 216, 218, 219, 220 }

B grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 52, 53, 60, 61, 62, 66, 85, 86, 87, 88, 89, 101, 102, 118, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 159, 160, 171, 187, 188, 189, 190, 192, 193, 194, 209, 210, 211, 212, 214, 238 }

C grade { 213 }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 45, 54, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 191, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 215, 217, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 241 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 70, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 118, 122, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 158, 159, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 175, 187, 188, 189, 190, 192, 193, 194, 195, 196, 197, 198, 200, 201, 209, 211, 212, 213, 214, 215, 216, 218, 219, 220, 221, 223, 238 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 96, 103, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 132, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 191, 199, 202, 203, 204, 205, 206, 207, 208, 210, 217, 222, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 241 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15, 17, 21, 23, 50, 52, 56, 57, 60, 64, 135, 136, 137, 138, 140, 141, 142, 143, 145, 149, 157, 159, 163, 164, 167, 171, 238 }

B grade { 51, 53, 55, 58, 59, 61, 62, 63, 66, 70, 158, 160, 162, 165, 166, 168, 169, 170, 175, 192, 193, 194, 200, 201, 209, 211, 212 }

C grade { }

F normal fail { 7, 14, 16, 18, 19, 20, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 45, 54, 65, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 96, 103, 120, 127, 139, 144, 146, 147, 148, 150, 151, 152, 153, 154, 155, 156, 161, 172, 173, 174, 176, 177, 178, 179, 180, 181, 184, 187, 188, 189, 190, 191, 195, 196, 197, 198, 199, 202, 203, 204, 205, 206, 207, 208, 210, 224, 225, 226, 227, 228, 229, 230, 233, 236, 237, 241 }

F(-1) timedout fail { 89, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 186, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 239, 243 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	57	81	59	70	67	63	442	52
N.S.	1	0.97	1.37	1.00	1.19	1.14	1.07	7.49	0.88
time (sec)	N/A	0.228	0.018	0.200	0.211	0.257	0.367	0.300	3.324

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	62	58	55	69	68	403	53
N.S.	1	1.00	1.09	1.02	0.96	1.21	1.19	7.07	0.93
time (sec)	N/A	0.239	0.020	0.086	0.191	0.260	0.331	0.294	3.290

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	47	70	50	61	58	53	296	43
N.S.	1	0.98	1.46	1.04	1.27	1.21	1.10	6.17	0.90
time (sec)	N/A	0.231	0.019	0.069	0.179	0.257	0.326	0.297	3.253

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	47	51	50	44	58	58	258	44
N.S.	1	1.02	1.11	1.09	0.96	1.26	1.26	5.61	0.96
time (sec)	N/A	0.226	0.018	0.059	0.193	0.250	0.251	0.296	3.203

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	59	38	50	48	42	148	35
N.S.	1	1.00	1.59	1.03	1.35	1.30	1.14	4.00	0.95
time (sec)	N/A	0.196	0.015	0.052	0.196	0.247	0.223	0.291	3.195

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	30	29	30	42	27	156	27
N.S.	1	1.00	1.00	0.97	1.00	1.40	0.90	5.20	0.90
time (sec)	N/A	0.163	0.003	0.046	0.197	0.256	0.121	0.284	3.472

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	24	28	0	0	0	0	0
N.S.	1	1.00	0.92	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.186	0.018	0.094	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	38	39	42	39	47	41	94	33
N.S.	1	1.06	1.08	1.17	1.08	1.31	1.14	2.61	0.92
time (sec)	N/A	0.203	0.017	0.069	0.196	0.262	0.265	0.284	3.200

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	34	59	39	45	43	36	135	46
N.S.	1	0.92	1.59	1.05	1.22	1.16	0.97	3.65	1.24
time (sec)	N/A	0.200	0.018	0.063	0.192	0.252	0.232	0.295	3.182

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	52	59	56	49	59	70	251	46
N.S.	1	0.96	1.09	1.04	0.91	1.09	1.30	4.65	0.85
time (sec)	N/A	0.234	0.025	0.069	0.192	0.256	0.371	0.295	3.184

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	46	70	43	60	52	46	292	59
N.S.	1	0.96	1.46	0.90	1.25	1.08	0.96	6.08	1.23
time (sec)	N/A	0.221	0.024	0.079	0.207	0.268	0.284	0.290	3.527

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	62	70	64	61	70	80	397	71
N.S.	1	0.95	1.08	0.98	0.94	1.08	1.23	6.11	1.09
time (sec)	N/A	0.246	0.018	0.093	0.181	0.308	0.462	0.293	3.373

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	204	164	182	215	193	211	889	171
N.S.	1	1.41	1.13	1.26	1.48	1.33	1.46	6.13	1.18
time (sec)	N/A	1.328	0.051	0.394	0.203	0.268	0.472	0.302	3.886

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	199	161	229	0	0	0	0	0
N.S.	1	1.23	0.99	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.307	0.341	1.092	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	137	132	143	189	160	168	603	134
N.S.	1	1.21	1.17	1.27	1.67	1.42	1.49	5.34	1.19
time (sec)	N/A	0.849	0.043	0.107	0.190	0.256	0.362	0.302	3.365

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	142	122	201	0	0	0	0	0
N.S.	1	1.09	0.94	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.839	0.226	0.896	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	78	90	106	158	122	114	301	89
N.S.	1	1.04	1.20	1.41	2.11	1.63	1.52	4.01	1.19
time (sec)	N/A	0.448	0.086	0.094	0.203	0.265	0.291	0.302	3.275

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	88	82	115	0	0	0	0	0
N.S.	1	1.19	1.11	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.491	0.144	0.960	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	117	143	151	630	0	0	0	0	0
N.S.	1	1.22	1.29	5.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.762	0.443	6.100	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	77	94	195	0	0	0	0	0
N.S.	1	1.08	1.32	2.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.486	0.115	0.939	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	77	101	134	151	135	126	278	246
N.S.	1	0.96	1.26	1.68	1.89	1.69	1.58	3.48	3.08
time (sec)	N/A	0.543	0.052	0.139	0.203	0.272	0.343	0.310	3.964

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	119	145	247	0	0	0	0	0
N.S.	1	0.92	1.12	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.792	0.264	1.016	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	136	164	171	224	173	184	612	303
N.S.	1	1.16	1.40	1.46	1.91	1.48	1.57	5.23	2.59
time (sec)	N/A	0.914	0.054	0.125	0.206	0.261	0.433	0.294	4.229

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	499	305	803	0	0	0	0	0
N.S.	1	2.02	1.23	3.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	3.451	0.523	7.901	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	262	365	383	1038	0	0	0	0	0
N.S.	1	1.39	1.46	3.96	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.817	0.596	13.380	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	290	245	666	0	0	0	0	0
N.S.	1	1.57	1.32	3.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.091	0.363	3.635	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	197	216	250	951	0	0	0	0	0
N.S.	1	1.10	1.27	4.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.639	0.385	5.527	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	138	161	488	0	0	0	0	0
N.S.	1	1.12	1.31	3.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.960	0.265	15.904	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	123	161	230	0	0	0	0	0
N.S.	1	1.14	1.49	2.13	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.718	0.231	1.529	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	184	212	315	1304	0	0	0	0	0
N.S.	1	1.15	1.71	7.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.139	0.514	16.520	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	102	115	196	1329	0	0	0	0	0
N.S.	1	1.13	1.92	13.03	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.788	0.270	30.596	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	123	119	192	4599	0	0	0	0	0
N.S.	1	0.97	1.56	37.39	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.983	0.207	16.000	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	200	197	305	1542	0	0	0	0	0
N.S.	1	0.98	1.52	7.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.852	0.739	58.156	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	187	242	295	1136	0	0	0	0	0
N.S.	1	1.29	1.58	6.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.823	0.477	18.944	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	142	128	107	134	296	0	0	0
N.S.	1	1.15	1.03	0.86	1.08	2.39	0.00	0.00	0.00
time (sec)	N/A	0.310	0.089	1.298	0.277	0.283	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	123	115	93	118	255	0	0	0
N.S.	1	1.16	1.08	0.88	1.11	2.41	0.00	0.00	0.00
time (sec)	N/A	0.284	0.066	0.560	0.282	0.288	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	119	114	89	119	223	0	0	0
N.S.	1	1.12	1.08	0.84	1.12	2.10	0.00	0.00	0.00
time (sec)	N/A	0.275	0.063	0.498	0.290	0.283	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	92	98	68	103	211	0	88	0
N.S.	1	1.08	1.15	0.80	1.21	2.48	0.00	1.04	0.00
time (sec)	N/A	0.256	0.035	0.570	0.278	0.283	0.000	0.278	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	95	99	69	94	221	0	93	0
N.S.	1	1.12	1.16	0.81	1.11	2.60	0.00	1.09	0.00
time (sec)	N/A	0.252	0.053	0.586	0.289	0.287	0.000	0.290	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	120	107	93	101	243	0	117	0
N.S.	1	1.12	1.00	0.87	0.94	2.27	0.00	1.09	0.00
time (sec)	N/A	0.276	0.066	0.578	0.266	0.283	0.000	0.335	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	122	108	93	112	253	0	117	0
N.S.	1	1.14	1.01	0.87	1.05	2.36	0.00	1.09	0.00
time (sec)	N/A	0.261	0.057	0.578	0.280	0.288	0.000	0.322	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	142	122	107	130	272	0	135	0
N.S.	1	1.14	0.98	0.86	1.04	2.18	0.00	1.08	0.00
time (sec)	N/A	0.297	0.069	0.586	0.281	0.275	0.000	0.342	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	379	44	15	18	18
N.S.	1	1.00	1.12	1.00	23.69	2.75	0.94	1.12	1.12
time (sec)	N/A	0.201	3.147	1.556	2.408	0.253	4.912	0.337	3.890

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	218	30	15	18	18
N.S.	1	1.00	1.12	1.00	13.62	1.88	0.94	1.12	1.12
time (sec)	N/A	0.199	2.016	1.574	1.319	0.259	2.670	0.294	3.490

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	59	0	0	0	0	0	0
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.069	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	14	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.88	1.12	1.12
time (sec)	N/A	0.202	0.229	1.625	0.245	0.251	1.152	0.287	3.071

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	116	32	15	18	18
N.S.	1	1.00	1.12	1.00	7.25	2.00	0.94	1.12	1.12
time (sec)	N/A	0.206	0.487	1.632	0.284	0.248	8.433	0.304	3.176

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.173	0.369	0.422	0.328	0.258	1.530	0.317	3.286

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	1.12
time (sec)	N/A	0.194	0.286	0.843	0.374	0.258	130.299	0.339	3.598

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	53	78	56	69	64	58	78	69
N.S.	1	0.98	1.44	1.04	1.28	1.19	1.07	1.44	1.28
time (sec)	N/A	0.247	0.034	0.282	0.185	0.245	5.166	0.300	3.790

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	49	53	45	46	62	85	57	61
N.S.	1	1.02	1.10	0.94	0.96	1.29	1.77	1.19	1.27
time (sec)	N/A	0.238	0.033	0.408	0.190	0.258	4.140	0.280	3.411

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	67	46	58	54	48	181	60
N.S.	1	1.00	1.56	1.07	1.35	1.26	1.12	4.21	1.40
time (sec)	N/A	0.222	0.030	0.383	0.198	0.253	2.908	0.299	3.441

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	37	50	71	188	52
N.S.	1	1.00	1.14	1.00	1.00	1.35	1.92	5.08	1.41
time (sec)	N/A	0.198	0.018	0.316	0.194	0.256	2.588	0.288	3.256

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	36	28	126	0	0	0	0	0
N.S.	1	1.20	0.93	4.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.247	0.027	0.122	0.000	0.000	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	42	45	47	41	55	80	51	55
N.S.	1	1.05	1.12	1.18	1.02	1.38	2.00	1.28	1.38
time (sec)	N/A	0.223	0.025	0.112	0.183	0.286	4.111	0.283	3.457

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	38	65	45	51	49	41	67	52
N.S.	1	0.93	1.59	1.10	1.24	1.20	1.00	1.63	1.27
time (sec)	N/A	0.219	0.026	0.144	0.201	0.269	2.843	0.300	3.705

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	54	61	63	51	65	97	65	67
N.S.	1	0.96	1.09	1.12	0.91	1.16	1.73	1.16	1.20
time (sec)	N/A	0.244	0.024	0.178	0.181	0.253	7.375	0.289	3.648

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	74	93	53	69	197	944	73	72
N.S.	1	1.14	1.43	0.82	1.06	3.03	14.52	1.12	1.11
time (sec)	N/A	0.254	0.031	0.647	0.285	0.280	3.669	0.368	3.460

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	70	91	51	66	186	670	75	70
N.S.	1	1.11	1.44	0.81	1.05	2.95	10.63	1.19	1.11
time (sec)	N/A	0.234	0.030	0.365	0.272	0.275	2.823	0.323	3.342

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	57	37	55	160	702	83	55
N.S.	1	1.00	1.30	0.84	1.25	3.64	15.95	1.89	1.25
time (sec)	N/A	0.178	0.018	0.282	0.270	0.273	2.093	0.278	3.594

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	55	75	42	61	157	1374	79	62
N.S.	1	1.20	1.63	0.91	1.33	3.41	29.87	1.72	1.35
time (sec)	N/A	0.218	0.030	0.368	0.270	0.275	3.861	0.313	3.407

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	69	91	51	65	181	1904	95	71
N.S.	1	1.10	1.44	0.81	1.03	2.87	30.22	1.51	1.13
time (sec)	N/A	0.239	0.034	0.313	0.299	0.267	5.123	0.325	3.485

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	71	91	51	66	187	1948	91	71
N.S.	1	1.13	1.44	0.81	1.05	2.97	30.92	1.44	1.13
time (sec)	N/A	0.230	0.035	0.317	0.282	0.275	6.914	0.327	3.791

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	153	146	163	217	176	206	175	335
N.S.	1	1.22	1.17	1.30	1.74	1.41	1.65	1.40	2.68
time (sec)	N/A	0.910	0.056	1.000	0.204	0.257	6.867	0.346	4.220

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	162	132	380	0	0	0	0	0
N.S.	1	1.11	0.90	2.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.922	0.231	1.348	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	92	106	121	186	138	163	361	275
N.S.	1	1.01	1.16	1.33	2.04	1.52	1.79	3.97	3.02
time (sec)	N/A	0.533	0.044	0.866	0.193	0.255	4.007	0.311	3.850

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	104	99	134	0	0	0	0	0
N.S.	1	1.11	1.05	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.563	0.130	1.333	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	163	181	0	0	0	0	0	0
N.S.	1	1.19	1.32	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.832	0.353	0.000	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	87	91	119	820	0	0	0	0	0
N.S.	1	1.05	1.37	9.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.571	0.126	1.028	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	87	111	152	175	151	175	0	278
N.S.	1	0.99	1.26	1.73	1.99	1.72	1.99	0.00	3.16
time (sec)	N/A	0.631	0.066	0.367	0.222	0.271	5.762	0.000	4.445

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1173	1173	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.498	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1129	1129	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.194	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	958	958	566	0	0	0	0	0	0
N.S.	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.735	2.131	0.000	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	942	942	566	0	0	0	0	0	0
N.S.	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.689	2.816	0.000	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1102	1102	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.108	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1176	1176	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.225	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	141	158	185	798	0	0	0	0	0
N.S.	1	1.12	1.31	5.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.055	0.362	1.352	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	143	213	265	0	0	0	0	0
N.S.	1	1.07	1.59	1.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.823	0.210	2.586	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	240	371	0	0	0	0	0	0
N.S.	1	1.16	1.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.255	0.346	0.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	125	133	222	0	0	0	0	0	0
N.S.	1	1.06	1.78	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.877	0.320	0.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	139	137	218	0	0	0	0	0	0
N.S.	1	0.99	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.060	0.228	0.000	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	356	241	256	316	434	0	0	0
N.S.	1	1.12	0.76	0.81	1.00	1.37	0.00	0.00	0.00
time (sec)	N/A	0.661	0.139	4.033	0.286	0.285	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	350	240	248	310	376	0	0	0
N.S.	1	1.10	0.76	0.78	0.98	1.19	0.00	0.00	0.00
time (sec)	N/A	0.623	0.098	0.648	0.299	0.289	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	328	227	240	301	385	0	0	0
N.S.	1	1.09	0.75	0.80	1.00	1.28	0.00	0.00	0.00
time (sec)	N/A	0.609	0.077	0.626	0.271	0.275	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	320	227	242	296	354	0	493	0
N.S.	1	1.12	0.80	0.85	1.04	1.24	0.00	1.73	0.00
time (sec)	N/A	0.582	0.053	0.607	0.286	0.280	0.000	0.292	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	324	268	246	296	388	0	505	0
N.S.	1	1.14	0.94	0.86	1.04	1.36	0.00	1.77	0.00
time (sec)	N/A	0.606	0.107	0.635	0.274	0.271	0.000	0.496	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	326	268	238	277	421	0	516	0
N.S.	1	1.08	0.89	0.79	0.92	1.40	0.00	1.71	0.00
time (sec)	N/A	0.570	0.103	0.643	0.304	0.285	0.000	0.676	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	351	275	250	298	460	0	532	0
N.S.	1	1.11	0.87	0.79	0.94	1.45	0.00	1.68	0.00
time (sec)	N/A	0.624	0.087	0.641	0.275	0.280	0.000	1.789	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	349	281	250	297	447	0	519	0
N.S.	1	1.10	0.89	0.79	0.94	1.41	0.00	1.64	0.00
time (sec)	N/A	0.611	0.107	0.622	0.290	0.292	0.000	5.375	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6327	5360	0	0	0	0	0	0	0
N.S.	1	0.85	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	11.556	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6177	5216	0	0	0	0	0	0	0
N.S.	1	0.84	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	9.876	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6334	5171	0	0	0	0	0	0	0
N.S.	1	0.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	9.857	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	6520	5305	0	0	0	0	0	0	0
N.S.	1	0.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	9.761	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	413	50	17	20	20
N.S.	1	1.00	1.11	1.00	22.94	2.78	0.94	1.11	1.11
time (sec)	N/A	0.193	1.612	0.132	2.647	0.268	52.141	0.374	3.792

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	236	34	17	20	20
N.S.	1	1.00	1.11	1.00	13.11	1.89	0.94	1.11	1.11
time (sec)	N/A	0.199	1.033	0.150	1.466	0.250	33.332	0.321	3.469

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.242	0.070	0.000	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.198	0.330	0.120	0.236	0.247	42.005	0.312	3.040

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	134	36	0	20	20
N.S.	1	1.00	1.11	1.00	7.44	2.00	0.00	1.11	1.11
time (sec)	N/A	0.197	0.347	0.153	0.308	0.253	0.000	0.321	3.362

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	53	78	56	69	64	0	78	69
N.S.	1	0.98	1.44	1.04	1.28	1.19	0.00	1.44	1.28
time (sec)	N/A	0.243	0.029	0.560	0.180	0.256	0.000	0.284	3.850

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	49	53	45	46	62	0	57	61
N.S.	1	1.02	1.10	0.94	0.96	1.29	0.00	1.19	1.27
time (sec)	N/A	0.230	0.027	0.594	0.193	0.245	0.000	0.269	3.364

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	67	46	58	54	0	181	60
N.S.	1	1.00	1.56	1.07	1.35	1.26	0.00	4.21	1.40
time (sec)	N/A	0.217	0.029	0.492	0.188	0.253	0.000	0.298	3.365

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	37	50	0	188	52
N.S.	1	1.00	1.14	1.00	1.00	1.35	0.00	5.08	1.41
time (sec)	N/A	0.198	0.015	0.385	0.180	0.250	0.000	0.284	3.493

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	36	28	92	0	0	0	0	0
N.S.	1	1.20	0.93	3.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	0.026	0.248	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	42	45	47	41	55	0	51	55
N.S.	1	1.05	1.12	1.18	1.02	1.38	0.00	1.28	1.38
time (sec)	N/A	0.209	0.023	0.167	0.197	0.260	0.000	0.274	3.320

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	38	65	45	51	49	0	67	52
N.S.	1	0.93	1.59	1.10	1.24	1.20	0.00	1.63	1.27
time (sec)	N/A	0.210	0.024	0.238	0.184	0.248	0.000	0.282	3.436

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	54	61	63	51	65	0	65	67
N.S.	1	0.96	1.09	1.12	0.91	1.16	0.00	1.16	1.20
time (sec)	N/A	0.238	0.028	0.364	0.189	0.260	0.000	0.269	3.237

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	177	196	184	162	981	0	207	125
N.S.	1	1.02	1.13	1.06	0.93	5.64	0.00	1.19	0.72
time (sec)	N/A	0.402	0.049	0.415	0.276	0.281	0.000	0.325	4.030

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	136	99	90	260	0	109	107
N.S.	1	1.00	1.35	0.98	0.89	2.57	0.00	1.08	1.06
time (sec)	N/A	0.266	0.032	0.214	0.283	0.280	0.000	0.283	5.604

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	166	187	159	155	228	0	165	118
N.S.	1	1.01	1.13	0.96	0.94	1.38	0.00	1.00	0.72
time (sec)	N/A	0.370	0.056	0.318	0.272	0.273	0.000	0.311	4.193

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	127	196	172	100	151	0	125	135
N.S.	1	1.10	1.70	1.50	0.87	1.31	0.00	1.09	1.17
time (sec)	N/A	0.335	0.051	0.456	0.274	0.251	0.000	0.285	6.231

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	191	198	186	164	248	0	208	127
N.S.	1	1.09	1.12	1.06	0.93	1.41	0.00	1.18	0.72
time (sec)	N/A	0.403	0.038	0.755	0.285	0.277	0.000	0.363	3.684

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	125	198	114	103	166	0	126	124
N.S.	1	1.07	1.69	0.97	0.88	1.42	0.00	1.08	1.06
time (sec)	N/A	0.325	0.033	0.464	0.272	0.255	0.000	0.275	4.953

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	176	187	177	155	238	0	173	118
N.S.	1	1.07	1.13	1.07	0.94	1.44	0.00	1.05	0.72
time (sec)	N/A	0.368	0.027	0.346	0.278	0.268	0.000	0.356	3.727

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	110	183	105	94	117	0	106	117
N.S.	1	1.06	1.76	1.01	0.90	1.12	0.00	1.02	1.12
time (sec)	N/A	0.304	0.033	0.307	0.289	0.266	0.000	0.283	4.948

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	186	196	172	160	196	0	193	125
N.S.	1	1.07	1.13	0.99	0.92	1.13	0.00	1.11	0.72
time (sec)	N/A	0.398	0.049	0.378	0.272	0.263	0.000	0.512	3.716

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	153	146	163	217	176	0	175	335
N.S.	1	1.22	1.17	1.30	1.74	1.41	0.00	1.40	2.68
time (sec)	N/A	0.907	0.058	1.770	0.192	0.265	0.000	0.353	4.387

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	146	162	132	2906	0	0	0	0	0
N.S.	1	1.11	0.90	19.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.897	0.212	1.348	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	92	106	121	186	138	0	361	275
N.S.	1	1.01	1.16	1.33	2.04	1.52	0.00	3.97	3.02
time (sec)	N/A	0.509	0.042	1.276	0.186	0.274	0.000	0.299	3.666

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	104	99	134	0	0	0	0	0
N.S.	1	1.08	1.03	1.40	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.543	0.118	1.518	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	140	163	181	0	0	0	0	0	0
N.S.	1	1.16	1.29	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.819	0.325	0.000	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	90	91	117	2993	0	0	0	0	0
N.S.	1	1.01	1.30	33.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.555	0.126	1.197	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	87	111	152	175	151	0	0	278
N.S.	1	0.99	1.26	1.73	1.99	1.72	0.00	0.00	3.16
time (sec)	N/A	0.631	0.067	0.681	0.207	0.273	0.000	0.000	3.964

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	144	137	159	3062	0	0	0	0	0
N.S.	1	0.95	1.10	21.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.866	0.275	1.436	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	231	244	334	0	0	0	0	0	0
N.S.	1	1.06	1.45	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.744	0.361	0.000	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	139	158	185	800	0	0	0	0	0
N.S.	1	1.14	1.33	5.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.048	0.199	1.800	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	143	191	265	0	0	0	0	0
N.S.	1	1.10	1.47	2.04	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.819	0.189	2.925	0.000	0.000	0.000	0.000	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	210	240	368	0	0	0	0	0	0
N.S.	1	1.14	1.75	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.187	0.349	0.000	0.000	0.000	0.000	0.000	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	120	133	223	0	0	0	0	0	0
N.S.	1	1.11	1.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.812	0.307	0.000	0.000	0.000	0.000	0.000	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	136	137	218	0	0	0	0	0	0
N.S.	1	1.01	1.60	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.028	0.229	0.000	0.000	0.000	0.000	0.000	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	416	50	0	20	20
N.S.	1	1.00	1.11	1.00	23.11	2.78	0.00	1.11	1.11
time (sec)	N/A	0.214	1.599	0.154	2.666	0.244	0.000	0.336	3.141

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	239	34	0	20	20
N.S.	1	1.00	1.11	1.00	13.28	1.89	0.00	1.11	1.11
time (sec)	N/A	0.205	0.994	0.145	1.469	0.255	0.000	0.331	3.120

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	74	74	64	0	0	0	0	0	0
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.251	0.061	0.000	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	0	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.00	1.11	1.11
time (sec)	N/A	0.213	0.327	0.134	0.256	0.244	0.000	0.315	3.039

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	141	36	0	20	20
N.S.	1	1.00	1.11	1.00	7.83	2.00	0.00	1.11	1.11
time (sec)	N/A	0.204	0.335	0.136	0.313	0.248	0.000	0.298	3.464

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	48	67	46	57	48	46	262	45
N.S.	1	0.96	1.34	0.92	1.14	0.96	0.92	5.24	0.90
time (sec)	N/A	0.237	0.020	0.806	0.193	0.236	0.165	0.280	3.235

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	42	50	57	42	49	49	227	42
N.S.	1	0.93	1.11	1.27	0.93	1.09	1.09	5.04	0.93
time (sec)	N/A	0.238	0.017	0.806	0.182	0.246	0.140	0.258	3.202

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	35	56	43	44	39	36	130	36
N.S.	1	0.90	1.44	1.10	1.13	1.00	0.92	3.33	0.92
time (sec)	N/A	0.210	0.015	0.654	0.188	0.247	0.136	0.259	3.197

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	32	29	35	24	150	27
N.S.	1	1.00	1.00	1.10	1.00	1.21	0.83	5.17	0.93
time (sec)	N/A	0.162	0.003	0.599	0.181	0.242	0.134	0.259	3.184

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	33	28	57	0	0	0	0	0
N.S.	1	1.10	0.93	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.239	0.024	0.681	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	38	37	37	48	39	87	43
N.S.	1	1.00	1.09	1.06	1.06	1.37	1.11	2.49	1.23
time (sec)	N/A	0.201	0.018	0.406	0.192	0.260	0.297	0.275	3.462

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	60	42	52	46	44	123	49
N.S.	1	1.00	1.40	0.98	1.21	1.07	1.02	2.86	1.14
time (sec)	N/A	0.211	0.016	0.718	0.189	0.251	0.304	0.276	3.176

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	55	62	57	55	62	68	234	59
N.S.	1	0.96	1.09	1.00	0.96	1.09	1.19	4.11	1.04
time (sec)	N/A	0.248	0.019	0.694	0.193	0.262	0.359	0.269	3.402

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	140	131	161	189	149	158	552	142
N.S.	1	1.14	1.07	1.31	1.54	1.21	1.28	4.49	1.15
time (sec)	N/A	1.005	0.049	3.577	0.208	0.262	0.229	0.273	3.359

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	131	145	298	0	0	0	0	0
N.S.	1	0.92	1.02	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.884	0.228	3.534	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	82	92	118	136	111	104	268	101
N.S.	1	0.99	1.11	1.42	1.64	1.34	1.25	3.23	1.22
time (sec)	N/A	0.601	0.032	2.845	0.202	0.257	0.180	0.292	3.273

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	80	97	235	0	0	0	0	0
N.S.	1	1.08	1.31	3.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.522	0.095	1.295	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	133	159	177	704	0	0	0	0	0
N.S.	1	1.20	1.33	5.29	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.846	0.296	56.963	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	101	101	134	0	0	0	0	0
N.S.	1	1.16	1.16	1.54	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.573	0.076	1.970	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	119	132	165	130	124	255	235
N.S.	1	1.00	1.37	1.52	1.90	1.49	1.43	2.93	2.70
time (sec)	N/A	0.525	0.049	2.635	0.204	0.255	0.415	0.292	3.991

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	203	266	286	1267	0	0	0	0	0
N.S.	1	1.31	1.41	6.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.911	0.434	88.033	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	217	216	322	1773	0	0	0	0	0
N.S.	1	1.00	1.48	8.17	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.974	0.615	303.077	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	135	131	193	5036	0	0	0	0	0
N.S.	1	0.97	1.43	37.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.026	0.229	1.138	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	108	121	198	1475	0	0	0	0	0
N.S.	1	1.12	1.83	13.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.777	0.208	338.366	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	208	236	373	1452	0	0	0	0	0
N.S.	1	1.13	1.79	6.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.209	0.324	246.330	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	140	205	265	0	0	0	0	0
N.S.	1	1.11	1.63	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.850	0.142	3.641	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	139	154	195	6081	0	0	0	0	0
N.S.	1	1.11	1.40	43.75	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.079	0.227	0.700	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	52	73	48	62	53	51	71	66
N.S.	1	0.96	1.35	0.89	1.15	0.98	0.94	1.31	1.22
time (sec)	N/A	0.257	0.025	0.947	0.189	0.248	3.222	0.258	3.533

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	42	50	59	42	52	75	52	56
N.S.	1	0.93	1.11	1.31	0.93	1.16	1.67	1.16	1.24
time (sec)	N/A	0.245	0.022	1.256	0.196	0.242	2.572	0.285	3.428

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	39	62	45	49	44	41	162	57
N.S.	1	0.91	1.44	1.05	1.14	1.02	0.95	3.77	1.33
time (sec)	N/A	0.230	0.021	0.974	0.198	0.249	2.005	0.275	3.597

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	39	42	34	43	61	184	47
N.S.	1	1.00	1.15	1.24	1.00	1.26	1.79	5.41	1.38
time (sec)	N/A	0.207	0.015	0.734	0.187	0.267	1.374	0.276	3.543

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	37	28	149	0	0	0	0	0
N.S.	1	1.23	0.93	4.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.244	0.024	0.666	0.000	0.000	0.000	0.000	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	42	37	37	55	76	52	56
N.S.	1	1.00	1.14	1.00	1.00	1.49	2.05	1.41	1.51
time (sec)	N/A	0.212	0.015	0.773	0.184	0.260	4.205	0.274	3.280

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	64	44	56	52	49	66	59
N.S.	1	1.00	1.42	0.98	1.24	1.16	1.09	1.47	1.31
time (sec)	N/A	0.246	0.022	0.666	0.185	0.239	5.623	0.279	3.440

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	55	62	45	55	67	94	65	66
N.S.	1	0.96	1.09	0.79	0.96	1.18	1.65	1.14	1.16
time (sec)	N/A	0.255	0.022	0.665	0.174	0.242	7.757	0.271	3.298

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	72	88	53	62	170	845	67	67
N.S.	1	1.14	1.40	0.84	0.98	2.70	13.41	1.06	1.06
time (sec)	N/A	0.248	0.025	1.273	0.273	0.261	3.550	0.306	3.419

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	65	86	51	61	162	830	69	65
N.S.	1	1.07	1.41	0.84	1.00	2.66	13.61	1.13	1.07
time (sec)	N/A	0.236	0.025	1.063	0.260	0.267	2.663	0.285	3.615

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	54	39	51	138	632	57	52
N.S.	1	1.00	1.23	0.89	1.16	3.14	14.36	1.30	1.18
time (sec)	N/A	0.177	0.014	0.795	0.260	0.267	2.039	0.266	3.235

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	55	72	44	57	159	886	62	59
N.S.	1	1.20	1.57	0.96	1.24	3.46	19.26	1.35	1.28
time (sec)	N/A	0.226	0.028	0.671	0.263	0.269	3.538	0.306	3.416

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	72	90	55	64	189	1046	72	69
N.S.	1	1.11	1.38	0.85	0.98	2.91	16.09	1.11	1.06
time (sec)	N/A	0.244	0.030	0.762	0.270	0.261	4.824	0.324	3.600

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	74	90	55	65	196	994	74	69
N.S.	1	1.14	1.38	0.85	1.00	3.02	15.29	1.14	1.06
time (sec)	N/A	0.243	0.029	0.905	0.277	0.276	6.797	0.364	3.541

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	94	88	104	805	157	126	151	327	247
N.S.	1	0.94	1.11	8.56	1.67	1.34	1.61	3.48	2.63
time (sec)	N/A	0.600	0.044	0.498	0.197	0.297	2.000	0.301	3.807

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	94	90	107	889	0	0	0	0	0
N.S.	1	0.96	1.14	9.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.554	0.099	10.813	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	163	183	0	0	0	0	0	0
N.S.	1	1.13	1.27	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.842	0.303	0.000	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	105	114	134	0	0	0	0	0
N.S.	1	1.06	1.15	1.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.582	0.049	4.845	0.000	0.000	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	91	131	138	183	143	172	0	262
N.S.	1	0.94	1.35	1.42	1.89	1.47	1.77	0.00	2.70
time (sec)	N/A	0.526	0.070	242.624	0.186	0.261	6.177	0.000	3.927

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1214	1214	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.463	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1172	1172	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.197	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1549	1050	565	0	0	0	0	0	0
N.S.	1	0.68	0.36	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.751	2.407	0.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1117	1117	568	0	0	0	0	0	0
N.S.	1	1.00	0.51	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.443	2.126	0.000	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1263	1263	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.487	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	1337	1337	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.644	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	398	50	17	20	20
N.S.	1	1.00	1.11	1.00	22.11	2.78	0.94	1.11	1.11
time (sec)	N/A	0.200	2.052	0.266	2.641	0.244	57.815	0.326	3.175

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	228	34	17	20	20
N.S.	1	1.00	1.11	1.00	12.67	1.89	0.94	1.11	1.11
time (sec)	N/A	0.200	1.321	0.264	1.445	0.251	36.349	0.304	3.114

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	75	78	68	0	0	0	0	0	0
N.S.	1	1.04	0.91	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.275	0.068	0.000	0.000	0.000	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.205	0.317	0.234	0.233	0.247	45.419	0.291	3.396

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	128	36	0	20	20
N.S.	1	1.00	1.11	1.00	7.11	2.00	0.00	1.11	1.11
time (sec)	N/A	0.201	0.333	0.246	0.332	0.260	0.000	0.293	3.185

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	101	114	79	86	89	0	359	86
N.S.	1	1.15	1.30	0.90	0.98	1.01	0.00	4.08	0.98
time (sec)	N/A	0.247	0.033	0.840	0.206	0.256	0.000	0.287	3.913

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	84	101	71	78	80	0	301	58
N.S.	1	1.12	1.35	0.95	1.04	1.07	0.00	4.01	0.77
time (sec)	N/A	0.240	0.028	0.773	0.190	0.270	0.000	0.292	3.642

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	67	88	63	69	70	0	239	49
N.S.	1	1.08	1.42	1.02	1.11	1.13	0.00	3.85	0.79
time (sec)	N/A	0.218	0.026	0.697	0.183	0.253	0.000	0.280	3.588

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	42	50	53	56	0	174	32
N.S.	1	1.00	1.08	1.28	1.36	1.44	0.00	4.46	0.82
time (sec)	N/A	0.170	0.040	0.718	0.205	0.248	0.000	0.319	3.601

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	40	29	57	61	0	0	0	0
N.S.	1	1.38	1.00	1.97	2.10	0.00	0.00	0.00	0.00
time (sec)	N/A	0.245	0.025	0.900	0.302	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	43	67	57	51	53	231	168	52
N.S.	1	1.08	1.68	1.42	1.28	1.32	5.78	4.20	1.30
time (sec)	N/A	0.214	0.027	0.753	0.196	0.264	2.538	0.287	3.521

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	59	86	65	64	64	342	356	61
N.S.	1	0.98	1.43	1.08	1.07	1.07	5.70	5.93	1.02
time (sec)	N/A	0.228	0.030	0.806	0.195	0.252	8.151	0.298	3.784

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	99	73	72	74	371	534	69
N.S.	1	1.00	1.36	1.00	0.99	1.01	5.08	7.32	0.95
time (sec)	N/A	0.229	0.031	0.725	0.200	0.257	19.807	0.300	3.798

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	305	224	309	265	273	0	0	453
N.S.	1	1.45	1.06	1.46	1.26	1.29	0.00	0.00	2.15
time (sec)	N/A	2.041	0.076	7.759	0.197	0.273	0.000	0.000	6.554

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	226	194	279	241	241	0	0	185
N.S.	1	1.31	1.12	1.61	1.39	1.39	0.00	0.00	1.07
time (sec)	N/A	1.434	0.075	7.550	0.201	0.272	0.000	0.000	3.913

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	159	160	249	215	207	0	0	143
N.S.	1	1.23	1.24	1.93	1.67	1.60	0.00	0.00	1.11
time (sec)	N/A	0.941	0.062	7.595	0.199	0.282	0.000	0.000	3.651

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	96	115	213	175	165	0	0	94
N.S.	1	1.13	1.35	2.51	2.06	1.94	0.00	0.00	1.11
time (sec)	N/A	0.500	0.041	7.606	0.196	0.268	0.000	0.000	3.508

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	145	177	203	662	0	0	0	0	0
N.S.	1	1.22	1.40	4.57	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.888	0.336	562.753	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	89	129	231	174	157	680	0	278
N.S.	1	1.05	1.52	2.72	2.05	1.85	8.00	0.00	3.27
time (sec)	N/A	0.609	0.084	1.005	0.198	0.269	2.618	0.000	4.233

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	A	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	152	178	263	234	201	972	0	341
N.S.	1	1.14	1.34	1.98	1.76	1.51	7.31	0.00	2.56
time (sec)	N/A	1.045	0.092	0.993	0.223	0.269	7.297	0.000	5.358

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	374	949	418	1341	1972	0	0	0	0
N.S.	1	2.54	1.12	3.59	5.27	0.00	0.00	0.00	0.00
time (sec)	N/A	5.464	0.791	3.407	0.846	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	B	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	304	611	351	1264	1579	0	0	0	0
N.S.	1	2.01	1.15	4.16	5.19	0.00	0.00	0.00	0.00
time (sec)	N/A	3.525	0.533	0.765	0.788	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	356	285	1145	1184	0	0	0	0
N.S.	1	1.52	1.22	4.89	5.06	0.00	0.00	0.00	0.00
time (sec)	N/A	2.095	0.376	30.441	0.712	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	170	201	5673	0	0	0	0	0
N.S.	1	1.20	1.42	39.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.018	0.194	5.192	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	224	262	423	1363	0	0	0	0	0
N.S.	1	1.17	1.89	6.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.204	0.333	13.888	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	142	151	230	4700	528	0	0	0	0
N.S.	1	1.06	1.62	33.10	3.72	0.00	0.00	0.00	0.00
time (sec)	N/A	1.057	0.222	5.786	1.156	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	234	306	333	1219	703	0	0	0	0
N.S.	1	1.31	1.42	5.21	3.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.913	0.506	4.702	1.326	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	35	31	35	24	36	121	170	24
N.S.	1	0.92	0.82	0.92	0.63	0.95	3.18	4.47	0.63
time (sec)	N/A	0.201	0.014	0.131	0.210	0.252	1.137	0.276	3.491

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	28	25	30	19	31	0	121	0
N.S.	1	0.90	0.81	0.97	0.61	1.00	0.00	3.90	0.00
time (sec)	N/A	0.194	0.010	0.020	0.185	0.255	0.000	0.268	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	20	17	16	25	87	72	14
N.S.	1	1.00	1.00	0.85	0.80	1.25	4.35	3.60	0.70
time (sec)	N/A	0.174	0.007	0.033	0.183	0.249	0.209	0.277	3.349

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	29	18	37	126	72	22
N.S.	1	1.00	1.00	1.21	0.75	1.54	5.25	3.00	0.92
time (sec)	N/A	0.180	0.017	0.153	0.191	0.258	0.406	0.266	3.825

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	C	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	212	222	194	172	2039	0	227	231
N.S.	1	1.12	1.17	1.02	0.91	10.73	0.00	1.19	1.22
time (sec)	N/A	0.429	0.066	0.244	0.278	0.958	0.000	0.364	16.758

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	54	75	55	58	64	0	97	110
N.S.	1	1.10	1.53	1.12	1.18	1.31	0.00	1.98	2.24
time (sec)	N/A	0.242	0.030	0.658	0.197	0.278	0.000	0.290	4.873

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	200	222	194	172	1488	0	0	247
N.S.	1	1.05	1.17	1.02	0.91	7.83	0.00	0.00	1.30
time (sec)	N/A	0.426	0.043	0.033	0.277	0.909	0.000	0.000	15.196

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	114	179	158	1914	0	186	107
N.S.	1	1.00	0.67	1.05	0.93	11.26	0.00	1.09	0.63
time (sec)	N/A	0.423	0.113	0.026	0.296	0.917	0.000	0.322	8.384

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	42	32	57	62	0	0	0	0
N.S.	1	1.24	0.94	1.68	1.82	0.00	0.00	0.00	0.00
time (sec)	N/A	0.243	0.025	0.175	0.305	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	180	205	167	163	234	0	172	220
N.S.	1	1.05	1.19	0.97	0.95	1.36	0.00	1.00	1.28
time (sec)	N/A	0.380	0.048	0.032	0.270	0.276	0.000	0.374	11.404

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	207	220	180	168	214	0	200	228
N.S.	1	1.10	1.17	0.96	0.89	1.14	0.00	1.06	1.21
time (sec)	N/A	0.406	0.055	0.039	0.280	0.269	0.000	0.383	10.363

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	49	140	55	51	59	0	67	114
N.S.	1	1.04	2.98	1.17	1.09	1.26	0.00	1.43	2.43
time (sec)	N/A	0.238	0.038	0.037	0.197	0.256	0.000	0.308	3.829

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	102	122	220	186	179	0	0	105
N.S.	1	1.01	1.21	2.18	1.84	1.77	0.00	0.00	1.04
time (sec)	N/A	0.515	0.072	1.832	0.191	0.282	0.000	0.000	4.061

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	156	179	207	706	0	0	0	0	0
N.S.	1	1.15	1.33	4.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.812	0.352	31.967	0.000	0.000	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	C	B	B	F(-1)	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	96	95	210	3062	175	173	0	0	281
N.S.	1	0.99	2.19	31.90	1.82	1.80	0.00	0.00	2.93
time (sec)	N/A	0.610	0.100	3.267	0.192	0.268	0.000	0.000	4.557

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	66	66	73	0	0	0	0	0	0
N.S.	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.226	0.048	0.000	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	73	0	0	0	0	0	0
N.S.	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.215	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	58	58	58	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.091	0.000	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	39	35	0	141	0	0	0
N.S.	1	1.00	1.08	0.97	0.00	3.92	0.00	0.00	0.00
time (sec)	N/A	0.253	0.079	0.838	0.000	0.260	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	67	67	66	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.229	0.057	0.000	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	70	71	73	0	0	0	0	0	0
N.S.	1	1.01	1.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	0.044	0.000	0.000	0.000	0.000	0.000	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	72	72	73	0	0	0	0	0	0
N.S.	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.235	0.041	0.000	0.000	0.000	0.000	0.000	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	155	32	14	16	16
N.S.	1	1.00	1.14	1.00	11.07	2.29	1.00	1.14	1.14
time (sec)	N/A	0.185	12.408	0.012	1.429	0.252	28.846	0.576	3.401

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	143	28	12	14	14
N.S.	1	1.00	1.17	1.00	11.92	2.33	1.00	1.17	1.17
time (sec)	N/A	0.173	1.553	0.006	0.566	0.238	17.643	0.406	3.540

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	163	181	750	0	0	0	0	0
N.S.	1	1.10	1.22	5.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.821	0.314	23.717	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	154	32	15	18	18
N.S.	1	1.00	1.12	1.00	9.62	2.00	0.94	1.12	1.12
time (sec)	N/A	0.195	12.225	0.041	0.652	0.261	19.271	0.557	3.720

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	153	32	15	18	18
N.S.	1	1.00	1.12	1.00	9.56	2.00	0.94	1.12	1.12
time (sec)	N/A	0.195	17.193	0.013	0.652	0.298	46.574	0.545	3.408

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	28	33	29	147	129	0	0	0
N.S.	1	0.93	1.10	0.97	4.90	4.30	0.00	0.00	0.00
time (sec)	N/A	0.221	0.031	0.556	0.249	0.256	0.000	0.000	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	28	22	61	104	0	0	0	0
N.S.	1	1.17	0.92	2.54	4.33	0.00	0.00	0.00	0.00
time (sec)	N/A	0.223	0.011	0.315	0.191	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	17	16	15	22	15	101	15
N.S.	1	1.00	0.89	0.84	0.79	1.16	0.79	5.32	0.79
time (sec)	N/A	0.176	0.002	0.408	0.182	0.236	0.089	0.261	0.068

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	468	65	0	20	20
N.S.	1	1.00	1.11	1.00	26.00	3.61	0.00	1.11	1.11
time (sec)	N/A	0.196	6.881	0.092	1.663	0.253	0.000	0.943	3.440

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	265	44	17	20	20
N.S.	1	1.00	1.11	1.00	14.72	2.44	0.94	1.11	1.11
time (sec)	N/A	0.191	14.322	0.104	1.013	0.258	177.901	0.647	3.640

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	84	90	77	0	0	0	0	0	0
N.S.	1	1.07	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.080	0.000	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	15	20	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.83	1.11	1.11
time (sec)	N/A	0.195	0.349	0.020	0.249	0.252	34.155	0.405	3.168

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	169	36	0	20	20
N.S.	1	1.00	1.11	1.00	9.39	2.00	0.00	1.11	1.11
time (sec)	N/A	0.194	1.840	0.010	0.412	0.264	0.000	0.541	3.208

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [202] had the largest ratio of [1.22222000000000008]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	0.97	12	0.250
2	A	5	4	1.00	12	0.333
3	A	3	3	0.98	12	0.250
4	A	5	4	1.02	12	0.333
5	A	3	3	1.00	10	0.300
6	A	1	1	1.00	8	0.125
7	A	1	1	1.00	12	0.083
8	A	6	5	1.06	12	0.417
9	A	3	3	0.92	12	0.250
10	A	5	4	0.96	12	0.333
11	A	4	4	0.96	12	0.333
12	A	5	4	0.95	12	0.333
13	A	15	14	1.41	14	1.000
14	A	14	13	1.23	14	0.929
15	A	10	9	1.21	14	0.643
16	A	10	9	1.09	14	0.643
17	A	4	4	1.04	12	0.333
18	A	6	5	1.19	10	0.500
19	A	4	4	1.22	14	0.286
20	A	4	4	1.08	14	0.286
21	A	9	8	0.96	14	0.571
22	A	8	8	0.92	14	0.571

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	14	13	1.16	14	0.929
24	B	18	17	2.02	14	1.214
25	A	16	15	1.39	14	1.071
26	A	14	13	1.57	14	0.929
27	A	10	10	1.10	14	0.714
28	A	9	8	1.12	12	0.667
29	A	5	5	1.14	10	0.500
30	A	5	5	1.15	14	0.357
31	A	5	5	1.13	14	0.357
32	A	7	7	0.97	14	0.500
33	A	15	14	0.98	14	1.000
34	A	11	11	1.29	14	0.786
35	A	8	7	1.15	16	0.438
36	A	8	7	1.16	16	0.438
37	A	7	6	1.12	16	0.375
38	A	7	6	1.08	16	0.375
39	A	6	5	1.12	16	0.312
40	A	8	7	1.12	16	0.438
41	A	7	6	1.14	16	0.375
42	A	9	8	1.14	16	0.500
43	N/A	1	0	1.00	16	0.000
44	N/A	1	0	1.00	16	0.000
45	A	2	2	1.00	14	0.143
46	N/A	1	0	1.00	16	0.000
47	N/A	1	0	1.00	16	0.000
48	N/A	1	0	1.00	10	0.000
49	N/A	1	0	1.00	16	0.000
50	A	5	4	0.98	14	0.286
51	A	5	4	1.02	14	0.286
52	A	5	4	1.00	14	0.286
53	A	2	2	1.00	12	0.167
54	A	3	2	1.20	14	0.143
55	A	6	5	1.05	14	0.357

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
56	A	5	4	0.93	14	0.286
57	A	5	4	0.96	14	0.286
58	A	5	5	1.14	14	0.357
59	A	5	5	1.11	14	0.357
60	A	1	1	1.00	10	0.100
61	A	4	4	1.20	14	0.286
62	A	5	5	1.10	14	0.357
63	A	5	5	1.13	14	0.357
64	A	11	10	1.22	16	0.625
65	A	11	10	1.11	16	0.625
66	A	6	5	1.01	16	0.312
67	A	7	6	1.11	14	0.429
68	A	6	5	1.19	16	0.312
69	A	6	5	1.05	16	0.312
70	A	10	9	0.99	16	0.562
71	A	2	2	1.00	16	0.125
72	A	2	2	1.00	16	0.125
73	A	2	2	1.00	12	0.167
74	A	2	2	1.00	16	0.125
75	A	2	2	1.00	16	0.125
76	A	2	2	1.00	16	0.125
77	A	10	9	1.12	16	0.562
78	A	7	6	1.07	14	0.429
79	A	7	6	1.16	16	0.375
80	A	7	6	1.06	16	0.375
81	A	9	8	0.99	16	0.500
82	A	17	16	1.12	18	0.889
83	A	16	15	1.10	18	0.833
84	A	16	15	1.09	18	0.833
85	A	16	15	1.12	18	0.833
86	A	16	15	1.14	18	0.833
87	A	15	14	1.08	18	0.778
88	A	17	16	1.11	18	0.889
89	A	17	16	1.10	18	0.889

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
90	A	5	4	0.85	20	0.200
91	A	5	4	0.84	20	0.200
92	A	5	4	0.82	20	0.200
93	A	5	4	0.81	20	0.200
94	N/A	1	0	1.00	18	0.000
95	N/A	1	0	1.00	18	0.000
96	A	2	2	1.00	16	0.125
97	N/A	1	0	1.00	18	0.000
98	N/A	1	0	1.00	18	0.000
99	A	5	4	0.98	14	0.286
100	A	5	4	1.02	14	0.286
101	A	5	4	1.00	14	0.286
102	A	2	2	1.00	14	0.143
103	A	3	2	1.20	14	0.143
104	A	6	5	1.05	14	0.357
105	A	5	4	0.93	14	0.286
106	A	5	4	0.96	14	0.286
107	A	12	11	1.02	14	0.786
108	A	1	1	1.00	10	0.100
109	A	11	10	1.01	14	0.714
110	A	11	10	1.10	14	0.714
111	A	12	11	1.09	14	0.786
112	A	11	10	1.07	14	0.714
113	A	11	10	1.07	12	0.833
114	A	10	9	1.06	14	0.643
115	A	12	11	1.07	14	0.786
116	A	11	10	1.22	16	0.625
117	A	11	10	1.11	16	0.625
118	A	6	5	1.01	16	0.312
119	A	7	6	1.08	16	0.375
120	A	6	5	1.16	16	0.312
121	A	6	5	1.01	16	0.312
122	A	10	9	0.99	16	0.562

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
123	A	10	9	0.95	16	0.562
124	A	12	11	1.06	16	0.688
125	A	10	9	1.14	16	0.562
126	A	7	6	1.10	16	0.375
127	A	7	6	1.14	16	0.375
128	A	7	6	1.11	16	0.375
129	A	9	8	1.01	16	0.500
130	N/A	1	0	1.00	18	0.000
131	N/A	1	0	1.00	18	0.000
132	A	2	2	1.00	16	0.125
133	N/A	1	0	1.00	18	0.000
134	N/A	1	0	1.00	18	0.000
135	A	4	4	0.96	14	0.286
136	A	7	6	0.93	14	0.429
137	A	4	4	0.90	12	0.333
138	A	1	1	1.00	10	0.100
139	A	3	2	1.10	14	0.143
140	A	2	2	1.00	14	0.143
141	A	4	4	1.00	14	0.286
142	A	7	6	0.96	14	0.429
143	A	15	14	1.14	16	0.875
144	A	10	9	0.92	16	0.562
145	A	10	9	0.99	14	0.643
146	A	10	9	1.08	12	0.750
147	A	6	5	1.20	16	0.312
148	A	7	6	1.16	16	0.375
149	A	6	5	1.00	16	0.312
150	A	13	12	1.31	16	0.750
151	A	16	15	1.00	16	0.938
152	A	9	8	0.97	14	0.571
153	A	9	9	1.12	12	0.750
154	A	7	6	1.13	16	0.375
155	A	7	6	1.11	16	0.375

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
156	A	10	9	1.11	16	0.562
157	A	7	6	0.96	14	0.429
158	A	7	6	0.93	14	0.429
159	A	7	6	0.91	14	0.429
160	A	3	3	1.00	12	0.250
161	A	3	2	1.23	14	0.143
162	A	2	2	1.00	14	0.143
163	A	7	6	1.00	14	0.429
164	A	7	6	0.96	14	0.429
165	A	7	7	1.14	14	0.500
166	A	6	6	1.07	14	0.429
167	A	1	1	1.00	10	0.100
168	A	5	5	1.20	14	0.357
169	A	7	7	1.11	14	0.500
170	A	6	6	1.14	14	0.429
171	A	10	9	0.94	16	0.562
172	A	6	5	0.96	14	0.357
173	A	6	5	1.13	16	0.312
174	A	7	6	1.06	16	0.375
175	A	6	5	0.94	16	0.312
176	A	3	3	1.00	16	0.188
177	A	3	3	1.00	16	0.188
178	A	3	3	0.68	12	0.250
179	A	3	3	1.00	16	0.188
180	A	3	3	1.00	16	0.188
181	A	3	3	1.00	16	0.188
182	N/A	1	0	1.00	18	0.000
183	N/A	1	0	1.00	18	0.000
184	A	4	3	1.04	16	0.188
185	N/A	1	0	1.00	18	0.000
186	N/A	1	0	1.00	18	0.000
187	A	8	7	1.15	16	0.438
188	A	7	6	1.12	16	0.375

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
189	A	6	5	1.08	14	0.357
190	A	1	1	1.00	12	0.083
191	A	3	2	1.38	16	0.125
192	A	5	4	1.08	16	0.250
193	A	6	5	0.98	16	0.312
194	A	7	6	1.00	16	0.375
195	A	21	20	1.45	18	1.111
196	A	16	15	1.31	18	0.833
197	A	11	10	1.23	16	0.625
198	A	6	5	1.13	14	0.357
199	A	6	5	1.22	18	0.278
200	A	10	9	1.05	18	0.500
201	A	15	14	1.14	18	0.778
202	B	23	22	2.54	18	1.222
203	B	19	18	2.01	18	1.000
204	A	15	14	1.52	16	0.875
205	A	10	9	1.20	14	0.643
206	A	7	6	1.17	18	0.333
207	A	9	8	1.06	18	0.444
208	A	13	12	1.31	18	0.667
209	A	3	3	0.92	12	0.250
210	A	3	3	0.90	12	0.250
211	A	2	2	1.00	12	0.167
212	A	4	4	1.00	12	0.333
213	A	13	12	1.12	16	0.750
214	A	6	5	1.10	16	0.312
215	A	13	12	1.05	14	0.857
216	A	1	1	1.00	12	0.083
217	A	3	2	1.24	16	0.125
218	A	12	11	1.05	16	0.688
219	A	13	12	1.10	16	0.750
220	A	6	5	1.04	16	0.312
221	A	6	5	1.01	18	0.278
222	A	6	5	1.15	18	0.278

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
223	A	10	9	0.99	18	0.500
224	A	2	2	1.00	14	0.143
225	A	2	2	1.00	12	0.167
226	A	1	1	1.00	10	0.100
227	A	3	2	1.00	14	0.143
228	A	2	2	1.00	14	0.143
229	A	2	2	1.01	14	0.143
230	A	2	2	1.00	14	0.143
231	N/A	1	0	1.00	14	0.000
232	N/A	1	0	1.00	12	0.000
233	A	6	5	1.10	16	0.312
234	N/A	1	0	1.00	16	0.000
235	N/A	1	0	1.00	16	0.000
236	A	3	2	0.93	10	0.200
237	A	3	2	1.17	10	0.200
238	A	3	3	1.00	4	0.750
239	N/A	1	0	1.00	18	0.000
240	N/A	1	0	1.00	18	0.000
241	A	3	3	1.07	16	0.188
242	N/A	1	0	1.00	18	0.000
243	N/A	1	0	1.00	18	0.000

2.3. Detailed conclusion table specific for Rubi results

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^5(a + b\,\mathrm{arctanh}(cx))\,dx$

3.1.1 Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^5(a+b\,\mathrm{arctanh}(cx))\,dx=\frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}-\frac{b\,\mathrm{arctanh}(cx)}{6c^6}+\frac{1}{6}x^6(a+b\,\mathrm{arctanh}(cx))$$

output
```
1/6*b*x/c^5+1/18*b*x^3/c^3+1/30*b*x^5/c-1/6*b*arctanh(c*x)/c^6+1/6*x^6*(a+
b*arctanh(c*x))
```

3.1.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int x^5(a+b\,\mathrm{arctanh}(cx))\,dx=\frac{bx}{6c^5}+\frac{bx^3}{18c^3}+\frac{bx^5}{30c}+\frac{ax^6}{6}+\frac{1}{6}bx^6\mathrm{arctanh}(cx)+\frac{b\log(1-cx)}{12c^6}-\frac{b\log(1+cx)}{12c^6}$$

input
```
Integrate[x^5*(a + b*ArcTanh[c*x]),x]
```

output
```
(b*x)/(6*c^5) + (b*x^3)/(18*c^3) + (b*x^5)/(30*c) + (a*x^6)/6 + (b*x^6*Arc
Tanh[c*x])/6 + (b*Log[1 - c*x])/(12*c^6) - (b*Log[1 + c*x])/(12*c^6)
```

3.1.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6452, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+b\operatorname{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{6}x^6(a+b\operatorname{arctanh}(cx))-\frac{1}{6}bc\int \frac{x^6}{1-c^2x^2}dx$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{6}x^6(a+b\operatorname{arctanh}(cx))-\frac{1}{6}bc\int \left(-\frac{x^4}{c^2}-\frac{x^2}{c^4}+\frac{1}{c^6\left(1-c^2x^2\right)}-\frac{1}{c^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6(a+b\operatorname{arctanh}(cx))-\frac{1}{6}bc\left(\frac{\operatorname{arctanh}(cx)}{c^7}-\frac{x}{c^6}-\frac{x^3}{3c^4}-\frac{x^5}{5c^2}\right)$$

input
```
Int[x^5*(a + b*ArcTanh[c*x]),x]
```

output
```
(x^6*(a + b*ArcTanh[c*x]))/6 - (b*c*(-(x/c^6) - x^3/(3*c^4) - x^5/(5*c^2)
+ ArcTanh[c*x]/c^7))/6
```

3.1.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.1.4 Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$-\frac{-15b\,\operatorname{arctanh}(cx)x^6c^6-15a\,c^6x^6-3b\,c^5x^5-5b\,c^3x^3-15bcx+15b\,\operatorname{arctanh}(cx)}{90c^6}$	59
parts	$\frac{a\,x^6}{6}+\frac{b\left(\frac{c^6x^6\,\operatorname{arctanh}(cx)}{6}+\frac{c^5x^5}{30}+\frac{c^3x^3}{18}+\frac{cx}{6}+\frac{\ln(cx-1)}{12}-\frac{\ln(cx+1)}{12}\right)}{c^6}$	62
derivativedivides	$\frac{\frac{a\,c^6x^6}{6}+b\left(\frac{c^6x^6\,\operatorname{arctanh}(cx)}{6}+\frac{c^5x^5}{30}+\frac{c^3x^3}{18}+\frac{cx}{6}+\frac{\ln(cx-1)}{12}-\frac{\ln(cx+1)}{12}\right)}{c^6}$	66
default	$\frac{\frac{a\,c^6x^6}{6}+b\left(\frac{c^6x^6\,\operatorname{arctanh}(cx)}{6}+\frac{c^5x^5}{30}+\frac{c^3x^3}{18}+\frac{cx}{6}+\frac{\ln(cx-1)}{12}-\frac{\ln(cx+1)}{12}\right)}{c^6}$	66
risch	$\frac{b\,x^6\ln(cx+1)}{12}-\frac{b\,x^6\ln(-cx+1)}{12}+\frac{a\,x^6}{6}+\frac{b\,x^5}{30c}+\frac{b\,x^3}{18c^3}+\frac{bx}{6c^5}-\frac{b\ln(cx+1)}{12c^6}+\frac{b\ln(-cx+1)}{12c^6}$	83

input
```
int(x^5*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/90*(-15*b*arctanh(c*x)*x^6*c^6-15*a*c^6*x^6-3*b*c^5*x^5-5*b*c^3*x^3-15*
b*c*x+15*b*arctanh(c*x))/c^6
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14

$$\int x^5(a+b\operatorname{arctanh}(cx))\,dx = \frac{30\,ac^6x^6+6\,bc^5x^5+10\,bc^3x^3+30\,bcx+15\,(bc^6x^6-b)\log\left(-\frac{cx+1}{cx-1}\right)}{180\,c^6}$$

input
```
integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/180*(30*a*c^6*x^6 + 6*b*c^5*x^5 + 10*b*c^3*x^3 + 30*b*c*x + 15*(b*c^6*x^
6 - b)*log(-(c*x + 1)/(c*x - 1)))/c^6
```

3.1.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int x^5(a+b\text{arctanh}(cx))\,dx = \begin{cases} \frac{ax^6}{6} + \frac{bx^6\,\text{atanh}\,(cx)}{6} + \frac{bx^5}{30c} + \frac{bx^3}{18c^3} + \frac{bx}{6c^5} - \frac{b\,\text{atanh}\,(cx)}{6c^6} & \text{for } c \neq 0 \\ \frac{ax^6}{6} & \text{otherwise} \end{cases}$$

input
```
integrate(x**5*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*x**6/6 + b*x**6*atanh(c*x)/6 + b*x**5/(30*c) + b*x**3/(18*c**
3) + b*x/(6*c**5) - b*atanh(c*x)/(6*c**6), Ne(c, 0)), (a*x**6/6, True))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.19

$$\int x^5(a+b\text{arctanh}(cx))\,dx = \frac{1}{6}\,ax^6 + \frac{1}{180}\left(30\,x^6\,\text{artanh}\,(cx) + c\left(\frac{2\,(3\,c^4x^5 + 5\,c^2x^3 + 15\,x)}{c^6} - \frac{15\,\log\,(cx+1)}{c^7} + \frac{15\,\log\,(cx-1)}{c^7}\right)\right)b$$

input
```
integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/180*(30*x^6*arctanh(c*x) + c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*
x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7))*b
```

3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(49) = 98$.

Time = 0.30 (sec) , antiderivative size = 442, normalized size of antiderivative = 7.49

$$\int x^5(a + b\text{arctanh}(cx))\,dx$$
$$= \frac{1}{45}c\left(\frac{15\left(\frac{3(cx+1)^5b}{(cx-1)^5} + \frac{10(cx+1)^3b}{(cx-1)^3} + \frac{3(cx+1)b}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^6c^7}{(cx-1)^6} - \frac{6(cx+1)^5c^7}{(cx-1)^5} + \frac{15(cx+1)^4c^7}{(cx-1)^4} - \frac{20(cx+1)^3c^7}{(cx-1)^3} + \frac{15(cx+1)^2c^7}{(cx-1)^2} - \frac{6(cx+1)c^7}{cx-1} + c^7} + \frac{\frac{90(cx+1)^5a}{(cx-1)^5} + \frac{300(cx+1}{(cx-1)}}{\frac{(cx+1)^6}{(cx-1}}\right.$$

input
```
integrate(x^5*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
1/45*c*(15*(3*(c*x + 1)^5*b/(c*x - 1)^5 + 10*(c*x + 1)^3*b/(c*x - 1)^3 + 3
*(c*x + 1)*b/(c*x - 1))*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6*c^7/(c*x -
1)^6 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 - 20
*(c*x + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x + 1
)*c^7/(c*x - 1) + c^7) + (90*(c*x + 1)^5*a/(c*x - 1)^5 + 300*(c*x + 1)^3*a
/(c*x - 1)^3 + 90*(c*x + 1)*a/(c*x - 1) + 45*(c*x + 1)^5*b/(c*x - 1)^5 - 1
35*(c*x + 1)^4*b/(c*x - 1)^4 + 230*(c*x + 1)^3*b/(c*x - 1)^3 - 210*(c*x +
1)^2*b/(c*x - 1)^2 + 93*(c*x + 1)*b/(c*x - 1) - 23*b)/((c*x + 1)^6*c^7/(c*
x - 1)^6 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4
- 20*(c*x + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x
 + 1)*c^7/(c*x - 1) + c^7))
```

3.1.9 Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int x^5(a + b\text{arctanh}(cx))\,dx = \frac{\frac{b\,c^3\,x^3}{18} - \frac{b\,\text{atanh}(c\,x)}{6} + \frac{b\,c^5\,x^5}{30} + \frac{b\,c\,x}{6}}{c^6} + \frac{a\,x^6}{6} + \frac{b\,x^6\,\text{atanh}(c\,x)}{6}$$

input
```
int(x^5*(a + b*atanh(c*x)),x)
```

output
```
((b*c^3*x^3)/18 - (b*atanh(c*x))/6 + (b*c^5*x^5)/30 + (b*c*x)/6)/c^6 + (a*
x^6)/6 + (b*x^6*atanh(c*x))/6
```

3.2 $\int x^4(a + b\text{arctanh}(cx))\,dx$

3.2.1 Optimal result

Integrand size = 12, antiderivative size = 57

$$\int x^4(a+b\text{arctanh}(cx))\,dx=\frac{bx^2}{10c^3}+\frac{bx^4}{20c}+\frac{1}{5}x^5(a+b\text{arctanh}(cx))+\frac{b\log(1-c^2x^2)}{10c^5}$$

output
```
1/10*b*x^2/c^3+1/20*b*x^4/c+1/5*x^5*(a+b*arctanh(c*x))+1/10*b*ln(-c^2*x^2+
1)/c^5
```

3.2.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int x^4(a+b\text{arctanh}(cx))\,dx=\frac{bx^2}{10c^3}+\frac{bx^4}{20c}+\frac{ax^5}{5}+\frac{1}{5}bx^5\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{10c^5}$$

input
```
Integrate[x^4*(a + b*ArcTanh[c*x]),x]
```

output
```
(b*x^2)/(10*c^3) + (b*x^4)/(20*c) + (a*x^5)/5 + (b*x^5*ArcTanh[c*x])/5 + (
b*Log[1 - c^2*x^2])/(10*c^5)
```

3.2.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6452, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{5}bc\int \frac{x^5}{1-c^2x^2}dx$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\int \frac{x^4}{1-c^2x^2}dx^2$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\int \left(-\frac{x^2}{c^2}-\frac{1}{c^4\left(c^2x^2-1\right)}-\frac{1}{c^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}-\frac{x^4}{2c^2}-\frac{\log\left(1-c^2x^2\right)}{c^6}\right)$$

input
```
Int[x^4*(a + b*ArcTanh[c*x]),x]
```

output
```
(x^5*(a + b*ArcTanh[c*x]))/5 - (b*c*(-(x^2/c^4) - x^4/(2*c^2) - Log[1 - c^
2*x^2]/c^6))/10
```

3.2.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.2.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

method	result	size
parts	$\frac{a\,x^{5}}{5}+\frac{b\left(\frac{c^{5} x^{5} \operatorname{arctanh}\left(c x \right)}{5}+\frac{c^{4} x^{4}}{20}+\frac{c^{2} x^{2}}{10}+\frac{\ln \left(c x -1\right)}{10}+\frac{\ln \left(c x +1\right)}{10}\right)}{c^{5}}$	58
derivativedivides	$\frac{\frac{c^{5} x^{5} a}{5}+b \left(\frac{c^{5} x^{5} \operatorname{arctanh}\left(c x \right)}{5}+\frac{c^{4} x^{4}}{20}+\frac{c^{2} x^{2}}{10}+\frac{\ln \left(c x -1\right)}{10}+\frac{\ln \left(c x +1\right)}{10}\right)}{c^{5}}$	62
default	$\frac{\frac{c^{5} x^{5} a}{5}+b \left(\frac{c^{5} x^{5} \operatorname{arctanh}\left(c x \right)}{5}+\frac{c^{4} x^{4}}{20}+\frac{c^{2} x^{2}}{10}+\frac{\ln \left(c x -1\right)}{10}+\frac{\ln \left(c x +1\right)}{10}\right)}{c^{5}}$	62
parallelrisch	$\frac{4 b \operatorname{arctanh}\left(c x \right) x^{5} c^{5}+4 c^{5} x^{5} a +b \,c^{4} x^{4}+2 b \,c^{2} x^{2}+4 \ln \left(c x -1\right) b +4 b \operatorname{arctanh}\left(c x \right)+2 b}{20 c^{5}}$	65
risch	$\frac{b\,x^{5} \ln \left(c x +1\right)}{10}-\frac{b\,x^{5} \ln \left(-c x +1\right)}{10}+\frac{a\,x^{5}}{5}+\frac{b\,x^{4}}{20 c}+\frac{b\,x^{2}}{10 c^{3}}+\frac{b \ln \left(c^{2} x^{2}-1\right)}{10 c^{5}}$	67

input
```
int(x^4*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+b/c^5*(1/5*c^5*x^5*arctanh(c*x)+1/20*c^4*x^4+1/10*c^2*x^2+1/10*l
n(c*x-1)+1/10*ln(c*x+1))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.21

$$\int x^4(a+b\text{arctanh}(cx))\,dx=\frac{2\,bc^5x^5\log\left(-\frac{cx+1}{cx-1}\right)+4\,ac^5x^5+bc^4x^4+2\,bc^2x^2+2\,b\log\left(c^2x^2-1\right)}{20\,c^5}$$

input
```
integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/20*(2*b*c^5*x^5*log(-(c*x + 1)/(c*x - 1)) + 4*a*c^5*x^5 + b*c^4*x^4 + 2*
b*c^2*x^2 + 2*b*log(c^2*x^2 - 1))/c^5
```

3.2.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int x^4(a+b\text{arctanh}(cx))\,dx$$
$$=\begin{cases}\frac{ax^5}{5}+\frac{bx^5\,\text{atanh}\,(cx)}{5}+\frac{bx^4}{20c}+\frac{bx^2}{10c^3}+\frac{b\log\left(x-\frac{1}{c}\right)}{5c^5}+\frac{b\,\text{atanh}\,(cx)}{5c^5} & \text{for } c\neq 0\\ \frac{ax^5}{5} & \text{otherwise}\end{cases}$$

input
```
integrate(x**4*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*x**5/5 + b*x**5*atanh(c*x)/5 + b*x**4/(20*c) + b*x**2/(10*c**
3) + b*log(x - 1/c)/(5*c**5) + b*atanh(c*x)/(5*c**5), Ne(c, 0)), (a*x**5/5
, True))
```

3.2.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int x^4(a + b\text{arctanh}(cx))\,dx = \frac{1}{5}\,ax^5 + \frac{1}{20}\left(4\,x^5\,\text{artanh}\left(cx\right) + c\left(\frac{c^2x^4 + 2\,x^2}{c^4} + \frac{2\,\log\left(c^2x^2 - 1\right)}{c^6}\right)\right)b$$

input

```
integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output

```
1/5*a*x^5 + 1/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(c^
2*x^2 - 1)/c^6))*b
```

3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(49) = 98.

Time = 0.29 (sec) , antiderivative size = 403, normalized size of antiderivative = 7.07

$$\int x^4(a + b\text{arctanh}(cx))\,dx$$
$$= \frac{1}{5}\,c\left(\frac{\left(\frac{5\,(cx+1)^4b}{(cx-1)^4} + \frac{10\,(cx+1)^2b}{(cx-1)^2} + b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5c^6}{(cx-1)^5} - \frac{5\,(cx+1)^4c^6}{(cx-1)^4} + \frac{10\,(cx+1)^3c^6}{(cx-1)^3} - \frac{10\,(cx+1)^2c^6}{(cx-1)^2} + \frac{5\,(cx+1)c^6}{cx-1} - c^6} + \frac{2\left(\frac{5\,(cx+1)^4a}{(cx-1)^4} + \frac{10\,(cx+1)^2a}{(cx-1)^2} + a + \frac{2\,(c}{(c}\right.}{\frac{(cx+1)^5c^6}{(cx-1)^5} - \frac{5\,(cx+1)^4c^6}{(cx-1)^4} + \frac{10\,(cx}{(cx}}\right.$$

input

```
integrate(x^4*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output

```
1/5*c*((5*(c*x + 1)^4*b/(c*x - 1)^4 + 10*(c*x + 1)^2*b/(c*x - 1)^2 + b)*lo
g(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5*c^6/(c*x - 1)^5 - 5*(c*x + 1)^4*c^6/(
c*x - 1)^4 + 10*(c*x + 1)^3*c^6/(c*x - 1)^3 - 10*(c*x + 1)^2*c^6/(c*x - 1)
^2 + 5*(c*x + 1)*c^6/(c*x - 1) - c^6) + 2*(5*(c*x + 1)^4*a/(c*x - 1)^4 + 1
0*(c*x + 1)^2*a/(c*x - 1)^2 + a + 2*(c*x + 1)^4*b/(c*x - 1)^4 - 4*(c*x + 1
)^3*b/(c*x - 1)^3 + 4*(c*x + 1)^2*b/(c*x - 1)^2 - 2*(c*x + 1)*b/(c*x - 1))
/((c*x + 1)^5*c^6/(c*x - 1)^5 - 5*(c*x + 1)^4*c^6/(c*x - 1)^4 + 10*(c*x +
1)^3*c^6/(c*x - 1)^3 - 10*(c*x + 1)^2*c^6/(c*x - 1)^2 + 5*(c*x + 1)*c^6/(c
*x - 1) - c^6) - b*log(-(c*x + 1)/(c*x - 1) + 1)/c^6 + b*log(-(c*x + 1)/(c
*x - 1))/c^6)
```

3.2.9 Mupad [B] (verification not implemented)

Time = 3.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int x^4(a+b\operatorname{arctanh}(cx))\,dx=\frac{a\,x^5}{5}+\frac{\frac{b\,\ln\left(c^2\,x^2-1\right)}{10}+\frac{b\,c^2\,x^2}{10}+\frac{b\,c^4\,x^4}{20}}{c^5}+\frac{b\,x^5\,\mathrm{atanh}(c\,x)}{5}$$

input

```
int(x^4*(a + b*atanh(c*x)),x)
```

output

```
(a*x^5)/5 + ((b*log(c^2*x^2 - 1))/10 + (b*c^2*x^2)/10 + (b*c^4*x^4)/20)/c^
5 + (b*x^5*atanh(c*x))/5
```

3.3 $\int x^3(a + b\,\text{arctanh}(cx))\,dx$

3.3.1 Optimal result

Integrand size = 12, antiderivative size = 48

$$\int x^3(a + b\,\text{arctanh}(cx))\,dx = \frac{bx}{4c^3} + \frac{bx^3}{12c} - \frac{b\,\text{arctanh}(cx)}{4c^4} + \frac{1}{4}x^4(a + b\,\text{arctanh}(cx))$$

output
```
1/4*b*x/c^3+1/12*b*x^3/c-1/4*b*arctanh(c*x)/c^4+1/4*x^4*(a+b*arctanh(c*x))
```

3.3.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.46

$$\int x^3(a + b\,\text{arctanh}(cx))\,dx = \frac{bx}{4c^3} + \frac{bx^3}{12c} + \frac{ax^4}{4} + \frac{1}{4}bx^4\text{arctanh}(cx) + \frac{b\log(1 - cx)}{8c^4} - \frac{b\log(1 + cx)}{8c^4}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*x]),x]
```

output
```
(b*x)/(4*c^3) + (b*x^3)/(12*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*x])/4 + (b*L
og[1 - c*x])/(8*c^4) - (b*Log[1 + c*x])/(8*c^4)
```

3.3.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6452, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\int \frac{x^4}{1-c^2x^2}dx$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\int \left(-\frac{x^2}{c^2}+\frac{1}{c^4(1-c^2x^2)}-\frac{1}{c^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\left(\frac{\text{arctanh}(cx)}{c^5}-\frac{x}{c^4}-\frac{x^3}{3c^2}\right)$$

input
```
Int[x^3*(a + b*ArcTanh[c*x]),x]
```

output
```
(x^4*(a + b*ArcTanh[c*x]))/4 - (b*c*(-(x/c^4) - x^3/(3*c^2) + ArcTanh[c*x]
/c^5))/4
```

3.3.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.3.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$-\frac{-3b\,\mathrm{arctanh}(cx)x^4c^4-3c^4x^4a-b\,c^3x^3-3bcx+3b\,\mathrm{arctanh}(cx)}{12c^4}$	50
parts	$\frac{a\,x^4}{4}+\frac{b\left(\frac{c^4x^4\,\mathrm{arctanh}(cx)}{4}+\frac{c^3x^3}{12}+\frac{cx}{4}+\frac{\ln(cx-1)}{8}-\frac{\ln(cx+1)}{8}\right)}{c^4}$	54
derivativedivides	$\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\,\mathrm{arctanh}(cx)}{4}+\frac{c^3x^3}{12}+\frac{cx}{4}+\frac{\ln(cx-1)}{8}-\frac{\ln(cx+1)}{8}\right)}{c^4}$	58
default	$\frac{\frac{c^4x^4a}{4}+b\left(\frac{c^4x^4\,\mathrm{arctanh}(cx)}{4}+\frac{c^3x^3}{12}+\frac{cx}{4}+\frac{\ln(cx-1)}{8}-\frac{\ln(cx+1)}{8}\right)}{c^4}$	58
risch	$\frac{b\,x^4\ln(cx+1)}{8}-\frac{b\,x^4\ln(-cx+1)}{8}+\frac{a\,x^4}{4}+\frac{b\,x^3}{12c}+\frac{bx}{4c^3}+\frac{b\ln(-cx+1)}{8c^4}-\frac{b\ln(cx+1)}{8c^4}$	74

input
```
int(x^3*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*b*arctanh(c*x)*x^4*c^4-3*c^4*x^4*a-b*c^3*x^3-3*b*c*x+3*b*arctanh
(c*x))/c^4
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.21

$$\int x^3(a+b\mathrm{arctanh}(cx))\,dx=\frac{6\,ac^4x^4+2\,bc^3x^3+6\,bcx+3\,(bc^4x^4-b)\log\left(-\frac{cx+1}{cx-1}\right)}{24\,c^4}$$

input
```
integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/24*(6*a*c^4*x^4 + 2*b*c^3*x^3 + 6*b*c*x + 3*(b*c^4*x^4 - b)*log(-(c*x +
1)/(c*x - 1)))/c^4
```

3.3.6 Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int x^3(a+b\operatorname{arctanh}(cx))\,dx = \begin{cases} \frac{ax^4}{4} + \frac{bx^4\operatorname{atanh}(cx)}{4} + \frac{bx^3}{12c} + \frac{bx}{4c^3} - \frac{b\operatorname{atanh}(cx)}{4c^4} & \text{for } c \neq 0 \\ \frac{ax^4}{4} & \text{otherwise} \end{cases}$$

input
```
integrate(x**3*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*x**4/4 + b*x**4*atanh(c*x)/4 + b*x**3/(12*c) + b*x/(4*c**3) -
 b*atanh(c*x)/(4*c**4), Ne(c, 0)), (a*x**4/4, True))
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int x^3(a+b\operatorname{arctanh}(cx))\,dx = \frac{1}{4}\,ax^4 + \frac{1}{24}\left(6\,x^4\operatorname{artanh}(cx) + c\left(\frac{2\,(c^2x^3+3\,x)}{c^4} - \frac{3\,\log(cx+1)}{c^5} + \frac{3\,\log(cx-1)}{c^5}\right)\right)b$$

input
```
integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/24*(6*x^4*arctanh(c*x) + c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*
x + 1)/c^5 + 3*log(c*x - 1)/c^5))*b
```

3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. 2(40) = 80.

Time = 0.30 (sec) , antiderivative size = 296, normalized size of antiderivative = 6.17

$$\int x^3(a+b\operatorname{arctanh}(cx))\,dx = \frac{1}{3}\,c\left(\frac{3\left(\frac{(cx+1)^3b}{(cx-1)^3} + \frac{(cx+1)b}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4c^5}{(cx-1)^4} - \frac{4\,(cx+1)^3c^5}{(cx-1)^3} + \frac{6\,(cx+1)^2c^5}{(cx-1)^2} - \frac{4\,(cx+1)c^5}{cx-1} + c^5} + \frac{\frac{6\,(cx+1)^3a}{(cx-1)^3} + \frac{6\,(cx+1)a}{cx-1} + \frac{3\,(cx+1)^3b}{(cx-1)^3} - \frac{6\,(cx+1)^2b}{(cx-1)^2} + \frac{5\,(c}{c}}{\frac{(cx+1)^4c^5}{(cx-1)^4} - \frac{4\,(cx+1)^3c^5}{(cx-1)^3} + \frac{6\,(cx+1)^2c^5}{(cx-1)^2} - \frac{4\,(cx+1)c^5}{cx-1}}\right.$$

input
```
integrate(x^3*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
1/3*c*(3*((c*x + 1)^3*b/(c*x - 1)^3 + (c*x + 1)*b/(c*x - 1))*log(-(c*x + 1
)/(c*x - 1))/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^3
+ 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5) + (6*(c
*x + 1)^3*a/(c*x - 1)^3 + 6*(c*x + 1)*a/(c*x - 1) + 3*(c*x + 1)^3*b/(c*x -
 1)^3 - 6*(c*x + 1)^2*b/(c*x - 1)^2 + 5*(c*x + 1)*b/(c*x - 1) - 2*b)/((c*x
 + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^
5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5))
```

3.3.9 Mupad [B] (verification not implemented)

Time = 3.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int x^3(a+b\text{arctanh}(cx))\,dx=\frac{a\,x^4}{4}+\frac{\frac{b\,c^3\,x^3}{12}-\frac{b\,\text{atanh}(c\,x)}{4}+\frac{b\,c\,x}{4}}{c^4}+\frac{b\,x^4\,\text{atanh}(c\,x)}{4}$$

input
```
int(x^3*(a + b*atanh(c*x)),x)
```

output
```
(a*x^4)/4 + ((b*c^3*x^3)/12 - (b*atanh(c*x))/4 + (b*c*x)/4)/c^4 + (b*x^4*a
tanh(c*x))/4
```

3.4 $\int x^2(a+b\,\mathbf{arctanh}(cx))\,dx$

3.4.1 Optimal result

Integrand size = 12, antiderivative size = 46

$$\int x^2(a+b\,\mathrm{arctanh}(cx))\,dx=\frac{bx^2}{6c}+\frac{1}{3}x^3(a+b\,\mathrm{arctanh}(cx))+\frac{b\log(1-c^2x^2)}{6c^3}$$

output
```
1/6*b*x^2/c+1/3*x^3*(a+b*arctanh(c*x))+1/6*b*ln(-c^2*x^2+1)/c^3
```

3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11

$$\int x^2(a+b\,\mathrm{arctanh}(cx))\,dx=\frac{bx^2}{6c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\mathrm{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{6c^3}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x]),x]
```

output
```
(b*x^2)/(6*c) + (a*x^3)/3 + (b*x^3*ArcTanh[c*x])/3 + (b*Log[1 - c^2*x^2])/
(6*c^3)
```

3.4.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6452, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \; 6452$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{3}bc\int \frac{x^3}{1-c^2x^2}dx$$

$$\Big\downarrow \; 243$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\int \frac{x^2}{1-c^2x^2}dx^2$$

$$\Big\downarrow \; 49$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\int \left(-\frac{1}{c^2}-\frac{1}{c^2\left(c^2x^2-1\right)}\right)dx^2$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log\left(1-c^2x^2\right)}{c^4}\right)$$

input
```
Int[x^2*(a + b*ArcTanh[c*x]),x]
```

output
```
(x^3*(a + b*ArcTanh[c*x]))/3 - (b*c*(-(x^2/c^2) - Log[1 - c^2*x^2]/c^4))/6
```

3.4.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.4.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.09

method	result	size
parts	$\frac{a\,x^3}{3}+\frac{b\left(\frac{c^3x^3\,\mathrm{arctanh}(cx)}{3}+\frac{c^2x^2}{6}+\frac{\ln(cx-1)}{6}+\frac{\ln(cx+1)}{6}\right)}{c^3}$	50
parallelrisch	$\frac{2b\,\mathrm{arctanh}(cx)x^3c^3+2c^3x^3a+b\,c^2x^2+2\ln(cx-1)b+2b\,\mathrm{arctanh}(cx)}{6c^3}$	53
derivativedivides	$\frac{\frac{c^3x^3a}{3}+b\left(\frac{c^3x^3\,\mathrm{arctanh}(cx)}{3}+\frac{c^2x^2}{6}+\frac{\ln(cx-1)}{6}+\frac{\ln(cx+1)}{6}\right)}{c^3}$	54
default	$\frac{\frac{c^3x^3a}{3}+b\left(\frac{c^3x^3\,\mathrm{arctanh}(cx)}{3}+\frac{c^2x^2}{6}+\frac{\ln(cx-1)}{6}+\frac{\ln(cx+1)}{6}\right)}{c^3}$	54
risch	$\frac{b\,x^3\ln(cx+1)}{6}-\frac{b\,x^3\ln(-cx+1)}{6}+\frac{a\,x^3}{3}+\frac{b\,x^2}{6c}+\frac{b\ln(c^2x^2-1)}{6c^3}$	58

input
```
int(x^2*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+b/c^3*(1/3*c^3*x^3*arctanh(c*x)+1/6*c^2*x^2+1/6*ln(c*x-1)+1/6*ln
(c*x+1))
```

3.4.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int x^2(a+b\text{arctanh}(cx))\,dx=\frac{bc^3x^3\log\left(-\frac{cx+1}{cx-1}\right)+2\,ac^3x^3+bc^2x^2+b\log\left(c^2x^2-1\right)}{6\,c^3}$$

input
```
integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/6*(b*c^3*x^3*log(-(c*x + 1)/(c*x - 1)) + 2*a*c^3*x^3 + b*c^2*x^2 + b*log
(c^2*x^2 - 1))/c^3
```

3.4.6 Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int x^2(a+b\text{arctanh}(cx))\,dx=\begin{cases}\frac{ax^3}{3}+\frac{bx^3\,\text{atanh}\,(cx)}{3}+\frac{bx^2}{6c}+\frac{b\log\left(x-\frac{1}{c}\right)}{3c^3}+\frac{b\,\text{atanh}\,(cx)}{3c^3} & \text{for } c\neq 0\\ \frac{ax^3}{3} & \text{otherwise}\end{cases}$$

input
```
integrate(x**2*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*x**3/3 + b*x**3*atanh(c*x)/3 + b*x**2/(6*c) + b*log(x - 1/c)/
(3*c**3) + b*atanh(c*x)/(3*c**3), Ne(c, 0)), (a*x**3/3, True))
```

3.4.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2(a+b\text{arctanh}(cx))\,dx=\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\,\text{artanh}\,(cx)+c\left(\frac{x^2}{c^2}+\frac{\log\left(c^2x^2-1\right)}{c^4}\right)\right)b$$

input
```
integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4))*
b
```

3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(40) = 80$.

Time = 0.30 (sec) , antiderivative size = 258, normalized size of antiderivative = 5.61

$$\int x^2(a + b\text{arctanh}(cx))\,dx$$
$$= \frac{1}{3}c\left(\frac{\left(\frac{3(cx+1)^2b}{(cx-1)^2} + b\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3c^4}{(cx-1)^3} - \frac{3(cx+1)^2c^4}{(cx-1)^2} + \frac{3(cx+1)c^4}{cx-1} - c^4} + \frac{2\left(\frac{3(cx+1)^2a}{(cx-1)^2} + a + \frac{(cx+1)^2b}{(cx-1)^2} - \frac{(cx+1)b}{cx-1}\right)}{\frac{(cx+1)^3c^4}{(cx-1)^3} - \frac{3(cx+1)^2c^4}{(cx-1)^2} + \frac{3(cx+1)c^4}{cx-1} - c^4} - \frac{b\log\left(-\frac{cx+1}{cx-1} + 1\right)}{c^4} +$$

input
```
integrate(x^2*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
1/3*c*((3*(c*x + 1)^2*b/(c*x - 1)^2 + b)*log(-(c*x + 1)/(c*x - 1))/((c*x +
 1)^3*c^4/(c*x - 1)^3 - 3*(c*x + 1)^2*c^4/(c*x - 1)^2 + 3*(c*x + 1)*c^4/(c
*x - 1) - c^4) + 2*(3*(c*x + 1)^2*a/(c*x - 1)^2 + a + (c*x + 1)^2*b/(c*x -
 1)^2 - (c*x + 1)*b/(c*x - 1))/((c*x + 1)^3*c^4/(c*x - 1)^3 - 3*(c*x + 1)^
2*c^4/(c*x - 1)^2 + 3*(c*x + 1)*c^4/(c*x - 1) - c^4) - b*log(-(c*x + 1)/(c
*x - 1) + 1)/c^4 + b*log(-(c*x + 1)/(c*x - 1))/c^4)
```

3.4.9 Mupad [B] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int x^2(a + b\text{arctanh}(cx))\,dx = \frac{\frac{b\ln(c^2x^2-1)}{6} + \frac{b\,c^2\,x^2}{6}}{c^3} + \frac{a\,x^3}{3} + \frac{b\,x^3\,\text{atanh}(c\,x)}{3}$$

input
```
int(x^2*(a + b*atanh(c*x)),x)
```

output
```
((b*log(c^2*x^2 - 1))/6 + (b*c^2*x^2)/6)/c^3 + (a*x^3)/3 + (b*x^3*atanh(c*
x))/3
```

3.5 $\int x(a + b\text{arctanh}(cx))\,dx$

3.5.1 Optimal result

Integrand size = 10, antiderivative size = 37

$$\int x(a + b\text{arctanh}(cx))\,dx = \frac{bx}{2c} - \frac{b\text{arctanh}(cx)}{2c^2} + \frac{1}{2}x^2(a + b\text{arctanh}(cx))$$

output
```
1/2*b*x/c-1/2*b*arctanh(c*x)/c^2+1/2*x^2*(a+b*arctanh(c*x))
```

3.5.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int x(a + b\text{arctanh}(cx))\,dx = \frac{bx}{2c} + \frac{ax^2}{2} + \frac{1}{2}bx^2\text{arctanh}(cx) + \frac{b\log(1 - cx)}{4c^2} - \frac{b\log(1 + cx)}{4c^2}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x]),x]
```

output
```
(b*x)/(2*c) + (a*x^2)/2 + (b*x^2*ArcTanh[c*x])/2 + (b*Log[1 - c*x])/(4*c^2
) - (b*Log[1 + c*x])/(4*c^2)
```

3.5.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {6452, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\int\frac{x^2}{1-c^2x^2}dx$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\int\frac{1}{1-c^2x^2}dx}{c^2}-\frac{x}{c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)$$

input
```
Int[x*(a + b*ArcTanh[c*x]),x]
```

output
```
(x^2*(a + b*ArcTanh[c*x]))/2 - (b*c*(-(x/c^2) + ArcTanh[c*x]/c^3))/2
```

3.5.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.5.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

method	result	size
parallelrisch	$\frac{\operatorname{arctanh}(cx) b\,c^{2} x^{2}+a\,c^{2} x^{2}+bcx-b\operatorname{arctanh}(cx)}{2c^{2}}$	38
parts	$\frac{a\,x^{2}}{2}+\frac{b\left(\frac{c^{2} x^{2} \operatorname{arctanh}(cx)}{2}+\frac{cx}{2}+\frac{\ln(cx-1)}{4}-\frac{\ln(cx+1)}{4}\right)}{c^{2}}$	46
derivativedivides	$\frac{\frac{a\,c^{2} x^{2}}{2}+b\left(\frac{c^{2} x^{2} \operatorname{arctanh}(cx)}{2}+\frac{cx}{2}+\frac{\ln(cx-1)}{4}-\frac{\ln(cx+1)}{4}\right)}{c^{2}}$	50
default	$\frac{\frac{a\,c^{2} x^{2}}{2}+b\left(\frac{c^{2} x^{2} \operatorname{arctanh}(cx)}{2}+\frac{cx}{2}+\frac{\ln(cx-1)}{4}-\frac{\ln(cx+1)}{4}\right)}{c^{2}}$	50
risch	$\frac{b\,x^{2} \ln(cx+1)}{4}-\frac{b\,x^{2} \ln(-cx+1)}{4}+\frac{a\,x^{2}}{2}+\frac{bx}{2c}+\frac{b\ln(-cx+1)}{4c^{2}}-\frac{b\ln(cx+1)}{4c^{2}}$	65

input
```
int(x*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(arctanh(c*x)*b*c^2*x^2+a*c^2*x^2+b*c*x-b*arctanh(c*x))/c^2
```

3.5.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int x(a+b\operatorname{arctanh}(cx))\,dx=\frac{2\,ac^{2}x^{2}+2\,bcx+(bc^{2}x^{2}-b)\log\left(-\frac{cx+1}{cx-1}\right)}{4\,c^{2}}$$

input
```
integrate(x*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
1/4*(2*a*c^2*x^2 + 2*b*c*x + (b*c^2*x^2 - b)*log(-(c*x + 1)/(c*x - 1)))/c^
2
```

3.5.6 Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x(a+b\text{arctanh}(cx))\,dx = \begin{cases} \frac{ax^2}{2} + \frac{bx^2\,\text{atanh}\,(cx)}{2} + \frac{bx}{2c} - \frac{b\,\text{atanh}\,(cx)}{2c^2} & \text{for } c \neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*atanh(c*x)),x)
```

output
```
Piecewise((a*x**2/2 + b*x**2*atanh(c*x)/2 + b*x/(2*c) - b*atanh(c*x)/(2*c*
*2), Ne(c, 0)), (a*x**2/2, True))
```

3.5.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int x(a+b\text{arctanh}(cx))\,dx = \frac{1}{2}\,ax^2 + \frac{1}{4}\left(2\,x^2\,\text{artanh}\,(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log\,(cx+1)}{c^3} + \frac{\log\,(cx-1)}{c^3}\right)\right)b$$

input
```
integrate(x*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/4*(2*x^2*arctanh(c*x) + c*(2*x/c^2 - log(c*x + 1)/c^3 + log(
c*x - 1)/c^3))*b
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(31) = 62$.

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 4.00

$$\int x(a+b\text{arctanh}(cx))\,dx = c\left(\frac{(cx+1)b\log\left(-\frac{cx+1}{cx-1}\right)}{\left(\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2\,(cx+1)c^3}{cx-1} + c^3\right)(cx-1)} + \frac{\frac{2\,(cx+1)a}{cx-1} + \frac{(cx+1)b}{cx-1} - b}{\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2\,(cx+1)c^3}{cx-1} + c^3}\right)$$

input
```
integrate(x*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
c*((c*x + 1)*b*log(-(c*x + 1)/(c*x - 1))/(((c*x + 1)^2*c^3/(c*x - 1)^2 - 2
*(c*x + 1)*c^3/(c*x - 1) + c^3)*(c*x - 1)) + (2*(c*x + 1)*a/(c*x - 1) + (c
*x + 1)*b/(c*x - 1) - b)/((c*x + 1)^2*c^3/(c*x - 1)^2 - 2*(c*x + 1)*c^3/(c
*x - 1) + c^3))
```

3.5.9 Mupad [B] (verification not implemented)

Time = 3.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int x(a + b\text{arctanh}(cx))\,dx = \frac{a\,x^2}{2} - \frac{\frac{b\,\text{atanh}(c\,x)}{2} - \frac{b\,c\,x}{2}}{c^2} + \frac{b\,x^2\,\text{atanh}(c\,x)}{2}$$

input
```
int(x*(a + b*atanh(c*x)),x)
```

output
```
(a*x^2)/2 - ((b*atanh(c*x))/2 - (b*c*x)/2)/c^2 + (b*x^2*atanh(c*x))/2
```

3.6 $\int (a + b\text{arctanh}(cx))\,dx$

3.6.1 Optimal result

Integrand size = 8, antiderivative size = 30

$$\int (a + b\text{arctanh}(cx))\,dx = ax + bx\text{arctanh}(cx) + \frac{b\log(1 - c^2x^2)}{2c}$$

output
```
a*x+b*x*arctanh(c*x)+1/2*b*ln(-c^2*x^2+1)/c
```

3.6.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\text{arctanh}(cx))\,dx = ax + bx\text{arctanh}(cx) + \frac{b\log(1 - c^2x^2)}{2c}$$

input
```
Integrate[a + b*ArcTanh[c*x],x]
```

output
```
a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c)
```

3.6.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}$$

input
```
Int[a + b*ArcTanh[c*x],x]
```

output
```
a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c)
```

3.6.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.6.4 Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
default	$ax+bx\,\text{arctanh}\left(cx\right)+\frac{b\ln\left(-c^2x^2+1\right)}{2c}$	29
parts	$ax+bx\,\text{arctanh}\left(cx\right)+\frac{b\ln\left(-c^2x^2+1\right)}{2c}$	29
derivativedivides	$\frac{cxa+b\left(cx\,\text{arctanh}(cx)+\frac{\ln\left(-c^2x^2+1\right)}{2}\right)}{c}$	33
parallelrisch	$-\frac{b(-cx\,\text{arctanh}(cx)-\ln(cx-1)-\text{arctanh}(cx))}{c}+ax$	34
risch	$ax+\frac{bx\ln(cx+1)}{2}-\frac{b\ln(-cx+1)x}{2}+\frac{b\ln(c^2x^2-1)}{2c}$	42

input
```
int(a+b*arctanh(c*x),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arctanh(c*x)+1/2*b*ln(-c^2*x^2+1)/c
```

3.6.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.40

$$\int (a + b\text{arctanh}(cx))\, dx = \frac{bcx\log\left(-\frac{cx+1}{cx-1}\right) + 2\,acx + b\log\left(c^2x^2 - 1\right)}{2\,c}$$

input
```
integrate(a+b*arctanh(c*x),x, algorithm="fricas")
```

output
```
1/2*(b*c*x*log(-(c*x + 1)/(c*x - 1)) + 2*a*c*x + b*log(c^2*x^2 - 1))/c
```

3.6.6 Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int (a + b\text{arctanh}(cx))\, dx = ax + b\left(\begin{cases} x\,\text{atanh}\,(cx) + \frac{\log\,(cx+1)}{c} - \frac{\text{atanh}\,(cx)}{c} & \text{for } c \neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*atanh(c*x),x)
```

output
```
a*x + b*Piecewise((x*atanh(c*x) + log(c*x + 1)/c - atanh(c*x)/c, Ne(c, 0))
, (0, True))
```

3.6.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (a + b\text{arctanh}(cx))\, dx = ax + \frac{(2\,cx\,\text{artanh}\,(cx) + \log\,(-c^2x^2 + 1))b}{2\,c}$$

input
```
integrate(a+b*arctanh(c*x),x, algorithm="maxima")
```

output
```
a*x + 1/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*b/c
```

3.6.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(28) = 56.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 5.20

$$\int (a + b\text{arctanh}(cx))\,dx = bc\left(\frac{\log\left(\frac{|-cx-1|}{|cx-1|}\right)}{c^2} - \frac{\log\left(\left|-\frac{cx+1}{cx-1}+1\right|\right)}{c^2} + \frac{\log\left(-\frac{\frac{c\left(\frac{cx+1}{cx-1}+1\right)}{\frac{(cx+1)c}{cx-1}-c}+1}{\frac{c\left(\frac{cx+1}{cx-1}+1\right)}{\frac{(cx+1)c}{cx-1}-c}-1}\right)}{c^2\left(\frac{cx+1}{cx-1}-1\right)}\right) + ax$$

input
```
integrate(a+b*arctanh(c*x),x, algorithm="giac")
```

output
```
b*c*(log(abs(-c*x - 1)/abs(c*x - 1))/c^2 - log(abs(-(c*x + 1)/(c*x - 1) +
1))/c^2 + log(-(c*((c*x + 1)/(c*x - 1) + 1)/((c*x + 1)*c/(c*x - 1) - c) +
1)/(c*((c*x + 1)/(c*x - 1) + 1)/((c*x + 1)*c/(c*x - 1) - c) - 1))/(c^2*((c
*x + 1)/(c*x - 1) - 1))) + a*x
```

3.6.9 Mupad [B] (verification not implemented)

Time = 3.47 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.90

$$\int (a + b\text{arctanh}(cx))\,dx = a\,x + \frac{b\,\ln\left(c^2\,x^2-1\right)}{2\,c} + b\,x\,\text{atanh}(c\,x)$$

input
```
int(a + b*atanh(c*x),x)
```

output
```
a*x + (b*log(c^2*x^2 - 1))/(2*c) + b*x*atanh(c*x)
```

3.7 $\int \frac{a+b\operatorname{arctanh}(cx)}{x}\,dx$

3.7.1 Optimal result

Integrand size = 12, antiderivative size = 26

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x}\,dx = a\log(x) - \frac{1}{2}b\operatorname{PolyLog}(2,-cx) + \frac{1}{2}b\operatorname{PolyLog}(2,cx)$$

output
```
a*ln(x)-1/2*b*polylog(2,-c*x)+1/2*b*polylog(2,c*x)
```

3.7.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x}\,dx = a\log(x) + \frac{1}{2}b(-\operatorname{PolyLog}(2,-cx) + \operatorname{PolyLog}(2,cx))$$

input
```
Integrate[(a + b*ArcTanh[c*x])/x,x]
```

output
```
a*Log[x] + (b*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/2
```

3.7.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\text{arctanh}(cx)}{x}\,dx$$

$$\Big\downarrow \; 6446$$

$$a\log(x) - \frac{1}{2}b\,\text{PolyLog}(2,-cx) + \frac{1}{2}b\,\text{PolyLog}(2,cx)$$

input
```
Int[(a + b*ArcTanh[c*x])/x,x]
```

output
```
a*Log[x] - (b*PolyLog[2, -(c*x)])/2 + (b*PolyLog[2, c*x])/2
```

3.7.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

3.7.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08

method	result	size
risch	$a\ln(-cx) + \frac{b\,\text{dilog}(-cx+1)}{2} - \frac{b\,\text{dilog}(cx+1)}{2}$	28
parts	$a\ln(x) + b\left(\ln(cx)\,\text{arctanh}(cx) - \frac{\text{dilog}(cx+1)}{2} - \frac{\ln(cx)\ln(cx+1)}{2} - \frac{\text{dilog}(cx)}{2}\right)$	44
derivativedivides	$a\ln(cx) + b\left(\ln(cx)\,\text{arctanh}(cx) - \frac{\text{dilog}(cx+1)}{2} - \frac{\ln(cx)\ln(cx+1)}{2} - \frac{\text{dilog}(cx)}{2}\right)$	46
default	$a\ln(cx) + b\left(\ln(cx)\,\text{arctanh}(cx) - \frac{\text{dilog}(cx+1)}{2} - \frac{\ln(cx)\ln(cx+1)}{2} - \frac{\text{dilog}(cx)}{2}\right)$	46

input
```
int((a+b*arctanh(c*x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(-c*x)+1/2*b*dilog(-c*x+1)-1/2*b*dilog(c*x+1)
```

3.7.5 Fricas [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x} \, dx = \int \frac{b \operatorname{artanh}(cx) + a}{x} \, dx$$

input
```
integrate((a+b*arctanh(c*x))/x,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)/x, x)
```

3.7.6 Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x} \, dx = \int \frac{a + b \operatorname{atanh}(cx)}{x} \, dx$$

input
```
integrate((a+b*atanh(c*x))/x,x)
```

output
```
Integral((a + b*atanh(c*x))/x, x)
```

3.7.7 Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x} \, dx = \int \frac{b \operatorname{artanh}(cx) + a}{x} \, dx$$

input
```
integrate((a+b*arctanh(c*x))/x,x, algorithm="maxima")
```

output
```
1/2*b*integrate((log(c*x + 1) - log(-c*x + 1))/x, x) + a*log(x)
```

3.7.8 Giac [F]

$$\int \frac{a + b\text{arctanh}(cx)}{x}\,dx = \int \frac{b\,\text{artanh}\,(cx) + a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)/x, x)
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx)}{x}\,dx = \int \frac{a + b\,\text{atanh}(c\,x)}{x}\,dx$$

input
```
int((a + b*atanh(c*x))/x,x)
```

output
```
int((a + b*atanh(c*x))/x, x)
```

3.8 $\int \frac{a+b\,\mathrm{arctanh}(cx)}{x^2}\,dx$

3.8.1 Optimal result

Integrand size = 12, antiderivative size = 36

$$\int \frac{a+b\,\mathrm{arctanh}(cx)}{x^2}\,dx = -\frac{a+b\,\mathrm{arctanh}(cx)}{x} + bc\log(x) - \frac{1}{2}bc\log\left(1-c^2x^2\right)$$

output
```
(-a-b*arctanh(c*x))/x+b*c*ln(x)-1/2*b*c*ln(-c^2*x^2+1)
```

3.8.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{a+b\,\mathrm{arctanh}(cx)}{x^2}\,dx = -\frac{a}{x} - \frac{b\,\mathrm{arctanh}(cx)}{x} + bc\log(x) - \frac{1}{2}bc\log\left(1-c^2x^2\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x])/x^2,x]
```

output
```
-(a/x) - (b*ArcTanh[c*x])/x + b*c*Log[x] - (b*c*Log[1 - c^2*x^2])/2
```

3.8.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {6452, 243, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^2}\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$bc\int \frac{1}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{x}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{x}$$

$$\Big\downarrow \text{ 47}$$

$$\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{a+b\text{arctanh}(cx)}{x}$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\log(x^2)\right)-\frac{a+b\text{arctanh}(cx)}{x}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}bc(\log(x^2)-\log(1-c^2x^2))-\frac{a+b\text{arctanh}(cx)}{x}$$

input
```
Int[(a + b*ArcTanh[c*x])/x^2,x]
```

output
```
-((a + b*ArcTanh[c*x])/x) + (b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2
```

3.8.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.8.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

method	result	size
parallelrisch	$\frac{bc\ln(x)x-\ln(cx-1)xbc-x\,\operatorname{arctanh}(cx)bc-b\,\operatorname{arctanh}(cx)-a}{x}$	42
parts	$-\frac{a}{x}+bc\left(-\frac{\operatorname{arctanh}(cx)}{cx}+\ln(cx)-\frac{\ln(cx+1)}{2}-\frac{\ln(cx-1)}{2}\right)$	44
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arctanh}(cx)}{cx}+\ln(cx)-\frac{\ln(cx+1)}{2}-\frac{\ln(cx-1)}{2}\right)\right)$	48
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\operatorname{arctanh}(cx)}{cx}+\ln(cx)-\frac{\ln(cx+1)}{2}-\frac{\ln(cx-1)}{2}\right)\right)$	48
risch	$-\frac{b\ln(cx+1)}{2x}+\frac{2bc\ln(x)x-bc\ln(c^2x^2-1)x+b\ln(-cx+1)-2a}{2x}$	54

input
```
int((a+b*arctanh(c*x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
(b*c*ln(x)*x-ln(c*x-1)*x*b*c-x*arctanh(c*x)*b*c-b*arctanh(c*x)-a)/x
```

3.8.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.31

$$\int \frac{a+b\text{arctanh}(cx)}{x^2}\,dx = -\frac{bcx\log\left(c^2x^2-1\right)-2\,bcx\log\left(x\right)+b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a}{2\,x}$$

input
```
integrate((a+b*arctanh(c*x))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(b*c*x*log(c^2*x^2 - 1) - 2*b*c*x*log(x) + b*log(-(c*x + 1)/(c*x - 1)
) + 2*a)/x
```

3.8.6 Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{a+b\text{arctanh}(cx)}{x^2}\,dx$$
$$= \begin{cases} -\frac{a}{x}+bc\log\left(x\right)-bc\log\left(x-\frac{1}{c}\right)-bc\,\text{atanh}\left(cx\right)-\frac{b\,\text{atanh}\left(cx\right)}{x} & \text{for } c\neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atanh(c*x))/x**2,x)
```

output
```
Piecewise((-a/x + b*c*log(x) - b*c*log(x - 1/c) - b*c*atanh(c*x) - b*atanh
(c*x)/x, Ne(c, 0)), (-a/x, True))
```

3.8.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{arctanh}(cx)}{x^2}\,dx = -\frac{1}{2}\left(c\left(\log\left(c^2x^2-1\right)-\log\left(x^2\right)\right)+\frac{2\,\text{artanh}\left(cx\right)}{x}\right)b-\frac{a}{x}$$

input
```
integrate((a+b*arctanh(c*x))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*b - a/x
```

3.8.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(34) = 68$.

Time = 0.28 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.61

$$\int \frac{a+b\text{arctanh}(cx)}{x^2}\,dx = \left(b\log\left(-\frac{cx+1}{cx-1}-1\right)-b\log\left(-\frac{cx+1}{cx-1}\right)+\frac{b\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{cx+1}{cx-1}+1}+\frac{2\,a}{\frac{cx+1}{cx-1}+1}\right)c$$

input
```
integrate((a+b*arctanh(c*x))/x^2,x, algorithm="giac")
```

output
```
(b*log(-(c*x + 1)/(c*x - 1) - 1) - b*log(-(c*x + 1)/(c*x - 1)) + b*log(-(c
*x + 1)/(c*x - 1))/((c*x + 1)/(c*x - 1) + 1) + 2*a/((c*x + 1)/(c*x - 1) +
1))*c
```

3.8.9 Mupad [B] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.92

$$\int \frac{a+b\text{arctanh}(cx)}{x^2}\,dx = b\,c\,\ln\left(x\right)-\frac{a+b\,\text{atanh}(c\,x)}{x}-\frac{b\,c\,\ln\left(c^2\,x^2-1\right)}{2}$$

input
```
int((a + b*atanh(c*x))/x^2,x)
```

output
```
b*c*log(x) - (a + b*atanh(c*x))/x - (b*c*log(c^2*x^2 - 1))/2
```

3.9 $\int \frac{a+b\text{arctanh}(cx)}{x^3}\,dx$

3.9.1 Optimal result

Integrand size = 12, antiderivative size = 37

$$\int \frac{a+b\text{arctanh}(cx)}{x^3}\,dx = -\frac{bc}{2x}+\frac{1}{2}bc^2\text{arctanh}(cx)-\frac{a+b\text{arctanh}(cx)}{2x^2}$$

output
```
-1/2*b*c/x+1/2*b*c^2*arctanh(c*x)+1/2*(-a-b*arctanh(c*x))/x^2
```

3.9.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{a+b\text{arctanh}(cx)}{x^3}\,dx = -\frac{a}{2x^2}-\frac{bc}{2x}-\frac{b\text{arctanh}(cx)}{2x^2}-\frac{1}{4}bc^2\log(1-cx)+\frac{1}{4}bc^2\log(1+cx)$$

input
```
Integrate[(a + b*ArcTanh[c*x])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*c)/(2*x) - (b*ArcTanh[c*x])/(2*x^2) - (b*c^2*Log[1 - c*x])
/4 + (b*c^2*Log[1 + c*x])/4
```

3.9.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6452, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^3}dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}$$

input
```
Int[(a + b*ArcTanh[c*x])/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c*x])/x^2 + (b*c*(-x^(-1) + c*ArcTanh[c*x]))/2
```

3.9.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.9.4 Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$-\frac{-\operatorname{arctanh}(cx) b\, c^2 x^2+a\, c^2 x^2+bcx+b \operatorname{arctanh}(cx)+a}{2x^2}$	39
parts	$-\frac{a}{2x^2}+b\, c^2\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{1}{2cx}+\frac{\ln(cx+1)}{4}-\frac{\ln(cx-1)}{4}\right)$	50
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{1}{2cx}+\frac{\ln(cx+1)}{4}-\frac{\ln(cx-1)}{4}\right)\right)$	54
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\operatorname{arctanh}(cx)}{2c^2x^2}-\frac{1}{2cx}+\frac{\ln(cx+1)}{4}-\frac{\ln(cx-1)}{4}\right)\right)$	54
risch	$-\frac{b\ln(cx+1)}{4x^2}-\frac{b\,x^2\ln(-cx+1)c^2-b\,c^2\ln(-cx-1)x^2+2bcx-b\ln(-cx+1)+2a}{4x^2}$	69

input
```
int((a+b*arctanh(c*x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-arctanh(c*x)*b*c^2*x^2+a*c^2*x^2+b*c*x+b*arctanh(c*x)+a)/x^2
```

3.9.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16

$$\int \frac{a+b\operatorname{arctanh}(cx)}{x^3}\,dx=-\frac{2\,bcx-(bc^2x^2-b)\log\left(-\frac{cx+1}{cx-1}\right)+2\,a}{4\,x^2}$$

input
```
integrate((a+b*arctanh(c*x))/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b*c*x - (b*c^2*x^2 - b)*log(-(c*x + 1)/(c*x - 1)) + 2*a)/x^2
```

3.9.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{a + b\text{arctanh}(cx)}{x^3} \, dx = -\frac{a}{2x^2} + \frac{bc^2 \operatorname{atanh}(cx)}{2} - \frac{bc}{2x} - \frac{b \operatorname{atanh}(cx)}{2x^2}$$

input
```
integrate((a+b*atanh(c*x))/x**3,x)
```

output
```
-a/(2*x**2) + b*c**2*atanh(c*x)/2 - b*c/(2*x) - b*atanh(c*x)/(2*x**2)
```

3.9.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{a + b\text{arctanh}(cx)}{x^3} \, dx = \frac{1}{4} \left(\left(c \log(cx + 1) - c \log(cx - 1) - \frac{2}{x} \right) c - \frac{2 \operatorname{artanh}(cx)}{x^2} \right) b - \frac{a}{2x^2}$$

input
```
integrate((a+b*arctanh(c*x))/x^3,x, algorithm="maxima")
```

output
```
1/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*b - 1
/2*a/x^2
```

3.9.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(31) = 62$.

Time = 0.29 (sec) , antiderivative size = 135, normalized size of antiderivative = 3.65

$$\int \frac{a + b\text{arctanh}(cx)}{x^3} \, dx = \left(\frac{(cx + 1)bc \log\left(-\frac{cx+1}{cx-1}\right)}{(cx - 1)\left(\frac{(cx+1)^2}{(cx-1)^2} + \frac{2(cx+1)}{cx-1} + 1\right)} + \frac{\frac{2(cx+1)ac}{cx-1} + \frac{(cx+1)bc}{cx-1} + bc}{\frac{(cx+1)^2}{(cx-1)^2} + \frac{2(cx+1)}{cx-1} + 1} \right) c$$

input
```
integrate((a+b*arctanh(c*x))/x^3,x, algorithm="giac")
```

output
```
((c*x + 1)*b*c*log(-(c*x + 1)/(c*x - 1))/((c*x - 1)*((c*x + 1)^2/(c*x - 1)
^2 + 2*(c*x + 1)/(c*x - 1) + 1)) + (2*(c*x + 1)*a*c/(c*x - 1) + (c*x + 1)*
b*c/(c*x - 1) + b*c)/((c*x + 1)^2/(c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1)
)*c
```

3.9.9 Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{a + b\text{arctanh}(cx)}{x^3} \, dx = \frac{b\,c\,\text{atan}\left(\frac{c^2\,x}{\sqrt{-c^2}}\right)\,\sqrt{-c^2}}{2} - \frac{\frac{a}{2} + \frac{b\,\text{atanh}(c\,x)}{2} + \frac{b\,c\,x}{2}}{x^2}$$

input
```
int((a + b*atanh(c*x))/x^3,x)
```

output
```
(b*c*atan((c^2*x)/(-c^2)^(1/2))*(-c^2)^(1/2))/2 - (a/2 + (b*atanh(c*x))/2
+ (b*c*x)/2)/x^2
```

3.10 $\int \frac{a+b\text{arctanh}(cx)}{x^4}\,dx$

3.10.1 Optimal result

Integrand size = 12, antiderivative size = 54

$$\int \frac{a+b\text{arctanh}(cx)}{x^4}\,dx=-\frac{bc}{6x^2}-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{3}bc^3\log(x)-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)$$

output
```
-1/6*b*c/x^2+1/3*(-a-b*arctanh(c*x))/x^3+1/3*b*c^3*ln(x)-1/6*b*c^3*ln(-c^2
*x^2+1)
```

3.10.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}(cx)}{x^4}\,dx=-\frac{a}{3x^3}-\frac{bc}{6x^2}-\frac{b\text{arctanh}(cx)}{3x^3}+\frac{1}{3}bc^3\log(x)-\frac{1}{6}bc^3\log\left(1-c^2x^2\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*c)/(6*x^2) - (b*ArcTanh[c*x])/(3*x^3) + (b*c^3*Log[x])/3 -
 (b*c^3*Log[1 - c^2*x^2])/6
```

3.10.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6452, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^4}\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{3}bc\int \frac{1}{x^3\left(1-c^2x^2\right)}dx-\frac{a+b\text{arctanh}(cx)}{3x^3}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{6}bc\int \frac{1}{x^4\left(1-c^2x^2\right)}dx^2-\frac{a+b\text{arctanh}(cx)}{3x^3}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{6}bc\int \left(-\frac{c^4}{c^2x^2-1}+\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\text{arctanh}(cx)}{3x^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}$$

input
```
Int[(a + b*ArcTanh[c*x])/x^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c*x])/x^3 + (b*c*(-x^(-2) + c^2*Log[x^2] - c^2*Log[1 -
 c^2*x^2]))/6
```

3.10.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.10.4 Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{1}{6c^2x^2}+\frac{\ln(cx)}{3}-\frac{\ln(cx+1)}{6}-\frac{\ln(cx-1)}{6}\right)$	56
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{1}{6c^2x^2}+\frac{\ln(cx)}{3}-\frac{\ln(cx+1)}{6}-\frac{\ln(cx-1)}{6}\right)\right)$	60
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\operatorname{arctanh}(cx)}{3c^3x^3}-\frac{1}{6c^2x^2}+\frac{\ln(cx)}{3}-\frac{\ln(cx+1)}{6}-\frac{\ln(cx-1)}{6}\right)\right)$	60
risch	$-\frac{b\ln(cx+1)}{6x^3}+\frac{2b\,c^3\ln(x)x^3-b\,c^3\ln(c^2x^2-1)x^3-bcx+b\ln(-cx+1)-2a}{6x^3}$	67
parallelrisch	$\frac{2b\,c^3\ln(x)x^3-2\ln(cx-1)x^3b\,c^3-2b\operatorname{arctanh}(cx)x^3c^3-b\,c^3x^3-bcx-2b\operatorname{arctanh}(cx)-2a}{6x^3}$	70

input
```
int((a+b*arctanh(c*x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arctanh(c*x)-1/6/c^2/x^2+1/3*ln(c*x)-1/6*ln
(c*x+1)-1/6*ln(c*x-1))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{a + b\text{arctanh}(cx)}{x^4}\,dx = -\frac{bc^3x^3\log(c^2x^2-1) - 2\,bc^3x^3\log(x) + bcx + b\log\left(-\frac{cx+1}{cx-1}\right) + 2\,a}{6\,x^3}$$

input
```
integrate((a+b*arctanh(c*x))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(b*c^3*x^3*log(c^2*x^2 - 1) - 2*b*c^3*x^3*log(x) + b*c*x + b*log(-(c*
x + 1)/(c*x - 1)) + 2*a)/x^3
```

3.10.6 Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{a + b\text{arctanh}(cx)}{x^4}\,dx = \begin{cases} -\frac{a}{3x^3} + \frac{bc^3\log(x)}{3} - \frac{bc^3\log\left(x-\frac{1}{c}\right)}{3} - \frac{bc^3\,\text{atanh}(cx)}{3} - \frac{bc}{6x^2} - \frac{b\,\text{atanh}(cx)}{3x^3} & \text{for } c \neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atanh(c*x))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3) + b*c**3*log(x)/3 - b*c**3*log(x - 1/c)/3 - b*c**3*
atanh(c*x)/3 - b*c/(6*x**2) - b*atanh(c*x)/(3*x**3), Ne(c, 0)), (-a/(3*x**
3), True))
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

$$\int \frac{a + b\text{arctanh}(cx)}{x^4}\,dx = -\frac{1}{6}\left(\left(c^2\log(c^2x^2-1) - c^2\log(x^2) + \frac{1}{x^2}\right)c + \frac{2\,\text{artanh}(cx)}{x^3}\right)b - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctanh(c*x))/x^4,x, algorithm="maxima")
```

output
```
-1/6*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*b - 1/3*a/x^3
```

3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(46) = 92$.

Time = 0.30 (sec) , antiderivative size = 251, normalized size of antiderivative = 4.65

$$\int \frac{a + b\text{arctanh}(cx)}{x^4}\,dx$$
$$= \frac{1}{3}\left(bc^2\log\left(-\frac{cx+1}{cx-1}-1\right) - bc^2\log\left(-\frac{cx+1}{cx-1}\right) + \frac{\left(\frac{3(cx+1)^2bc^2}{(cx-1)^2}+bc^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3(cx+1)^2}{(cx-1)^2}+\frac{3(cx+1)}{cx-1}+1} + \frac{2\left(\frac{3(cx+1)^2ac^2}{(cx-1)^2}+ac\right.}{\frac{(cx+1)^3}{(cx-1)^3}+\frac{3(c}{(c}}\right.$$

input
```
integrate((a+b*arctanh(c*x))/x^4,x, algorithm="giac")
```

output
```
1/3*(b*c^2*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^2*log(-(c*x + 1)/(c*x - 1))
 + (3*(c*x + 1)^2*b*c^2/(c*x - 1)^2 + b*c^2)*log(-(c*x + 1)/(c*x - 1))/((c
*x + 1)^3/(c*x - 1)^3 + 3*(c*x + 1)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1)
+ 1) + 2*(3*(c*x + 1)^2*a*c^2/(c*x - 1)^2 + a*c^2 + (c*x + 1)^2*b*c^2/(c*x
 - 1)^2 + (c*x + 1)*b*c^2/(c*x - 1))/((c*x + 1)^3/(c*x - 1)^3 + 3*(c*x + 1
)^2/(c*x - 1)^2 + 3*(c*x + 1)/(c*x - 1) + 1))*c
```

3.10.9 Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{a + b\text{arctanh}(cx)}{x^4}\,dx = \frac{b\,c^3\,\ln(x)}{3} - \frac{b\,c^3\,\ln(c^2\,x^2-1)}{6} - \frac{\frac{a}{3}+\frac{b\,\text{atanh}(c\,x)}{3}+\frac{b\,c\,x}{6}}{x^3}$$

input
```
int((a + b*atanh(c*x))/x^4,x)
```

output
```
(b*c^3*log(x))/3 - (b*c^3*log(c^2*x^2 - 1))/6 - (a/3 + (b*atanh(c*x))/3 +
(b*c*x)/6)/x^3
```

3.11 $\int \frac{a+b\text{arctanh}(cx)}{x^5}\,dx$

3.11.1 Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \frac{a+b\text{arctanh}(cx)}{x^5}\,dx = -\frac{bc}{12x^3}-\frac{bc^3}{4x}+\frac{1}{4}bc^4\text{arctanh}(cx)-\frac{a+b\text{arctanh}(cx)}{4x^4}$$

output
```
-1/12*b*c/x^3-1/4*b*c^3/x+1/4*b*c^4*arctanh(c*x)+1/4*(-a-b*arctanh(c*x))/x
^4
```

3.11.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.46

$$\int \frac{a+b\text{arctanh}(cx)}{x^5}\,dx = -\frac{a}{4x^4}-\frac{bc}{12x^3}-\frac{bc^3}{4x}-\frac{b\text{arctanh}(cx)}{4x^4} - \frac{1}{8}bc^4\log(1-cx)+\frac{1}{8}bc^4\log(1+cx)$$

input
```
Integrate[(a + b*ArcTanh[c*x])/x^5,x]
```

output
```
-1/4*a/x^4 - (b*c)/(12*x^3) - (b*c^3)/(4*x) - (b*ArcTanh[c*x])/(4*x^4) - (
b*c^4*Log[1 - c*x])/8 + (b*c^4*Log[1 + c*x])/8
```

3.11.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6452, 264, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^5} \, dx$$

$$\Big\downarrow \ 6452$$

$$\frac{1}{4}bc\int \frac{1}{x^4(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{4x^4}$$

$$\Big\downarrow \ 264$$

$$\frac{1}{4}bc\left(c^2\int \frac{1}{x^2(1-c^2x^2)}dx-\frac{1}{3x^3}\right)-\frac{a+b\text{arctanh}(cx)}{4x^4}$$

$$\Big\downarrow \ 264$$

$$\frac{1}{4}bc\left(c^2\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{a+b\text{arctanh}(cx)}{4x^4}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{4}bc\left(c^2\left(c\text{arctanh}(cx)-\frac{1}{x}\right)-\frac{1}{3x^3}\right)-\frac{a+b\text{arctanh}(cx)}{4x^4}$$

input
```
Int[(a + b*ArcTanh[c*x])/x^5,x]
```

output
```
-1/4*(a + b*ArcTanh[c*x])/x^4 + (b*c*(-1/3*1/x^3 + c^2*(-x^(-1) + c*ArcTan
h[c*x])))/4
```

3.11.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.11.4 Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$-\frac{-3b\,\operatorname{arctanh}(cx)x^4c^4+3b\,c^3x^3+bcx+3b\,\operatorname{arctanh}(cx)+3a}{12x^4}$	43
parts	$-\frac{a}{4x^4}+b\,c^4\left(-\frac{\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{1}{12c^3x^3}-\frac{1}{4cx}+\frac{\ln(cx+1)}{8}-\frac{\ln(cx-1)}{8}\right)$	58
derivativedivides	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{1}{12c^3x^3}-\frac{1}{4cx}+\frac{\ln(cx+1)}{8}-\frac{\ln(cx-1)}{8}\right)\right)$	62
default	$c^4\left(-\frac{a}{4c^4x^4}+b\left(-\frac{\operatorname{arctanh}(cx)}{4c^4x^4}-\frac{1}{12c^3x^3}-\frac{1}{4cx}+\frac{\ln(cx+1)}{8}-\frac{\ln(cx-1)}{8}\right)\right)$	62
risch	$-\frac{b\ln(cx+1)}{8x^4}+\frac{3b\,c^4\ln(-cx-1)x^4-3b\,x^4\ln(-cx+1)c^4-6b\,c^3x^3-2bcx+3b\ln(-cx+1)-6a}{24x^4}$	79

input
```
int((a+b*arctanh(c*x))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-3*b*arctanh(c*x)*x^4*c^4+3*b*c^3*x^3+b*c*x+3*b*arctanh(c*x)+3*a)/x
^4
```

3.11.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.08

$$\int \frac{a + b\text{arctanh}(cx)}{x^5}\,dx = -\frac{6\,bc^3x^3 + 2\,bcx - 3\,(bc^4x^4 - b)\log\left(-\frac{cx+1}{cx-1}\right) + 6\,a}{24\,x^4}$$

input
```
integrate((a+b*arctanh(c*x))/x^5,x, algorithm="fricas")
```

output
```
-1/24*(6*b*c^3*x^3 + 2*b*c*x - 3*(b*c^4*x^4 - b)*log(-(c*x + 1)/(c*x - 1))
 + 6*a)/x^4
```

3.11.6 Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{a + b\text{arctanh}(cx)}{x^5}\,dx = -\frac{a}{4x^4} + \frac{bc^4\,\text{atanh}\,(cx)}{4} - \frac{bc^3}{4x} - \frac{bc}{12x^3} - \frac{b\,\text{atanh}\,(cx)}{4x^4}$$

input
```
integrate((a+b*atanh(c*x))/x**5,x)
```

output
```
-a/(4*x**4) + b*c**4*atanh(c*x)/4 - b*c**3/(4*x) - b*c/(12*x**3) - b*atanh
(c*x)/(4*x**4)
```

3.11.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.25

$$\int \frac{a + b\text{arctanh}(cx)}{x^5}\,dx = \frac{1}{24}\left(\left(3\,c^3\log\,(cx+1) - 3\,c^3\log\,(cx-1) - \frac{2\,(3\,c^2x^2+1)}{x^3}\right)c - \frac{6\,\text{artanh}\,(cx)}{x^4}\right)b - \frac{a}{4\,x^4}$$

input
```
integrate((a+b*arctanh(c*x))/x^5,x, algorithm="maxima")
```

output
```
1/24*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c
- 6*arctanh(c*x)/x^4)*b - 1/4*a/x^4
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(40) = 80$.

Time = 0.29 (sec) , antiderivative size = 292, normalized size of antiderivative = 6.08

$$\int \frac{a + b\text{arctanh}(cx)}{x^5} \, dx$$
$$= \frac{1}{3}c\left(\frac{3\left(\frac{(cx+1)^3bc^3}{(cx-1)^3} + \frac{(cx+1)bc^3}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^4}{(cx-1)^4} + \frac{4(cx+1)^3}{(cx-1)^3} + \frac{6(cx+1)^2}{(cx-1)^2} + \frac{4(cx+1)}{cx-1} + 1} + \frac{\frac{6(cx+1)^3ac^3}{(cx-1)^3} + \frac{6(cx+1)ac^3}{cx-1} + \frac{3(cx+1)^3bc^3}{(cx-1)^3} + \frac{6(cx+1)^2bc^3}{(cx-1)^2} + \frac{5(cx\cdots}{cx\cdots}}{\frac{(cx+1)^4}{(cx-1)^4} + \frac{4(cx+1)^3}{(cx-1)^3} + \frac{6(cx+1)^2}{(cx-1)^2} + \frac{4(cx+1)}{cx-1} + 1\cdots}\right.$$

input
```
integrate((a+b*arctanh(c*x))/x^5,x, algorithm="giac")
```

output
```
1/3*c*(3*((c*x + 1)^3*b*c^3/(c*x - 1)^3 + (c*x + 1)*b*c^3/(c*x - 1))*log(-
(c*x + 1)/(c*x - 1))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3
+ 6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + (6*(c*x + 1)^3*
a*c^3/(c*x - 1)^3 + 6*(c*x + 1)*a*c^3/(c*x - 1) + 3*(c*x + 1)^3*b*c^3/(c*x
 - 1)^3 + 6*(c*x + 1)^2*b*c^3/(c*x - 1)^2 + 5*(c*x + 1)*b*c^3/(c*x - 1) +
2*b*c^3)/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1
)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))
```

3.11.9 Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \frac{a + b\text{arctanh}(cx)}{x^5} \, dx = \frac{b\,\ln(1 - c\,x)}{8\,x^4} - \frac{b\,\ln(c\,x + 1)}{8\,x^4} - \frac{b\,c^3\,x^3 + \frac{b\,c\,x}{3} + a}{4\,x^4} - \frac{b\,c^4\,\text{atan}(c\,x\,1\text{i})\,1\text{i}}{4}$$

input
```
int((a + b*atanh(c*x))/x^5,x)
```

output
```
(b*log(1 - c*x))/(8*x^4) - (b*c^4*atan(c*x*1i)*1i)/4 - (b*log(c*x + 1))/(8
*x^4) - (a + b*c^3*x^3 + (b*c*x)/3)/(4*x^4)
```

3.12 $\int \frac{a+b\text{arctanh}(cx)}{x^6}\,dx$

3.12.1 Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{a+b\text{arctanh}(cx)}{x^6}\,dx = -\frac{bc}{20x^4}-\frac{bc^3}{10x^2}-\frac{a+b\text{arctanh}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)$$

output
```
-1/20*b*c/x^4-1/10*b*c^3/x^2+1/5*(-a-b*arctanh(c*x))/x^5+1/5*b*c^5*ln(x)-1
/10*b*c^5*ln(-c^2*x^2+1)
```

3.12.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{arctanh}(cx)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{bc}{20x^4}-\frac{bc^3}{10x^2}-\frac{b\text{arctanh}(cx)}{5x^5}+\frac{1}{5}bc^5\log(x)-\frac{1}{10}bc^5\log\left(1-c^2x^2\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x])/x^6,x]
```

output
```
-1/5*a/x^5 - (b*c)/(20*x^4) - (b*c^3)/(10*x^2) - (b*ArcTanh[c*x])/(5*x^5)
+ (b*c^5*Log[x])/5 - (b*c^5*Log[1 - c^2*x^2])/10
```

3.12.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6452, 243, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{x^6}\,dx$$

$$\Big\downarrow \; 6452$$

$$\frac{1}{5}bc\int \frac{1}{x^5\left(1-c^2x^2\right)}dx-\frac{a+b\text{arctanh}(cx)}{5x^5}$$

$$\Big\downarrow \; 243$$

$$\frac{1}{10}bc\int \frac{1}{x^6\left(1-c^2x^2\right)}dx^2-\frac{a+b\text{arctanh}(cx)}{5x^5}$$

$$\Big\downarrow \; 54$$

$$\frac{1}{10}bc\int \left(-\frac{c^6}{c^2x^2-1}+\frac{c^4}{x^2}+\frac{c^2}{x^4}+\frac{1}{x^6}\right)dx^2-\frac{a+b\text{arctanh}(cx)}{5x^5}$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{10}bc\left(c^4\log\left(x^2\right)-\frac{c^2}{x^2}-c^4\log\left(1-c^2x^2\right)-\frac{1}{2x^4}\right)-\frac{a+b\text{arctanh}(cx)}{5x^5}$$

input
```
Int[(a + b*ArcTanh[c*x])/x^6,x]
```

output
```
-1/5*(a + b*ArcTanh[c*x])/x^5 + (b*c*(-1/2*1/x^4 - c^2/x^2 + c^4*Log[x^2]
- c^4*Log[1 - c^2*x^2]))/10
```

3.12.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.12.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

method	result	size
parts	$-\frac{a}{5x^5}+b\,c^5\left(-\frac{\operatorname{arctanh}(cx)}{5c^5x^5}-\frac{1}{20c^4x^4}-\frac{1}{10c^2x^2}+\frac{\ln(cx)}{5}-\frac{\ln(cx+1)}{10}-\frac{\ln(cx-1)}{10}\right)$	64
derivativedivides	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arctanh}(cx)}{5c^5x^5}-\frac{1}{20c^4x^4}-\frac{1}{10c^2x^2}+\frac{\ln(cx)}{5}-\frac{\ln(cx+1)}{10}-\frac{\ln(cx-1)}{10}\right)\right)$	68
default	$c^5\left(-\frac{a}{5c^5x^5}+b\left(-\frac{\operatorname{arctanh}(cx)}{5c^5x^5}-\frac{1}{20c^4x^4}-\frac{1}{10c^2x^2}+\frac{\ln(cx)}{5}-\frac{\ln(cx+1)}{10}-\frac{\ln(cx-1)}{10}\right)\right)$	68
risch	$-\frac{b\ln(cx+1)}{10x^5}+\frac{4b\,c^5\ln(x)x^5-2b\,c^5\ln(c^2x^2-1)x^5-2b\,c^3x^3-bcx+2b\ln(-cx+1)-4a}{20x^5}$	77
parallelrisch	$\frac{4b\,c^5\ln(x)x^5-4\ln(cx-1)x^5b\,c^5-4b\,\operatorname{arctanh}(cx)x^5c^5-2b\,c^5x^5-2b\,c^3x^3-bcx-4b\,\operatorname{arctanh}(cx)-4a}{20x^5}$	79

input
```
int((a+b*arctanh(c*x))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5+b*c^5*(-1/5/c^5/x^5*arctanh(c*x)-1/20/c^4/x^4-1/10/c^2/x^2+1/5*
ln(c*x)-1/10*ln(c*x+1)-1/10*ln(c*x-1))
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{arctanh}(cx)}{x^6}\,dx = -\frac{2\,bc^5x^5\log\left(c^2x^2-1\right)-4\,bc^5x^5\log\left(x\right)+2\,bc^3x^3+bcx+2\,b\log\left(-\frac{cx+1}{cx-1}\right)+4\,a}{20\,x^5}$$

input
```
integrate((a+b*arctanh(c*x))/x^6,x, algorithm="fricas")
```

output
```
-1/20*(2*b*c^5*x^5*log(c^2*x^2 - 1) - 4*b*c^5*x^5*log(x) + 2*b*c^3*x^3 + b
*c*x + 2*b*log(-(c*x + 1)/(c*x - 1)) + 4*a)/x^5
```

3.12.6 Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.23

$$\int \frac{a+b\text{arctanh}(cx)}{x^6}\,dx = \begin{cases} -\frac{a}{5x^5}+\frac{bc^5\log(x)}{5}-\frac{bc^5\log\left(x-\frac{1}{c}\right)}{5}-\frac{bc^5\,\text{atanh}\,(cx)}{5}-\frac{bc^3}{10x^2}-\frac{bc}{20x^4}-\frac{b\,\text{atanh}\,(cx)}{5x^5} & \text{for}\ c\neq 0 \\ -\frac{a}{5x^5} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atanh(c*x))/x**6,x)
```

output
```
Piecewise((-a/(5*x**5) + b*c**5*log(x)/5 - b*c**5*log(x - 1/c)/5 - b*c**5*
atanh(c*x)/5 - b*c**3/(10*x**2) - b*c/(20*x**4) - b*atanh(c*x)/(5*x**5), N
e(c, 0)), (-a/(5*x**5), True))
```

3.12.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{arctanh}(cx)}{x^6}\,dx = -\frac{1}{20}\left(\left(2\,c^4\log\left(c^2x^2-1\right)-2\,c^4\log\left(x^2\right)+\frac{2\,c^2x^2+1}{x^4}\right)c+\frac{4\,\text{artanh}\,(cx)}{x^5}\right)b-\frac{a}{5\,x^5}$$

input
```
integrate((a+b*arctanh(c*x))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*c^4*log(c^2*x^2 - 1) - 2*c^4*log(x^2) + (2*c^2*x^2 + 1)/x^4)*c +
 4*arctanh(c*x)/x^5)*b - 1/5*a/x^5
```

3.12.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(55) = 110$.

Time = 0.29 (sec) , antiderivative size = 397, normalized size of antiderivative = 6.11

$$\int \frac{a + b\text{arctanh}(cx)}{x^6}\,dx = \frac{1}{5}\left(bc^4\log\left(-\frac{cx+1}{cx-1}-1\right) - bc^4\log\left(-\frac{cx+1}{cx-1}\right) + \frac{\left(\frac{5\,(cx+1)^4bc^4}{(cx-1)^4} + \frac{10\,(cx+1)^2bc^4}{(cx-1)^2} + bc^4\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^5}{(cx-1)^5} + \frac{5\,(cx+1)^4}{(cx-1)^4} + \frac{10\,(cx+1)^3}{(cx-1)^3} + \frac{10\,(cx+1)^2}{(cx-1)^2} + \frac{5\,(cx+1)}{cx-1} + }\right.$$

input
```
integrate((a+b*arctanh(c*x))/x^6,x, algorithm="giac")
```

output
```
1/5*(b*c^4*log(-(c*x + 1)/(c*x - 1) - 1) - b*c^4*log(-(c*x + 1)/(c*x - 1))
 + (5*(c*x + 1)^4*b*c^4/(c*x - 1)^4 + 10*(c*x + 1)^2*b*c^4/(c*x - 1)^2 + b
*c^4)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(
c*x - 1)^4 + 10*(c*x + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(
c*x + 1)/(c*x - 1) + 1) + 2*(5*(c*x + 1)^4*a*c^4/(c*x - 1)^4 + 10*(c*x + 1
)^2*a*c^4/(c*x - 1)^2 + a*c^4 + 2*(c*x + 1)^4*b*c^4/(c*x - 1)^4 + 4*(c*x +
 1)^3*b*c^4/(c*x - 1)^3 + 4*(c*x + 1)^2*b*c^4/(c*x - 1)^2 + 2*(c*x + 1)*b*
c^4/(c*x - 1))/((c*x + 1)^5/(c*x - 1)^5 + 5*(c*x + 1)^4/(c*x - 1)^4 + 10*(
c*x + 1)^3/(c*x - 1)^3 + 10*(c*x + 1)^2/(c*x - 1)^2 + 5*(c*x + 1)/(c*x - 1
) + 1))*c
```

3.12.9 Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \frac{a + b\text{arctanh}(cx)}{x^6}\,dx = \frac{b\,c^5\,\ln(x)}{5} - \frac{b\,c^5\,\ln(c^2\,x^2-1)}{10} - \frac{\frac{b\,c^3\,x^3}{2} + \frac{b\,c\,x}{4} + a}{5\,x^5} - \frac{b\,\ln(c\,x+1)}{10\,x^5} + \frac{b\,\ln(1-c\,x)}{10\,x^5}$$

input
```
int((a + b*atanh(c*x))/x^6,x)
```

output
```
(b*c^5*log(x))/5 - (b*c^5*log(c^2*x^2 - 1))/10 - (a + (b*c^3*x^3)/2 + (b*c
*x)/4)/(5*x^5) - (b*log(c*x + 1))/(10*x^5) + (b*log(1 - c*x))/(10*x^5)
```

3.12. $\int \frac{a+b\text{arctanh}(cx)}{x^6}\,dx$

3.13 $\int x^5(a+b\text{arctanh}(cx))^2\,dx$

3.13.1 Optimal result

Integrand size = 14, antiderivative size = 145

$$\int x^5(a+b\text{arctanh}(cx))^2\,dx=\frac{abx}{3c^5}+\frac{4b^2x^2}{45c^4}+\frac{b^2x^4}{60c^2}+\frac{b^2x\text{arctanh}(cx)}{3c^5}+\frac{bx^3(a+b\text{arctanh}(cx))}{9c^3}+\frac{bx^5(a+b\text{arctanh}(cx))}{15c}-\frac{(a+b\text{arctanh}(cx))^2}{6c^6}+\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2+\frac{23b^2\log(1-c^2x^2)}{90c^6}$$

output
```
1/3*a*b*x/c^5+4/45*b^2*x^2/c^4+1/60*b^2*x^4/c^2+1/3*b^2*x*arctanh(c*x)/c^5
+1/9*b*x^3*(a+b*arctanh(c*x))/c^3+1/15*b*x^5*(a+b*arctanh(c*x))/c-1/6*(a+b
*arctanh(c*x))^2/c^6+1/6*x^6*(a+b*arctanh(c*x))^2+23/90*b^2*ln(-c^2*x^2+1)
/c^6
```

3.13.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.13

$$\int x^5(a+b\text{arctanh}(cx))^2\,dx = \frac{60abcx+16b^2c^2x^2+20abc^3x^3+3b^2c^4x^4+12abc^5x^5+30a^2c^6x^6+4bcx(15ac^5x^5+b(15+5c^2x^2+3c^4x^4)}{}$$

input
```
Integrate[x^5*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(60*a*b*c*x + 16*b^2*c^2*x^2 + 20*a*b*c^3*x^3 + 3*b^2*c^4*x^4 + 12*a*b*c^5
*x^5 + 30*a^2*c^6*x^6 + 4*b*c*x*(15*a*c^5*x^5 + b*(15 + 5*c^2*x^2 + 3*c^4*
x^4))*ArcTanh[c*x] + 30*b^2*(-1 + c^6*x^6)*ArcTanh[c*x]^2 + 2*b*(15*a + 23
*b)*Log[1 - c*x] - 30*a*b*Log[1 + c*x] + 46*b^2*Log[1 + c*x])/(180*c^6)
```

3.13.3 Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.41, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6452, 6542, 6452, 243, 49, 2009, 6542, 6452, 243, 49, 2009, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+b\text{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2-\frac{1}{3}bc\int \frac{x^6(a+b\text{arctanh}(cx))}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2-\frac{1}{3}bc\left(\frac{\int \frac{x^4(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\int x^4(a+b\text{arctanh}(cx))dx}{c^2}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2-\frac{1}{3}bc\left(\frac{\int \frac{x^4(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{5}bc\int \frac{x^5}{1-c^2x^2}dx}{c^2}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2-\frac{1}{3}bc\left(\frac{\int \frac{x^4(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\int \frac{x^4}{1-c^2x^2}dx^2}{c^2}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2-\frac{1}{3}bc\left(\frac{\int\frac{x^4(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\int\left(-\frac{x^2}{c^2}-\frac{1}{c^4(c^2x^2-1)}-\frac{1}{c^4}\right)dx^2}{c^2}\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2-\frac{1}{3}bc\left(\frac{\int\frac{x^4(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}-\frac{x^4}{2c^2}-\frac{\log(1-c^2x^2)}{c^6}\right)}{c^2}\right)$$

$$\Big\downarrow \; 6542$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2-\frac{1}{3}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\int x^2(a+b\text{arctanh}(cx))dx}{c^2}}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}-\frac{x^4}{2c^2}-\frac{\log(1-c^2x^2)}{c^6}\right)}{c^2}\right)$$

$$\Big\downarrow \; 6452$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2-\frac{1}{3}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{3}bc\int\frac{x^3}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}-\frac{x^4}{2c^2}-\log\right.}{c^2}\right.$$

$$\Big\downarrow \; 243$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2-\frac{1}{3}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\int\frac{x^2}{1-c^2x^2}dx^2}{c^2}}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}-\frac{x^4}{2c^2}-\text{lo}\right.}{c^2}\right.$$

$$\Big\downarrow \; 49$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2-\frac{1}{3}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\int\left(-\frac{1}{c^2}-\frac{1}{c^2(c^2x^2-1)}\right)dx^2}{c^2}}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}\right.}{c^2}\right.$$

$$\Big\downarrow \; 2009$$

3.13. $\int x^5(a+b\text{arctanh}(cx))^2\,dx$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2 - \frac{1}{3}bc\left(\frac{\frac{\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2} - \frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{c^2}}{c^2} - \frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))-\frac{1}{10}bc\left(-\frac{x^2}{c^4}-\right.}{c^2}\right.$$

$$\Big\downarrow \textbf{6542}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2 - \frac{1}{3}bc\left(\frac{\frac{\frac{\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{\int(a+b\text{arctanh}(cx))dx}{c^2}}{c^2} - \frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{c^2}}{c^2} - \frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))}{}\right.$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2 - \frac{1}{3}bc\left(\frac{\frac{\frac{\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}}{c^2} - \frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{c^2}}{c^2} - \frac{\frac{1}{5}x^5(a+\text{barc}}{}\right.$$

$$\Big\downarrow \textbf{6510}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^2 - \frac{1}{3}bc\left(\frac{\frac{\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}}{c^2} - \frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{c^2}}{c^2} - \frac{\frac{1}{5}x^5(a+\text{barct}}{}\right.$$

input
```
Int[x^5*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(x^6*(a + b*ArcTanh[c*x])^2)/6 - (b*c*(-(((x^5*(a + b*ArcTanh[c*x]))/5 - (
b*c*(-(x^2/c^4) - x^4/(2*c^2) - Log[1 - c^2*x^2]/c^6))/10)/c^2) + (-(((x^3
*(a + b*ArcTanh[c*x]))/3 - (b*c*(-(x^2/c^2) - Log[1 - c^2*x^2]/c^4))/6)/c^
2) + ((a + b*ArcTanh[c*x])^2/(2*b*c^3) - (a*x + b*x*ArcTanh[c*x] + (b*Log[
1 - c^2*x^2])/(2*c))/c^2)/c^2)/c^2))/3
```

3.13. $\int x^5(a+b\text{arctanh}(cx))^2\,dx$

3.13.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.13.4 Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.26

method	result
parallelrisch	$\frac{30b^2 \operatorname{arctanh}(cx)^2 x^6 c^6+60ab \operatorname{arctanh}(cx) x^6 c^6+30a^2 c^6 x^6+12b^2 \operatorname{arctanh}(cx) x^5 c^5+12ab c^5 x^5+3b^2 c^4 x^4+20b^2 \operatorname{arctanh}(c}{}$
parts	$\frac{a^2 x^6}{6}+\frac{b^2\left(\frac{c^6 x^6 \operatorname{arctanh}(cx)^2}{6}+\frac{c^5 x^5 \operatorname{arctanh}(cx)}{15}+\frac{c^3 x^3 \operatorname{arctanh}(cx)}{9}+\frac{cx \operatorname{arctanh}(cx)}{3}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{6}-\frac{\operatorname{arctanh}(cx)\ln(}{6}\right.}{}$
derivativedivides	$\frac{\frac{a^2 c^6 x^6}{6}+b^2\left(\frac{c^6 x^6 \operatorname{arctanh}(cx)^2}{6}+\frac{c^5 x^5 \operatorname{arctanh}(cx)}{15}+\frac{c^3 x^3 \operatorname{arctanh}(cx)}{9}+\frac{cx \operatorname{arctanh}(cx)}{3}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{6}-\frac{\operatorname{arctanh}(cx)\ln(}{6}\right.}{}$
default	$\frac{\frac{a^2 c^6 x^6}{6}+b^2\left(\frac{c^6 x^6 \operatorname{arctanh}(cx)^2}{6}+\frac{c^5 x^5 \operatorname{arctanh}(cx)}{15}+\frac{c^3 x^3 \operatorname{arctanh}(cx)}{9}+\frac{cx \operatorname{arctanh}(cx)}{3}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{6}-\frac{\operatorname{arctanh}(cx)\ln(}{6}\right.}{}$
risch	$\frac{b^2(c^6x^6-1)\ln(cx+1)^2}{24c^6}+\frac{b(-15b x^6 \ln(-cx+1)c^6+30a c^6x^6+6b c^5x^5+10b c^3x^3+30bcx+15b\ln(-cx+1))\ln(cx+1)}{180c^6}+$

input
```
int(x^5*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/180*(30*b^2*arctanh(c*x)^2*x^6*c^6+60*a*b*arctanh(c*x)*x^6*c^6+30*a^2*c^
6*x^6+12*b^2*arctanh(c*x)*x^5*c^5+12*a*b*c^5*x^5+3*b^2*c^4*x^4+20*b^2*arct
anh(c*x)*x^3*c^3+20*a*b*c^3*x^3+16*b^2*c^2*x^2+60*b^2*arctanh(c*x)*x*c+60*
a*b*c*x-30*b^2*arctanh(c*x)^2+92*ln(c*x-1)*b^2-60*arctanh(c*x)*a*b+92*arct
anh(c*x)*b^2+16*b^2)/c^6
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.33

$$\int x^5(a+b\operatorname{arctanh}(cx))^2\,dx$$
$$=\frac{60\,a^2c^6x^6+24\,abc^5x^5+6\,b^2c^4x^4+40\,abc^3x^3+32\,b^2c^2x^2+120\,abcx+15\,(b^2c^6x^6-b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2-4\,(}{}$$

input
```
integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
1/360*(60*a^2*c^6*x^6 + 24*a*b*c^5*x^5 + 6*b^2*c^4*x^4 + 40*a*b*c^3*x^3 +
32*b^2*c^2*x^2 + 120*a*b*c*x + 15*(b^2*c^6*x^6 - b^2)*log(-(c*x + 1)/(c*x
- 1))^2 - 4*(15*a*b - 23*b^2)*log(c*x + 1) + 4*(15*a*b + 23*b^2)*log(c*x -
 1) + 4*(15*a*b*c^6*x^6 + 3*b^2*c^5*x^5 + 5*b^2*c^3*x^3 + 15*b^2*c*x)*log(
-(c*x + 1)/(c*x - 1)))/c^6
```

3.13.6 Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.46

$$\int x^5(a+b\text{arctanh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2x^6}{6}+\frac{abx^6\,\text{atanh}\,(cx)}{3}+\frac{abx^5}{15c}+\frac{abx^3}{9c^3}+\frac{abx}{3c^5}-\frac{ab\,\text{atanh}\,(cx)}{3c^6}+\frac{b^2x^6\,\text{atanh}^2\,(cx)}{6}+\frac{b^2x^5\,\text{atanh}\,(cx)}{15c}+\frac{b^2x^4}{60c^2}+\frac{b^2x^3\,\text{atanh}\,(cx)}{9c^3}\\ \frac{a^2x^6}{6}\end{cases}$$

input
```
integrate(x**5*(a+b*atanh(c*x))**2,x)
```

output
```
Piecewise((a**2*x**6/6 + a*b*x**6*atanh(c*x)/3 + a*b*x**5/(15*c) + a*b*x**
3/(9*c**3) + a*b*x/(3*c**5) - a*b*atanh(c*x)/(3*c**6) + b**2*x**6*atanh(c*
x)**2/6 + b**2*x**5*atanh(c*x)/(15*c) + b**2*x**4/(60*c**2) + b**2*x**3*at
anh(c*x)/(9*c**3) + 4*b**2*x**2/(45*c**4) + b**2*x*atanh(c*x)/(3*c**5) + 2
3*b**2*log(x - 1/c)/(45*c**6) - b**2*atanh(c*x)**2/(6*c**6) + 23*b**2*atan
h(c*x)/(45*c**6), Ne(c, 0)), (a**2*x**6/6, True))
```

3.13.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.48

$$\int x^5(a+b\text{arctanh}(cx))^2\,dx=\frac{1}{6}\,b^2x^6\,\text{artanh}\,(cx)^2+\frac{1}{6}\,a^2x^6$$
$$+\frac{1}{90}\left(30\,x^6\,\text{artanh}\,(cx)+c\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6}-\frac{15\,\log\,(cx+1)}{c^7}+\frac{15\,\log\,(cx-1)}{c^7}\right)\right)ab$$
$$+\frac{1}{360}\left(4\,c\left(\frac{2\,(3\,c^4x^5+5\,c^2x^3+15\,x)}{c^6}-\frac{15\,\log\,(cx+1)}{c^7}+\frac{15\,\log\,(cx-1)}{c^7}\right)\text{artanh}\,(cx)+\frac{6\,c^4x^4+32\,c^2}{}\right.$$

input
```
integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6*arctanh(c*x)^2 + 1/6*a^2*x^6 + 1/90*(30*x^6*arctanh(c*x) + c*(
2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x -
1)/c^7))*a*b + 1/360*(4*c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c
*x + 1)/c^7 + 15*log(c*x - 1)/c^7)*arctanh(c*x) + (6*c^4*x^4 + 32*c^2*x^2
- 2*(15*log(c*x - 1) - 46)*log(c*x + 1) + 15*log(c*x + 1)^2 + 15*log(c*x -
 1)^2 + 92*log(c*x - 1))/c^6)*b^2
```

3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(127) = 254$.

Time = 0.30 (sec) , antiderivative size = 889, normalized size of antiderivative = 6.13

$$\int x^5(a + b\text{arctanh}(cx))^2\, dx = \text{Too large to display}$$

input
```
integrate(x^5*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
1/90*(15*(3*(c*x + 1)^5*b^2/(c*x - 1)^5 + 10*(c*x + 1)^3*b^2/(c*x - 1)^3 +
 3*(c*x + 1)*b^2/(c*x - 1))*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^6*c^7/(
c*x - 1)^6 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^
4 - 20*(c*x + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c
*x + 1)*c^7/(c*x - 1) + c^7) + 2*(90*(c*x + 1)^5*a*b/(c*x - 1)^5 + 300*(c*
x + 1)^3*a*b/(c*x - 1)^3 + 90*(c*x + 1)*a*b/(c*x - 1) + 45*(c*x + 1)^5*b^2
/(c*x - 1)^5 - 135*(c*x + 1)^4*b^2/(c*x - 1)^4 + 230*(c*x + 1)^3*b^2/(c*x
- 1)^3 - 210*(c*x + 1)^2*b^2/(c*x - 1)^2 + 93*(c*x + 1)*b^2/(c*x - 1) - 23
*b^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^6*c^7/(c*x - 1)^6 - 6*(c*x + 1)
^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c^7/(c*x - 1)^4 - 20*(c*x + 1)^3*c^7/(
c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x - 1)^2 - 6*(c*x + 1)*c^7/(c*x - 1) +
c^7) + 4*(45*(c*x + 1)^5*a^2/(c*x - 1)^5 + 150*(c*x + 1)^3*a^2/(c*x - 1)^3
 + 45*(c*x + 1)*a^2/(c*x - 1) + 45*(c*x + 1)^5*a*b/(c*x - 1)^5 - 135*(c*x
+ 1)^4*a*b/(c*x - 1)^4 + 230*(c*x + 1)^3*a*b/(c*x - 1)^3 - 210*(c*x + 1)^2
*a*b/(c*x - 1)^2 + 93*(c*x + 1)*a*b/(c*x - 1) - 23*a*b + 11*(c*x + 1)^5*b^
2/(c*x - 1)^5 - 38*(c*x + 1)^4*b^2/(c*x - 1)^4 + 54*(c*x + 1)^3*b^2/(c*x -
 1)^3 - 38*(c*x + 1)^2*b^2/(c*x - 1)^2 + 11*(c*x + 1)*b^2/(c*x - 1))/((c*x
 + 1)^6*c^7/(c*x - 1)^6 - 6*(c*x + 1)^5*c^7/(c*x - 1)^5 + 15*(c*x + 1)^4*c
^7/(c*x - 1)^4 - 20*(c*x + 1)^3*c^7/(c*x - 1)^3 + 15*(c*x + 1)^2*c^7/(c*x
- 1)^2 - 6*(c*x + 1)*c^7/(c*x - 1) + c^7) - 46*b^2*log(-(c*x + 1)/(c*x ...
```

3.13.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.18

$$\int x^5(a + b\text{arctanh}(cx))^2\, dx$$
$$= \frac{46\, b^2\, \ln\left(c^2\, x^2 - 1\right) - 30\, b^2\, \text{atanh}(c\, x)^2 + 30\, a^2\, c^6\, x^6 + 16\, b^2\, c^2\, x^2 + 3\, b^2\, c^4\, x^4 - 60\, a\, b\, \text{atanh}(c\, x) + 20\, b^2\, c^3 \ldots}{}$$

input
```
int(x^5*(a + b*atanh(c*x))^2,x)
```

output
```
(46*b^2*log(c^2*x^2 - 1) - 30*b^2*atanh(c*x)^2 + 30*a^2*c^6*x^6 + 16*b^2*c
^2*x^2 + 3*b^2*c^4*x^4 - 60*a*b*atanh(c*x) + 20*b^2*c^3*x^3*atanh(c*x) + 1
2*b^2*c^5*x^5*atanh(c*x) + 60*b^2*c*x*atanh(c*x) + 30*b^2*c^6*x^6*atanh(c*
x)^2 + 20*a*b*c^3*x^3 + 12*a*b*c^5*x^5 + 60*a*b*c*x + 60*a*b*c^6*x^6*atanh
(c*x))/(180*c^6)
```

3.13. $\int x^5(a + b\text{arctanh}(cx))^2\,dx$

3.14 $\int x^4(a+b\text{arctanh}(cx))^2\,dx$

3.14.1 Optimal result

Integrand size = 14, antiderivative size = 162

$$\int x^4(a+b\text{arctanh}(cx))^2\,dx = \frac{3b^2x}{10c^4}+\frac{b^2x^3}{30c^2}-\frac{3b^2\text{arctanh}(cx)}{10c^5}+\frac{bx^2(a+b\text{arctanh}(cx))}{5c^3}+\frac{bx^4(a+b\text{arctanh}(cx))}{10c}+\frac{(a+b\text{arctanh}(cx))^2}{5c^5}+\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2b(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{5c^5}-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^5}$$

output
```
3/10*b^2*x/c^4+1/30*b^2*x^3/c^2-3/10*b^2*arctanh(c*x)/c^5+1/5*b*x^2*(a+b*a
rctanh(c*x))/c^3+1/10*b*x^4*(a+b*arctanh(c*x))/c+1/5*(a+b*arctanh(c*x))^2/
c^5+1/5*x^5*(a+b*arctanh(c*x))^2-2/5*b*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c
^5-1/5*b^2*polylog(2,1-2/(-c*x+1))/c^5
```

3.14.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.99

$$\int x^4(a+b\text{arctanh}(cx))^2\,dx$$

$$= \frac{-9ab+9b^2cx+6abc^2x^2+b^2c^3x^3+3abc^4x^4+6a^2c^5x^5+6b^2(-1+c^5x^5)\,\text{arctanh}(cx)^2+3b\text{arctanh}(cx)\,(\cdots}{\cdots}$$

input
```
Integrate[x^4*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(-9*a*b + 9*b^2*c*x + 6*a*b*c^2*x^2 + b^2*c^3*x^3 + 3*a*b*c^4*x^4 + 6*a^2*
c^5*x^5 + 6*b^2*(-1 + c^5*x^5)*ArcTanh[c*x]^2 + 3*b*ArcTanh[c*x]*(4*a*c^5*
x^5 + b*(-3 + 2*c^2*x^2 + c^4*x^4) - 4*b*Log[1 + E^(-2*ArcTanh[c*x])]) + 6
*a*b*Log[-1 + c^2*x^2] + 6*b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(30*c^5)
```

3.14.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.23, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6452, 6542, 6452, 254, 2009, 6542, 6452, 262, 219, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\text{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\int \frac{x^5(a+b\text{arctanh}(cx))}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\int \frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\int x^3(a+b\text{arctanh}(cx))dx}{c^2}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\int \frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\int \frac{x^4}{1-c^2x^2}dx}{c^2}\right)$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\int \frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\int \left(-\frac{x^2}{c^2}+\frac{1}{c^4(1-c^2x^2)}-\frac{1}{c^4}\right)dx}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\int \frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\left(\frac{\text{arctanh}(cx)}{c^5}-\frac{x}{c^4}-\frac{x^3}{3c^2}\right)}{c^2}\right)$$

$\downarrow$ 6542

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\frac{\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\int x(a+b\text{arctanh}(cx))dx}{c^2}}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\left(\frac{\text{arctanh}(cx)}{c^5}-\frac{x}{c^4}-\frac{x^3}{3c^2}\right)}{c^2}\right)$$

$\downarrow$ 6452

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\frac{\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\int\frac{x^2}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\left(\frac{\text{arctanh}(cx)}{c^5}-\frac{x}{c^4}-\frac{x^3}{3c^2}\right)}{c^2}\right)$$

$\downarrow$ 262

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\frac{\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\int\frac{1}{1-c^2x^2}dx}{c^2}-\frac{x}{c^2}\right)}{c^2}}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\left(\frac{\text{arctanh}(cx)}{c^5}-\frac{x}{c^4}-\frac{x^3}{3c^2}\right)}{c^2}\right)$$

$\downarrow$ 219

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\frac{\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\left(\frac{\text{arctanh}(cx)}{c^5}-\frac{x}{c^4}-\frac{x^3}{3c^2}\right)}{c^2}\right)$$

$\downarrow$ 6546

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\frac{\frac{\int \frac{a+b\text{arctanh}(cx)}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))-\frac{1}{4}bc\left(\frac{\text{arctanh}(cx)}{c^5}-\frac{x}{c^4}-\frac{x^3}{3c^2}\right)}{c^2}\right)$$

3.14. $\int x^4(a+b\text{arctanh}(cx))^2\,dx$

$\big\downarrow$ 6470

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-$$
$$\frac{2}{5}bc\left(\frac{\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}}{c^2}-\frac{\frac{1}{4}x^4(\ldots}{}\right.$$

$\big\downarrow$ 2849

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-$$
$$\frac{2}{5}bc\left(\frac{\frac{\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-\frac{2}{1-cx}}d\frac{1}{1-cx}}{c}+\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}}{c^2}-\ldots\right.$$

$\big\downarrow$ 2752

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-$$
$$\frac{2}{5}bc\left(\frac{\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}}{c^2}-\ldots\right.$$

input
```
Int[x^4*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(x^5*(a + b*ArcTanh[c*x])^2)/5 - (2*b*c*(-(((x^4*(a + b*ArcTanh[c*x]))/4 -
 (b*c*(-(x/c^4) - x^3/(3*c^2) + ArcTanh[c*x]/c^5))/4)/c^2) + (-(((x^2*(a +
 b*ArcTanh[c*x]))/2 - (b*c*(-(x/c^2) + ArcTanh[c*x]/c^3))/2)/c^2) + (-1/2*
(a + b*ArcTanh[c*x])^2/(b*c^2) + (((a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/
c + (b*PolyLog[2, 1 - 2/(1 - c*x)])/(2*c))/c)/c^2)/c^2))/5
```

3.14. $\int x^4(a+b\text{arctanh}(cx))^2\,dx$

3.14.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.14.4 Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.41

method	result
parts	$\frac{a^2x^5}{5} + \frac{b^2\left(\frac{c^5x^5\operatorname{arctanh}(cx)^2}{5}+\frac{c^4x^4\operatorname{arctanh}(cx)}{10}+\frac{c^2x^2\operatorname{arctanh}(cx)}{5}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{5}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{5}+\frac{c^3x^3}{30}+\cdots\right.}{\cdots}$
derivativedivides	$\frac{\frac{c^5x^5a^2}{5}+b^2\left(\frac{c^5x^5\operatorname{arctanh}(cx)^2}{5}+\frac{c^4x^4\operatorname{arctanh}(cx)}{10}+\frac{c^2x^2\operatorname{arctanh}(cx)}{5}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{5}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{5}+\frac{c^3x^3}{30}+\cdots\right.}{\cdots}$
default	$\frac{\frac{c^5x^5a^2}{5}+b^2\left(\frac{c^5x^5\operatorname{arctanh}(cx)^2}{5}+\frac{c^4x^4\operatorname{arctanh}(cx)}{10}+\frac{c^2x^2\operatorname{arctanh}(cx)}{5}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{5}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{5}+\frac{c^3x^3}{30}+\cdots\right.}{\cdots}$
risch	$\frac{b^2\ln(cx+1)x^2}{10c^3}-\frac{a^2}{5c^5}+\frac{b^2\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{5c^5}-\frac{b^2\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{5c^5}+\frac{ba\ln(cx+1)}{5c^5}+\frac{ba\ln(cx+1)x^5}{5}-\cdots$

input
```
int(x^4*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/5*a^2*x^5+b^2/c^5*(1/5*c^5*x^5*arctanh(c*x)^2+1/10*c^4*x^4*arctanh(c*x)+
1/5*c^2*x^2*arctanh(c*x)+1/5*arctanh(c*x)*ln(c*x-1)+1/5*arctanh(c*x)*ln(c*
x+1)+1/30*c^3*x^3+3/10*c*x+3/20*ln(c*x-1)-3/20*ln(c*x+1)-1/5*dilog(1/2*c*x
+1/2)-1/10*ln(c*x-1)*ln(1/2*c*x+1/2)+1/20*ln(c*x-1)^2+1/10*(ln(c*x+1)-ln(1
/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/20*ln(c*x+1)^2)+2*a*b/c^5*(1/5*c^5*x^5*arc
tanh(c*x)+1/20*c^4*x^4+1/10*c^2*x^2+1/10*ln(c*x-1)+1/10*ln(c*x+1))
```

3.14.5 Fricas [F]

$$\int x^4(a+b\text{arctanh}(cx))^2\,dx=\int (b\,\text{artanh}\,(cx)+a)^2x^4\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^4*arctanh(c*x)^2 + 2*a*b*x^4*arctanh(c*x) + a^2*x^4, x)
```

3.14.6 Sympy [F]

$$\int x^4(a+b\text{arctanh}(cx))^2\,dx=\int x^4(a+b\,\text{atanh}\,(cx))^2\;dx$$

input
```
integrate(x**4*(a+b*atanh(c*x))**2,x)
```

output
```
Integral(x**4*(a + b*atanh(c*x))**2, x)
```

3.14.7 Maxima [F]

$$\int x^4(a+b\text{arctanh}(cx))^2\,dx=\int (b\,\text{artanh}\,(cx)+a)^2x^4\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*x^5 + 1/10*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(
c^2*x^2 - 1)/c^6))*a*b - 1/36000*(24*c^6*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)
/c^10 - 15*log(c*x + 1)/c^11 + 15*log(c*x - 1)/c^11) - 45*c^5*((c^2*x^4 +
2*x^2)/c^8 + 2*log(c^2*x^2 - 1)/c^10) - 1080000*c^5*integrate(1/150*x^5*lo
g(c*x + 1)/(c^6*x^2 - c^4), x) + 50*c^4*(2*(c^2*x^3 + 3*x)/c^8 - 3*log(c*x
 + 1)/c^9 + 3*log(c*x - 1)/c^9) - 300*c^3*(x^2/c^6 + log(c^2*x^2 - 1)/c^8)
 + 900*c^2*(2*x/c^6 - log(c*x + 1)/c^7 + log(c*x - 1)/c^7) - 540000*c*inte
grate(1/150*x*log(c*x + 1)/(c^6*x^2 - c^4), x) - 60*(30*c^5*x^5*log(c*x +
1)^2 + (12*c^5*x^5 - 15*c^4*x^4 + 20*c^3*x^3 - 30*c^2*x^2 + 60*c*x - 60*(c
^5*x^5 + 1)*log(c*x + 1))*log(-c*x + 1))/c^5 - (72*(c*x - 1)^5*(25*log(-c*
x + 1)^2 - 10*log(-c*x + 1) + 2) + 1125*(c*x - 1)^4*(8*log(-c*x + 1)^2 - 4
*log(-c*x + 1) + 1) + 2000*(c*x - 1)^3*(9*log(-c*x + 1)^2 - 6*log(-c*x + 1
) + 2) + 9000*(c*x - 1)^2*(2*log(-c*x + 1)^2 - 2*log(-c*x + 1) + 1) + 9000
*(c*x - 1)*(log(-c*x + 1)^2 - 2*log(-c*x + 1) + 2))/c^5 + 1800*log(150*c^6
*x^2 - 150*c^4)/c^5 - 540000*integrate(1/150*log(c*x + 1)/(c^6*x^2 - c^4),
 x))*b^2
```

3.14.8 Giac [F]

$$\int x^4(a+b\text{arctanh}(cx))^2\,dx=\int (b\,\text{artanh}\,(cx)+a)^2x^4\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^4, x)
```

3.14.9 Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\text{arctanh}(cx))^2\,dx=\int x^4\,(a+b\,\text{atanh}(c\,x))^2\,dx$$

input
```
int(x^4*(a + b*atanh(c*x))^2,x)
```

output
```
int(x^4*(a + b*atanh(c*x))^2, x)
```

3.15 $\int x^3(a+b\,\mathrm{arctanh}(cx))^2\,dx$

3.15.1 Optimal result

Integrand size = 14, antiderivative size = 113

$$\int x^3(a+b\,\mathrm{arctanh}(cx))^2\,dx=\frac{abx}{2c^3}+\frac{b^2x^2}{12c^2}+\frac{b^2x\,\mathrm{arctanh}(cx)}{2c^3}+\frac{bx^3(a+b\,\mathrm{arctanh}(cx))}{6c}-\frac{(a+b\,\mathrm{arctanh}(cx))^2}{4c^4}+\frac{1}{4}x^4(a+b\,\mathrm{arctanh}(cx))^2+\frac{b^2\log(1-c^2x^2)}{3c^4}$$

output
```
1/2*a*b*x/c^3+1/12*b^2*x^2/c^2+1/2*b^2*x*arctanh(c*x)/c^3+1/6*b*x^3*(a+b*a
rctanh(c*x))/c-1/4*(a+b*arctanh(c*x))^2/c^4+1/4*x^4*(a+b*arctanh(c*x))^2+1
/3*b^2*ln(-c^2*x^2+1)/c^4
```

3.15.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int x^3(a+b\,\mathrm{arctanh}(cx))^2\,dx=\frac{6abcx+b^2c^2x^2+2abc^3x^3+3a^2c^4x^4+2bcx(3ac^3x^3+b(3+c^2x^2))\,\mathrm{arctanh}(cx)+3b^2(-1+c^4x^4)\,\mathrm{arctanh}}{12c^4}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(6*a*b*c*x + b^2*c^2*x^2 + 2*a*b*c^3*x^3 + 3*a^2*c^4*x^4 + 2*b*c*x*(3*a*c^
3*x^3 + b*(3 + c^2*x^2))*ArcTanh[c*x] + 3*b^2*(-1 + c^4*x^4)*ArcTanh[c*x]^
2 + b*(3*a + 4*b)*Log[1 - c*x] - 3*a*b*Log[1 + c*x] + 4*b^2*Log[1 + c*x])/
(12*c^4)
```

3.15.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6452, 6542, 6452, 243, 49, 2009, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+b\text{arctanh}(cx))^2 \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\int \frac{x^4(a+b\text{arctanh}(cx))}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\frac{\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\int x^2(a+b\text{arctanh}(cx))dx}{c^2}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\frac{\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{3}bc\int \frac{x^3}{1-c^2x^2}dx}{c^2}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\frac{\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\int \frac{x^2}{1-c^2x^2}dx^2}{c^2}\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\frac{\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\int\left(-\frac{1}{c^2}-\frac{1}{c^2(c^2x^2-1)}\right)dx^2}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\frac{\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{c^2}\right)$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\frac{\frac{\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{\int(a+b\text{arctanh}(cx))dx}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\frac{\frac{\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{c^2}\right)$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\frac{\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))-\frac{1}{6}bc\left(-\frac{x^2}{c^2}-\frac{\log(1-c^2x^2)}{c^4}\right)}{c^2}\right)$$

input
```
Int[x^3*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(x^4*(a + b*ArcTanh[c*x])^2)/4 - (b*c*(-(((x^3*(a + b*ArcTanh[c*x]))/3 - (
b*c*(-(x^2/c^2) - Log[1 - c^2*x^2]/c^4))/6)/c^2) + ((a + b*ArcTanh[c*x])^2
/(2*b*c^3) - (a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/c^2)/c^
2))/2
```

3.15. $\int x^3(a+b\text{arctanh}(cx))^2\,dx$

3.15.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.15.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{3b^2 \operatorname{arctanh}(cx)^2 x^4 c^4+6ab \operatorname{arctanh}(cx) x^4 c^4+3c^4 x^4 a^2+2b^2 \operatorname{arctanh}(cx) x^3 c^3+2ab\, c^3 x^3+b^2 c^2 x^2+6b^2 \operatorname{arctanh}(cx) xc+6}{12c^4}$
parts	$\frac{a^2 x^4}{4}+\frac{b^2\left(\frac{c^4 x^4 \operatorname{arctanh}(cx)^2}{4}+\frac{c^3 x^3 \operatorname{arctanh}(cx)}{6}+\frac{cx \operatorname{arctanh}(cx)}{2}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{4}-\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{4}-\frac{\ln(cx-1)\ln}{8}\right.}{c^4}$
derivativedivides	$\frac{\frac{c^4 x^4 a^2}{4}+b^2\left(\frac{c^4 x^4 \operatorname{arctanh}(cx)^2}{4}+\frac{c^3 x^3 \operatorname{arctanh}(cx)}{6}+\frac{cx \operatorname{arctanh}(cx)}{2}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{4}-\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{4}-\frac{\ln(cx-1)\ln}{8}\right.}{}$
default	$\frac{\frac{c^4 x^4 a^2}{4}+b^2\left(\frac{c^4 x^4 \operatorname{arctanh}(cx)^2}{4}+\frac{c^3 x^3 \operatorname{arctanh}(cx)}{6}+\frac{cx \operatorname{arctanh}(cx)}{2}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{4}-\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{4}-\frac{\ln(cx-1)\ln}{8}\right.}{}$
risch	$\frac{b^2(c^4x^4-1)\ln(cx+1)^2}{16c^4}+\frac{b(-3b x^4 \ln(-cx+1)c^4+6c^4x^4a+2b\, c^3x^3+6bcx+3b\ln(-cx+1))\ln(cx+1)}{24c^4}+\frac{\ln(-cx+1)^2 b^2}{16}$

input
```
int(x^3*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*b^2*arctanh(c*x)^2*x^4*c^4+6*a*b*arctanh(c*x)*x^4*c^4+3*c^4*x^4*a^
2+2*b^2*arctanh(c*x)*x^3*c^3+2*a*b*c^3*x^3+b^2*c^2*x^2+6*b^2*arctanh(c*x)*
x*c+6*a*b*c*x-3*b^2*arctanh(c*x)^2+8*ln(c*x-1)*b^2-6*arctanh(c*x)*a*b+8*ar
ctanh(c*x)*b^2+b^2)/c^4
```

3.15.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.42

$$\int x^3 (a + b\operatorname{arctanh}(cx))^2 \, dx$$
$$= \frac{12\, a^2 c^4 x^4 + 8\, abc^3 x^3 + 4\, b^2 c^2 x^2 + 24\, abcx + 3\left(b^2 c^4 x^4 - b^2\right) \log\left(-\frac{cx+1}{cx-1}\right)^2 - 4\left(3\, ab - 4\, b^2\right) \log\left(cx+1\right) + 4}{48\, c^4}$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
1/48*(12*a^2*c^4*x^4 + 8*a*b*c^3*x^3 + 4*b^2*c^2*x^2 + 24*a*b*c*x + 3*(b^2
*c^4*x^4 - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 4*(3*a*b - 4*b^2)*log(c*x +
1) + 4*(3*a*b + 4*b^2)*log(c*x - 1) + 4*(3*a*b*c^4*x^4 + b^2*c^3*x^3 + 3*b
^2*c*x)*log(-(c*x + 1)/(c*x - 1)))/c^4
```

3.15.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.49

$$\int x^3(a+b\text{arctanh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2x^4}{4}+\frac{abx^4\,\text{atanh}\,(cx)}{2}+\frac{abx^3}{6c}+\frac{abx}{2c^3}-\frac{ab\,\text{atanh}\,(cx)}{2c^4}+\frac{b^2x^4\,\text{atanh}^2\,(cx)}{4}+\frac{b^2x^3\,\text{atanh}\,(cx)}{6c}+\frac{b^2x^2}{12c^2}+\frac{b^2x\,\text{atanh}\,(cx)}{2c^3}+\frac{2b^2\log}{3} \\ \frac{a^2x^4}{4}\end{cases}$$

input
```
integrate(x**3*(a+b*atanh(c*x))**2,x)
```

output
```
Piecewise((a**2*x**4/4 + a*b*x**4*atanh(c*x)/2 + a*b*x**3/(6*c) + a*b*x/(2
*c**3) - a*b*atanh(c*x)/(2*c**4) + b**2*x**4*atanh(c*x)**2/4 + b**2*x**3*a
tanh(c*x)/(6*c) + b**2*x**2/(12*c**2) + b**2*x*atanh(c*x)/(2*c**3) + 2*b**
2*log(x - 1/c)/(3*c**4) - b**2*atanh(c*x)**2/(4*c**4) + 2*b**2*atanh(c*x)/
(3*c**4), Ne(c, 0)), (a**2*x**4/4, True))
```

3.15.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.67

$$\int x^3(a+b\text{arctanh}(cx))^2\,dx=\frac{1}{4}\,b^2x^4\,\text{artanh}\,(cx)^2+\frac{1}{4}\,a^2x^4$$
$$+\frac{1}{12}\left(6\,x^4\,\text{artanh}\,(cx)+c\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\,(cx+1)}{c^5}+\frac{3\,\log\,(cx-1)}{c^5}\right)\right)ab$$
$$+\frac{1}{48}\left(4\,c\left(\frac{2\,(c^2x^3+3\,x)}{c^4}-\frac{3\,\log\,(cx+1)}{c^5}+\frac{3\,\log\,(cx-1)}{c^5}\right)\text{artanh}\,(cx)+\frac{4\,c^2x^2-2\,(3\,\log\,(cx-1)-8}{}\right.$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctanh(c*x)^2 + 1/4*a^2*x^4 + 1/12*(6*x^4*arctanh(c*x) + c*(2
*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a*b + 1/4
8*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5)*a
rctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(c*x
 + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*b^2
```

3.15.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(99) = 198.

Time = 0.30 (sec) , antiderivative size = 603, normalized size of antiderivative = 5.34

$$\int x^3(a+b\operatorname{arctanh}(cx))^2\,dx$$
$$=\frac{1}{6}\left(\frac{3\left(\frac{(cx+1)^3b^2}{(cx-1)^3}+\frac{(cx+1)b^2}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2}{\frac{(cx+1)^4c^5}{(cx-1)^4}-\frac{4(cx+1)^3c^5}{(cx-1)^3}+\frac{6(cx+1)^2c^5}{(cx-1)^2}-\frac{4(cx+1)c^5}{cx-1}+c^5}+\frac{2\left(\frac{6(cx+1)^3ab}{(cx-1)^3}+\frac{6(cx+1)ab}{cx-1}+\frac{3(cx+1)^3b^2}{(cx-1)^3}-\frac{6(cx+1)^2b^2}{(cx-1)^2}\right.}{\frac{(cx+1)^4c^5}{(cx-1)^4}-\frac{4(cx+1)^3c^5}{(cx-1)^3}+\frac{6(cx+1)^2c^5}{(cx-1)^2}}\right.$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
1/6*(3*((c*x + 1)^3*b^2/(c*x - 1)^3 + (c*x + 1)*b^2/(c*x - 1))*log(-(c*x +
 1)/(c*x - 1))^2/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1
)^3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^5/(c*x - 1) + c^5) + 2
*(6*(c*x + 1)^3*a*b/(c*x - 1)^3 + 6*(c*x + 1)*a*b/(c*x - 1) + 3*(c*x + 1)^
3*b^2/(c*x - 1)^3 - 6*(c*x + 1)^2*b^2/(c*x - 1)^2 + 5*(c*x + 1)*b^2/(c*x -
 1) - 2*b^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4*c^5/(c*x - 1)^4 - 4*(c
*x + 1)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*(c*x + 1)*c^
5/(c*x - 1) + c^5) + 2*(6*(c*x + 1)^3*a^2/(c*x - 1)^3 + 6*(c*x + 1)*a^2/(c
*x - 1) + 6*(c*x + 1)^3*a*b/(c*x - 1)^3 - 12*(c*x + 1)^2*a*b/(c*x - 1)^2 +
 10*(c*x + 1)*a*b/(c*x - 1) - 4*a*b + (c*x + 1)^3*b^2/(c*x - 1)^3 - 2*(c*x
 + 1)^2*b^2/(c*x - 1)^2 + (c*x + 1)*b^2/(c*x - 1))/((c*x + 1)^4*c^5/(c*x -
 1)^4 - 4*(c*x + 1)^3*c^5/(c*x - 1)^3 + 6*(c*x + 1)^2*c^5/(c*x - 1)^2 - 4*
(c*x + 1)*c^5/(c*x - 1) + c^5) - 4*b^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^5 +
 4*b^2*log(-(c*x + 1)/(c*x - 1))/c^5)*c
```

3.15.9 Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.19

$$\int x^3(a+b\operatorname{arctanh}(cx))^2\,dx$$
$$=\frac{4\,b^2\,\ln\left(c^2\,x^2-1\right)-3\,b^2\,\operatorname{atanh}(c\,x)^2+3\,a^2\,c^4\,x^4+b^2\,c^2\,x^2-6\,a\,b\,\operatorname{atanh}(c\,x)+2\,b^2\,c^3\,x^3\,\operatorname{atanh}(c\,x)+6\,b^2}{12\,c^4}$$

input
```
int(x^3*(a + b*atanh(c*x))^2,x)
```

output
```
(4*b^2*log(c^2*x^2 - 1) - 3*b^2*atanh(c*x)^2 + 3*a^2*c^4*x^4 + b^2*c^2*x^2
 - 6*a*b*atanh(c*x) + 2*b^2*c^3*x^3*atanh(c*x) + 6*b^2*c*x*atanh(c*x) + 3*
b^2*c^4*x^4*atanh(c*x)^2 + 2*a*b*c^3*x^3 + 6*a*b*c*x + 6*a*b*c^4*x^4*atanh
(c*x))/(12*c^4)
```

3.15. $\int x^3(a + b\text{arctanh}(cx))^2\,dx$

3.16 $\int x^2(a+b\,\text{arctanh}(cx))^2\,dx$

3.16.1 Optimal result

Integrand size = 14, antiderivative size = 130

$$\int x^2(a+b\,\text{arctanh}(cx))^2\,dx=\frac{b^2x}{3c^2}-\frac{b^2\text{arctanh}(cx)}{3c^3}+\frac{bx^2(a+b\,\text{arctanh}(cx))}{3c}+\frac{(a+b\,\text{arctanh}(cx))^2}{3c^3}+\frac{1}{3}x^3(a+b\,\text{arctanh}(cx))^2-\frac{2b(a+b\,\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{3c^3}-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{3c^3}$$

output
```
1/3*b^2*x/c^2-1/3*b^2*arctanh(c*x)/c^3+1/3*b*x^2*(a+b*arctanh(c*x))/c+1/3*
(a+b*arctanh(c*x))^2/c^3+1/3*x^3*(a+b*arctanh(c*x))^2-2/3*b*(a+b*arctanh(c
*x))*ln(2/(-c*x+1))/c^3-1/3*b^2*polylog(2,1-2/(-c*x+1))/c^3
```

3.16.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.94

$$\int x^2(a+b\,\text{arctanh}(cx))^2\,dx=\frac{b^2cx+abc^2x^2+a^2c^3x^3+b^2(-1+c^3x^3)\,\text{arctanh}(cx)^2+b\,\text{arctanh}(cx)\left(-b+bc^2x^2+2ac^3x^3-2b\log\left(1+\right.\right.}{3c^3}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(b^2*c*x + a*b*c^2*x^2 + a^2*c^3*x^3 + b^2*(-1 + c^3*x^3)*ArcTanh[c*x]^2 +
 b*ArcTanh[c*x]*(-b + b*c^2*x^2 + 2*a*c^3*x^3 - 2*b*Log[1 + E^(-2*ArcTanh[
c*x])]) + a*b*Log[-1 + c^2*x^2] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(3
*c^3)
```

3.16.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6452, 6542, 6452, 262, 219, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\text{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int \frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\int x(a+b\text{arctanh}(cx))dx}{c^2}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\int \frac{x^2}{1-c^2x^2}dx}{c^2}\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\int \frac{1}{1-c^2x^2}dx}{c^2}-\frac{x}{c^2}\right)}{c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}\right)$$

$\downarrow$ 6546

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\frac{\int\frac{a+b\text{arctanh}(cx)}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}\right)$$

$\downarrow$ 6470

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}\right)$$

$\downarrow$ 2849

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-\frac{2}{1-cx}}d\frac{1}{1-cx}}{c}+\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}\right)$$

$\downarrow$ 2752

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx)}{c^3}-\frac{x}{c^2}\right)}{c^2}\right)$$

input
```
Int[x^2*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(x^3*(a + b*ArcTanh[c*x])^2)/3 - (2*b*c*(-(((x^2*(a + b*ArcTanh[c*x]))/2 -
 (b*c*(-(x/c^2) + ArcTanh[c*x]/c^3))/2)/c^2) + (-1/2*(a + b*ArcTanh[c*x])^
2/(b*c^2) + (((a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b*PolyLog[2, 1 -
 2/(1 - c*x)])/(2*c))/c)/c^2))/3
```

3.16. $\int x^2(a+b\text{arctanh}(cx))^2\,dx$

3.16.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.16.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.55

method	result
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\frac{c^3x^3\operatorname{arctanh}(cx)^2}{3}+\frac{c^2x^2\operatorname{arctanh}(cx)}{3}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{3}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{3}+\frac{cx}{3}+\frac{\ln(cx-1)}{6}-\frac{\ln(cx+1)}{6}-\right.}{c^3}$
derivativedivides	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{c^3x^3\operatorname{arctanh}(cx)^2}{3}+\frac{c^2x^2\operatorname{arctanh}(cx)}{3}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{3}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{3}+\frac{cx}{3}+\frac{\ln(cx-1)}{6}-\frac{\ln(cx+1)}{6}-\right.}{}$
default	$\frac{\frac{a^2c^3x^3}{3}+b^2\left(\frac{c^3x^3\operatorname{arctanh}(cx)^2}{3}+\frac{c^2x^2\operatorname{arctanh}(cx)}{3}+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{3}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{3}+\frac{cx}{3}+\frac{\ln(cx-1)}{6}-\frac{\ln(cx+1)}{6}-\right.}{}$
risch	$\frac{a^2x^3}{3}+\frac{b^2\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln(cx+1)}{3c^3}-\frac{b^2\ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{3c^3}+\frac{b^2\ln(cx+1)x^2}{6c}-\frac{a^2}{3c^3}-\frac{b^2\operatorname{dilog}\left(\frac{cx}{2}+\frac{1}{2}\right)}{3c^3}-$

input
```
int(x^2*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+b^2/c^3*(1/3*c^3*x^3*arctanh(c*x)^2+1/3*c^2*x^2*arctanh(c*x)+1
/3*arctanh(c*x)*ln(c*x-1)+1/3*arctanh(c*x)*ln(c*x+1)+1/3*c*x+1/6*ln(c*x-1)
-1/6*ln(c*x+1)-1/3*dilog(1/2*c*x+1/2)-1/6*ln(c*x-1)*ln(1/2*c*x+1/2)+1/12*l
n(c*x-1)^2+1/6*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/12*ln(c*x+1)
^2)+2*a*b/c^3*(1/3*c^3*x^3*arctanh(c*x)+1/6*c^2*x^2+1/6*ln(c*x-1)+1/6*ln(c
*x+1))
```

3.16.5 Fricas [F]

$$\int x^2(a+b\operatorname{arctanh}(cx))^2\,dx=\int(b\operatorname{artanh}(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctanh(c*x)^2 + 2*a*b*x^2*arctanh(c*x) + a^2*x^2, x)
```

3.16.6 Sympy [F]

$$\int x^2(a+b\text{arctanh}(cx))^2\,dx=\int x^2(a+b\,\text{atanh}\,(cx))^2\,dx$$

input
```
integrate(x**2*(a+b*atanh(c*x))**2,x)
```

output
```
Integral(x**2*(a + b*atanh(c*x))**2, x)
```

3.16.7 Maxima [F]

$$\int x^2(a+b\text{arctanh}(cx))^2\,dx=\int (b\,\text{artanh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4)
)*a*b - 1/216*(2*c^4*(2*(c^2*x^3 + 3*x)/c^6 - 3*log(c*x + 1)/c^7 + 3*log(c
*x - 1)/c^7) - 3*c^3*(x^2/c^4 + log(c^2*x^2 - 1)/c^6) - 648*c^3*integrate(
1/9*x^3*log(c*x + 1)/(c^4*x^2 - c^2), x) + 9*c^2*(2*x/c^4 - log(c*x + 1)/c
^5 + log(c*x - 1)/c^5) - 324*c*integrate(1/9*x*log(c*x + 1)/(c^4*x^2 - c^2
), x) - 6*(3*c^3*x^3*log(c*x + 1)^2 + (2*c^3*x^3 - 3*c^2*x^2 + 6*c*x - 6*(
c^3*x^3 + 1)*log(c*x + 1))*log(-c*x + 1))/c^3 - (2*(c*x - 1)^3*(9*log(-c*x
 + 1)^2 - 6*log(-c*x + 1) + 2) + 27*(c*x - 1)^2*(2*log(-c*x + 1)^2 - 2*log
(-c*x + 1) + 1) + 54*(c*x - 1)*(log(-c*x + 1)^2 - 2*log(-c*x + 1) + 2))/c^
3 + 18*log(9*c^4*x^2 - 9*c^2)/c^3 - 324*integrate(1/9*log(c*x + 1)/(c^4*x^
2 - c^2), x))*b^2
```

3.16.8 Giac [F]

$$\int x^2(a+b\text{arctanh}(cx))^2\,dx=\int (b\,\text{artanh}\,(cx)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*x^2, x)
```

3.16.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\operatorname{arctanh}(cx))^2\,dx=\int x^2\,\left(a+b\,\mathrm{atanh}(c\,x)\right)^2\,dx$$

input
```
int(x^2*(a + b*atanh(c*x))^2,x)
```

output
```
int(x^2*(a + b*atanh(c*x))^2, x)
```

3.17 $\int x(a+b\text{arctanh}(cx))^2\,dx$

3.17.1 Optimal result

Integrand size = 12, antiderivative size = 75

$$\int x(a+b\text{arctanh}(cx))^2\,dx=\frac{abx}{c}+\frac{b^2x\text{arctanh}(cx)}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2c^2}+\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2+\frac{b^2\log(1-c^2x^2)}{2c^2}$$

output
```
a*b*x/c+b^2*x*arctanh(c*x)/c-1/2*(a+b*arctanh(c*x))^2/c^2+1/2*x^2*(a+b*arc
tanh(c*x))^2+1/2*b^2*ln(-c^2*x^2+1)/c^2
```

3.17.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.20

$$\int x(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{2abcx+a^2c^2x^2+2bcx(b+acx)\text{arctanh}(cx)+b^2(-1+c^2x^2)\text{arctanh}(cx)^2+b(a+b)\log(1-cx)-ab\log}{2c^2}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(2*a*b*c*x + a^2*c^2*x^2 + 2*b*c*x*(b + a*c*x)*ArcTanh[c*x] + b^2*(-1 + c^
2*x^2)*ArcTanh[c*x]^2 + b*(a + b)*Log[1 - c*x] - a*b*Log[1 + c*x] + b^2*Lo
g[1 + c*x])/(2*c^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6452, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{\int(a+b\text{arctanh}(cx))dx}{c^2}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log(1-c^2x^2)}{2c}}{c^2}\right)$$

input
```
Int[x*(a + b*ArcTanh[c*x])^2,x]
```

output
```
(x^2*(a + b*ArcTanh[c*x])^2)/2 - b*c*((a + b*ArcTanh[c*x])^2/(2*b*c^3) - (
a*x + b*x*ArcTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/c^2)
```

3.17.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.17.4 Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.41

method	result
parallelrisch	$\frac{b^2 \operatorname{arctanh}(cx)^2 x^2 c^2+2x^2 \operatorname{arctanh}(cx) ab\, c^2+c^2x^2a^2+2b^2 \operatorname{arctanh}(cx) xc+2abcx-b^2 \operatorname{arctanh}(cx)^2+2\ln(cx-1)b^2-2\ \mathrm{arc}\cdots}{2c^2}$
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\frac{c^2x^2 \operatorname{arctanh}(cx)^2}{2}+cx\ \operatorname{arctanh}(cx)+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}-\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{2}-\frac{\ln(cx-1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(cx-\cdots}{8}\cdots\right.}{c^2}$
derivativedivides	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2 \operatorname{arctanh}(cx)^2}{2}+cx\ \operatorname{arctanh}(cx)+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}-\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{2}-\frac{\ln(cx-1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(cx-\cdots}{8}\cdots\right.}{\cdots}$
default	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2 \operatorname{arctanh}(cx)^2}{2}+cx\ \operatorname{arctanh}(cx)+\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}-\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{2}-\frac{\ln(cx-1)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{4}+\frac{\ln(cx-\cdots}{8}\cdots\right.}{\cdots}$
risch	$\frac{b^2\left(c^2x^2-1\right)\ln(cx+1)^2}{8c^2}+\frac{b\left(-b\,x^2\ln(-cx+1)c^2+2a\,c^2x^2+2bcx+b\ln(-cx+1)\right)\ln(cx+1)}{4c^2}+\frac{\ln(-cx+1)^2b^2x^2}{8}-\ln(\cdots$

input
```
int(x*(a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*arctanh(c*x)^2*x^2*c^2+2*x^2*arctanh(c*x)*a*b*c^2+c^2*x^2*a^2+2*b
^2*arctanh(c*x)*x*c+2*a*b*c*x-b^2*arctanh(c*x)^2+2*ln(c*x-1)*b^2-2*arctanh
(c*x)*a*b+2*arctanh(c*x)*b^2+a^2)/c^2
```

3.17.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.63

$$\int x(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{4\,a^2c^2x^2+8\,abcx+(b^2c^2x^2-b^2)\log\left(-\frac{cx+1}{cx-1}\right)^2-4\,(ab-b^2)\log\left(cx+1\right)+4\,(ab+b^2)\log\left(cx-1\right)+4\,(ab}{8\,c^2}$$

input
```
integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
1/8*(4*a^2*c^2*x^2 + 8*a*b*c*x + (b^2*c^2*x^2 - b^2)*log(-(c*x + 1)/(c*x -
 1))^2 - 4*(a*b - b^2)*log(c*x + 1) + 4*(a*b + b^2)*log(c*x - 1) + 4*(a*b*
c^2*x^2 + b^2*c*x)*log(-(c*x + 1)/(c*x - 1)))/c^2
```

3.17.6 Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.52

$$\int x(a+b\text{arctanh}(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2x^2}{2}+abx^2\,\text{atanh}\,(cx)+\frac{abx}{c}-\frac{ab\,\text{atanh}\,(cx)}{c^2}+\frac{b^2x^2\,\text{atanh}^2\,(cx)}{2}+\frac{b^2x\,\text{atanh}\,(cx)}{c}+\frac{b^2\log\left(x-\frac{1}{c}\right)}{c^2}-\frac{b^2\,\text{atanh}^2\,(cx)}{2c^2}+\frac{b^2\,\text{a}}{} \\ \frac{a^2x^2}{2}\end{cases}$$

input
```
integrate(x*(a+b*atanh(c*x))**2,x)
```

output
```
Piecewise((a**2*x**2/2 + a*b*x**2*atanh(c*x) + a*b*x/c - a*b*atanh(c*x)/c*
*2 + b**2*x**2*atanh(c*x)**2/2 + b**2*x*atanh(c*x)/c + b**2*log(x - 1/c)/c
**2 - b**2*atanh(c*x)**2/(2*c**2) + b**2*atanh(c*x)/c**2, Ne(c, 0)), (a**2
*x**2/2, True))
```

3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(69) = 138$.

Time = 0.20 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.11

$$\int x(a+b\text{arctanh}(cx))^2\,dx = \frac{1}{2}\,b^2x^2\,\text{artanh}\,(cx)^2 + \frac{1}{2}\,a^2x^2 + \frac{1}{2}\left(2\,x^2\,\text{artanh}\,(cx) + c\left(\frac{2\,x}{c^2} - \frac{\log\,(cx+1)}{c^3} + \frac{\log\,(cx-1)}{c^3}\right)\right)ab + \frac{1}{8}\left(4\,c\left(\frac{2\,x}{c^2} - \frac{\log\,(cx+1)}{c^3} + \frac{\log\,(cx-1)}{c^3}\right)\text{artanh}\,(cx) - \frac{2\,(\log\,(cx-1)-2)\log\,(cx+1) - \log\,(cx+}{c^2}\right.$$

input
```
integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arctanh(c*x)^2 + 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c*x) + c*(2*
x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a*b + 1/8*(4*c*(2*x/c^2 - lo
g(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x - 1) - 2)*lo
g(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^2)*b^2
```

3.17.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(69) = 138$.

Time = 0.30 (sec) , antiderivative size = 301, normalized size of antiderivative = 4.01

$$\int x(a+b\text{arctanh}(cx))^2\,dx = \frac{1}{2}\left(\frac{(cx+1)b^2\log\left(-\frac{cx+1}{cx-1}\right)^2}{\left(\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2\,(cx+1)c^3}{cx-1} + c^3\right)(cx-1)} + \frac{2\left(\frac{2\,(cx+1)ab}{cx-1} + \frac{(cx+1)b^2}{cx-1} - b^2\right)\log\left(-\frac{cx+1}{cx-1}\right)}{\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2\,(cx+1)c^3}{cx-1} + c^3} + \frac{4\left(\frac{(cx+1)a^2}{cx-1} + \frac{(cx+1}{cx-}\right.}{\frac{(cx+1)^2c^3}{(cx-1)^2} - \frac{2\,(cx+1}{cx-}}\right.$$

input
```
integrate(x*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
1/2*((c*x + 1)*b^2*log(-(c*x + 1)/(c*x - 1))^2/(((c*x + 1)^2*c^3/(c*x - 1)
^2 - 2*(c*x + 1)*c^3/(c*x - 1) + c^3)*(c*x - 1)) + 2*(2*(c*x + 1)*a*b/(c*x
 - 1) + (c*x + 1)*b^2/(c*x - 1) - b^2)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1
)^2*c^3/(c*x - 1)^2 - 2*(c*x + 1)*c^3/(c*x - 1) + c^3) + 4*((c*x + 1)*a^2/
(c*x - 1) + (c*x + 1)*a*b/(c*x - 1) - a*b)/((c*x + 1)^2*c^3/(c*x - 1)^2 -
2*(c*x + 1)*c^3/(c*x - 1) + c^3) - 2*b^2*log(-(c*x + 1)/(c*x - 1) + 1)/c^3
 + 2*b^2*log(-(c*x + 1)/(c*x - 1))/c^3)*c
```

3.17.9 Mupad [B] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.19

$$\int x(a+b\text{arctanh}(cx))^2\,dx$$
$$=\frac{a^2\,x^2}{2}-\frac{\frac{b^2\,\text{atanh}(c\,x)^2}{2}-\frac{b^2\,\ln(c^2\,x^2-1)}{2}-c\,(x\,\text{atanh}(c\,x)\;b^2+a\,x\,b)+a\,b\,\text{atanh}(c\,x)}{c^2}+\frac{b^2\,x^2\,\text{atanh}(c\,x)^2}{2}+a\,b\,x^2\,\text{atanh}(c\,x)$$

input
```
int(x*(a + b*atanh(c*x))^2,x)
```

output
```
(a^2*x^2)/2 - ((b^2*atanh(c*x)^2)/2 - (b^2*log(c^2*x^2 - 1))/2 - c*(b^2*x*
atanh(c*x) + a*b*x) + a*b*atanh(c*x))/c^2 + (b^2*x^2*atanh(c*x)^2)/2 + a*b
*x^2*atanh(c*x)
```

3.18 $\int (a + b\text{arctanh}(cx))^2 \, dx$

3.18.1 Optimal result

Integrand size = 10, antiderivative size = 74

$$\int (a + b\text{arctanh}(cx))^2 \, dx = \frac{(a + b\text{arctanh}(cx))^2}{c} + x(a + b\text{arctanh}(cx))^2 - \frac{2b(a + b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c} - \frac{b^2 \text{PolyLog}\left(2, 1 - \frac{2}{1-cx}\right)}{c}$$

output

```
(a+b*arctanh(c*x))^2/c+x*(a+b*arctanh(c*x))^2-2*b*(a+b*arctanh(c*x))*ln(2/
(-c*x+1))/c-b^2*polylog(2,1-2/(-c*x+1))/c
```

3.18.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int (a + b\text{arctanh}(cx))^2 \, dx = \frac{b^2(-1 + cx)\text{arctanh}(cx)^2 + 2b\text{arctanh}(cx)\left(acx - b\log\left(1 + e^{-2\text{arctanh}(cx)}\right)\right) + a(acx + b\log(1 - c^2x^2)) +}{c}$$

input

```
Integrate[(a + b*ArcTanh[c*x])^2,x]
```

output

```
(b^2*(-1 + c*x)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(a*c*x - b*Log[1 + E^(-2
*ArcTanh[c*x])]) + a*(a*c*x + b*Log[1 - c^2*x^2]) + b^2*PolyLog[2, -E^(-2*
ArcTanh[c*x])])/c
```

3.18.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6436, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\operatorname{arctanh}(cx))^2\,dx$$

$$\Big\downarrow \text{ 6436}$$

$$x(a+b\operatorname{arctanh}(cx))^2-2bc\int \frac{x(a+b\operatorname{arctanh}(cx))}{1-c^2x^2}dx$$

$$\Big\downarrow \text{ 6546}$$

$$x(a+b\operatorname{arctanh}(cx))^2-2bc\left(\frac{\int \frac{a+b\operatorname{arctanh}(cx)}{1-cx}dx}{c}-\frac{(a+b\operatorname{arctanh}(cx))^2}{2bc^2}\right)$$

$$\Big\downarrow \text{ 6470}$$

$$x(a+b\operatorname{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))}{c}-b\int \frac{\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\operatorname{arctanh}(cx))^2}{2bc^2}\right)$$

$$\Big\downarrow \text{ 2849}$$

$$x(a+b\operatorname{arctanh}(cx))^2-2bc\left(\frac{\frac{b\int \frac{\log\left(\frac{2}{1-cx}\right)}{1-\frac{2}{1-cx}}d\frac{1}{1-cx}}{c}+\frac{\log\left(\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))}{c}}{c}-\frac{(a+b\operatorname{arctanh}(cx))^2}{2bc^2}\right)$$

$$\Big\downarrow \text{ 2752}$$

$$x(a+b\operatorname{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\operatorname{arctanh}(cx))}{c}+\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\operatorname{arctanh}(cx))^2}{2bc^2}\right)$$

input
```
Int[(a + b*ArcTanh[c*x])^2,x]
```

output
```
x*(a + b*ArcTanh[c*x])^2 - 2*b*c*(-1/2*(a + b*ArcTanh[c*x])^2/(b*c^2) + ((
(a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b*PolyLog[2, 1 - 2/(1 - c*x)])
/(2*c))/c)
```

3.18.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.18.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{cx\,a^2+b^2\left(\operatorname{arctanh}(cx)^2(cx-1)+2\operatorname{arctanh}(cx)^2-2\operatorname{arctanh}(cx)\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)-\operatorname{polylog}\left(2,-\frac{(cx+1)^2}{-c^2x^2+1}\right)\right)+2cxab\,\text{a}}{c}$
default	$\frac{cx\,a^2+b^2\left(\operatorname{arctanh}(cx)^2(cx-1)+2\operatorname{arctanh}(cx)^2-2\operatorname{arctanh}(cx)\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)-\operatorname{polylog}\left(2,-\frac{(cx+1)^2}{-c^2x^2+1}\right)\right)+2cxab\,\text{a}}{c}$
parts	$a^2x+\frac{b^2\left(\operatorname{arctanh}(cx)^2(cx-1)+2\operatorname{arctanh}(cx)^2-2\operatorname{arctanh}(cx)\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)-\operatorname{polylog}\left(2,-\frac{(cx+1)^2}{-c^2x^2+1}\right)\right)}{c}+2a$
risch	$-\frac{b^2\operatorname{dilog}\left(\frac{cx}{2}+\frac{1}{2}\right)}{c}-\frac{b^2\ln(cx-1)}{c}+\frac{b^2\ln(cx+1)^2x}{4}+\frac{b^2\ln(cx+1)^2}{4c}+\frac{\ln(-cx+1)^2b^2x}{4}-\frac{\ln(-cx+1)^2b^2}{4c}+\ln($

input
```
int((a+b*arctanh(c*x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/c*(c*x*a^2+b^2*(arctanh(c*x)^2*(c*x-1)+2*arctanh(c*x)^2-2*arctanh(c*x)*l
n(1+(c*x+1)^2/(-c^2*x^2+1))-polylog(2,-(c*x+1)^2/(-c^2*x^2+1)))+2*c*x*a*b*
arctanh(c*x)+a*b*ln(-c^2*x^2+1))
```

3.18.5 Fricas [F]

$$\int (a+b\text{arctanh}(cx))^2\,dx=\int (b\,\text{artanh}\,(cx)+a)^2\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2, x)
```

3.18.6 Sympy [F]

$$\int (a+b\text{arctanh}(cx))^2\,dx=\int (a+b\,\text{atanh}\,(cx))^2\,dx$$

input
```
integrate((a+b*atanh(c*x))**2,x)
```

output
```
Integral((a + b*atanh(c*x))**2, x)
```

3.18.7 Maxima [F]

$$\int (a+b\text{arctanh}(cx))^2\,dx = \int (b\,\text{artanh}\,(cx)+a)^2\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
-1/4*(c^2*(2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3) - 6*c*integrate(
x*log(c*x + 1)/(c^2*x^2 - 1), x) - (c*x - 1)*(log(-c*x + 1)^2 - 2*log(-c*x
 + 1) + 2)/c - (c*x*log(c*x + 1)^2 + 2*(c*x - (c*x + 1)*log(c*x + 1))*log(
-c*x + 1))/c + log(c^2*x^2 - 1)/c - 2*integrate(log(c*x + 1)/(c^2*x^2 - 1)
, x))*b^2 + a^2*x + (2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a*b/c
```

3.18.8 Giac [F]

$$\int (a+b\text{arctanh}(cx))^2\,dx = \int (b\,\text{artanh}\,(cx)+a)^2\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2, x)
```

3.18.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arctanh}(cx))^2\,dx = \int (a+b\,\text{atanh}(c\,x))^2\,dx$$

input
```
int((a + b*atanh(c*x))^2,x)
```

output
```
int((a + b*atanh(c*x))^2, x)
```

3.19 $\int \frac{(a+b\text{arctanh}(cx))^2}{x} dx$

3.19.1 Optimal result

Integrand size = 14, antiderivative size = 117

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx))^2}{x} dx = & 2(a+b\text{arctanh}(cx))^2\text{arctanh}\left(1-\frac{2}{1-cx}\right) \\ & - b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right) \\ & + b(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1-cx}\right) \\ & + \frac{1}{2}b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right) \\ & - \frac{1}{2}b^2\,\text{PolyLog}\left(3,-1+\frac{2}{1-cx}\right)\end{aligned}$$

output
```
-2*(a+b*arctanh(c*x))^2*arctanh(-1+2/(-c*x+1))-b*(a+b*arctanh(c*x))*polylo
g(2,1-2/(-c*x+1))+b*(a+b*arctanh(c*x))*polylog(2,-1+2/(-c*x+1))+1/2*b^2*po
lylog(3,1-2/(-c*x+1))-1/2*b^2*polylog(3,-1+2/(-c*x+1))
```

3.19.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.29

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x} dx = a^2\log(cx)+ab(-\text{PolyLog}(2,-cx)+\text{PolyLog}(2,cx)) +b^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\text{arctanh}(cx)^3-\text{arctanh}(cx)^2\log\left(1+e^{-2\text{arctanh}(cx)}\right) +\text{arctanh}(cx)^2\log\left(1-e^{2\text{arctanh}(cx)}\right) +\text{arctanh}(cx)\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right) +\text{arctanh}(cx)\text{PolyLog}\left(2,e^{2\text{arctanh}(cx)}\right) +\frac{1}{2}\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\text{arctanh}(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/x,x]
```

output
```
a^2*Log[c*x] + a*b*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]) + b^2*((I/24)*P
i^3 - (2*ArcTanh[c*x]^3)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] +
 ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, -E^(
-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])] + PolyLog[
3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[3, E^(2*ArcTanh[c*x])]/2)
```

3.19.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.22, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6448, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x} dx$$

$$\Big\downarrow \text{6448}$$

$$2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2-4bc\int \frac{(a+b\text{arctanh}(cx))\text{arctanh}\left(1-\frac{2}{1-cx}\right)}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6614}$$

$$2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2-$$
$$4bc\left(\frac{1}{2}\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1-cx}\right)}{1-c^2x^2}dx-\frac{1}{2}\int \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx\right)$$

$\Big\downarrow$ 6620

$$2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2-$$
$$4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{2c}-\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{1-c^2x^2}dx\right)+\frac{1}{2}\left(\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,\right.}{1-c^2}\right.\right.$$

$\Big\downarrow$ 7164

$$2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2-$$
$$4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{4c}\right)+\frac{1}{2}\left(\frac{b\,\text{PolyLog}\left(3,\frac{2}{1-cx}-1\right)}{4c}-\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x])^2/x,x]
```

output
```
2*(a + b*ArcTanh[c*x])^2*ArcTanh[1 - 2/(1 - c*x)] - 4*b*c*((((a + b*ArcTan
h[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/(2*c) - (b*PolyLog[3, 1 - 2/(1 - c*x)
])/(4*c))/2 + (-1/2*((a + b*ArcTanh[c*x])*PolyLog[2, -1 + 2/(1 - c*x)])/c
+ (b*PolyLog[3, -1 + 2/(1 - c*x)])/(4*c))/2)
```

3.19.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.19.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 6.10 (sec) , antiderivative size = 630, normalized size of antiderivative = 5.38

method	result
parts	$a^2 \ln(x) + b^2 \left(\ln(cx) \operatorname{arctanh}(cx)^2 - \operatorname{arctanh}(cx) \operatorname{polylog}\left(2, -\frac{(cx+1)^2}{-c^2x^2+1}\right) + \frac{\operatorname{polylog}\left(3, -\right.}{2} \right.$
derivativedivides	$a^2 \ln(cx) + b^2 \left(\ln(cx) \operatorname{arctanh}(cx)^2 - \operatorname{arctanh}(cx) \operatorname{polylog}\left(2, -\frac{(cx+1)^2}{-c^2x^2+1}\right) + \frac{\operatorname{polylog}\left(3, -\right.}{2} \right.$
default	$a^2 \ln(cx) + b^2 \left(\ln(cx) \operatorname{arctanh}(cx)^2 - \operatorname{arctanh}(cx) \operatorname{polylog}\left(2, -\frac{(cx+1)^2}{-c^2x^2+1}\right) + \frac{\operatorname{polylog}\left(3, -\right.}{2} \right.$

input
```
int((a+b*arctanh(c*x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2*(ln(c*x)*arctanh(c*x)^2-arctanh(c*x)*polylog(2,-(c*x+1)^2/(-
c^2*x^2+1))+1/2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-arctanh(c*x)^2*ln((c*x+
1)^2/(-c^2*x^2+1)-1)+arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arc
tanh(c*x)*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,-(c*x+1)/(-c^
2*x^2+1)^(1/2))+arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(
c*x)*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,(c*x+1)/(-c^2*x^2+1
)^(1/2))+1/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-
1)))*(csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))
-csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(
c*x+1)^2/(c^2*x^2-1)))-csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2
*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))+csgn(I*(-(c*x+1)^2/(c^2*x^2-1)
-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2)*arctanh(c*x)^2)+2*a*b*(ln(c*x)*arctanh(c
*x)-1/2*dilog(c*x+1)-1/2*ln(c*x)*ln(c*x+1)-1/2*dilog(c*x))
```

3.19.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x, x)
```

3.19.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x}\,dx=\int \frac{(a+b\,\text{atanh}\,(cx))^2}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x))**2/x,x)
```

output
```
Integral((a + b*atanh(c*x))**2/x, x)
```

3.19.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + integrate(1/4*b^2*(log(c*x + 1) - log(-c*x + 1))^2/x + a*b*(l
og(c*x + 1) - log(-c*x + 1))/x, x)
```

3.19.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/x, x)
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x}\,dx = \int \frac{(a + b\,\text{atanh}(c\,x))^2}{x}\,dx$$

input
```
int((a + b*atanh(c*x))^2/x,x)
```

output
```
int((a + b*atanh(c*x))^2/x, x)
```

3.20 $\int \frac{(a+b\text{arctanh}(cx))^2}{x^2}\,dx$

3.20.1 Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2}\,dx = c(a+b\text{arctanh}(cx))^2 - \frac{(a+b\text{arctanh}(cx))^2}{x} + 2bc(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right) - b^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)$$

output
```
c*(a+b*arctanh(c*x))^2-(a+b*arctanh(c*x))^2/x+2*b*c*(a+b*arctanh(c*x))*ln(
2-2/(c*x+1))-b^2*c*polylog(2,-1+2/(c*x+1))
```

3.20.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2}\,dx = \frac{b^2(-1+cx)\text{arctanh}(cx)^2 + 2b\text{arctanh}(cx)\left(-a+bcx\log\left(1-e^{-2\text{arctanh}(cx)}\right)\right) - a(a-2bcx\log(cx)+bcx\,\text{l}}{x}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/x^2,x]
```

output
```
(b^2*(-1 + c*x)*ArcTanh[c*x]^2 + 2*b*ArcTanh[c*x]*(-a + b*c*x*Log[1 - E^(-
2*ArcTanh[c*x])]) - a*(a - 2*b*c*x*Log[c*x] + b*c*x*Log[1 - c^2*x^2]) - b^
2*c*x*PolyLog[2, E^(-2*ArcTanh[c*x])])/x
```

3.20.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6452, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^2}\,dx$$

$$\Big\downarrow \text{6452}$$

$$2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{x}$$

$$\Big\downarrow \text{6550}$$

$$2bc\left(\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}\right)-\frac{(a+b\text{arctanh}(cx))^2}{x}$$

$$\Big\downarrow \text{6494}$$

$$2bc\left(-bc\int \frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))\right)-\frac{(a+b\text{arctanh}(cx))^2}{x}$$

$$\Big\downarrow \text{2897}$$

$$2bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{x}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/x^2,x]
```

output
```
-((a + b*ArcTanh[c*x])^2/x) + 2*b*c*((a + b*ArcTanh[c*x])^2/(2*b) + (a + b
*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[2, -1 + 2/(1 + c*x)])/2)
```

3.20.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(71) = 142.

Time = 0.94 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.75

method	result
parts	$-\frac{a^2}{x}+b^2c\left(-\frac{\operatorname{arctanh}(cx)^2}{cx}+2\ln(cx)\operatorname{arctanh}(cx)-\operatorname{arctanh}(cx)\ln(cx+1)-\operatorname{arctanh}(c\right.$
derivativedivides	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{cx}+2\ln(cx)\operatorname{arctanh}(cx)-\operatorname{arctanh}(cx)\ln(cx+1)-\operatorname{arctanh}\right.\right.$
default	$c\left(-\frac{a^2}{cx}+b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{cx}+2\ln(cx)\operatorname{arctanh}(cx)-\operatorname{arctanh}(cx)\ln(cx+1)-\operatorname{arctanh}\right.\right.$

input
```
int((a+b*arctanh(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+b^2*c*(-1/c/x*arctanh(c*x)^2+2*ln(c*x)*arctanh(c*x)-arctanh(c*x)*ln
(c*x+1)-arctanh(c*x)*ln(c*x-1)+dilog(1/2*c*x+1/2)+1/2*ln(c*x-1)*ln(1/2*c*x
+1/2)-1/4*ln(c*x-1)^2-1/2*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)+1/4
*ln(c*x+1)^2-dilog(c*x+1)-ln(c*x)*ln(c*x+1)-dilog(c*x))+2*a*b*c*(-1/c/x*ar
ctanh(c*x)+ln(c*x)-1/2*ln(c*x+1)-1/2*ln(c*x-1))
```

3.20.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2}\,dx=\int \frac{(b\operatorname{artanh}(cx)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x^2, x)
```

3.20.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^2} dx = \int \frac{(a + b\,\text{atanh}\,(cx))^2}{x^2} dx$$

input
```
integrate((a+b*atanh(c*x))**2/x**2,x)
```

output
```
Integral((a + b*atanh(c*x))**2/x**2, x)
```

3.20.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^2} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{x^2} dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="maxima")
```

output
```
-(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a*b - 1/4*b^2*(log(-
c*x + 1)^2/x + integrate(-((c*x - 1)*log(c*x + 1)^2 + 2*(c*x - (c*x - 1)*l
og(c*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)) - a^2/x
```

3.20.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^2} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{x^2} dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/x^2, x)
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2}\,dx=\int \frac{{\left(a+b\,\mathrm{atanh}\left(c\,x\right)\right)}^2}{x^2}\,dx$$

input
```
int((a + b*atanh(c*x))^2/x^2,x)
```

output
```
int((a + b*atanh(c*x))^2/x^2, x)
```

3.21 $\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}\,dx$

3.21.1 Optimal result

Integrand size = 14, antiderivative size = 80

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}\,dx = -\frac{bc(a+b\text{arctanh}(cx))}{x}+\frac{1}{2}c^2(a+b\text{arctanh}(cx))^2 - \frac{(a+b\text{arctanh}(cx))^2}{2x^2}+b^2c^2\log(x)-\frac{1}{2}b^2c^2\log\left(1-c^2x^2\right)$$

output
```
-b*c*(a+b*arctanh(c*x))/x+1/2*c^2*(a+b*arctanh(c*x))^2-1/2*(a+b*arctanh(c*
x))^2/x^2+b^2*c^2*ln(x)-1/2*b^2*c^2*ln(-c^2*x^2+1)
```

3.21.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}\,dx = -\frac{a^2+2abcx+2b(a+bcx)\text{arctanh}(cx)-b^2(-1+c^2x^2)\,\text{arctanh}(cx)^2-2b^2c^2x^2\log(x)+b(a+b)c^2x^2\log}{2x^2}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/x^3,x]
```

output
```
-1/2*(a^2 + 2*a*b*c*x + 2*b*(a + b*c*x)*ArcTanh[c*x] - b^2*(-1 + c^2*x^2)*
ArcTanh[c*x]^2 - 2*b^2*c^2*x^2*Log[x] + b*(a + b)*c^2*x^2*Log[1 - c*x] - (
a - b)*b*c^2*x^2*Log[1 + c*x])/x^2
```

3.21.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {6452, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}dx$$

$$\Big\downarrow \text{6452}$$

$$bc\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6544}$$

$$bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\int \frac{a+b\text{arctanh}(cx)}{x^2}dx\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6452}$$

$$bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+bc\int \frac{1}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{243}$$

$$bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}$$

$$\Big\downarrow \text{6510}$$

$$bc\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log\left(x^2\right)-\log\left(1-c^2x^2\right)\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c*x])^2/x^2 + b*c*(-((a + b*ArcTanh[c*x])/x) + (c*(a +
 b*ArcTanh[c*x])^2)/(2*b) + (b*c*(Log[x^2] - Log[1 - c^2*x^2]))/2)
```

3.21.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.21.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.68

method	result
parallelrisch	$\frac{b^2 \operatorname{arctanh}(cx)^2 x^2 c^2+2b^2c^2\ln(x)x^2-2\ln(cx-1)x^2b^2c^2+2x^2\operatorname{arctanh}(cx)abc^2-2x^2\operatorname{arctanh}(cx)b^2c^2-c^2x^2a^2-2b^2 \mathrm{arct}}{2x^2}$
parts	$-\frac{a^2}{2x^2}+b^2c^2\left(-\frac{\operatorname{arctanh}(cx)^2}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)}{cx}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{2}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}+\frac{\ln(cx-1)\ln}{4}\right.$
derivativedivides	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)}{cx}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{2}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}+\frac{\ln(cx-}{}\right.\right.$
default	$c^2\left(-\frac{a^2}{2c^2x^2}+b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{2c^2x^2}-\frac{\operatorname{arctanh}(cx)}{cx}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{2}-\frac{\operatorname{arctanh}(cx)\ln(cx-1)}{2}+\frac{\ln(cx-}{}\right.\right.$
risch	$\frac{b^2(c^2x^2-1)\ln(cx+1)^2}{8x^2}-\frac{b(bx^2\ln(-cx+1)c^2+2bcx-b\ln(-cx+1)+2a)\ln(cx+1)}{4x^2}+\frac{b^2c^2x^2\ln(-cx+1)^2+8b^2c^2\ln(x)}{}$

input
```
int((a+b*arctanh(c*x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*arctanh(c*x)^2*x^2*c^2+2*b^2*c^2*ln(x)*x^2-2*ln(c*x-1)*x^2*b^2*c^
2+2*x^2*arctanh(c*x)*a*b*c^2-2*x^2*arctanh(c*x)*b^2*c^2-c^2*x^2*a^2-2*b^2*
arctanh(c*x)*x*c-2*a*b*c*x-b^2*arctanh(c*x)^2-2*arctanh(c*x)*a*b-a^2)/x^2
```

3.21.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.69

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}\,dx$$
$$= \frac{8\,b^2c^2x^2\log{(x)} + 4\,(ab-b^2)c^2x^2\log{(cx+1)} - 4\,(ab+b^2)c^2x^2\log{(cx-1)} - 8\,abcx + (b^2c^2x^2-b^2)\log{(}}{8\,x^2}$$

input
```
integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="fricas")
```

output
```
1/8*(8*b^2*c^2*x^2*log(x) + 4*(a*b - b^2)*c^2*x^2*log(c*x + 1) - 4*(a*b +
b^2)*c^2*x^2*log(c*x - 1) - 8*a*b*c*x + (b^2*c^2*x^2 - b^2)*log(-(c*x + 1)
/(c*x - 1))^2 - 4*a^2 - 4*(b^2*c*x + a*b)*log(-(c*x + 1)/(c*x - 1)))/x^2
```

3.21.6 Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.58

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}\,dx$$
$$= \begin{cases} -\frac{a^2}{2x^2} + abc^2\,\text{atanh}\,(cx) - \frac{abc}{x} - \frac{ab\,\text{atanh}\,(cx)}{x^2} + b^2c^2\log{(x)} - b^2c^2\log{\left(x-\frac{1}{c}\right)} + \frac{b^2c^2\,\text{atanh}^2\,(cx)}{2} - b^2c^2\,\text{atanh} \\ -\frac{a^2}{2x^2} \end{cases}$$

input
```
integrate((a+b*atanh(c*x))**2/x**3,x)
```

output
```
Piecewise((-a**2/(2*x**2) + a*b*c**2*atanh(c*x) - a*b*c/x - a*b*atanh(c*x)
/x**2 + b**2*c**2*log(x) - b**2*c**2*log(x - 1/c) + b**2*c**2*atanh(c*x)**
2/2 - b**2*c**2*atanh(c*x) - b**2*c*atanh(c*x)/x - b**2*atanh(c*x)**2/(2*x
**2), Ne(c, 0)), (-a**2/(2*x**2), True))
```

3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. 2(74) = 148.

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.89

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^3}\,dx = \frac{1}{2}\left(\left(c\log(cx+1) - c\log(cx-1) - \frac{2}{x}\right)c - \frac{2\,\text{artanh}(cx)}{x^2}\right)ab + \frac{1}{8}\left(\left(2(\log(cx-1)-2)\log(cx+1) - \log(cx+1)^2 - \log(cx-1)^2 - 4\log(cx-1) + 8\log(x)\right)c^2 + 4 \right. - \frac{b^2\,\text{artanh}(cx)^2}{2\,x^2} - \frac{a^2}{2\,x^2}$$

input
```
integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="maxima")
```

output
```
1/2*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a*b +
 1/8*((2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2
 - 4*log(c*x - 1) + 8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) - 2
/x)*c*arctanh(c*x))*b^2 - 1/2*b^2*arctanh(c*x)^2/x^2 - 1/2*a^2/x^2
```

3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. 2(74) = 148.

Time = 0.31 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.48

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^3}\,dx$$

$$= \frac{1}{2}\left(2\,b^2c\log\left(-\frac{cx+1}{cx-1} - 1\right) - 2\,b^2c\log\left(-\frac{cx+1}{cx-1}\right) + \frac{(cx+1)b^2c\log\left(-\frac{cx+1}{cx-1}\right)^2}{(cx-1)\left(\frac{(cx+1)^2}{(cx-1)^2} + \frac{2\,(cx+1)}{cx-1} + 1\right)} + \frac{2\left(\frac{2\,(cx+1)abc}{cx-1} + \right.}{(cx-}\right.$$

input
```
integrate((a+b*arctanh(c*x))^2/x^3,x, algorithm="giac")
```

output
```
1/2*(2*b^2*c*log(-(c*x + 1)/(c*x - 1) - 1) - 2*b^2*c*log(-(c*x + 1)/(c*x -
 1)) + (c*x + 1)*b^2*c*log(-(c*x + 1)/(c*x - 1))^2/((c*x - 1)*((c*x + 1)^2
/(c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1)) + 2*(2*(c*x + 1)*a*b*c/(c*x - 1
) + (c*x + 1)*b^2*c/(c*x - 1) + b^2*c)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1
)^2/(c*x - 1)^2 + 2*(c*x + 1)/(c*x - 1) + 1) + 4*((c*x + 1)*a^2*c/(c*x - 1
) + (c*x + 1)*a*b*c/(c*x - 1) + a*b*c)/((c*x + 1)^2/(c*x - 1)^2 + 2*(c*x +
 1)/(c*x - 1) + 1))*c
```

3.21.9 Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.08

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}\,dx = \frac{b^2\,c^2\ln\left(c\,x+1\right)^2}{8}-\frac{a^2}{2\,x^2}+\frac{b^2\,c^2\ln\left(1-c\,x\right)^2}{8}-\frac{b^2\ln\left(c\,x+1\right)^2}{8\,x^2}-\frac{b^2\ln\left(1-c\,x\right)^2}{8\,x^2}+b^2\,c^2\,\ln\left(x\right)-\frac{b^2\,c^2\,\ln\left(c\,x-1\right)}{2}-\frac{b^2\,c^2\,\ln\left(c\,x+1\right)}{2}-\frac{a\,b\,\ln\left(c\,x+1\right)}{2\,x^2}+\frac{a\,b\,\ln\left(1-c\,x\right)}{2\,x^2}+\frac{b^2\,\ln\left(c\,x+1\right)\,\ln\left(1-c\,x\right)}{4\,x^2}-\frac{a\,b\,c}{x}-\frac{b^2\,c\,\ln\left(c\,x+1\right)}{2\,x}+\frac{b^2\,c\,\ln\left(1-c\,x\right)}{2\,x}-\frac{a\,b\,c^2\,\ln\left(c\,x-1\right)}{2}+\frac{a\,b\,c^2\,\ln\left(c\,x+1\right)}{2}-\frac{b^2\,c^2\,\ln\left(c\,x+1\right)\,\ln\left(1-c\,x\right)}{4}$$

input
```
int((a + b*atanh(c*x))^2/x^3,x)
```

output
```
(b^2*c^2*log(c*x + 1)^2)/8 - a^2/(2*x^2) + (b^2*c^2*log(1 - c*x)^2)/8 - (b
^2*log(c*x + 1)^2)/(8*x^2) - (b^2*log(1 - c*x)^2)/(8*x^2) + b^2*c^2*log(x)
 - (b^2*c^2*log(c*x - 1))/2 - (b^2*c^2*log(c*x + 1))/2 - (a*b*log(c*x + 1)
)/(2*x^2) + (a*b*log(1 - c*x))/(2*x^2) + (b^2*log(c*x + 1)*log(1 - c*x))/(
4*x^2) - (a*b*c)/x - (b^2*c*log(c*x + 1))/(2*x) + (b^2*c*log(1 - c*x))/(2*
x) - (a*b*c^2*log(c*x - 1))/2 + (a*b*c^2*log(c*x + 1))/2 - (b^2*c^2*log(c*
x + 1)*log(1 - c*x))/4
```

3.22 $\int \frac{(a+b\,\text{arctanh}(cx))^2}{x^4}\,dx$

3.22.1 Optimal result

Integrand size = 14, antiderivative size = 130

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx))^2}{x^4}\,dx &= -\frac{b^2c^2}{3x}+\frac{1}{3}b^2c^3\text{arctanh}(cx)-\frac{bc(a+b\text{arctanh}(cx))}{3x^2}\\ &\quad+\frac{1}{3}c^3(a+b\text{arctanh}(cx))^2-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}\\ &\quad+\frac{2}{3}bc^3(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)\\ &\quad-\frac{1}{3}b^2c^3\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)\end{aligned}$$

output
```
-1/3*b^2*c^2/x+1/3*b^2*c^3*arctanh(c*x)-1/3*b*c*(a+b*arctanh(c*x))/x^2+1/3
*c^3*(a+b*arctanh(c*x))^2-1/3*(a+b*arctanh(c*x))^2/x^3+2/3*b*c^3*(a+b*arct
anh(c*x))*ln(2-2/(c*x+1))-1/3*b^2*c^3*polylog(2,-1+2/(c*x+1))
```

3.22.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^4}\,dx = -\frac{a^2+abcx+b^2c^2x^2+b^2(1-c^3x^3)\,\text{arctanh}(cx)^2+b\text{arctanh}(cx)\left(2a+bcx-bc^3x^3-2bc^3x^3\log\left(1-e^{-}\right.\right.}{3x^3}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/x^4,x]
```

output
```
-1/3*(a^2 + a*b*c*x + b^2*c^2*x^2 + b^2*(1 - c^3*x^3)*ArcTanh[c*x]^2 + b*A
rcTanh[c*x]*(2*a + b*c*x - b*c^3*x^3 - 2*b*c^3*x^3*Log[1 - E^(-2*ArcTanh[c
*x])]) - 2*a*b*c^3*x^3*Log[c*x] + a*b*c^3*x^3*Log[1 - c^2*x^2] + b^2*c^3*x
^3*PolyLog[2, E^(-2*ArcTanh[c*x])])/x^3
```

3.22.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6452, 6544, 6452, 264, 219, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^4}dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{2}{3}bc\int \frac{a+b\text{arctanh}(cx)}{x^3(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 6544}$$

$$\frac{2}{3}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\int \frac{a+b\text{arctanh}(cx)}{x^3}dx\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{2}{3}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2}{3}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2}{3}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 6550

$$\frac{2}{3}bc\left(c^2\left(\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 6494

$$\frac{2}{3}bc\left(c^2\left(-bc\int \frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))\right)-\frac{a+b\text{arcta}}{2x^2}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^2}{3x^3}$$

$\Big\downarrow$ 2897

$$\frac{2}{3}bc\left(c^2\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{a+b\text{arc}}{2}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^2}{3x^3}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/x^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c*x])^2/x^3 + (2*b*c*(-1/2*(a + b*ArcTanh[c*x])/x^2 +
(b*c*(-x^(-1) + c*ArcTanh[c*x]))/2 + c^2*((a + b*ArcTanh[c*x])^2/(2*b) + (
a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[2, -1 + 2/(1 + c*x)]
)/2)))/3
```

3.22.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.22. $\int \frac{(a+b\text{arctanh}(cx))^2}{x^4}\,dx$

3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(116) = 232$.

Time = 1.02 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.90

method	result
parts	$-\frac{a^2}{3x^3} + b^2c^3\Big(-\frac{\operatorname{arctanh}(cx)^2}{3c^3x^3} - \frac{\operatorname{arctanh}(cx)}{3c^2x^2} + \frac{2\ln(cx)\operatorname{arctanh}(cx)}{3} - \frac{\operatorname{arctanh}(cx)\ln(cx+1)}{3} - \frac{\operatorname{arctanh}(cx)}{3}$
derivativedivides	$c^3\Big(-\frac{a^2}{3c^3x^3} + b^2\Big(-\frac{\operatorname{arctanh}(cx)^2}{3c^3x^3} - \frac{\operatorname{arctanh}(cx)}{3c^2x^2} + \frac{2\ln(cx)\operatorname{arctanh}(cx)}{3} - \frac{\operatorname{arctanh}(cx)\ln(cx+1)}{3} - \operatorname{arctanh}$
default	$c^3\Big(-\frac{a^2}{3c^3x^3} + b^2\Big(-\frac{\operatorname{arctanh}(cx)^2}{3c^3x^3} - \frac{\operatorname{arctanh}(cx)}{3c^2x^2} + \frac{2\ln(cx)\operatorname{arctanh}(cx)}{3} - \frac{\operatorname{arctanh}(cx)\ln(cx+1)}{3} - \operatorname{arctanh}$

input
```
int((a+b*arctanh(c*x))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+b^2*c^3*(-1/3/c^3/x^3*arctanh(c*x)^2-1/3/c^2/x^2*arctanh(c*x)
+2/3*ln(c*x)*arctanh(c*x)-1/3*arctanh(c*x)*ln(c*x+1)-1/3*arctanh(c*x)*ln(c
*x-1)-1/3/c/x+1/6*ln(c*x+1)-1/6*ln(c*x-1)+1/3*dilog(1/2*c*x+1/2)+1/6*ln(c*
x-1)*ln(1/2*c*x+1/2)-1/12*ln(c*x-1)^2-1/6*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-
1/2*c*x+1/2)+1/12*ln(c*x+1)^2-1/3*dilog(c*x+1)-1/3*ln(c*x)*ln(c*x+1)-1/3*d
ilog(c*x))+2*a*b*c^3*(-1/3/c^3/x^3*arctanh(c*x)-1/6/c^2/x^2+1/3*ln(c*x)-1/
6*ln(c*x+1)-1/6*ln(c*x-1))
```

3.22.5 Fricas [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx))^2}{x^4}\, dx = \int \frac{(b \operatorname{artanh}(cx) + a)^2}{x^4}\, dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)/x^4, x)
```

3.22.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^4} dx = \int \frac{(a + b\,\text{atanh}\,(cx))^2}{x^4} dx$$

input
```
integrate((a+b*atanh(c*x))**2/x**4,x)
```

output
```
Integral((a + b*atanh(c*x))**2/x**4, x)
```

3.22.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^4} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{x^4} dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*a*b - 1/12*b^2*(log(-c*x + 1)^2/x^3 + 3*integrate(-1/3*(3*(c*x - 1)*log(
c*x + 1)^2 + 2*(c*x - 3*(c*x - 1)*log(c*x + 1))*log(-c*x + 1))/(c*x^5 - x^
4), x)) - 1/3*a^2/x^3
```

3.22.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^4} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^2}{x^4} dx$$

input
```
integrate((a+b*arctanh(c*x))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2/x^4, x)
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^4}\,dx = \int \frac{\left(a+b\,\mathrm{atanh}(c\,x)\right)^2}{x^4}\,dx$$

input
```
int((a + b*atanh(c*x))^2/x^4,x)
```

output
```
int((a + b*atanh(c*x))^2/x^4, x)
```

3.23 $\int \frac{(a+b\text{arctanh}(cx))^2}{x^5} \, dx$

3.23.1 Optimal result

Integrand size = 14, antiderivative size = 117

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx))^2}{x^5} \, dx = & -\frac{b^2c^2}{12x^2} - \frac{bc(a+b\text{arctanh}(cx))}{6x^3} - \frac{bc^3(a+b\text{arctanh}(cx))}{2x} \\ & + \frac{1}{4}c^4(a+b\text{arctanh}(cx))^2 - \frac{(a+b\text{arctanh}(cx))^2}{4x^4} \\ & + \frac{2}{3}b^2c^4\log(x) - \frac{1}{3}b^2c^4\log\left(1-c^2x^2\right)\end{aligned}$$

output
```
-1/12*b^2*c^2/x^2-1/6*b*c*(a+b*arctanh(c*x))/x^3-1/2*b*c^3*(a+b*arctanh(c*
x))/x+1/4*c^4*(a+b*arctanh(c*x))^2-1/4*(a+b*arctanh(c*x))^2/x^4+2/3*b^2*c^
4*ln(x)-1/3*b^2*c^4*ln(-c^2*x^2+1)
```

3.23.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.40

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^5} \, dx =$$

$$-\frac{3a^2+2abcx+b^2c^2x^2+6abc^3x^3+2b(3a+bcx+3bc^3x^3)\text{arctanh}(cx)-3b^2(-1+c^4x^4)\text{arctanh}(cx)^2 - \cdots}{12\cdots}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^2/x^5,x]
```

output
```
-1/12*(3*a^2 + 2*a*b*c*x + b^2*c^2*x^2 + 6*a*b*c^3*x^3 + 2*b*(3*a + b*c*x
+ 3*b*c^3*x^3)*ArcTanh[c*x] - 3*b^2*(-1 + c^4*x^4)*ArcTanh[c*x]^2 - 8*b^2*
c^4*x^4*Log[x] + 3*a*b*c^4*x^4*Log[1 - c*x] + 4*b^2*c^4*x^4*Log[1 - c*x] -
 3*a*b*c^4*x^4*Log[1 + c*x] + 4*b^2*c^4*x^4*Log[1 + c*x])/x^4
```

3.23.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.16, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {6452, 6544, 6452, 243, 54, 2009, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^5}dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}bc\int \frac{a+b\text{arctanh}(cx)}{x^4(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{2}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx+\int \frac{a+b\text{arctanh}(cx)}{x^4}dx\right)-\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{3}bc\int \frac{1}{x^3(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{3x^3}\right)-\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{6}bc\int \frac{1}{x^4(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{3x^3}\right)-\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{2}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx+\frac{1}{6}bc\int \left(-\frac{c^4}{c^2x^2-1}+\frac{c^2}{x^2}+\frac{1}{x^4}\right)dx^2-\frac{a+b\text{arctanh}(cx)}{3x^3}\right)-\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\downarrow 2009$$

$$\frac{1}{2}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x^2(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-c^2x^2\right)-\frac{1}{x^2}\right)\right)-\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\downarrow 6544$$

$$\frac{1}{2}bc\left(c^2\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\int \frac{a+b\text{arctanh}(cx)}{x^2}dx\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\log\left(x^2\right)-c^2\log\left(1-\right.\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\downarrow 6452$$

$$\frac{1}{2}bc\left(c^2\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+bc\int \frac{1}{x(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{6}bc\left(c^2\log\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\downarrow 243$$

$$\frac{1}{2}bc\left(c^2\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{1}{x^2(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{6}bc\left(c\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\downarrow 47$$

$$\frac{1}{2}bc\left(c^2\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arctanh}(}{3x^3}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\downarrow 14$$

$$\frac{1}{2}bc\left(c^2\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\log\left(x^2\right)\right)-\frac{a+b\text{arctanh}(cx)}{x}\right)-\frac{a+b\text{arctanh}(c}{3x^3}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\downarrow 16$$

3.23. $\int \frac{(a+b\text{arctanh}(cx))^2}{x^5}\,dx$

$$\frac{1}{2}bc\left(c^2\left(c^2\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\log(1-c^2x^2))\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}+\right.$$
$$\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{2}bc\left(c^2\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc(\log(x^2)-\log(1-c^2x^2))\right)-\frac{a+b\text{arctanh}(cx)}{3x^3}+\frac{1}{6}bc\right.$$
$$\frac{(a+b\text{arctanh}(cx))^2}{4x^4}$$

input
```
Int[(a + b*ArcTanh[c*x])^2/x^5,x]
```

output
```
-1/4*(a + b*ArcTanh[c*x])^2/x^4 + (b*c*(-1/3*(a + b*ArcTanh[c*x])/x^3 + c^
2*(-((a + b*ArcTanh[c*x])/x) + (c*(a + b*ArcTanh[c*x])^2)/(2*b) + (b*c*(Lo
g[x^2] - Log[1 - c^2*x^2]))/2) + (b*c*(-x^(-2) + c^2*Log[x^2] - c^2*Log[1
- c^2*x^2]))/6))/2
```

3.23.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.23.4 Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.46

method	result
parallelrisch	$\frac{3b^2 \operatorname{arctanh}(cx)^2 x^4 c^4+8b^2c^4 \ln(x) x^4-8\ln(cx-1)x^4b^2c^4+6ab \operatorname{arctanh}(cx)x^4c^4-8 \operatorname{arctanh}(cx)x^4b^2c^4-b^2c^4x^4-6b^2 \operatorname{arc}}{12x^4}$
parts	$-\frac{a^2}{4x^4}+b^2c^4\left(-\frac{\operatorname{arctanh}(cx)^2}{4c^4x^4}-\frac{\operatorname{arctanh}(cx)}{6c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2cx}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{4}-\frac{\operatorname{arctanh}(cx)\ln(cx-1}{4}\right.$
derivativedivides	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{4c^4x^4}-\frac{\operatorname{arctanh}(cx)}{6c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2cx}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{4}-\frac{\operatorname{arctanh}(cx)\ln(c}{4}\right.\right.$
default	$c^4\left(-\frac{a^2}{4c^4x^4}+b^2\left(-\frac{\operatorname{arctanh}(cx)^2}{4c^4x^4}-\frac{\operatorname{arctanh}(cx)}{6c^3x^3}-\frac{\operatorname{arctanh}(cx)}{2cx}+\frac{\operatorname{arctanh}(cx)\ln(cx+1)}{4}-\frac{\operatorname{arctanh}(cx)\ln(c}{4}\right.\right.$
risch	$\frac{b^2(c^4x^4-1)\ln(cx+1)^2}{16x^4}-\frac{b(3b\,x^4\ln(-cx+1)c^4+6b\,c^3x^3+2bcx-3b\ln(-cx+1)+6a)\ln(cx+1)}{24x^4}+\frac{3b^2c^4x^4\ln(-cx+1)^2}{}$

input
```
int((a+b*arctanh(c*x))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/12*(3*b^2*arctanh(c*x)^2*x^4*c^4+8*b^2*c^4*ln(x)*x^4-8*ln(c*x-1)*x^4*b^2
*c^4+6*a*b*arctanh(c*x)*x^4*c^4-8*arctanh(c*x)*x^4*b^2*c^4-b^2*c^4*x^4-6*b
^2*arctanh(c*x)*x^3*c^3-6*a*b*c^3*x^3-b^2*c^2*x^2-2*b^2*arctanh(c*x)*x*c-2
*a*b*c*x-3*b^2*arctanh(c*x)^2-6*arctanh(c*x)*a*b-3*a^2)/x^4
```

3.23.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.48

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^5}\,dx$$
$$= \frac{32\,b^2c^4x^4\log(x) + 4\,(3\,ab - 4\,b^2)c^4x^4\log(cx+1) - 4\,(3\,ab + 4\,b^2)c^4x^4\log(cx-1) - 24\,abc^3x^3 - 4\,b^2c^2x^2}{48\,x^4}$$

input
```
integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="fricas")
```

output
```
1/48*(32*b^2*c^4*x^4*log(x) + 4*(3*a*b - 4*b^2)*c^4*x^4*log(c*x + 1) - 4*(
3*a*b + 4*b^2)*c^4*x^4*log(c*x - 1) - 24*a*b*c^3*x^3 - 4*b^2*c^2*x^2 - 8*a
*b*c*x + 3*(b^2*c^4*x^4 - b^2)*log(-(c*x + 1)/(c*x - 1))^2 - 12*a^2 - 4*(3
*b^2*c^3*x^3 + b^2*c*x + 3*a*b)*log(-(c*x + 1)/(c*x - 1)))/x^4
```

3.23.6 Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.57

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^5}\,dx$$
$$= \begin{cases} -\frac{a^2}{4x^4} + \frac{abc^4\operatorname{atanh}(cx)}{2} - \frac{abc^3}{2x} - \frac{abc}{6x^3} - \frac{ab\operatorname{atanh}(cx)}{2x^4} + \frac{2b^2c^4\log(x)}{3} - \frac{2b^2c^4\log\left(x-\frac{1}{c}\right)}{3} + \frac{b^2c^4\operatorname{atanh}^2(cx)}{4} - \frac{2b^2c^4\operatorname{atanh}(cx)}{3} \\ -\frac{a^2}{4x^4} \end{cases}$$

input
```
integrate((a+b*atanh(c*x))**2/x**5,x)
```

output
```
Piecewise((-a**2/(4*x**4) + a*b*c**4*atanh(c*x)/2 - a*b*c**3/(2*x) - a*b*c
/(6*x**3) - a*b*atanh(c*x)/(2*x**4) + 2*b**2*c**4*log(x)/3 - 2*b**2*c**4*l
og(x - 1/c)/3 + b**2*c**4*atanh(c*x)**2/4 - 2*b**2*c**4*atanh(c*x)/3 - b**
2*c**3*atanh(c*x)/(2*x) - b**2*c**2/(12*x**2) - b**2*c*atanh(c*x)/(6*x**3)
 - b**2*atanh(c*x)**2/(4*x**4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(103) = 206$.

Time = 0.21 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.91

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^5}\,dx$$
$$= \frac{1}{12}\left(\left(3\,c^3\log(cx+1) - 3\,c^3\log(cx-1) - \frac{2\,(3\,c^2x^2+1)}{x^3}\right)c - \frac{6\,\text{artanh}\,(cx)}{x^4}\right)ab$$
$$+ \frac{1}{48}\left(\left(32\,c^2\log(x) - \frac{3\,c^2x^2\log(cx+1)^2 + 3\,c^2x^2\log(cx-1)^2 + 16\,c^2x^2\log(cx-1) - 2\,(3\,c^2x^2\log(ca}{x^2}\right.\right.$$
$$- \frac{b^2\,\text{artanh}\,(cx)^2}{4\,x^4} - \frac{a^2}{4\,x^4}$$

input
```
integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="maxima")
```

output
```
1/12*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c
- 6*arctanh(c*x)/x^4)*a*b + 1/48*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1)
^2 + 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*log
(c*x - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1)
- 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*b^2 - 1/4*b^
2*arctanh(c*x)^2/x^4 - 1/4*a^2/x^4
```

3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 612 vs. $2(103) = 206$.

Time = 0.29 (sec) , antiderivative size = 612, normalized size of antiderivative = 5.23

$$\int \frac{(a + b\text{arctanh}(cx))^2}{x^5}\,dx$$
$$= \frac{1}{6}\left(4\,b^2c^3\log\left(-\frac{cx+1}{cx-1} - 1\right) - 4\,b^2c^3\log\left(-\frac{cx+1}{cx-1}\right) + \frac{3\left(\frac{(cx+1)^3b^2c^3}{(cx-1)^3} + \frac{(cx+1)b^2c^3}{cx-1}\right)\log\left(-\frac{cx+1}{cx-1}\right)^2}{\frac{(cx+1)^4}{(cx-1)^4} + \frac{4\,(cx+1)^3}{(cx-1)^3} + \frac{6\,(cx+1)^2}{(cx-1)^2} + \frac{4\,(cx+1)}{cx-1} + 1} + \frac{2\,(}{}\right.$$

input
```
integrate((a+b*arctanh(c*x))^2/x^5,x, algorithm="giac")
```

output
```
1/6*(4*b^2*c^3*log(-(c*x + 1)/(c*x - 1) - 1) - 4*b^2*c^3*log(-(c*x + 1)/(c
*x - 1)) + 3*((c*x + 1)^3*b^2*c^3/(c*x - 1)^3 + (c*x + 1)*b^2*c^3/(c*x - 1
))*log(-(c*x + 1)/(c*x - 1))^2/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c
*x - 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1) + 2*(6*
(c*x + 1)^3*a*b*c^3/(c*x - 1)^3 + 6*(c*x + 1)*a*b*c^3/(c*x - 1) + 3*(c*x +
 1)^3*b^2*c^3/(c*x - 1)^3 + 6*(c*x + 1)^2*b^2*c^3/(c*x - 1)^2 + 5*(c*x + 1
)*b^2*c^3/(c*x - 1) + 2*b^2*c^3)*log(-(c*x + 1)/(c*x - 1))/((c*x + 1)^4/(c
*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c*x + 1)^2/(c*x - 1)^2 + 4*(c*x
 + 1)/(c*x - 1) + 1) + 2*(6*(c*x + 1)^3*a^2*c^3/(c*x - 1)^3 + 6*(c*x + 1)*
a^2*c^3/(c*x - 1) + 6*(c*x + 1)^3*a*b*c^3/(c*x - 1)^3 + 12*(c*x + 1)^2*a*b
*c^3/(c*x - 1)^2 + 10*(c*x + 1)*a*b*c^3/(c*x - 1) + 4*a*b*c^3 + (c*x + 1)^
3*b^2*c^3/(c*x - 1)^3 + 2*(c*x + 1)^2*b^2*c^3/(c*x - 1)^2 + (c*x + 1)*b^2*
c^3/(c*x - 1))/((c*x + 1)^4/(c*x - 1)^4 + 4*(c*x + 1)^3/(c*x - 1)^3 + 6*(c
*x + 1)^2/(c*x - 1)^2 + 4*(c*x + 1)/(c*x - 1) + 1))*c
```

3.23.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.59

$$\int \frac{(a+b\text{arctanh}(cx))^2}{x^5}\,dx = \frac{b^2\,c^4\ln{(c\,x+1)}^2}{16} - \frac{a^2}{4\,x^4} + \frac{b^2\,c^4\ln{(1-c\,x)}^2}{16} - \frac{b^2\ln{(c\,x+1)}^2}{16\,x^4} - \frac{b^2\ln{(1-c\,x)}^2}{16\,x^4} - \frac{b^2\,c^2}{12\,x^2} + \frac{2\,b^2\,c^4\,\ln{(x)}}{3} - \frac{b^2\,c^4\,\ln{(c\,x-1)}}{3} - \frac{b^2\,c^4\,\ln{(c\,x+1)}}{3} - \frac{a\,b\,\ln{(c\,x+1)}}{4\,x^4} + \frac{a\,b\,\ln{(1-c\,x)}}{4\,x^4} + \frac{b^2\,\ln{(c\,x+1)}\,\ln{(1-c\,x)}}{8\,x^4} - \frac{a\,b\,c}{6\,x^3} - \frac{b^2\,c\,\ln{(c\,x+1)}}{12\,x^3} + \frac{b^2\,c\,\ln{(1-c\,x)}}{12\,x^3} - \frac{a\,b\,c^3}{2\,x} - \frac{b^2\,c^3\,\ln{(c\,x+1)}}{4\,x} + \frac{b^2\,c^3\,\ln{(1-c\,x)}}{4\,x} - \frac{a\,b\,c^4\,\ln{(c\,x-1)}}{4} + \frac{a\,b\,c^4\,\ln{(c\,x+1)}}{4} - \frac{b^2\,c^4\,\ln{(c\,x+1)}\,\ln{(1-c\,x)}}{8}$$

input
```
int((a + b*atanh(c*x))^2/x^5,x)
```

output
```
(b^2*c^4*log(c*x + 1)^2)/16 - a^2/(4*x^4) + (b^2*c^4*log(1 - c*x)^2)/16 -
(b^2*log(c*x + 1)^2)/(16*x^4) - (b^2*log(1 - c*x)^2)/(16*x^4) - (b^2*c^2)/
(12*x^2) + (2*b^2*c^4*log(x))/3 - (b^2*c^4*log(c*x - 1))/3 - (b^2*c^4*log(
c*x + 1))/3 - (a*b*log(c*x + 1))/(4*x^4) + (a*b*log(1 - c*x))/(4*x^4) + (b
^2*log(c*x + 1)*log(1 - c*x))/(8*x^4) - (a*b*c)/(6*x^3) - (b^2*c*log(c*x +
 1))/(12*x^3) + (b^2*c*log(1 - c*x))/(12*x^3) - (a*b*c^3)/(2*x) - (b^2*c^3
*log(c*x + 1))/(4*x) + (b^2*c^3*log(1 - c*x))/(4*x) - (a*b*c^4*log(c*x - 1
))/4 + (a*b*c^4*log(c*x + 1))/4 - (b^2*c^4*log(c*x + 1)*log(1 - c*x))/8
```

3.23. $\int \frac{(a+b\text{arctanh}(cx))^2}{x^5}\,dx$

3.24 $\int x^5(a+b\text{arctanh}(cx))^3\,dx$

3.24.1 Optimal result

Integrand size = 14, antiderivative size = 247

$$\int x^5(a+b\text{arctanh}(cx))^3\,dx=\frac{19b^3x}{60c^5}+\frac{b^3x^3}{60c^3}-\frac{19b^3\text{arctanh}(cx)}{60c^6}+\frac{4b^2x^2(a+b\text{arctanh}(cx))}{15c^4}+\frac{b^2x^4(a+b\text{arctanh}(cx))}{20c^2}+\frac{23b(a+b\text{arctanh}(cx))^2}{30c^6}+\frac{bx(a+b\text{arctanh}(cx))^2}{2c^5}+\frac{bx^3(a+b\text{arctanh}(cx))^2}{6c^3}+\frac{bx^5(a+b\text{arctanh}(cx))^2}{10c}-\frac{(a+b\text{arctanh}(cx))^3}{6c^6}+\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-\frac{23b^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{15c^6}-\frac{23b^3\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{30c^6}$$

output
```
19/60*b^3*x/c^5+1/60*b^3*x^3/c^3-19/60*b^3*arctanh(c*x)/c^6+4/15*b^2*x^2*(
a+b*arctanh(c*x))/c^4+1/20*b^2*x^4*(a+b*arctanh(c*x))/c^2+23/30*b*(a+b*arc
tanh(c*x))^2/c^6+1/2*b*x*(a+b*arctanh(c*x))^2/c^5+1/6*b*x^3*(a+b*arctanh(c
*x))^2/c^3+1/10*b*x^5*(a+b*arctanh(c*x))^2/c-1/6*(a+b*arctanh(c*x))^3/c^6+
1/6*x^6*(a+b*arctanh(c*x))^3-23/15*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c
^6-23/30*b^3*polylog(2,1-2/(-c*x+1))/c^6
```

3.24.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.23

$$\int x^5(a+b\text{arctanh}(cx))^3\,dx$$
$$=\frac{-19ab^2+30a^2bcx+19b^3cx+16ab^2c^2x^2+10a^2bc^3x^3+b^3c^3x^3+3ab^2c^4x^4+6a^2bc^5x^5+10a^3c^6x^6+2b^2(b}{}$$

input
```
Integrate[x^5*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(-19*a*b^2 + 30*a^2*b*c*x + 19*b^3*c*x + 16*a*b^2*c^2*x^2 + 10*a^2*b*c^3*x
^3 + b^3*c^3*x^3 + 3*a*b^2*c^4*x^4 + 6*a^2*b*c^5*x^5 + 10*a^3*c^6*x^6 + 2*
b^2*(b*(-23 + 15*c*x + 5*c^3*x^3 + 3*c^5*x^5) + 15*a*(-1 + c^6*x^6))*ArcTa
nh[c*x]^2 + 10*b^3*(-1 + c^6*x^6)*ArcTanh[c*x]^3 + b*ArcTanh[c*x]*(30*a^2*
c^6*x^6 + 4*a*b*c*x*(15 + 5*c^2*x^2 + 3*c^4*x^4) + b^2*(-19 + 16*c^2*x^2 +
 3*c^4*x^4) - 92*b^2*Log[1 + E^(-2*ArcTanh[c*x])]) + 15*a^2*b*Log[1 - c*x]
 - 15*a^2*b*Log[1 + c*x] + 46*a*b^2*Log[1 - c^2*x^2] + 46*b^3*PolyLog[2, -
E^(-2*ArcTanh[c*x])])/(60*c^6)
```

3.24.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 499 vs. 2(247) = 494.

Time = 3.45 (sec) , antiderivative size = 499, normalized size of antiderivative = 2.02, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.214, Rules used = {6452, 6542, 6452, 6542, 6452, 254, 2009, 6542, 6436, 6452, 262, 219, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5(a+b\text{arctanh}(cx))^3\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-\frac{1}{2}bc\int \frac{x^6(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-\frac{1}{2}bc\left(\frac{\int\frac{x^4(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\int x^4(a+b\text{arctanh}(cx))^2dx}{c^2}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-\frac{1}{2}bc\left(\frac{\int\frac{x^4(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\int\frac{x^5(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}\right)$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-\frac{1}{2}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\int x^2(a+b\text{arctanh}(cx))^2dx}{c^2}}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\int\frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\int\right.}{c^2}\right.$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-\frac{1}{2}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\int\right.}{}\right.$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-\frac{1}{2}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\int\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

3.24. $\int x^5(a+b\text{arctanh}(cx))^3\,dx$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-$$

$$\frac{1}{2}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{5}x^5(a+b\text{arctanh}(cx))^2-\frac{2}{5}bc\left(\frac{\int\right.}{}}{}\right.$$

$$\downarrow \text{6542}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-$$

$$\frac{1}{2}bc\left(\frac{\frac{\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\int(a+b\text{arctanh}(cx))^2dx}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\int x(a+b\text{arctanh}(cx))}{c^2}\right.}{c^2}}{c^2}\right.$$

$$\downarrow \text{6436}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-$$

$$\frac{1}{2}bc\left(\frac{\frac{\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}}{c^2}\right.}{c^2}}{c^2}\right.$$

$$\downarrow \text{6452}$$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-$$

$$\frac{1}{2}bc\left(\frac{\frac{\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}}{c^2}\right.}{c^2}}{c^2}\right.$$

$$\downarrow \text{262}$$

3.24. $\int x^5(a+b\text{arctanh}(cx))^3\,dx$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3 - \frac{1}{2}bc\left(\frac{\frac{\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2} - \frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2} - \frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}}{c^2}\right.}{c}}{c^2}\right.$$

$\downarrow$ 219

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3 - \frac{1}{2}bc\left(\frac{\frac{\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2} - \frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2} - \frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}}{c^2}\right.}{}}{c^2}\right.$$

$\downarrow$ 6510

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3 - \frac{1}{2}bc\left(\frac{\frac{\frac{(a+b\text{arctanh}(cx))^3}{3bc^3} - \frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2} - \frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2} - \right.}{c^2}}{c^2}\right.$$

$\downarrow$ 6546

3.24. $\int x^5(a+b\text{arctanh}(cx))^3\,dx$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-$$
$$\frac{1}{2}bc\left(\frac{\frac{\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\int\cdots\right.}{c^2}\right.$$

$\downarrow$ 6470

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-$$
$$\frac{1}{2}bc\left(\frac{\frac{\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{a}\cdots}{\cdots}\right.$$

$\downarrow$ 2849

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3-$$
$$\frac{1}{2}bc\left(\frac{\frac{\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-\frac{2}{1-cx}}d\frac{1}{1-cx}}{c}+\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a\cdots}{\cdots}\right.$$

$\downarrow$ 2752

3.24. $\int x^5(a+b\text{arctanh}(cx))^3\,dx$

$$\frac{1}{6}x^6(a+b\text{arctanh}(cx))^3 - \frac{1}{2}bc\left(\frac{\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c^2}}{c^2} - \frac{\frac{1}{3}x^3(a \cdots}{\cdots}\right.$$

input
```
Int[x^5*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(x^6*(a + b*ArcTanh[c*x])^3)/6 - (b*c*(-(((x^5*(a + b*ArcTanh[c*x])^2)/5 -
 (2*b*c*(-(((x^4*(a + b*ArcTanh[c*x]))/4 - (b*c*(-(x/c^4) - x^3/(3*c^2) +
ArcTanh[c*x]/c^5))/4)/c^2) + (-(((x^2*(a + b*ArcTanh[c*x]))/2 - (b*c*(-(x/
c^2) + ArcTanh[c*x]/c^3))/2)/c^2) + (-1/2*(a + b*ArcTanh[c*x])^2/(b*c^2) +
 (((a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b*PolyLog[2, 1 - 2/(1 - c*x
)])/(2*c))/c)/c^2)/c^2))/5)/c^2) + (-(((x^3*(a + b*ArcTanh[c*x])^2)/3 - (2
*b*c*(-(((x^2*(a + b*ArcTanh[c*x]))/2 - (b*c*(-(x/c^2) + ArcTanh[c*x]/c^3)
)/2)/c^2) + (-1/2*(a + b*ArcTanh[c*x])^2/(b*c^2) + (((a + b*ArcTanh[c*x])*
Log[2/(1 - c*x)])/c + (b*PolyLog[2, 1 - 2/(1 - c*x)])/(2*c))/c)/c^2))/3)/c
^2) + ((a + b*ArcTanh[c*x])^3/(3*b*c^3) - (x*(a + b*ArcTanh[c*x])^2 - 2*b*
c*(-1/2*(a + b*ArcTanh[c*x])^2/(b*c^2) + (((a + b*ArcTanh[c*x])*Log[2/(1 -
 c*x)])/c + (b*PolyLog[2, 1 - 2/(1 - c*x)])/(2*c))/c))/c^2)/c^2)/c^2))/2
```

3.24.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

3.24. $\int x^5(a+b\text{arctanh}(cx))^3\,dx$

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. $2(221) = 442$.

Time = 7.90 (sec) , antiderivative size = 803, normalized size of antiderivative = 3.25

method	result
risch	$-\frac{19b^3\ln(-cx-1)}{120c^6}+\frac{a^3x^6}{6}+\frac{19b^3\ln(-cx+1)}{120c^6}+\frac{19b^3x}{60c^5}+\frac{b^3x^3}{60c^3}-\frac{23b^3\ln(-cx+1)^2}{120c^6}-\frac{b^3\ln(-cx+1)^3x^6}{48}+\frac{b^3}{}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^5*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
-19/120/c^6*b^3*ln(-c*x-1)+1/6*a^3*x^6+19/120/c^6*b^3*ln(-c*x+1)+19/60*b^3
*x/c^5+1/60*b^3*x^3/c^3-23/120/c^6*b^3*ln(-c*x+1)^2-1/48*b^3*ln(-c*x+1)^3*
x^6+1/48/c^6*b^3*ln(-c*x+1)^3-1/4*b/c^6*ln(-c*x-1)*a^2+23/30*b^3/c^6*ln(-1
/2*c*x+1/2)*ln(1/2*c*x+1/2)-23/30*b^3/c^6*ln(1/2*c*x+1/2)*ln(-c*x+1)+1/48*
b^3*(c^6*x^6-1)/c^6*ln(c*x+1)^3+1/240*b^2*(-15*b*x^6*ln(-c*x+1)*c^6+30*a*c
^6*x^6+6*b*c^5*x^5+10*b*c^3*x^3+30*b*c*x+15*b*ln(-c*x+1)-30*a+46*b)/c^6*ln
(c*x+1)^2-1/2/c^5*a*b^2*ln(-c*x+1)*x+1/4/c^6*a^2*b*ln(-c*x+1)+1/40/c*b^3*l
n(-c*x+1)^2*x^5+1/24/c^3*b^3*ln(-c*x+1)^2*x^3-1/8/c^6*a*b^2*ln(-c*x+1)^2+2
3/30/c^6*a*b^2*ln(-c*x+1)+1/8/c^5*b^3*ln(-c*x+1)^2*x-1/40/c^2*b^3*ln(-c*x+
1)*x^4-2/15/c^4*b^3*ln(-c*x+1)*x^2-1/4*a^2*b*ln(-c*x+1)*x^6+1/8*a*b^2*ln(-
c*x+1)^2*x^6+23/30*b^3/c^6*dilog(-1/2*c*x+1/2)-19/60/c^6*a*b^2-23/30/c^6*a
^2*b+23/30*b^2/c^6*ln(-c*x-1)*a+1/20/c^2*a*b^2*x^4+4/15/c^4*a*b^2*x^2+1/10
/c*a^2*b*x^5+1/6/c^3*a^2*b*x^3+1/2/c^5*a^2*b*x-1/6/c^6*a^3-1/3*b^3/c^6+(1/
16*b^3*(c^6*x^6-1)/c^6*ln(-c*x+1)^2-1/60*b^2*x*(15*a*c^5*x^5+3*b*c^4*x^4+5
*b*c^2*x^2+15*b)/c^5*ln(-c*x+1)+1/120*b*(30*a^2*c^6*x^6+12*a*b*c^5*x^5+3*b
^2*c^4*x^4+20*a*b*c^3*x^3+16*b^2*c^2*x^2+60*a*b*c*x+30*b*ln(-c*x+1)*a+46*b
^2*ln(-c*x+1))/c^6)*ln(c*x+1)-1/10/c*a*b^2*ln(-c*x+1)*x^5-1/6/c^3*a*b^2*ln
(-c*x+1)*x^3
```

3.24.5 Fricas [F]

$$\int x^5(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^5*arctanh(c*x)^3 + 3*a*b^2*x^5*arctanh(c*x)^2 + 3*a^2*b*x^5
*arctanh(c*x) + a^3*x^5, x)
```

3.24.6 Sympy [F]

$$\int x^5(a+b\text{arctanh}(cx))^3\,dx=\int x^5(a+b\,\text{atanh}\,(cx))^3\,dx$$

input
```
integrate(x**5*(a+b*atanh(c*x))**3,x)
```

output
```
Integral(x**5*(a + b*atanh(c*x))**3, x)
```

3.24.7 Maxima [F]

$$\int x^5(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

output
```
1/2*a*b^2*x^6*arctanh(c*x)^2 + 1/6*a^3*x^6 + 1/60*(30*x^6*arctanh(c*x) + c
*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*log(c*x + 1)/c^7 + 15*log(c*x
- 1)/c^7))*a^2*b + 1/120*(4*c*(2*(3*c^4*x^5 + 5*c^2*x^3 + 15*x)/c^6 - 15*l
og(c*x + 1)/c^7 + 15*log(c*x - 1)/c^7)*arctanh(c*x) + (6*c^4*x^4 + 32*c^2*
x^2 - 2*(15*log(c*x - 1) - 46)*log(c*x + 1) + 15*log(c*x + 1)^2 + 15*log(c
*x - 1)^2 + 92*log(c*x - 1))/c^6)*a*b^2 - 1/1728000*(500*c^7*((2*c^4*x^6 +
 3*c^2*x^4 + 6*x^2)/c^11 + 6*log(c^2*x^2 - 1)/c^13) + 728*c^6*(2*(3*c^4*x^
5 + 5*c^2*x^3 + 15*x)/c^11 - 15*log(c*x + 1)/c^12 + 15*log(c*x - 1)/c^12)
+ 1485*c^5*((c^2*x^4 + 2*x^2)/c^9 + 2*log(c^2*x^2 - 1)/c^11) - 622080000*c
^5*integrate(1/3600*x^5*log(c*x + 1)/(c^7*x^2 - c^5), x) + 9750*c^4*(2*(c^
2*x^3 + 3*x)/c^9 - 3*log(c*x + 1)/c^10 + 3*log(c*x - 1)/c^10) - 2700*c^3*(
x^2/c^7 + log(c^2*x^2 - 1)/c^9) - 1036800000*c^3*integrate(1/3600*x^3*log(
c*x + 1)/(c^7*x^2 - c^5), x) + 227700*c^2*(2*x/c^7 - log(c*x + 1)/c^8 + lo
g(c*x - 1)/c^8) - 5495040000*c*integrate(1/3600*x*log(c*x + 1)/(c^7*x^2 -
c^5), x) + (1000*(36*log(-c*x + 1)^3 - 18*log(-c*x + 1)^2 + 6*log(-c*x + 1
) - 1)*(c*x - 1)^6 + 1728*(125*log(-c*x + 1)^3 - 75*log(-c*x + 1)^2 + 30*l
og(-c*x + 1) - 6)*(c*x - 1)^5 + 16875*(32*log(-c*x + 1)^3 - 24*log(-c*x +
1)^2 + 12*log(-c*x + 1) - 3)*(c*x - 1)^4 + 80000*(9*log(-c*x + 1)^3 - 9*lo
g(-c*x + 1)^2 + 6*log(-c*x + 1) - 2)*(c*x - 1)^3 + 135000*(4*log(-c*x + 1)
^3 - 6*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 3)*(c*x - 1)^2 + 216000*(log...
```

3.24.8 Giac [F]

$$\int x^5(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3*x^5, x)
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int x^5(a+b\text{arctanh}(cx))^3\,dx=\int x^5\,(a+b\,\text{atanh}(c\,x))^3\,dx$$

input
```
int(x^5*(a + b*atanh(c*x))^3,x)
```

output
```
int(x^5*(a + b*atanh(c*x))^3, x)
```

3.25 $\int x^4(a+b\text{arctanh}(cx))^3\,dx$

3.25.1 Optimal result

Integrand size = 14, antiderivative size = 262

$$
\begin{aligned}
\int x^4(a+b\text{arctanh}(cx))^3\,dx &= \frac{9ab^2x}{10c^4}+\frac{b^3x^2}{20c^3}+\frac{9b^3x\text{arctanh}(cx)}{10c^4} \\
&\quad+\frac{b^2x^3(a+b\text{arctanh}(cx))}{10c^2}-\frac{9b(a+b\text{arctanh}(cx))^2}{20c^5} \\
&\quad+\frac{3bx^2(a+b\text{arctanh}(cx))^2}{10c^3}+\frac{3bx^4(a+b\text{arctanh}(cx))^2}{20c} \\
&\quad+\frac{(a+b\text{arctanh}(cx))^3}{5c^5}+\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3 \\
&\quad-\frac{3b(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1-cx}\right)}{5c^5}+\frac{b^3\log(1-c^2x^2)}{2c^5} \\
&\quad-\frac{3b^2(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{5c^5} \\
&\quad+\frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{10c^5}
\end{aligned}
$$

output
```
9/10*a*b^2*x/c^4+1/20*b^3*x^2/c^3+9/10*b^3*x*arctanh(c*x)/c^4+1/10*b^2*x^3
*(a+b*arctanh(c*x))/c^2-9/20*b*(a+b*arctanh(c*x))^2/c^5+3/10*b*x^2*(a+b*ar
ctanh(c*x))^2/c^3+3/20*b*x^4*(a+b*arctanh(c*x))^2/c+1/5*(a+b*arctanh(c*x))
^3/c^5+1/5*x^5*(a+b*arctanh(c*x))^3-3/5*b*(a+b*arctanh(c*x))^2*ln(2/(-c*x+
1))/c^5+1/2*b^3*ln(-c^2*x^2+1)/c^5-3/5*b^2*(a+b*arctanh(c*x))*polylog(2,1-
2/(-c*x+1))/c^5+3/10*b^3*polylog(3,1-2/(-c*x+1))/c^5
```

3.25.2 Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.46

$$\int x^4(a+b\text{arctanh}(cx))^3\,dx$$
$$=\frac{-b^3+18ab^2cx+6a^2bc^2x^2+b^3c^2x^2+2ab^2c^3x^3+3a^2bc^4x^4+4a^3c^5x^5-18ab^2\text{arctanh}(cx)+18b^3cx\text{arctan}}{}$$

input
```
Integrate[x^4*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(-b^3 + 18*a*b^2*c*x + 6*a^2*b*c^2*x^2 + b^3*c^2*x^2 + 2*a*b^2*c^3*x^3 + 3
*a^2*b*c^4*x^4 + 4*a^3*c^5*x^5 - 18*a*b^2*ArcTanh[c*x] + 18*b^3*c*x*ArcTan
h[c*x] + 12*a*b^2*c^2*x^2*ArcTanh[c*x] + 2*b^3*c^3*x^3*ArcTanh[c*x] + 6*a*
b^2*c^4*x^4*ArcTanh[c*x] + 12*a^2*b*c^5*x^5*ArcTanh[c*x] - 12*a*b^2*ArcTan
h[c*x]^2 - 9*b^3*ArcTanh[c*x]^2 + 6*b^3*c^2*x^2*ArcTanh[c*x]^2 + 3*b^3*c^4
*x^4*ArcTanh[c*x]^2 + 12*a*b^2*c^5*x^5*ArcTanh[c*x]^2 - 4*b^3*ArcTanh[c*x]
^3 + 4*b^3*c^5*x^5*ArcTanh[c*x]^3 - 24*a*b^2*ArcTanh[c*x]*Log[1 + E^(-2*Ar
cTanh[c*x])] - 12*b^3*ArcTanh[c*x]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 6*a^2*
b*Log[1 - c^2*x^2] + 10*b^3*Log[1 - c^2*x^2] + 12*b^2*(a + b*ArcTanh[c*x])
*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 6*b^3*PolyLog[3, -E^(-2*ArcTanh[c*x])]
)/(20*c^5)
```

3.25.3 Rubi [A] (verified)

Time = 2.82 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.39, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.071, Rules used = {6452, 6542, 6452, 6542, 6452, 243, 49, 2009, 6542, 2009, 6510, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4(a+b\text{arctanh}(cx))^3\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-\frac{3}{5}bc\int\frac{x^5(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-\frac{3}{5}bc\left(\frac{\int\frac{x^3(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\int x^3(a+b\text{arctanh}(cx))^2dx}{c^2}\right)$$

$\big\downarrow$ 6452

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{5}bc\left(\frac{\int\frac{x^3(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\int\frac{x^4(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}\right)$$

$\big\downarrow$ 6542

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{5}bc\left(\frac{\frac{\int\frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\int x(a+b\text{arctanh}(cx))^2dx}{c^2}}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\frac{\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\int x\right.}{c^2}\right.$$

$\big\downarrow$ 6452

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{5}bc\left(\frac{\frac{\int\frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\int\frac{x^2}{}\right.}{}\right.$$

$\big\downarrow$ 243

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{5}bc\left(\frac{\frac{\int\frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2-\frac{1}{2}bc\left(\int\frac{x^2}{}\right.}{}\right.$$

$\big\downarrow$ 49

3.25. $\int x^4(a+b\text{arctanh}(cx))^3\,dx$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3 - \frac{3}{5}bc\left(\frac{\frac{\int \frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2} - \frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2 - bc\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2} - \frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2 - \frac{1}{2}bc\left(\frac{\int \frac{x^2}{}}{}\right.}{}\right.$$

$\downarrow$ 2009

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3 - \frac{3}{5}bc\left(\frac{\frac{\int \frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2} - \frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2 - bc\int \frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2} - \frac{\frac{1}{4}x^4(a+b\text{arctanh}(cx))^2 - \frac{1}{2}bc\left(\frac{\int \frac{x^2}{}}{}\right.}{}\right.$$

$\downarrow$ 6542

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3 - \frac{3}{5}bc\left(\frac{\frac{\int \frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2} - \frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2 - bc\left(\frac{\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2} - \frac{\int (a+b\text{arctanh}(cx))dx}{c^2}\right)}{c^2}}{c^2} - \frac{\frac{1}{4}x^4(a+b\text{arctanl}}{}\right.$$

$\downarrow$ 2009

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3 - \frac{3}{5}bc\left(\frac{\frac{\int \frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2} - \frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2 - bc\left(\frac{\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2} - \frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c^2}\right)}{c^2}}{c^2} - \frac{\frac{1}{4}x^4(a\ -}{}\right.$$

$\downarrow$ 6510

3.25. $\int x^4(a+b\text{arctanh}(cx))^3\,dx$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{5}bc\left(\frac{\frac{\int\frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c^2}\right)}{c^2}}{c^2}-\frac{\frac{1}{4}x^4(a+\cdots}{}\right.$$

$\downarrow$ 6546

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{5}bc\left(\frac{\frac{\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^3}{3bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2\cdots\right.}{2c}}{c^2}\right.}{c^2}}{c^2}\cdots\right.$$

$\downarrow$ 6470

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{5}bc\left(\frac{\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{c}-2b\int\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(cx))^3}{3bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arct}\cdots}{2b\cdots}\right.}{\cdots}}{c^2}\cdots\right.$$

$\downarrow$ 6620

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{5}bc\left(\frac{\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{c}-2b\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{1-c^2x^2}dx-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{2c}\right)}{c}-\frac{(a+b\text{arctanh}(cx))^3}{3bc^2}}{c^2}-\frac{\frac{1}{2}x^2(\cdots}{\cdots}}{c^2}\cdots\right.$$

$\downarrow$ 7164

3.25. $\int x^4(a+b\text{arctanh}(cx))^3\,dx$

$$\frac{1}{5}x^5(a+b\text{arctanh}(cx))^3-$$

$$\frac{3}{5}bc\left(\frac{\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{c}-2b\left(\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{4c}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{2c}\right)}{c}-\frac{(a+b\text{arctanh}(cx))^3}{3bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{ar}}{}}{c^2}\right.$$

input
```
Int[x^4*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(x^5*(a + b*ArcTanh[c*x])^3)/5 - (3*b*c*(-(((x^4*(a + b*ArcTanh[c*x])^2)/4
 - (b*c*(-(((x^3*(a + b*ArcTanh[c*x]))/3 - (b*c*(-(x^2/c^2) - Log[1 - c^2*
x^2]/c^4))/6)/c^2) + ((a + b*ArcTanh[c*x])^2/(2*b*c^3) - (a*x + b*x*ArcTan
h[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/c^2)/c^2))/2)/c^2) + (-(((x^2*(a + b*
ArcTanh[c*x])^2)/2 - b*c*((a + b*ArcTanh[c*x])^2/(2*b*c^3) - (a*x + b*x*Ar
cTanh[c*x] + (b*Log[1 - c^2*x^2])/(2*c))/c^2))/c^2) + (-1/3*(a + b*ArcTanh
[c*x])^3/(b*c^2) + (((a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c - 2*b*(-1/
2*((a + b*ArcTanh[c*x])*PolyLog[2, 1 - 2/(1 - c*x)])/c + (b*PolyLog[3, 1 -
 2/(1 - c*x)])/(4*c)))/c)/c^2)/c^2))/5
```

3.25.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.25. $\int x^4(a+b\operatorname{arctanh}(cx))^3\,dx$

3.25.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.38 (sec) , antiderivative size = 1038, normalized size of antiderivative = 3.96

method	result	size
derivativedivides	Expression too large to display	1038
default	Expression too large to display	1038
parts	Expression too large to display	1040

input
```
int(x^4*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^5*(1/5*c^5*x^5*a^3+b^3*(9/10*c*x*arctanh(c*x)-3/10*I*Pi*arctanh(c*x)^2
+1/10*c^3*x^3*arctanh(c*x)+1/20*c^2*x^2+3/20*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2
*x^2-1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c
*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)^2-1/20+3/10*c^2*x^2*arctanh(c*x)^2+3/10
*arctanh(c*x)^2*ln(c*x-1)+3/10*arctanh(c*x)^2*ln(c*x+1)-3/5*arctanh(c*x)^2
*ln(2)-3/5*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-9/20*arctanh(c*x)
^2+3/20*c^4*x^4*arctanh(c*x)^2-ln(1+(c*x+1)^2/(-c^2*x^2+1))+arctanh(c*x)+1
/5*arctanh(c*x)^3+3/10*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))+1/5*c^5*x^5*arct
anh(c*x)^3-3/5*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))-3/20*I*Pi*c
sgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2
/(c^2*x^2-1)))^2*arctanh(c*x)^2+3/20*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*cs
gn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-3/2
0*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*
arctanh(c*x)^2-3/10*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)
^2/(c^2*x^2-1))^2*arctanh(c*x)^2+3/10*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)
))^2*arctanh(c*x)^2-3/10*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(
c*x)^2-3/20*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-3/20*I*Pi*
csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)^2)+
3*a*b^2*(1/5*c^5*x^5*arctanh(c*x)^2+1/10*c^4*x^4*arctanh(c*x)+1/5*c^2*x^2*
arctanh(c*x)+1/5*arctanh(c*x)*ln(c*x-1)+1/5*arctanh(c*x)*ln(c*x+1)+1/30...
```

3.25.5 Fricas [F]

$$\int x^4(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x^4\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^4*arctanh(c*x)^3 + 3*a*b^2*x^4*arctanh(c*x)^2 + 3*a^2*b*x^4
*arctanh(c*x) + a^3*x^4, x)
```

3.25.6 Sympy [F]

$$\int x^4(a+b\text{arctanh}(cx))^3\,dx=\int x^4(a+b\,\text{atanh}\,(cx))^3\,\,dx$$

input
```
integrate(x**4*(a+b*atanh(c*x))**3,x)
```

output
```
Integral(x**4*(a + b*atanh(c*x))**3, x)
```

3.25.7 Maxima [F]

$$\int x^4(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x^4\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

output
```
1/5*a^3*x^5 + 3/20*(4*x^5*arctanh(c*x) + c*((c^2*x^4 + 2*x^2)/c^4 + 2*log(
c^2*x^2 - 1)/c^6))*a^2*b - 1/80*(2*(b^3*c^5*x^5 - b^3)*log(-c*x + 1)^3 - 3
*(4*a*b^2*c^5*x^5 + b^3*c^4*x^4 + 2*b^3*c^2*x^2 + 2*(b^3*c^5*x^5 + b^3)*lo
g(c*x + 1))*log(-c*x + 1)^2)/c^5 - integrate(-1/40*(5*(b^3*c^5*x^5 - b^3*c
^4*x^4)*log(c*x + 1)^3 + 30*(a*b^2*c^5*x^5 - a*b^2*c^4*x^4)*log(c*x + 1)^2
 - 3*(4*a*b^2*c^5*x^5 + b^3*c^4*x^4 + 2*b^3*c^2*x^2 + 5*(b^3*c^5*x^5 - b^3
*c^4*x^4)*log(c*x + 1)^2 - 2*(10*a*b^2*c^4*x^4 - (10*a*b^2*c^5 + b^3*c^5)*
x^5 - b^3)*log(c*x + 1))*log(-c*x + 1))/(c^5*x - c^4), x)
```

3.25.8 Giac [F]

$$\int x^4(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x^4\,dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3*x^4, x)
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int x^4(a+b\text{arctanh}(cx))^3\,dx=\int x^4\,(a+b\,\text{atanh}(c\,x))^3\,dx$$

input
```
int(x^4*(a + b*atanh(c*x))^3,x)
```

output
```
int(x^4*(a + b*atanh(c*x))^3, x)
```

3.26 $\int x^3(a+b\text{arctanh}(cx))^3\,dx$

3.26.1 Optimal result

Integrand size = 14, antiderivative size = 185

$$\begin{aligned}\int x^3(a+b\text{arctanh}(cx))^3\,dx &= \frac{b^3x}{4c^3}-\frac{b^3\text{arctanh}(cx)}{4c^4}+\frac{b^2x^2(a+b\text{arctanh}(cx))}{4c^2}\\&+\frac{b(a+b\text{arctanh}(cx))^2}{c^4}+\frac{3bx(a+b\text{arctanh}(cx))^2}{4c^3}\\&+\frac{bx^3(a+b\text{arctanh}(cx))^2}{4c}-\frac{(a+b\text{arctanh}(cx))^3}{4c^4}\\&+\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-\frac{2b^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c^4}\\&-\frac{b^3\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^4}\end{aligned}$$

output
```
1/4*b^3*x/c^3-1/4*b^3*arctanh(c*x)/c^4+1/4*b^2*x^2*(a+b*arctanh(c*x))/c^2+
b*(a+b*arctanh(c*x))^2/c^4+3/4*b*x*(a+b*arctanh(c*x))^2/c^3+1/4*b*x^3*(a+b
*arctanh(c*x))^2/c-1/4*(a+b*arctanh(c*x))^3/c^4+1/4*x^4*(a+b*arctanh(c*x))
^3-2*b^2*(a+b*arctanh(c*x))*ln(2/(-c*x+1))/c^4-b^3*polylog(2,1-2/(-c*x+1))
/c^4
```

3.26.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.32

$$\int x^3(a+b\text{arctanh}(cx))^3\,dx$$
$$= \frac{-2ab^2+6a^2bcx+2b^3cx+2ab^2c^2x^2+2a^2bc^3x^3+2a^3c^4x^4+2b^2(b(-4+3cx+c^3x^3)+3a(-1+c^4x^4))\,\text{ar}}{}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(-2*a*b^2 + 6*a^2*b*c*x + 2*b^3*c*x + 2*a*b^2*c^2*x^2 + 2*a^2*b*c^3*x^3 +
2*a^3*c^4*x^4 + 2*b^2*(b*(-4 + 3*c*x + c^3*x^3) + 3*a*(-1 + c^4*x^4))*ArcT
anh[c*x]^2 + 2*b^3*(-1 + c^4*x^4)*ArcTanh[c*x]^3 + 2*b*ArcTanh[c*x]*(3*a^2
*c^4*x^4 + b^2*(-1 + c^2*x^2) + 2*a*b*c*x*(3 + c^2*x^2) - 8*b^2*Log[1 + E^
(-2*ArcTanh[c*x])]) + 3*a^2*b*Log[1 - c*x] - 3*a^2*b*Log[1 + c*x] + 8*a*b^
2*Log[1 - c^2*x^2] + 8*b^3*PolyLog[2, -E^(-2*ArcTanh[c*x])])/(8*c^4)
```

3.26.3 Rubi [A] (verified)

Time = 2.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.57, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.929, Rules used = {6452, 6542, 6452, 6542, 6436, 6452, 262, 219, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\text{arctanh}(cx))^3\,dx$$
$$\Big\downarrow \text{6452}$$
$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-\frac{3}{4}bc\int \frac{x^4(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx$$
$$\Big\downarrow \text{6542}$$
$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-\frac{3}{4}bc\left(\frac{\int \frac{x^2(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\int x^2(a+b\text{arctanh}(cx))^2dx}{c^2}\right)$$
$$\Big\downarrow \text{6452}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-\frac{3}{4}bc\left(\frac{\int\frac{x^2(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\int\frac{x^3(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}\right)$$

$\downarrow$ 6542

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-\frac{3}{4}bc\left(\frac{\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\int(a+b\text{arctanh}(cx))^2dx}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}-\frac{\int x(a+\cdots}{\cdots}\right.}{c^2}\right.$$

$\downarrow$ 6436

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-\frac{3}{4}bc\left(\frac{\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{a}\cdots}{}}{}\right.}{c^2}\right.$$

$\downarrow$ 6452

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-\frac{3}{4}bc\left(\frac{\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{a}\cdots}{}}{}\right.}{}\right.$$

$\downarrow$ 262

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-\frac{3}{4}bc\left(\frac{\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{a}\cdots}{}}{}\right.}{}\right.$$

$\downarrow$ 219

3.26. $\int x^3(a+b\text{arctanh}(cx))^3\,dx$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{4}bc\left(\frac{\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\cdots}{}}{}\cdots\right.}{}\cdots\right.$$

$\downarrow$ 6510

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{4}bc\left(\frac{\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int\frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2-\frac{2}{3}bc\left(\frac{\int\frac{x(a+b\text{arct}\cdots}{1-c\cdots}}{c\cdots}\cdots\right.}{}\cdots\right.$$

$\downarrow$ 6546

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{4}bc\left(\frac{\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c^2}}{c^2}-\frac{\frac{1}{3}x^3(a+b\text{arctanh}(cx))^2\cdots}{}\cdots\right.$$

$\downarrow$ 6470

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-$$
$$\frac{3}{4}bc\left(\frac{\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c^2}}{c^2}-\frac{\frac{1}{3}x^3\cdots}{}\cdots\right.$$

$\downarrow$ 2849

3.26. $\int x^3(a+b\text{arctanh}(cx))^3\,dx$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-\frac{3}{4}bc\left(\frac{\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-\frac{2}{1-cx}}d\frac{1}{1-cx}}{c}+\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c^2}}{c^2}-\right.$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx))^3-\frac{3}{4}bc\left(\frac{\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c^2}}{c^2}-\right.$$

input
```
Int[x^3*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(x^4*(a + b*ArcTanh[c*x])^3)/4 - (3*b*c*(-(((x^3*(a + b*ArcTanh[c*x])^2)/3
 - (2*b*c*(-(((x^2*(a + b*ArcTanh[c*x]))/2 - (b*c*(-(x/c^2) + ArcTanh[c*x]
/c^3))/2)/c^2) + (-1/2*(a + b*ArcTanh[c*x])^2/(b*c^2) + (((a + b*ArcTanh[c
*x])*Log[2/(1 - c*x)])/c + (b*PolyLog[2, 1 - 2/(1 - c*x)])/(2*c))/c)/c^2))
/3)/c^2) + ((a + b*ArcTanh[c*x])^3/(3*b*c^3) - (x*(a + b*ArcTanh[c*x])^2 -
 2*b*c*(-1/2*(a + b*ArcTanh[c*x])^2/(b*c^2) + (((a + b*ArcTanh[c*x])*Log[2
/(1 - c*x)])/c + (b*PolyLog[2, 1 - 2/(1 - c*x)])/(2*c))/c))/c^2)/c^2))/4
```

3.26.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

3.26. $\int x^3(a+b\text{arctanh}(cx))^3\,dx$

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.26.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(171) = 342$.

Time = 3.64 (sec) , antiderivative size = 666, normalized size of antiderivative = 3.60

method	result
risch	$\frac{3\ln(-cx+1)^2 a\, b^2 x^4}{16} - \frac{3\ln(-cx+1)a^2 b\, x^4}{8} + \frac{a^3 x^4}{4} + \frac{b^3 \ln\left(-\frac{cx}{2}+\frac{1}{2}\right)\ln\left(\frac{cx}{2}+\frac{1}{2}\right)}{c^4} - \frac{b^3 \ln\left(\frac{cx}{2}+\frac{1}{2}\right)\ln(-cx+1)}{c^4} - 3c$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
3/16*ln(-c*x+1)^2*a*b^2*x^4-3/8*ln(-c*x+1)*a^2*b*x^4+1/4*a^3*x^4+b^3/c^4*l
n(-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-b^3/c^4*ln(1/2*c*x+1/2)*ln(-c*x+1)-3/4/c^3
*a*b^2*ln(-c*x+1)*x-3/8*b/c^4*ln(-c*x-1)*a^2+b^2/c^4*ln(-c*x-1)*a+1/4*b^3*
x/c^3+(3/32*b^3*(c^4*x^4-1)/c^4*ln(-c*x+1)^2-1/8*b^2*x*(3*a*c^3*x^3+b*c^2*
x^2+3*b)/c^3*ln(-c*x+1)+1/8*b*(3*c^4*x^4*a^2+2*a*b*c^3*x^3+b^2*c^2*x^2+6*a
*b*c*x+3*b*ln(-c*x+1)*a+4*b^2*ln(-c*x+1))/c^4)*ln(c*x+1)-1/32*ln(-c*x+1)^3
*b^3*x^4-1/4/c^4*a^3+b^3/c^4*dilog(-1/2*c*x+1/2)-1/4/c*a*b^2*ln(-c*x+1)*x^
3+1/32/c^4*b^3*ln(-c*x+1)^3-1/4/c^4*b^3*ln(-c*x+1)^2-1/c^4*a^2*b-1/4/c^4*a
*b^2-1/4*b^3/c^4-3/16/c^4*a*b^2*ln(-c*x+1)^2+1/c^4*a*b^2*ln(-c*x+1)+1/4/c*
a^2*b*x^3+3/4/c^3*a^2*b*x+1/4/c^2*a*b^2*x^2+1/16/c*b^3*ln(-c*x+1)^2*x^3+3/
16/c^3*b^3*ln(-c*x+1)^2*x-1/8/c^2*b^3*ln(-c*x+1)*x^2+3/8/c^4*a^2*b*ln(-c*x
+1)+1/8/c^4*b^3*ln(-c*x+1)+1/32*b^3*(c^4*x^4-1)/c^4*ln(c*x+1)^3+1/32*b^2*(
-3*b*x^4*ln(-c*x+1)*c^4+6*c^4*x^4*a+2*b*c^3*x^3+6*b*c*x+3*b*ln(-c*x+1)-6*a
+8*b)/c^4*ln(c*x+1)^2-1/8/c^4*b^3*ln(-c*x-1)
```

3.26.5 Fricas [F]

$$\int x^3(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arctanh(c*x)^3 + 3*a*b^2*x^3*arctanh(c*x)^2 + 3*a^2*b*x^3
*arctanh(c*x) + a^3*x^3, x)
```

3.26.6 Sympy [F]

$$\int x^3(a+b\text{arctanh}(cx))^3\,dx=\int x^3(a+b\,\text{atanh}\,(cx))^3\,dx$$

input
```
integrate(x**3*(a+b*atanh(c*x))**3,x)
```

output
```
Integral(x**3*(a + b*atanh(c*x))**3, x)
```

3.26.7 Maxima [F]

$$\int x^3(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arctanh(c*x)^2 + 1/4*a^3*x^4 + 1/8*(6*x^4*arctanh(c*x) + c*(
2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5))*a^2*b +
1/16*(4*c*(2*(c^2*x^3 + 3*x)/c^4 - 3*log(c*x + 1)/c^5 + 3*log(c*x - 1)/c^5
)*arctanh(c*x) + (4*c^2*x^2 - 2*(3*log(c*x - 1) - 8)*log(c*x + 1) + 3*log(
c*x + 1)^2 + 3*log(c*x - 1)^2 + 16*log(c*x - 1))/c^4)*a*b^2 - 1/9216*(27*c
^5*((c^2*x^4 + 2*x^2)/c^7 + 2*log(c^2*x^2 - 1)/c^9) + 74*c^4*(2*(c^2*x^3 +
 3*x)/c^7 - 3*log(c*x + 1)/c^8 + 3*log(c*x - 1)/c^8) + 60*c^3*(x^2/c^5 + l
og(c^2*x^2 - 1)/c^7) - 221184*c^3*integrate(1/96*x^3*log(c*x + 1)/(c^5*x^2
 - c^3), x) + 1692*c^2*(2*x/c^5 - log(c*x + 1)/c^6 + log(c*x - 1)/c^6) - 1
105920*c*integrate(1/96*x*log(c*x + 1)/(c^5*x^2 - c^3), x) + (9*(32*log(-c
*x + 1)^3 - 24*log(-c*x + 1)^2 + 12*log(-c*x + 1) - 3)*(c*x - 1)^4 + 128*(
9*log(-c*x + 1)^3 - 9*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 2)*(c*x - 1)^3 +
 432*(4*log(-c*x + 1)^3 - 6*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 3)*(c*x -
1)^2 + 1152*(log(-c*x + 1)^3 - 3*log(-c*x + 1)^2 + 6*log(-c*x + 1) - 6)*(c
*x - 1))/c^4 - 12*(24*(c^4*x^4 - 1)*log(c*x + 1)^3 + 48*(c^3*x^3 + 3*c*x)*
log(c*x + 1)^2 - 6*(3*c^4*x^4 - 4*c^3*x^3 + 6*c^2*x^2 - 12*c*x - 12*(c^4*x
^4 - 1)*log(c*x + 1) + 7)*log(-c*x + 1)^2 + (9*c^4*x^4 + 28*c^3*x^3 - 18*c
^2*x^2 - 72*(c^4*x^4 - 1)*log(c*x + 1)^2 + 300*c*x - 96*(c^3*x^3 + 3*c*x +
 4)*log(c*x + 1))*log(-c*x + 1))/c^4 + 1800*log(96*c^5*x^2 - 96*c^3)/c^4 -
 442368*integrate(1/96*log(c*x + 1)/(c^5*x^2 - c^3), x))*b^3
```

3.26.8 Giac [F]

$$\int x^3(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3*x^3, x)
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int x^3(a + b\text{arctanh}(cx))^3 \, dx = \int x^3 \, (a + b \, \text{atanh}(c \, x))^3 \, dx$$

input
```
int(x^3*(a + b*atanh(c*x))^3,x)
```

output
```
int(x^3*(a + b*atanh(c*x))^3, x)
```

3.27 $\int x^2(a+b\operatorname{arctanh}(cx))^3\,dx$

3.27.1 Optimal result

Integrand size = 14, antiderivative size = 197

$$\begin{aligned}\int x^2(a+b\operatorname{arctanh}(cx))^3\,dx &= \frac{ab^2x}{c^2}+\frac{b^3x\operatorname{arctanh}(cx)}{c^2}-\frac{b(a+b\operatorname{arctanh}(cx))^2}{2c^3}\\&+\frac{bx^2(a+b\operatorname{arctanh}(cx))^2}{2c}+\frac{(a+b\operatorname{arctanh}(cx))^3}{3c^3}\\&+\frac{1}{3}x^3(a+b\operatorname{arctanh}(cx))^3\\&-\frac{b(a+b\operatorname{arctanh}(cx))^2\log\left(\frac{2}{1-cx}\right)}{c^3}+\frac{b^3\log(1-c^2x^2)}{2c^3}\\&-\frac{b^2(a+b\operatorname{arctanh}(cx))\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c^3}\\&+\frac{b^3\operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c^3}\end{aligned}$$

output
```
a*b^2*x/c^2+b^3*x*arctanh(c*x)/c^2-1/2*b*(a+b*arctanh(c*x))^2/c^3+1/2*b*x^
2*(a+b*arctanh(c*x))^2/c+1/3*(a+b*arctanh(c*x))^3/c^3+1/3*x^3*(a+b*arctanh
(c*x))^3-b*(a+b*arctanh(c*x))^2*ln(2/(-c*x+1))/c^3+1/2*b^3*ln(-c^2*x^2+1)/
c^3-b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c^3+1/2*b^3*polylog(3,1
-2/(-c*x+1))/c^3
```

3.27.2 Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.27

$$\int x^2(a+b\text{arctanh}(cx))^3\,dx$$
$$=\frac{3a^2bc^2x^2+2a^3c^3x^3+6a^2bc^3x^3\text{arctanh}(cx)+3a^2b\log\left(1-c^2x^2\right)+6ab^2\left(cx+\left(-1+c^3x^3\right)\text{arctanh}(cx)^2+\right.}{}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(3*a^2*b*c^2*x^2 + 2*a^3*c^3*x^3 + 6*a^2*b*c^3*x^3*ArcTanh[c*x] + 3*a^2*b*
Log[1 - c^2*x^2] + 6*a*b^2*(c*x + (-1 + c^3*x^3)*ArcTanh[c*x]^2 + ArcTanh[
c*x]*(-1 + c^2*x^2 - 2*Log[1 + E^(-2*ArcTanh[c*x])]) + PolyLog[2, -E^(-2*A
rcTanh[c*x])]) + b^3*(6*c*x*ArcTanh[c*x] - 3*ArcTanh[c*x]^2 + 3*c^2*x^2*Ar
cTanh[c*x]^2 - 2*ArcTanh[c*x]^3 + 2*c^3*x^3*ArcTanh[c*x]^3 - 6*ArcTanh[c*x
]^2*Log[1 + E^(-2*ArcTanh[c*x])] + 3*Log[1 - c^2*x^2] + 6*ArcTanh[c*x]*Pol
yLog[2, -E^(-2*ArcTanh[c*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(6*c^
3)
```

3.27.3 Rubi [A] (verified)

Time = 1.64 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6452, 6542, 6452, 6542, 2009, 6510, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\text{arctanh}(cx))^3\,dx$$
$$\Big\downarrow \text{6452}$$
$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^3-bc\int\frac{x^3(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx$$
$$\Big\downarrow \text{6542}$$
$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^3-bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\int x(a+b\text{arctanh}(cx))^2dx}{c^2}\right)$$
$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^3-bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\int\frac{x^2(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}\right)$$

$\downarrow$ 6542

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^3-bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{\int(a+b\text{arctanh}(cx))dx}{c^2}\right)}{c^2}\right)$$

$\downarrow$ 2009

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^3-bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx}{c^2}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c^2}\right)}{c^2}\right)$$

$\downarrow$ 6510

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^3-bc\left(\frac{\int\frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(cx)+\frac{b\log\left(1-c^2x^2\right)}{2c}}{c^2}\right)}{c^2}\right)$$

$\downarrow$ 6546

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^3-bc\left(\frac{\frac{\int\frac{(a+b\text{arctanh}(cx))^2}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^3}{3bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2bc^3}-\frac{ax+bx\text{arctanh}(c}{c^2}\right.}{c^2}\right.$$

$\downarrow$ 6470

3.27. $\int x^2(a+b\text{arctanh}(cx))^3\,dx$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^3-$$
$$bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{c}-2b\int\frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(cx))^3}{3bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a+b\text{arctanh}(cx))^2-bc\Bigg(}{}\right.$$

$\downarrow$ 6620

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^3-$$
$$bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{c}-2b\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{1-c^2x^2}dx-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{2c}\right)}{c}-\frac{(a+b\text{arctanh}(cx))^3}{3bc^2}}{c^2}-\frac{\frac{1}{2}x}{}\right.$$

$\downarrow$ 7164

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx))^3-$$
$$bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{c}-2b\left(\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{4c}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{2c}\right)}{c}-\frac{(a+b\text{arctanh}(cx))^3}{3bc^2}}{c^2}-\frac{\frac{1}{2}x^2(a-}{}\right.$$

input
```
Int[x^2*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(x^3*(a + b*ArcTanh[c*x])^3)/3 - b*c*(-(((x^2*(a + b*ArcTanh[c*x])^2)/2 -
b*c*((a + b*ArcTanh[c*x])^2/(2*b*c^3) - (a*x + b*x*ArcTanh[c*x] + (b*Log[1
 - c^2*x^2])/(2*c))/c^2))/c^2) + (-1/3*(a + b*ArcTanh[c*x])^3/(b*c^2) + ((
(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c - 2*b*(-1/2*((a + b*ArcTanh[c*x
])*PolyLog[2, 1 - 2/(1 - c*x)])/c + (b*PolyLog[3, 1 - 2/(1 - c*x)])/(4*c))
)/c)/c^2)
```

3.27. $\int x^2(a+b\text{arctanh}(cx))^3\,dx$

3.27.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.27.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.53 (sec) , antiderivative size = 951, normalized size of antiderivative = 4.83

method	result	size
derivativedivides	Expression too large to display	951
default	Expression too large to display	951
parts	Expression too large to display	953

input
```
int(x^2*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c^3*(1/3*c^3*x^3*a^3+b^3*(1/3*c^3*x^3*arctanh(c*x)^3+1/2*c^2*x^2*arctanh
(c*x)^2+1/2*arctanh(c*x)^2*ln(c*x-1)+1/2*arctanh(c*x)^2*ln(c*x+1)-arctanh(
c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)*polylog(2,-(c*x+1)^2/(-
c^2*x^2+1))+1/2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-1/12*arctanh(c*x)*(3*I*
csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh
(c*x)*Pi+6*I*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-
1))^2*arctanh(c*x)*Pi+3*I*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)*Pi-
3*I*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^
2/(c^2*x^2-1)))^2*arctanh(c*x)*Pi-3*I*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I
*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2
*x^2-1)))*arctanh(c*x)*Pi+3*I*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c
^2*x^2-1)))^3*arctanh(c*x)*Pi+3*I*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^
2/(c^2*x^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)*Pi+6*I*cs
gn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)*Pi-6*I*csgn(I/(1-(c*x+1)^2/
(c^2*x^2-1)))^2*arctanh(c*x)*Pi+6*I*arctanh(c*x)*Pi-4*arctanh(c*x)^2+12*ar
ctanh(c*x)*ln(2)+6*arctanh(c*x)-12*c*x-12)-ln(1+(c*x+1)^2/(-c^2*x^2+1)))+3
*a*b^2*(1/3*c^3*x^3*arctanh(c*x)^2+1/3*c^2*x^2*arctanh(c*x)+1/3*arctanh(c*
x)*ln(c*x-1)+1/3*arctanh(c*x)*ln(c*x+1)+1/3*c*x+1/6*ln(c*x-1)-1/6*ln(c*x+1
)-1/3*dilog(1/2*c*x+1/2)-1/6*ln(c*x-1)*ln(1/2*c*x+1/2)+1/12*ln(c*x-1)^2+1/
6*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1/12*ln(c*x+1)^2)+3*a^2*...
```

3.27.5 Fricas [F]

$$\int x^2(a + b\text{arctanh}(cx))^3\,dx = \int (b\,\text{artanh}\,(cx) + a)^3 x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arctanh(c*x)^3 + 3*a*b^2*x^2*arctanh(c*x)^2 + 3*a^2*b*x^2
*arctanh(c*x) + a^3*x^2, x)
```

3.27.6 Sympy [F]

$$\int x^2(a + b\text{arctanh}(cx))^3\,dx = \int x^2(a + b\,\text{atanh}\,(cx))^3\,dx$$

input
```
integrate(x**2*(a+b*atanh(c*x))**3,x)
```

output
```
Integral(x**2*(a + b*atanh(c*x))**3, x)
```

3.27.7 Maxima [F]

$$\int x^2(a + b\text{arctanh}(cx))^3\,dx = \int (b\,\text{artanh}\,(cx) + a)^3 x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*x^3 + 1/2*(2*x^3*arctanh(c*x) + c*(x^2/c^2 + log(c^2*x^2 - 1)/c^4)
)*a^2*b - 1/24*((b^3*c^3*x^3 - b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c^3*x^3 +
 b^3*c^2*x^2 + (b^3*c^3*x^3 + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/c^3 - in
tegrate(-1/8*((b^3*c^3*x^3 - b^3*c^2*x^2)*log(c*x + 1)^3 + 6*(a*b^2*c^3*x^
3 - a*b^2*c^2*x^2)*log(c*x + 1)^2 - (4*a*b^2*c^3*x^3 + 2*b^3*c^2*x^2 + 3*(
b^3*c^3*x^3 - b^3*c^2*x^2)*log(c*x + 1)^2 - 2*(6*a*b^2*c^2*x^2 - (6*a*b^2*
c^3 + b^3*c^3)*x^3 - b^3)*log(c*x + 1))*log(-c*x + 1))/(c^3*x - c^2), x)
```

3.27.8 Giac [F]

$$\int x^2(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3*x^2, x)
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\text{arctanh}(cx))^3\,dx=\int x^2\,\left(a+b\,\text{atanh}(c\,x)\right)^3\,dx$$

input
```
int(x^2*(a + b*atanh(c*x))^3,x)
```

output
```
int(x^2*(a + b*atanh(c*x))^3, x)
```

3.28 $\int x(a+b\text{arctanh}(cx))^3\,dx$

3.28.1 Optimal result

Integrand size = 12, antiderivative size = 123

$$\int x(a+b\text{arctanh}(cx))^3\,dx=\frac{3b(a+b\text{arctanh}(cx))^2}{2c^2}+\frac{3bx(a+b\text{arctanh}(cx))^2}{2c}-\frac{(a+b\text{arctanh}(cx))^3}{2c^2}+\frac{1}{2}x^2(a+b\text{arctanh}(cx))^3-\frac{3b^2(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{c^2}-\frac{3b^3\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c^2}$$

output
```
3/2*b*(a+b*arctanh(c*x))^2/c^2+3/2*b*x*(a+b*arctanh(c*x))^2/c-1/2*(a+b*arc
tanh(c*x))^3/c^2+1/2*x^2*(a+b*arctanh(c*x))^3-3*b^2*(a+b*arctanh(c*x))*ln(
2/(-c*x+1))/c^2-3/2*b^3*polylog(2,1-2/(-c*x+1))/c^2
```

3.28.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.31

$$\int x(a+b\text{arctanh}(cx))^3\,dx$$
$$=\frac{6b^2(-1+cx)(a+b+acx)\text{arctanh}(cx)^2+2b^3(-1+c^2x^2)\,\text{arctanh}(cx)^3+6b\text{arctanh}(cx)\,\left(acx(2b+acx)\right.}{}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x])^3,x]
```

output
```
(6*b^2*(-1 + c*x)*(a + b + a*c*x)*ArcTanh[c*x]^2 + 2*b^3*(-1 + c^2*x^2)*Ar
cTanh[c*x]^3 + 6*b*ArcTanh[c*x]*(a*c*x*(2*b + a*c*x) - 2*b^2*Log[1 + E^(-2
*ArcTanh[c*x])]) + a*(6*a*b*c*x + 2*a^2*c^2*x^2 + 3*a*b*Log[1 - c*x] - 3*a
*b*Log[1 + c*x] + 6*b^2*Log[1 - c^2*x^2]) + 6*b^3*PolyLog[2, -E^(-2*ArcTan
h[c*x])])/(4*c^2)
```

3.28.3 Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6452, 6542, 6436, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arctanh}(cx))^3\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^3-\frac{3}{2}bc\int \frac{x^2(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^3-\frac{3}{2}bc\left(\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{\int (a+b\text{arctanh}(cx))^2dx}{c^2}\right)$$

$$\Big\downarrow \text{6436}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^3-\frac{3}{2}bc\left(\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx}{c^2}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}\right)$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^3-\frac{3}{2}bc\left(\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\int \frac{x(a+b\text{arctanh}(cx))}{1-c^2x^2}dx}{c^2}\right)$$

$$\Big\downarrow \text{6546}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^3-\frac{3}{2}bc\left(\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\int\frac{a+b\text{arctanh}(cx)}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^2}{2bc^2}\right)}{c^2}\right)$$

$$\Big\downarrow \text{6470}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^3-\frac{3}{2}bc\left(\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}-b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(c}{2bc^2}\right.}{c^2}\right.$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^3-\frac{3}{2}bc\left(\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx}\right)}{1-\frac{2}{1-cx}}d\frac{1}{1-cx}}{c}+\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}}{c}-\frac{(a+b\text{arctan}}{2bc^2}\right.}{c^2}\right.$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx))^3-\frac{3}{2}bc\left(\frac{(a+b\text{arctanh}(cx))^3}{3bc^3}-\frac{x(a+b\text{arctanh}(cx))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{2c}}{c}-\frac{(a+b\text{arctan}}{2bc^2}\right.}{c^2}\right.$$

input
```
Int[x*(a + b*ArcTanh[c*x])^3,x]
```

3.28. $\int x(a+b\text{arctanh}(cx))^3\,dx$

output
```
(x^2*(a + b*ArcTanh[c*x])^3)/2 - (3*b*c*((a + b*ArcTanh[c*x])^3/(3*b*c^3)
- (x*(a + b*ArcTanh[c*x])^2 - 2*b*c*(-1/2*(a + b*ArcTanh[c*x])^2/(b*c^2) +
 (((a + b*ArcTanh[c*x])*Log[2/(1 - c*x)])/c + (b*PolyLog[2, 1 - 2/(1 - c*x
)])/(2*c))/c))/c^2))/2
```

3.28.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.28.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(113) = 226$.

Time = 15.90 (sec) , antiderivative size = 488, normalized size of antiderivative = 3.97

method	result
risch	$\frac{a^3x^2}{2}+\frac{b^3(c^2x^2-1)\ln(cx+1)^3}{16c^2}+\frac{3b^2(-b\,x^2\ln(-cx+1)c^2+2a\,c^2x^2+2bcx+b\ln(-cx+1)-2a+2b)\ln(cx+1)^2}{16c^2}-\frac{3a^2}{2c^2}$
derivativedivides	Expression too large to display
default	Expression too large to display
parts	Expression too large to display

input
```
int(x*(a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^3*x^2+1/16*b^3*(c^2*x^2-1)/c^2*ln(c*x+1)^3+3/16*b^2*(-b*x^2*ln(-c*x+
1)*c^2+2*a*c^2*x^2+2*b*c*x+b*ln(-c*x+1)-2*a+2*b)/c^2*ln(c*x+1)^2-3/2/c^2*a
^2*b+1/16/c^2*b^3*ln(-c*x+1)^3-3/8/c^2*b^3*ln(-c*x+1)^2+(3/16*b^3*(c^2*x^2
-1)/c^2*ln(-c*x+1)^2-3/4*b^2*x*(a*c*x+b)/c*ln(-c*x+1)-3/4*b*(-c^2*x^2*a^2-
2*a*b*c*x-b*ln(-c*x+1)*a-b^2*ln(-c*x+1))/c^2)*ln(c*x+1)-1/2/c^2*a^3-1/16*l
n(-c*x+1)^3*b^3*x^2-3/2*b^3/c^2*ln(-c*x+1)*ln(1/2*c*x+1/2)+3/2*b^3/c^2*ln(
-1/2*c*x+1/2)*ln(1/2*c*x+1/2)-3/4*a^2*b/c^2*ln(-c*x-1)+3/2*a*b^2/c^2*ln(-c
*x-1)+3/2/c*a^2*b*x-3/8/c^2*a*b^2*ln(-c*x+1)^2+3/2/c^2*a*b^2*ln(-c*x+1)+3/
4/c^2*a^2*b*ln(-c*x+1)+3/8/c*b^3*ln(-c*x+1)^2*x+3/2*b^3/c^2*dilog(-1/2*c*x
+1/2)+3/8*ln(-c*x+1)^2*a*b^2*x^2-3/4*ln(-c*x+1)*a^2*b*x^2-3/2/c*a*b^2*ln(-
c*x+1)*x
```

3.28.5 Fricas [F]

$$\int x(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arctanh(c*x)^3 + 3*a*b^2*x*arctanh(c*x)^2 + 3*a^2*b*x*arcta
nh(c*x) + a^3*x, x)
```

3.28.6 Sympy [F]

$$\int x(a+b\text{arctanh}(cx))^3\,dx=\int x(a+b\,\text{atanh}\,(cx))^3\,dx$$

input
```
integrate(x*(a+b*atanh(c*x))**3,x)
```

output
```
Integral(x*(a + b*atanh(c*x))**3, x)
```

3.28.7 Maxima [F]

$$\int x(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arctanh(c*x)^2 + 1/2*a^3*x^2 + 3/4*(2*x^2*arctanh(c*x) + c*(
2*x/c^2 - log(c*x + 1)/c^3 + log(c*x - 1)/c^3))*a^2*b + 3/8*(4*c*(2*x/c^2
- log(c*x + 1)/c^3 + log(c*x - 1)/c^3)*arctanh(c*x) - (2*(log(c*x - 1) - 2
)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)^2 - 4*log(c*x - 1))/c^2)*a*
b^2 - 1/64*(3*c^3*(x^2/c^3 + log(c^2*x^2 - 1)/c^5) + 21*c^2*(2*x/c^3 - log
(c*x + 1)/c^4 + log(c*x - 1)/c^4) - 576*c*integrate(1/4*x*log(c*x + 1)/(c^
3*x^2 - c), x) - 2*(12*c*x*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)^3
 - 3*(c^2*x^2 - 2*c*x - 2*(c^2*x^2 - 1)*log(c*x + 1) + 1)*log(-c*x + 1)^2
+ 3*(c^2*x^2 - 2*(c^2*x^2 - 1)*log(c*x + 1)^2 + 6*c*x - 8*(c*x + 1)*log(c*
x + 1))*log(-c*x + 1))/c^2 + ((4*log(-c*x + 1)^3 - 6*log(-c*x + 1)^2 + 6*l
og(-c*x + 1) - 3)*(c*x - 1)^2 + 8*(log(-c*x + 1)^3 - 3*log(-c*x + 1)^2 + 6
*log(-c*x + 1) - 6)*(c*x - 1))/c^2 + 18*log(4*c^3*x^2 - 4*c)/c^2 - 192*int
egrate(1/4*log(c*x + 1)/(c^3*x^2 - c), x))*b^3
```

3.28.8 Giac [F]

$$\int x(a + b\text{arctanh}(cx))^3\,dx = \int (b\,\text{artanh}\,(cx) + a)^3 x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3*x, x)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int x(a + b\text{arctanh}(cx))^3\,dx = \int x\,(a + b\,\text{atanh}(c\,x))^3\,dx$$

input
```
int(x*(a + b*atanh(c*x))^3,x)
```

output
```
int(x*(a + b*atanh(c*x))^3, x)
```

3.29 $\int (a+b\text{arctanh}(cx))^3\,dx$

3.29.1 Optimal result

Integrand size = 10, antiderivative size = 108

$$\int (a+b\text{arctanh}(cx))^3\,dx=\frac{(a+b\text{arctanh}(cx))^3}{c}+x(a+b\text{arctanh}(cx))^3-\frac{3b(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1-cx}\right)}{c}-\frac{3b^2(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{c}+\frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{2c}$$

output
```
(a+b*arctanh(c*x))^3/c+x*(a+b*arctanh(c*x))^3-3*b*(a+b*arctanh(c*x))^2*ln(
2/(-c*x+1))/c-3*b^2*(a+b*arctanh(c*x))*polylog(2,1-2/(-c*x+1))/c+3/2*b^3*p
olylog(3,1-2/(-c*x+1))/c
```

3.29.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.49

$$\int (a+b\text{arctanh}(cx))^3\,dx$$
$$=\frac{2a^3cx+6a^2bcx\text{arctanh}(cx)+3a^2b\log\left(1-c^2x^2\right)+6ab^2\left(\text{arctanh}(cx)\left((-1+cx)\text{arctanh}(cx)-2\log\left(1+\right.\right.\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^3,x]
```

output
```
(2*a^3*c*x + 6*a^2*b*c*x*ArcTanh[c*x] + 3*a^2*b*Log[1 - c^2*x^2] + 6*a*b^2
*(ArcTanh[c*x]*((-1 + c*x)*ArcTanh[c*x] - 2*Log[1 + E^(-2*ArcTanh[c*x])])
+ PolyLog[2, -E^(-2*ArcTanh[c*x])]) + b^3*(2*ArcTanh[c*x]^2*((-1 + c*x)*Ar
cTanh[c*x] - 3*Log[1 + E^(-2*ArcTanh[c*x])]) + 6*ArcTanh[c*x]*PolyLog[2, -
E^(-2*ArcTanh[c*x])] + 3*PolyLog[3, -E^(-2*ArcTanh[c*x])]))/(2*c)
```

3.29.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6436, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arctanh}(cx))^3\,dx$$

$$\Big\downarrow \text{6436}$$

$$x(a+b\text{arctanh}(cx))^3-3bc\int \frac{x(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6546}$$

$$x(a+b\text{arctanh}(cx))^3-3bc\left(\frac{\int \frac{(a+b\text{arctanh}(cx))^2}{1-cx}dx}{c}-\frac{(a+b\text{arctanh}(cx))^3}{3bc^2}\right)$$

$$\Big\downarrow \text{6470}$$

$$x(a+b\text{arctanh}(cx))^3-3bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{c}-2b\int \frac{(a+b\text{arctanh}(cx))\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx}{c}-\frac{(a+b\text{arctanh}(cx))^3}{3bc^2}\right)$$

$$\Big\downarrow \text{6620}$$

$$x(a+b\text{arctanh}(cx))^3-3bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{c}-2b\left(\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{1-c^2x^2}dx-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{2c}\right)}{c}-\frac{(a+b\text{arct}}{3b}\right.$$

$$\Big\downarrow \text{7164}$$

$$x(a+b\text{arctanh}(cx))^3-3bc\left(\frac{\frac{\log\left(\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{c}-2b\left(\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)}{4c}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{2c}\right)}{c}-\frac{(a+b\text{arctanh}(c$$

input
```
Int[(a + b*ArcTanh[c*x])^3,x]
```

output
```
x*(a + b*ArcTanh[c*x])^3 - 3*b*c*(-1/3*(a + b*ArcTanh[c*x])^3/(b*c^2) + ((
(a + b*ArcTanh[c*x])^2*Log[2/(1 - c*x)])/c - 2*b*(-1/2*((a + b*ArcTanh[c*x
])*PolyLog[2, 1 - 2/(1 - c*x)])/c + (b*PolyLog[3, 1 - 2/(1 - c*x)])/(4*c))
)/c)
```

3.29.3.1 Defintions of rubi rules used

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(106) = 212$.

Time = 1.53 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.13

method	result
derivativedivides	$\frac{a^3cx+b^3\left(\operatorname{arctanh}(cx)^3(cx-1)+2\operatorname{arctanh}(cx)^3-3\operatorname{arctanh}(cx)^2\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)-3\operatorname{arctanh}(cx)\operatorname{polylog}\left(2,-\frac{(cx+1)^2}{-c^2x^2+\cdots}\right.\right.}{\cdots}$
default	$\frac{a^3cx+b^3\left(\operatorname{arctanh}(cx)^3(cx-1)+2\operatorname{arctanh}(cx)^3-3\operatorname{arctanh}(cx)^2\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)-3\operatorname{arctanh}(cx)\operatorname{polylog}\left(2,-\frac{(cx+1)^2}{-c^2x^2+\cdots}\right.\right.}{\cdots}$
parts	$a^3x+\frac{b^3\left(\operatorname{arctanh}(cx)^3(cx-1)+2\operatorname{arctanh}(cx)^3-3\operatorname{arctanh}(cx)^2\ln\left(1+\frac{(cx+1)^2}{-c^2x^2+1}\right)-3\operatorname{arctanh}(cx)\operatorname{polylog}\left(2,-\frac{(cx+\cdots}{-c^2x\cdots}\right.\right.}{c}$

input
```
int((a+b*arctanh(c*x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/c*(a^3*c*x+b^3*(arctanh(c*x)^3*(c*x-1)+2*arctanh(c*x)^3-3*arctanh(c*x)^2
*ln(1+(c*x+1)^2/(-c^2*x^2+1))-3*arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^
2+1))+3/2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1)))+3*a*b^2*(arctanh(c*x)^2*(c*x
-1)+2*arctanh(c*x)^2-2*arctanh(c*x)*ln(1+(c*x+1)^2/(-c^2*x^2+1))-polylog(2
,-(c*x+1)^2/(-c^2*x^2+1)))+3*a^2*b*(c*x*arctanh(c*x)+1/2*ln(-c^2*x^2+1)))
```

3.29.5 Fricas [F]

$$\int (a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*x
) + a^3, x)
```

3.29.6 Sympy [F]

$$\int (a+b\text{arctanh}(cx))^3\,dx=\int (a+b\,\text{atanh}\,(cx))^3\,dx$$

input
```
integrate((a+b*atanh(c*x))**3,x)
```

output
```
Integral((a + b*atanh(c*x))**3, x)
```

3.29.7 Maxima [F]

$$\int (a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

output
```
a^3*x + 3/2*(2*c*x*arctanh(c*x) + log(-c^2*x^2 + 1))*a^2*b/c - 1/8*((b^3*c
*x - b^3)*log(-c*x + 1)^3 - 3*(2*a*b^2*c*x + (b^3*c*x + b^3)*log(c*x + 1))
*log(-c*x + 1)^2)/c - integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(
a*b^2*c*x - a*b^2)*log(c*x + 1)^2 - 3*(4*a*b^2*c*x + (b^3*c*x - b^3)*log(c
*x + 1)^2 - 2*(2*a*b^2 - b^3 - (2*a*b^2*c + b^3*c)*x)*log(c*x + 1))*log(-c
*x + 1))/(c*x - 1), x)
```

3.29.8 Giac [F]

$$\int (a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3, x)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int (a+b\text{arctanh}(cx))^3\,dx=\int (a+b\,\text{atanh}(c\,x))^3\,dx$$

input
```
int((a + b*atanh(c*x))^3,x)
```

output
```
int((a + b*atanh(c*x))^3, x)
```

3.30 $\int \frac{(a+b\text{arctanh}(cx))^3}{x} \, dx$

3.30.1 Optimal result

Integrand size = 14, antiderivative size = 184

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx))^3}{x} \, dx = {} & 2(a+b\text{arctanh}(cx))^3\text{arctanh}\left(1-\frac{2}{1-cx}\right) \\ & - \frac{3}{2}b(a+b\text{arctanh}(cx))^2 \operatorname{PolyLog}\left(2,1-\frac{2}{1-cx}\right) \\ & + \frac{3}{2}b(a+b\text{arctanh}(cx))^2 \operatorname{PolyLog}\left(2,-1+\frac{2}{1-cx}\right) \\ & + \frac{3}{2}b^2(a+b\text{arctanh}(cx)) \operatorname{PolyLog}\left(3,1-\frac{2}{1-cx}\right) \\ & - \frac{3}{2}b^2(a+b\text{arctanh}(cx)) \operatorname{PolyLog}\left(3,-1+\frac{2}{1-cx}\right) \\ & - \frac{3}{4}b^3 \operatorname{PolyLog}\left(4,1-\frac{2}{1-cx}\right) \\ & + \frac{3}{4}b^3 \operatorname{PolyLog}\left(4,-1+\frac{2}{1-cx}\right)\end{aligned}$$

output
```
-2*(a+b*arctanh(c*x))^3*arctanh(-1+2/(-c*x+1))-3/2*b*(a+b*arctanh(c*x))^2*
polylog(2,1-2/(-c*x+1))+3/2*b*(a+b*arctanh(c*x))^2*polylog(2,-1+2/(-c*x+1)
)+3/2*b^2*(a+b*arctanh(c*x))*polylog(3,1-2/(-c*x+1))-3/2*b^2*(a+b*arctanh(
c*x))*polylog(3,-1+2/(-c*x+1))-3/4*b^3*polylog(4,1-2/(-c*x+1))+3/4*b^3*pol
ylog(4,-1+2/(-c*x+1))
```

3.30.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.71

$$
\begin{aligned}
\int \frac{(a+b\text{arctanh}(cx))^3}{x}\,dx = {} & a^3\log(cx)+\frac{3}{2}a^2b(-\text{PolyLog}(2,-cx)+\text{PolyLog}(2,cx)) \\
& + 3ab^2\bigg(\frac{i\pi^3}{24}-\frac{2}{3}\text{arctanh}(cx)^3 \\
& - \text{arctanh}(cx)^2\log\left(1+e^{-2\text{arctanh}(cx)}\right) \\
& + \text{arctanh}(cx)^2\log\left(1-e^{2\text{arctanh}(cx)}\right) \\
& + \text{arctanh}(cx)\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right) \\
& + \text{arctanh}(cx)\,\text{PolyLog}\left(2,e^{2\text{arctanh}(cx)}\right) \\
& + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\text{arctanh}(cx)}\right)\bigg) \\
& + \frac{1}{64}b^3\big(\pi^4-32\text{arctanh}(cx)^4 \\
& - 64\text{arctanh}(cx)^3\log\left(1+e^{-2\text{arctanh}(cx)}\right) \\
& + 64\text{arctanh}(cx)^3\log\left(1-e^{2\text{arctanh}(cx)}\right) \\
& + 96\text{arctanh}(cx)^2\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx)}\right) \\
& + 96\text{arctanh}(cx)^2\,\text{PolyLog}\left(2,e^{2\text{arctanh}(cx)}\right) \\
& + 96\text{arctanh}(cx)\,\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx)}\right) \\
& - 96\text{arctanh}(cx)\,\text{PolyLog}\left(3,e^{2\text{arctanh}(cx)}\right) \\
& + 48\,\text{PolyLog}\left(4,-e^{-2\text{arctanh}(cx)}\right)+48\,\text{PolyLog}\left(4,e^{2\text{arctanh}(cx)}\right)\big)
\end{aligned}
$$

input
```
Integrate[(a + b*ArcTanh[c*x])^3/x,x]
```

output
```
a^3*Log[c*x] + (3*a^2*b*(-PolyLog[2, -(c*x)] + PolyLog[2, c*x]))/2 + 3*a*b
^2*((I/24)*Pi^3 - (2*ArcTanh[c*x]^3)/3 - ArcTanh[c*x]^2*Log[1 + E^(-2*ArcT
anh[c*x])] + ArcTanh[c*x]^2*Log[1 - E^(2*ArcTanh[c*x])] + ArcTanh[c*x]*Pol
yLog[2, -E^(-2*ArcTanh[c*x])] + ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x])
] + PolyLog[3, -E^(-2*ArcTanh[c*x])]/2 - PolyLog[3, E^(2*ArcTanh[c*x])]/2)
 + (b^3*(Pi^4 - 32*ArcTanh[c*x]^4 - 64*ArcTanh[c*x]^3*Log[1 + E^(-2*ArcTan
h[c*x])] + 64*ArcTanh[c*x]^3*Log[1 - E^(2*ArcTanh[c*x])] + 96*ArcTanh[c*x]
^2*PolyLog[2, -E^(-2*ArcTanh[c*x])] + 96*ArcTanh[c*x]^2*PolyLog[2, E^(2*Ar
cTanh[c*x])] + 96*ArcTanh[c*x]*PolyLog[3, -E^(-2*ArcTanh[c*x])] - 96*ArcTa
nh[c*x]*PolyLog[3, E^(2*ArcTanh[c*x])] + 48*PolyLog[4, -E^(-2*ArcTanh[c*x]
)] + 48*PolyLog[4, E^(2*ArcTanh[c*x])]))/64
```

3.30.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6448, 6614, 6620, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x}\,dx$$

$$\Big\downarrow \text{6448}$$

$$2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^3-6bc\int \frac{(a+b\text{arctanh}(cx))^2\text{arctanh}\left(1-\frac{2}{1-cx}\right)}{1-c^2x^2}dx$$

$$\Big\downarrow \text{6614}$$

$$2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^3-$$
$$6bc\left(\frac{1}{2}\int \frac{(a+b\text{arctanh}(cx))^2\log\left(2-\frac{2}{1-cx}\right)}{1-c^2x^2}dx-\frac{1}{2}\int \frac{(a+b\text{arctanh}(cx))^2\log\left(\frac{2}{1-cx}\right)}{1-c^2x^2}dx\right)$$

$$\Big\downarrow \text{6620}$$

$$2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^3-$$
$$6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{2c}-b\int \frac{(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)}{1-c^2x^2}dx\right)+\frac{1}{2}\Big(\right.$$

$$\Big\downarrow \text{6624}$$

$$2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^3-$$
$$6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{2c}-b\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{2c}-\frac{1}{2}b\int \text{Pol}\right.\right.\right.$$

$$\Big\downarrow \text{7164}$$

$$2\text{arctanh}\left(1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^3-$$
$$6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))^2}{2c}-b\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-cx}\right)(a+b\text{arctanh}(cx))}{2c}-\frac{b\,\text{PolyLog}}{}\right.\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x])^3/x,x]
```

output
```
2*(a + b*ArcTanh[c*x])^3*ArcTanh[1 - 2/(1 - c*x)] - 6*b*c*((((a + b*ArcTan
h[c*x])^2*PolyLog[2, 1 - 2/(1 - c*x)])/(2*c) - b*(((a + b*ArcTanh[c*x])*Po
lyLog[3, 1 - 2/(1 - c*x)])/(2*c) - (b*PolyLog[4, 1 - 2/(1 - c*x)])/(4*c)))
/2 + (-1/2*((a + b*ArcTanh[c*x])^2*PolyLog[2, -1 + 2/(1 - c*x)])/c + b*(((
a + b*ArcTanh[c*x])*PolyLog[3, -1 + 2/(1 - c*x)])/(2*c) - (b*PolyLog[4, -1
 + 2/(1 - c*x)])/(4*c)))/2)
```

3.30.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.30.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.52 (sec) , antiderivative size = 1304, normalized size of antiderivative = 7.09

method	result	size
parts	Expression too large to display	1304
derivativedivides	Expression too large to display	1306
default	Expression too large to display	1306

input
```
int((a+b*arctanh(c*x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(ln(c*x)*arctanh(c*x)^3-arctanh(c*x)^3*ln((c*x+1)^2/(-c^2*x^
2+1)-1)+arctanh(c*x)^3*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3*arctanh(c*x)^2*p
olylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-6*arctanh(c*x)*polylog(3,-(c*x+1)/(-
c^2*x^2+1)^(1/2))+6*polylog(4,-(c*x+1)/(-c^2*x^2+1)^(1/2))+arctanh(c*x)^3*
ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+3*arctanh(c*x)^2*polylog(2,(c*x+1)/(-c^2*
x^2+1)^(1/2))-6*arctanh(c*x)*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+6*polyl
og(4,(c*x+1)/(-c^2*x^2+1)^(1/2))+1/2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1
)/(1-(c*x+1)^2/(c^2*x^2-1)))*(csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I/(1
-(c*x+1)^2/(c^2*x^2-1)))-csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(c*x+
1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))-csgn(I*(-(c*x+1)^2/(c^2*x^2
-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))+csgn(I
*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2)*arctanh(c*x)^3-3
/2*arctanh(c*x)^2*polylog(2,-(c*x+1)^2/(-c^2*x^2+1))+3/2*arctanh(c*x)*poly
log(3,-(c*x+1)^2/(-c^2*x^2+1))-3/4*polylog(4,-(c*x+1)^2/(-c^2*x^2+1)))+3*a
*b^2*(ln(c*x)*arctanh(c*x)^2-arctanh(c*x)*polylog(2,-(c*x+1)^2/(-c^2*x^2+1
))+1/2*polylog(3,-(c*x+1)^2/(-c^2*x^2+1))-arctanh(c*x)^2*ln((c*x+1)^2/(-c^
2*x^2+1)-1)+arctanh(c*x)^2*ln(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)
*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,-(c*x+1)/(-c^2*x^2+1)^
(1/2))+arctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+2*arctanh(c*x)*poly
log(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-2*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2...
```

3.30.5 Fricas [F]

$$\int \frac{(a + b\text{arctanh}(cx))^3}{x} \, dx = \int \frac{(b \operatorname{artanh}(cx) + a)^3}{x} \, dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*
x) + a^3)/x, x)
```

3.30.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^3}{x} \, dx = \int \frac{(a + b \operatorname{atanh}(cx))^3}{x} \, dx$$

input
```
integrate((a+b*atanh(c*x))**3/x,x)
```

output
```
Integral((a + b*atanh(c*x))**3/x, x)
```

3.30.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx))^3}{x} \, dx = \int \frac{(b \operatorname{artanh}(cx) + a)^3}{x} \, dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + integrate(1/8*b^3*(log(c*x + 1) - log(-c*x + 1))^3/x + 3/4*a*
b^2*(log(c*x + 1) - log(-c*x + 1))^2/x + 3/2*a^2*b*(log(c*x + 1) - log(-c*
x + 1))/x, x)
```

3.30.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx))^3}{x}\,dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^3}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3/x, x)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx))^3}{x}\,dx = \int \frac{(a + b\,\text{atanh}(c\,x))^3}{x}\,dx$$

input
```
int((a + b*atanh(c*x))^3/x,x)
```

output
```
int((a + b*atanh(c*x))^3/x, x)
```

3.31 $\int \frac{(a+b\text{arctanh}(cx))^3}{x^2}\,dx$

3.31.1 Optimal result

Integrand size = 14, antiderivative size = 102

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx))^3}{x^2}\,dx &= c(a+b\text{arctanh}(cx))^3-\frac{(a+b\text{arctanh}(cx))^3}{x}\\&\quad+3bc(a+b\text{arctanh}(cx))^2\log\left(2-\frac{2}{1+cx}\right)\\&\quad-3b^2c(a+b\text{arctanh}(cx))\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)\\&\quad-\frac{3}{2}b^3c\,\text{PolyLog}\left(3,-1+\frac{2}{1+cx}\right)\end{aligned}$$

output
```
c*(a+b*arctanh(c*x))^3-(a+b*arctanh(c*x))^3/x+3*b*c*(a+b*arctanh(c*x))^2*l
n(2-2/(c*x+1))-3*b^2*c*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1))-3/2*b^3*
c*polylog(3,-1+2/(c*x+1))
```

3.31.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.92

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^2}\,dx = -\frac{a^3}{x}-\frac{3a^2b\text{arctanh}(cx)}{x}+3a^2bc\log(x)-\frac{3}{2}a^2bc\log\left(1-c^2x^2\right)+3ab^2c\left(\text{arctanh}(cx)\left(\text{arctanh}(cx)-\frac{\text{arctanh}(cx)}{cx}+2\log\left(1-e^{-2\text{arctanh}(cx)}\right)\right)-\text{PolyLog}\left(2,e^{-2\text{arctanh}(cx)}\right)\right)+b^3c\left(\frac{i\pi^3}{8}-\text{arctanh}(cx)^3-\frac{\text{arctanh}(cx)^3}{cx}+3\text{arctanh}(cx)^2\log\left(1-e^{2\text{arctanh}(cx)}\right)+3\text{arctanh}(cx)\,\text{PolyLog}\left(2,e^{2\text{arctanh}(cx)}\right)-\frac{3}{2}\text{PolyLog}\left(3,e^{2\text{arctanh}(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x])^3/x^2,x]
```

output
```
-(a^3/x) - (3*a^2*b*ArcTanh[c*x])/x + 3*a^2*b*c*Log[x] - (3*a^2*b*c*Log[1
- c^2*x^2])/2 + 3*a*b^2*c*(ArcTanh[c*x]*(ArcTanh[c*x] - ArcTanh[c*x]/(c*x)
 + 2*Log[1 - E^(-2*ArcTanh[c*x])]) - PolyLog[2, E^(-2*ArcTanh[c*x])]) + b^
3*c*((I/8)*Pi^3 - ArcTanh[c*x]^3 - ArcTanh[c*x]^3/(c*x) + 3*ArcTanh[c*x]^2
*Log[1 - E^(2*ArcTanh[c*x])] + 3*ArcTanh[c*x]*PolyLog[2, E^(2*ArcTanh[c*x]
)] - (3*PolyLog[3, E^(2*ArcTanh[c*x])])/2)
```

3.31.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6452, 6550, 6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^2}\,dx$$

$$\Big\downarrow \; 6452$$

$$3bc\int \frac{(a+b\text{arctanh}(cx))^2}{x\left(1-c^2x^2\right)}dx-\frac{(a+b\text{arctanh}(cx))^3}{x}$$

$$\Big\downarrow \text{6550}$$

$$3bc\left(\int \frac{(a+b\text{arctanh}(cx))^2}{x(cx+1)}dx+\frac{(a+b\text{arctanh}(cx))^3}{3b}\right)-\frac{(a+b\text{arctanh}(cx))^3}{x}$$

$$\Big\downarrow \text{6494}$$

$$3bc\left(-2bc\int \frac{(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx+\frac{(a+b\text{arctanh}(cx))^3}{3b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx)\right.$$

$$\frac{(a+b\text{arctanh}(cx))^3}{x}$$

$$\Big\downarrow \text{6618}$$

$$3bc\left(-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)(a+b\text{arctanh}(cx))}{2c}-\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{1-c^2x^2}dx\right)+\frac{(a+b\text{arctanh}(cx))^3}{3b}\right.$$

$$\frac{(a+b\text{arctanh}(cx))^3}{x}$$

$$\Big\downarrow \text{7164}$$

$$3bc\left(-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)(a+b\text{arctanh}(cx))}{2c}+\frac{b\,\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{4c}\right)+\frac{(a+b\text{arctanh}(cx))^3}{3b}+\log\right.$$

$$\frac{(a+b\text{arctanh}(cx))^3}{x}$$

input
```
Int[(a + b*ArcTanh[c*x])^3/x^2,x]
```

output
```
-((a + b*ArcTanh[c*x])^3/x) + 3*b*c*((a + b*ArcTanh[c*x])^3/(3*b) + (a + b
*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - 2*b*c*(((a + b*ArcTanh[c*x])*PolyL
og[2, -1 + 2/(1 + c*x)])/(2*c) + (b*PolyLog[3, -1 + 2/(1 + c*x)])/(4*c)))
```

3.31. $\int \frac{(a+b\text{arctanh}(cx))^3}{x^2}\,dx$

3.31.3.1 Defintions of rubi rules used

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.31.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 30.60 (sec) , antiderivative size = 1329, normalized size of antiderivative = 13.03

method	result	size
parts	Expression too large to display	1329
derivativedivides	Expression too large to display	1331
default	Expression too large to display	1331

input
```
int((a+b*arctanh(c*x))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/x*a^3+b^3*c*(-1/c/x*arctanh(c*x)^3-3/2*arctanh(c*x)^2*ln(c*x-1)+3*ln(c*
x)*arctanh(c*x)^2-3/2*arctanh(c*x)^2*ln(c*x+1)+3*arctanh(c*x)^2*ln((c*x+1)
/(-c^2*x^2+1)^(1/2))-arctanh(c*x)^3+3/4*(2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^
2-1)-1))*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-
1)/(1-(c*x+1)^2/(c^2*x^2-1)))+2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^3-I
*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^
2/(c^2*x^2-1)))^2+I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/
(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2-I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2
-1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1
)^2/(c^2*x^2-1)))-2*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1
)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2+I*Pi*csgn(I*(c*x+1)/(-c^2*
x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))-2*I*Pi*csgn(I/(1-(c*x+1)^2/(
c^2*x^2-1)))^2+2*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/
(c^2*x^2-1))^2+2*I*Pi-2*I*Pi*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I*(-(
c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2+2*I*Pi*csgn(I*(-(c*x+
1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3+I*Pi*csgn(I*(c*x+1)^2/(c^
2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3+I*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3
+4*ln(2))*arctanh(c*x)^2-3*arctanh(c*x)^2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+3*a
rctanh(c*x)^2*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+6*arctanh(c*x)*polylog(2,(c
*x+1)/(-c^2*x^2+1)^(1/2))-6*polylog(3,(c*x+1)/(-c^2*x^2+1)^(1/2))+3*arc...
```

3.31.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*
x) + a^3)/x^2, x)
```

3.31.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx))^3}{x^2} dx = \int \frac{(a + b\,\text{atanh}\,(cx))^3}{x^2} dx$$

input
```
integrate((a+b*atanh(c*x))**3/x**2,x)
```

output
```
Integral((a + b*atanh(c*x))**3/x**2, x)
```

3.31.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx))^3}{x^2} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^3}{x^2} dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="maxima")
```

output
```
-3/2*(c*(log(c^2*x^2 - 1) - log(x^2)) + 2*arctanh(c*x)/x)*a^2*b - a^3/x -
1/8*((b^3*c*x - b^3)*log(-c*x + 1)^3 + 3*(2*a*b^2 + (b^3*c*x + b^3)*log(c*
x + 1))*log(-c*x + 1)^2)/x - integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^
3 + 6*(a*b^2*c*x - a*b^2)*log(c*x + 1)^2 + 3*(4*a*b^2*c*x - (b^3*c*x - b^3
)*log(c*x + 1)^2 + 2*(b^3*c^2*x^2 + 2*a*b^2 - (2*a*b^2*c - b^3*c)*x)*log(c
*x + 1))*log(-c*x + 1))/(c*x^3 - x^2), x)
```

3.31.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx))^3}{x^2} dx = \int \frac{(b\,\text{artanh}\,(cx) + a)^3}{x^2} dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3/x^2, x)
```

3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\operatorname{arctanh}(cx))^3}{x^2} \, dx = \int \frac{\left(a + b\,\operatorname{atanh}(c\,x)\right)^3}{x^2} \, dx$$

input
```
int((a + b*atanh(c*x))^3/x^2,x)
```

output
```
int((a + b*atanh(c*x))^3/x^2, x)
```

3.32 $\int \frac{(a+b\,\mathrm{arctanh}(cx))^3}{x^3}\,dx$

3.32.1 Optimal result

Integrand size = 14, antiderivative size = 123

$$\begin{aligned}\int \frac{(a+b\,\mathrm{arctanh}(cx))^3}{x^3}\,dx &= \frac{3}{2}bc^2(a+b\,\mathrm{arctanh}(cx))^2-\frac{3bc(a+b\,\mathrm{arctanh}(cx))^2}{2x}\\&\quad+\frac{1}{2}c^2(a+b\,\mathrm{arctanh}(cx))^3-\frac{(a+b\,\mathrm{arctanh}(cx))^3}{2x^2}\\&\quad+3b^2c^2(a+b\,\mathrm{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)\\&\quad-\frac{3}{2}b^3c^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)\end{aligned}$$

output
```
3/2*b*c^2*(a+b*arctanh(c*x))^2-3/2*b*c*(a+b*arctanh(c*x))^2/x+1/2*c^2*(a+b
*arctanh(c*x))^3-1/2*(a+b*arctanh(c*x))^3/x^2+3*b^2*c^2*(a+b*arctanh(c*x))
*ln(2-2/(c*x+1))-3/2*b^3*c^2*polylog(2,-1+2/(c*x+1))
```

3.32.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.56

$$\int \frac{(a+b\,\mathrm{arctanh}(cx))^3}{x^3}\,dx$$

$$= \frac{6b^2(-1+cx)(a+acx+bcx)\mathrm{arctanh}(cx)^2+2b^3(-1+c^2x^2)\,\mathrm{arctanh}(cx)^3-6b\,\mathrm{arctanh}(cx)\left(a^2+2abcx-\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x])^3/x^3,x]
```

output
```
(6*b^2*(-1 + c*x)*(a + a*c*x + b*c*x)*ArcTanh[c*x]^2 + 2*b^3*(-1 + c^2*x^2
)*ArcTanh[c*x]^3 - 6*b*ArcTanh[c*x]*(a^2 + 2*a*b*c*x - 2*b^2*c^2*x^2*Log[1
 - E^(-2*ArcTanh[c*x])]) + a*(-2*a^2 - 6*a*b*c*x - 3*a*b*c^2*x^2*Log[1 - c
*x] + 3*a*b*c^2*x^2*Log[1 + c*x] + 12*b^2*c^2*x^2*Log[(c*x)/Sqrt[1 - c^2*x
^2]]) - 6*b^3*c^2*x^2*PolyLog[2, E^(-2*ArcTanh[c*x])])/(4*x^2)
```

3.32.3 Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6452, 6544, 6452, 6510, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^3}dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{3}{2}bc\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^3}{2x^2}$$

$$\Big\downarrow \text{6544}$$

$$\frac{3}{2}bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx+\int \frac{(a+b\text{arctanh}(cx))^2}{x^2}dx\right)-\frac{(a+b\text{arctanh}(cx))^3}{2x^2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{3}{2}bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx+2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{x}\right)-\frac{(a+b\text{arctanh}(cx))^3}{2x^2}$$

$$\Big\downarrow \text{6510}$$

$$\frac{3}{2}bc\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\frac{c(a+b\text{arctanh}(cx))^3}{3b}-\frac{(a+b\text{arctanh}(cx))^2}{x}\right)-\frac{(a+b\text{arctanh}(cx))^3}{2x^2}$$

$$\Big\downarrow \text{6550}$$

$$\frac{3}{2}bc\left(2bc\left(\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}\right)+\frac{c(a+b\text{arctanh}(cx))^3}{3b}-\frac{(a+b\text{arctanh}(cx))^2}{x}\right)-\frac{(a+b\text{arctanh}(cx))^3}{2x^2}$$

$\Big\downarrow$ 6494

$$\frac{3}{2}bc\left(2bc\left(-bc\int \frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))\right)+\frac{c(a+bar}{}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{2x^2}$$

$\Big\downarrow$ 2897

$$\frac{3}{2}bc\left(2bc\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)+\frac{c(a+}{}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{2x^2}$$

input
```
Int[(a + b*ArcTanh[c*x])^3/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c*x])^3/x^2 + (3*b*c*(-((a + b*ArcTanh[c*x])^2/x) + (c
*(a + b*ArcTanh[c*x])^3)/(3*b) + 2*b*c*((a + b*ArcTanh[c*x])^2/(2*b) + (a
+ b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[2, -1 + 2/(1 + c*x)])/
2)))/2
```

3.32.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.32.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 16.00 (sec) , antiderivative size = 4599, normalized size of antiderivative = 37.39

method	result	size
derivativedivides	Expression too large to display	4599
default	Expression too large to display	4599
parts	Expression too large to display	4601

input
```
int((a+b*arctanh(c*x))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
c^2*(-1/2*a^3/c^2/x^2+b^3*(-3/4*I*Pi*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+3/8
*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c
*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)*ln(1-(c*x+1)/(-c^2*x^2+1)^(1/2))+3/2*
dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/8*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-
1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x
)^2+3/8*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1
)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*I
*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x
+1)^2/(c^2*x^2-1)))^2*polylog(2,-(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*I*Pi*csgn
(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x
)^2+3/4*I*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-
1))^2*polylog(2,(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*arctanh(c*x)^2*ln(c*x-1)+3
/4*arctanh(c*x)^2*ln(c*x+1)-3/2*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/
2))-3/2*arctanh(c*x)^2-3/2/c/x*arctanh(c*x)^2-3/2*dilog((c*x+1)/(-c^2*x^2+
1)^(1/2))-3/4*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^3*polylog(2,(c*x+1)/(
-c^2*x^2+1)^(1/2))-3/4*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^3*polylog(2,
-(c*x+1)/(-c^2*x^2+1)^(1/2))+3/4*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^2*
dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*I*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)
))^2*dilog(1+(c*x+1)/(-c^2*x^2+1)^(1/2))+3/8*I*Pi*csgn(I*(c*x+1)^2/(c^2*x^
2-1))^3*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))-3/8*I*Pi*csgn(I*(c*x+1)^2/(c^...
```

3.32.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^3}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*
x) + a^3)/x^3, x)
```

3.32.6 Sympy [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx))^3}{x^3}\, dx = \int \frac{(a + b\operatorname{atanh}(cx))^3}{x^3}\, dx$$

input
```
integrate((a+b*atanh(c*x))**3/x**3,x)
```

output
```
Integral((a + b*atanh(c*x))**3/x**3, x)
```

3.32.7 Maxima [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx))^3}{x^3}\, dx = \int \frac{(b\operatorname{artanh}(cx) + a)^3}{x^3}\, dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="maxima")
```

output
```
3/4*((c*log(c*x + 1) - c*log(c*x - 1) - 2/x)*c - 2*arctanh(c*x)/x^2)*a^2*b
 + 3/8*((2*(log(c*x - 1) - 2)*log(c*x + 1) - log(c*x + 1)^2 - log(c*x - 1)
^2 - 4*log(c*x - 1) + 8*log(x))*c^2 + 4*(c*log(c*x + 1) - c*log(c*x - 1) -
 2/x)*c*arctanh(c*x))*a*b^2 - 1/16*b^3*(((c^2*x^2 - 1)*log(-c*x + 1)^3 + 3
*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1))*log(-c*x + 1)^2)/x^2 + 2*integrate(-
((c*x - 1)*log(c*x + 1)^3 + 3*(2*c^2*x^2 - (c*x - 1)*log(c*x + 1)^2 - (c^3
*x^3 - c*x)*log(c*x + 1))*log(-c*x + 1))/(c*x^4 - x^3), x)) - 3/2*a*b^2*ar
ctanh(c*x)^2/x^2 - 1/2*a^3/x^2
```

3.32.8 Giac [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx))^3}{x^3}\, dx = \int \frac{(b\operatorname{artanh}(cx) + a)^3}{x^3}\, dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3/x^3, x)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^3}{x^3}\,dx = \int \frac{\left(a+b\operatorname{atanh}(c\,x)\right)^3}{x^3}\,dx$$

input
```
int((a + b*atanh(c*x))^3/x^3,x)
```

output
```
int((a + b*atanh(c*x))^3/x^3, x)
```

3.33 $\int \frac{(a+b\,\mathrm{arctanh}(cx))^3}{x^4}\,dx$

3.33.1 Optimal result

Integrand size = 14, antiderivative size = 200

$$\begin{aligned}\int \frac{(a+b\,\mathrm{arctanh}(cx))^3}{x^4}\,dx =& -\frac{b^2c^2(a+b\,\mathrm{arctanh}(cx))}{x}+\frac{1}{2}bc^3(a+b\,\mathrm{arctanh}(cx))^2\\ &-\frac{bc(a+b\,\mathrm{arctanh}(cx))^2}{2x^2}+\frac{1}{3}c^3(a+b\,\mathrm{arctanh}(cx))^3\\ &-\frac{(a+b\,\mathrm{arctanh}(cx))^3}{3x^3}+b^3c^3\log(x)-\frac{1}{2}b^3c^3\log\left(1-c^2x^2\right)\\ &+bc^3(a+b\,\mathrm{arctanh}(cx))^2\log\left(2-\frac{2}{1+cx}\right)\\ &-b^2c^3(a+b\,\mathrm{arctanh}(cx))\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)\\ &-\frac{1}{2}b^3c^3\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1+cx}\right)\end{aligned}$$

output
```
-b^2*c^2*(a+b*arctanh(c*x))/x+1/2*b*c^3*(a+b*arctanh(c*x))^2-1/2*b*c*(a+b*
arctanh(c*x))^2/x^2+1/3*c^3*(a+b*arctanh(c*x))^3-1/3*(a+b*arctanh(c*x))^3/
x^3+b^3*c^3*ln(x)-1/2*b^3*c^3*ln(-c^2*x^2+1)+b*c^3*(a+b*arctanh(c*x))^2*ln
(2-2/(c*x+1))-b^2*c^3*(a+b*arctanh(c*x))*polylog(2,-1+2/(c*x+1))-1/2*b^3*c
^3*polylog(3,-1+2/(c*x+1))
```

3.33.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.74 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.52

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^4}\,dx$$
$$= -\frac{a^3}{3x^3} - \frac{a^2bc}{2x^2} - \frac{a^2b\text{arctanh}(cx)}{x^3} + a^2bc^3\log(x) - \frac{1}{2}a^2bc^3\log\left(1-c^2x^2\right)$$
$$+ \frac{ab^2\left(-c^2x^2 + (-1+c^3x^3)\,\text{arctanh}(cx)^2 + cx\text{arctanh}(cx)\left(-1+c^2x^2+2c^2x^2\log\left(1-e^{-2\text{arctanh}(cx)}\right)\right) -}{x^3}$$
$$+ b^3c^3\left(\frac{i\pi^3}{24} - \frac{\text{arctanh}(cx)}{cx} + \frac{1}{2}\text{arctanh}(cx)^2 - \frac{\text{arctanh}(cx)^2}{2c^2x^2} - \frac{1}{3}\text{arctanh}(cx)^3\right.$$
$$- \frac{\text{arctanh}(cx)^3}{3c^3x^3} + \text{arctanh}(cx)^2\log\left(1-e^{2\text{arctanh}(cx)}\right) + \log(cx) - \frac{1}{2}\log\left(1-c^2x^2\right)$$
$$\left. + \text{arctanh}(cx)\,\text{PolyLog}\left(2, e^{2\text{arctanh}(cx)}\right) - \frac{1}{2}\,\text{PolyLog}\left(3, e^{2\text{arctanh}(cx)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x])^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (a^2*b*c)/(2*x^2) - (a^2*b*ArcTanh[c*x])/x^3 + a^2*b*c^3*Lo
g[x] - (a^2*b*c^3*Log[1 - c^2*x^2])/2 + (a*b^2*(-(c^2*x^2) + (-1 + c^3*x^3
)*ArcTanh[c*x]^2 + c*x*ArcTanh[c*x]*(-1 + c^2*x^2 + 2*c^2*x^2*Log[1 - E^(-
2*ArcTanh[c*x])]) - c^3*x^3*PolyLog[2, E^(-2*ArcTanh[c*x])]))/x^3 + b^3*c^
3*((I/24)*Pi^3 - ArcTanh[c*x]/(c*x) + ArcTanh[c*x]^2/2 - ArcTanh[c*x]^2/(2
*c^2*x^2) - ArcTanh[c*x]^3/3 - ArcTanh[c*x]^3/(3*c^3*x^3) + ArcTanh[c*x]^2
*Log[1 - E^(2*ArcTanh[c*x])] + Log[c*x] - Log[1 - c^2*x^2]/2 + ArcTanh[c*x
]*PolyLog[2, E^(2*ArcTanh[c*x])] - PolyLog[3, E^(2*ArcTanh[c*x])]/2)
```

3.33.3 Rubi [A] (verified)

Time = 1.85 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6452, 6544, 6452, 6544, 6452, 243, 47, 14, 16, 6510, 6550, 6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^4}\,dx$$

$$\Big\downarrow \; 6452$$

$$bc\int \frac{(a+b\text{arctanh}(cx))^2}{x^3\,(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$$\Big\downarrow \; 6544$$

$$bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{x\,(1-c^2x^2)}dx+\int \frac{(a+b\text{arctanh}(cx))^2}{x^3}dx\right)-\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$$\Big\downarrow \; 6452$$

$$bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{x\,(1-c^2x^2)}dx+bc\int \frac{a+b\text{arctanh}(cx)}{x^2\,(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{2x^2}\right)-\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$$\Big\downarrow \; 6544$$

$$bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{x\,(1-c^2x^2)}dx+bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\int \frac{a+b\text{arctanh}(cx)}{x^2}dx\right)-\frac{(a+b\text{arctanh}(cx))}{2x^2}\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$$\Big\downarrow \; 6452$$

$$bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{x\,(1-c^2x^2)}dx+bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+bc\int \frac{1}{x\,(1-c^2x^2)}dx-\frac{a+b\text{arctanh}(cx)}{x}\right)-\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$$\Big\downarrow \; 243$$

$$bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{x\,(1-c^2x^2)}dx+bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\int \frac{1}{x^2\,(1-c^2x^2)}dx^2-\frac{a+b\text{arctanh}(cx)}{x}\right)\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$$\Big\downarrow \; 47$$

$$bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{x\,(1-c^2x^2)}dx+bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\int \frac{1}{x^2}dx^2\right)-\frac{a+ba}{}\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$$\Big\downarrow \; 14$$

3.33. $\int \frac{(a+b\text{arctanh}(cx))^3}{x^4}\,dx$

$$bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{x\left(1-c^2x^2\right)}dx+bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^2}dx^2+\log \left(x^2\right)\right)-\frac{a+b\text{arc}}{}\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$\Big\downarrow$ 16

$$bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{x\left(1-c^2x^2\right)}dx+bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{1-c^2x^2}dx-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log \left(x^2\right)-\log \left(1-c^2\right.\right.\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$\Big\downarrow$ 6510

$$bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{x\left(1-c^2x^2\right)}dx+bc\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\left(\log \left(x^2\right)-\log \left(1-c^2x^2\right)\right.\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$\Big\downarrow$ 6550

$$bc\left(c^2\left(\int \frac{(a+b\text{arctanh}(cx))^2}{x(cx+1)}dx+\frac{(a+b\text{arctanh}(cx))^3}{3b}\right)+bc\left(\frac{c(a+b\text{arctanh}(cx))^2}{2b}-\frac{a+b\text{arctanh}(cx)}{x}+\frac{1}{2}bc\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$\Big\downarrow$ 6494

$$bc\left(c^2\left(-2bc\int \frac{(a+b\text{arctanh}(cx))\log \left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx+\frac{(a+b\text{arctanh}(cx))^3}{3b}+\log \left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(c\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$\Big\downarrow$ 6618

$$bc\left(c^2\left(-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)(a+b\text{arctanh}(cx))}{2c}-\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)}{1-c^2x^2}dx\right)+\frac{(a+b\text{arctanh}(c}{3b}\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

$\Big\downarrow$ 7164

3.33. $\int \frac{(a+b\text{arctanh}(cx))^3}{x^4}\,dx$

$$bc\left(c^2\left(-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)(a+b\text{arctanh}(cx))}{2c}+\frac{b\,\text{PolyLog}\left(3,\frac{2}{cx+1}-1\right)}{4c}\right)+\frac{(a+b\text{arctanh}(cx))^3}{3b}+\right.\right.$$

$$\frac{(a+b\text{arctanh}(cx))^3}{3x^3}$$

input
```
Int[(a + b*ArcTanh[c*x])^3/x^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c*x])^3/x^3 + b*c*(-1/2*(a + b*ArcTanh[c*x])^2/x^2 + b
*c*(-((a + b*ArcTanh[c*x])/x) + (c*(a + b*ArcTanh[c*x])^2)/(2*b) + (b*c*(L
og[x^2] - Log[1 - c^2*x^2]))/2) + c^2*((a + b*ArcTanh[c*x])^3/(3*b) + (a +
 b*ArcTanh[c*x])^2*Log[2 - 2/(1 + c*x)] - 2*b*c*(((a + b*ArcTanh[c*x])*Pol
yLog[2, -1 + 2/(1 + c*x)])/(2*c) + (b*PolyLog[3, -1 + 2/(1 + c*x)])/(4*c))
))
```

3.33.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.33. $\int \frac{(a+b\operatorname{arctanh}(cx))^3}{x^4}\,dx$

3.33.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 58.16 (sec) , antiderivative size = 1542, normalized size of antiderivative = 7.71

method	result	size
derivativedivides	Expression too large to display	1542
default	Expression too large to display	1542
parts	Expression too large to display	1544

input
```
int((a+b*arctanh(c*x))^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
c^3*(-1/3*a^3/c^3/x^3+b^3*(-1/3/c^3/x^3*arctanh(c*x)^3-1/2*arctanh(c*x)^2*
ln(c*x+1)-1/2*arctanh(c*x)^2*ln(c*x-1)-1/2/c^2/x^2*arctanh(c*x)^2+ln(c*x)*
arctanh(c*x)^2+arctanh(c*x)^2*ln((c*x+1)/(-c^2*x^2+1)^(1/2))-arctanh(c*x)^
2*ln((c*x+1)^2/(-c^2*x^2+1)-1)+1/12*arctanh(c*x)*(-6*I*csgn(I/(1-(c*x+1)^2
/(c^2*x^2-1)))^2*arctanh(c*x)*Pi*c*x+6*I*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)
/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*csgn(I/(1-(
c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)*Pi*c*x-6*I*csgn(I*(-(c*x+1)^2/(c^2*x^2
-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1))*arc
tanh(c*x)*Pi*c*x-6*I*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x
^2-1)))^2*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c*x)*Pi*c*x+3*I*csgn(I
*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)*Pi*c*x+6*
I*arctanh(c*x)*Pi*c*x+3*I*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)*Pi*
c*x+3*I*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))
*arctanh(c*x)*Pi*c*x+3*I*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^
2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)*Pi*c*x+6*I*csgn(I/
(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)*Pi*c*x-3*I*csgn(I*(c*x+1)^2/(c^2
*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh
(c*x)*Pi*c*x-3*I*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/(c^2*x
^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/(c^2*x^2-1))*arctanh(c*x
)*Pi*c*x+6*I*csgn(I*(-(c*x+1)^2/(c^2*x^2-1)-1)/(1-(c*x+1)^2/(c^2*x^2-1)...
```

3.33.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^4}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*
x) + a^3)/x^4, x)
```

3.33.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^4}\,dx=\int \frac{(a+b\,\text{atanh}\,(cx))^3}{x^4}\,dx$$

input
```
integrate((a+b*atanh(c*x))**3/x**4,x)
```

output
```
Integral((a + b*atanh(c*x))**3/x**4, x)
```

3.33.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^4}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="maxima")
```

output
```
-1/2*((c^2*log(c^2*x^2 - 1) - c^2*log(x^2) + 1/x^2)*c + 2*arctanh(c*x)/x^3
)*a^2*b - 1/3*a^3/x^3 - 1/24*((b^3*c^3*x^3 - b^3)*log(-c*x + 1)^3 + 3*(b^3
*c*x + 2*a*b^2 + (b^3*c^3*x^3 + b^3)*log(c*x + 1))*log(-c*x + 1)^2)/x^3 -
integrate(-1/8*((b^3*c*x - b^3)*log(c*x + 1)^3 + 6*(a*b^2*c*x - a*b^2)*log
(c*x + 1)^2 + (2*b^3*c^2*x^2 + 4*a*b^2*c*x - 3*(b^3*c*x - b^3)*log(c*x + 1
)^2 + 2*(b^3*c^4*x^4 + 6*a*b^2 - (6*a*b^2*c - b^3*c)*x)*log(c*x + 1))*log(
-c*x + 1))/(c*x^5 - x^4), x)
```

3.33.8 Giac [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx))^3}{x^4}\,dx = \int \frac{(b\operatorname{artanh}(cx)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3/x^4, x)
```

3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^3}{x^4}\,dx = \int \frac{(a+b\operatorname{atanh}(c\,x))^3}{x^4}\,dx$$

input
```
int((a + b*atanh(c*x))^3/x^4,x)
```

output
```
int((a + b*atanh(c*x))^3/x^4, x)
```

3.34 $\int \frac{(a+b\,\text{arctanh}(cx))^3}{x^5}\,dx$

3.34.1 Optimal result

Integrand size = 14, antiderivative size = 187

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx))^3}{x^5}\,dx = &-\frac{b^3c^3}{4x}+\frac{1}{4}b^3c^4\text{arctanh}(cx)-\frac{b^2c^2(a+b\text{arctanh}(cx))}{4x^2}\\&+bc^4(a+b\text{arctanh}(cx))^2-\frac{bc(a+b\text{arctanh}(cx))^2}{4x^3}\\&-\frac{3bc^3(a+b\text{arctanh}(cx))^2}{4x}\\&+\frac{1}{4}c^4(a+b\text{arctanh}(cx))^3-\frac{(a+b\text{arctanh}(cx))^3}{4x^4}\\&+2b^2c^4(a+b\text{arctanh}(cx))\log\left(2-\frac{2}{1+cx}\right)\\&-b^3c^4\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx}\right)\end{aligned}$$

output
```
-1/4*b^3*c^3/x+1/4*b^3*c^4*arctanh(c*x)-1/4*b^2*c^2*(a+b*arctanh(c*x))/x^2
+b*c^4*(a+b*arctanh(c*x))^2-1/4*b*c*(a+b*arctanh(c*x))^2/x^3-3/4*b*c^3*(a+
b*arctanh(c*x))^2/x+1/4*c^4*(a+b*arctanh(c*x))^3-1/4*(a+b*arctanh(c*x))^3/
x^4+2*b^2*c^4*(a+b*arctanh(c*x))*ln(2-2/(c*x+1))-b^3*c^4*polylog(2,-1+2/(c
*x+1))
```

3.34.2 Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.58

$$\int \frac{(a+b\operatorname{arctanh}(cx))^3}{x^5}\,dx =$$

$$-\frac{2a^3+2a^2bcx+2ab^2c^2x^2+6a^2bc^3x^3+2b^3c^3x^3-2ab^2c^4x^4+2b^2(bcx(1+3c^2x^2-4c^3x^3)+a(3-3c^4x^4$$

input
```
Integrate[(a + b*ArcTanh[c*x])^3/x^5,x]
```

output
```
-1/8*(2*a^3 + 2*a^2*b*c*x + 2*a*b^2*c^2*x^2 + 6*a^2*b*c^3*x^3 + 2*b^3*c^3*
x^3 - 2*a*b^2*c^4*x^4 + 2*b^2*(b*c*x*(1 + 3*c^2*x^2 - 4*c^3*x^3) + a*(3 -
3*c^4*x^4))*ArcTanh[c*x]^2 - 2*b^3*(-1 + c^4*x^4)*ArcTanh[c*x]^3 + 2*b*Arc
Tanh[c*x]*(3*a^2 + b^2*c^2*x^2*(1 - c^2*x^2) + 2*a*b*c*x*(1 + 3*c^2*x^2) -
 8*b^2*c^4*x^4*Log[1 - E^(-2*ArcTanh[c*x])]) + 3*a^2*b*c^4*x^4*Log[1 - c*x
] - 3*a^2*b*c^4*x^4*Log[1 + c*x] - 16*a*b^2*c^4*x^4*Log[(c*x)/Sqrt[1 - c^2
*x^2]] + 8*b^3*c^4*x^4*PolyLog[2, E^(-2*ArcTanh[c*x])])/x^4
```

3.34.3 Rubi [A] (verified)

Time = 1.82 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.29, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6452, 6544, 6452, 6544, 6452, 264, 219, 6510, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arctanh}(cx))^3}{x^5}dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{3}{4}bc\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^4(1-c^2x^2)}dx-\frac{(a+b\operatorname{arctanh}(cx))^3}{4x^4}$$

$$\Big\downarrow \text{ 6544}$$

$$\frac{3}{4}bc\left(c^2\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^2(1-c^2x^2)}dx+\int \frac{(a+b\operatorname{arctanh}(cx))^2}{x^4}dx\right)-\frac{(a+b\operatorname{arctanh}(cx))^3}{4x^4}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{3}{4}bc\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{x^2(1-c^2x^2)}dx+\frac{2}{3}bc\int \frac{a+b\text{arctanh}(cx)}{x^3(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{3x^3}\right)-\frac{(a+b\text{arctanh}(cx))^3}{4x^4}$$

$\downarrow$ 6544

$$\frac{3}{4}bc\left(c^2\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx+\int \frac{(a+b\text{arctanh}(cx))^2}{x^2}dx\right)+\frac{2}{3}bc\left(c^2\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\int \frac{a+ba\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{4x^4}$$

$\downarrow$ 6452

$$\frac{3}{4}bc\left(c^2\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx+2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{x}\right)+\frac{2}{3}bc\left(c^2\int \frac{a+barc}{x(1-}\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{4x^4}$$

$\downarrow$ 264

$$\frac{3}{4}bc\left(c^2\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx+2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{x}\right)+\frac{2}{3}bc\left(c^2\int \frac{a+barc}{x(1-}\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{4x^4}$$

$\downarrow$ 219

$$\frac{3}{4}bc\left(c^2\left(c^2\int \frac{(a+b\text{arctanh}(cx))^2}{1-c^2x^2}dx+2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx-\frac{(a+b\text{arctanh}(cx))^2}{x}\right)+\frac{2}{3}bc\left(c^2\int \frac{a+barc}{x(1-}\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{4x^4}$$

$\downarrow$ 6510

$$\frac{3}{4}bc\left(c^2\left(2bc\int \frac{a+b\text{arctanh}(cx)}{x(1-c^2x^2)}dx+\frac{c(a+b\text{arctanh}(cx))^3}{3b}-\frac{(a+b\text{arctanh}(cx))^2}{x}\right)+\frac{2}{3}bc\left(c^2\int \frac{a+b\text{arctanh}(c}{x(1-c^2x^2)}\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{4x^4}$$

$\downarrow$ 6550

$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\left(\int \frac{a+b\text{arctanh}(cx)}{x(cx+1)}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}\right)-\frac{a+b\text{arctanh}(cx)}{2x^2}+\frac{1}{2}bc\left(c\text{arctanh}(cx)-\frac{1}{x}\right)\right)\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{4x^4}$$

3.34. $\int \frac{(a+b\text{arctanh}(cx))^3}{x^5}\,dx$

$$\Big\downarrow \text{6494}$$

$$\frac{3}{4}bc\left(c^2\left(2bc\left(-bc\int \frac{\log\left(2-\frac{2}{cx+1}\right)}{1-c^2x^2}dx+\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))\right)+\frac{c(a+\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{4x^4}$$

$$\Big\downarrow \text{2897}$$

$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\left(\frac{(a+b\text{arctanh}(cx))^2}{2b}+\log\left(2-\frac{2}{cx+1}\right)(a+b\text{arctanh}(cx))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx+1}-1\right)\right)-\frac{a}{}\right.\right.$$
$$\frac{(a+b\text{arctanh}(cx))^3}{4x^4}$$

input
```
Int[(a + b*ArcTanh[c*x])^3/x^5,x]
```

output
```
-1/4*(a + b*ArcTanh[c*x])^3/x^4 + (3*b*c*(-1/3*(a + b*ArcTanh[c*x])^2/x^3
+ c^2*(-((a + b*ArcTanh[c*x])^2/x) + (c*(a + b*ArcTanh[c*x])^3)/(3*b) + 2*
b*c*((a + b*ArcTanh[c*x])^2/(2*b) + (a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*
x)] - (b*PolyLog[2, -1 + 2/(1 + c*x)])/2)) + (2*b*c*(-1/2*(a + b*ArcTanh[c
*x])/x^2 + (b*c*(-x^(-1) + c*ArcTanh[c*x]))/2 + c^2*((a + b*ArcTanh[c*x])^
2/(2*b) + (a + b*ArcTanh[c*x])*Log[2 - 2/(1 + c*x)] - (b*PolyLog[2, -1 + 2
/(1 + c*x)])/2)))/3))/4
```

3.34.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.34. $\int \frac{(a+b\text{arctanh}(cx))^3}{x^5} \, dx$

3.34.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 18.94 (sec) , antiderivative size = 1136, normalized size of antiderivative = 6.07

method	result	size
derivativedivides	Expression too large to display	1136
default	Expression too large to display	1136
parts	Expression too large to display	1193

input
```
int((a+b*arctanh(c*x))^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
c^4*(-3/16*I*b^3*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/(c^
2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-3/8*I*b^3*Pi*csgn(I*(
c*x+1)/(-c^2*x^2+1)^(1/2))*csgn(I*(c*x+1)^2/(c^2*x^2-1))^2*arctanh(c*x)^2-
3/16*I*b^3*Pi*csgn(I*(c*x+1)/(-c^2*x^2+1)^(1/2))^2*csgn(I*(c*x+1)^2/(c^2*x
^2-1))*arctanh(c*x)^2+3/16*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))*csgn(I*(
c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2+2*b^3*dil
og(1+(c*x+1)/(-c^2*x^2+1)^(1/2))-3/4*b^3*arctanh(c*x)^2*ln((c*x+1)/(-c^2*x
^2+1)^(1/2))-3/8*b^3*arctanh(c*x)^2*ln(c*x-1)+3/8*b^3*arctanh(c*x)^2*ln(c*
x+1)+3/16*I*b^3*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))*csgn(I*(c*x+1)^2/(c^2
*x^2-1))*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))*arctanh(c
*x)^2-3/4*b^3/c/x*arctanh(c*x)^2+1/4*b^3*arctanh(c*x)^3-b^3*arctanh(c*x)^2
+1/4*b^3*arctanh(c*x)-2*b^3*dilog((c*x+1)/(-c^2*x^2+1)^(1/2))+3*a*b^2*(-1/
4/c^4/x^4*arctanh(c*x)^2-1/6/c^3/x^3*arctanh(c*x)-1/2/c/x*arctanh(c*x)+1/4
*arctanh(c*x)*ln(c*x+1)-1/4*arctanh(c*x)*ln(c*x-1)+1/8*ln(c*x-1)*ln(1/2*c*
x+1/2)-1/16*ln(c*x-1)^2+1/8*(ln(c*x+1)-ln(1/2*c*x+1/2))*ln(-1/2*c*x+1/2)-1
/16*ln(c*x+1)^2-1/12/c^2/x^2+2/3*ln(c*x)-1/3*ln(c*x+1)-1/3*ln(c*x-1))-3/16
*I*b^3*Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1)/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctan
h(c*x)^2+3/8*I*b^3*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^3*arctanh(c*x)^2-3
/8*I*b^3*Pi*csgn(I/(1-(c*x+1)^2/(c^2*x^2-1)))^2*arctanh(c*x)^2-3/16*I*b^3*
Pi*csgn(I*(c*x+1)^2/(c^2*x^2-1))^3*arctanh(c*x)^2-1/4*a^3/c^4/x^4+2*b^3...
```

3.34.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^5}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*
x) + a^3)/x^5, x)
```

3.34.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^5}\,dx=\int \frac{(a+b\,\text{atanh}\,(cx))^3}{x^5}\,dx$$

input
```
integrate((a+b*atanh(c*x))**3/x**5,x)
```

output
```
Integral((a + b*atanh(c*x))**3/x**5, x)
```

3.34.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^5}\,dx=\int \frac{(b\,\text{artanh}\,(cx)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="maxima")
```

output
```
1/8*((3*c^3*log(c*x + 1) - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c -
 6*arctanh(c*x)/x^4)*a^2*b + 1/16*((32*c^2*log(x) - (3*c^2*x^2*log(c*x + 1
)^2 + 3*c^2*x^2*log(c*x - 1)^2 + 16*c^2*x^2*log(c*x - 1) - 2*(3*c^2*x^2*lo
g(c*x - 1) - 8*c^2*x^2)*log(c*x + 1) + 4)/x^2)*c^2 + 4*(3*c^3*log(c*x + 1)
 - 3*c^3*log(c*x - 1) - 2*(3*c^2*x^2 + 1)/x^3)*c*arctanh(c*x))*a*b^2 - 1/3
2*b^3*(((c^4*x^4 - 1)*log(-c*x + 1)^3 + (6*c^3*x^3 + 2*c*x - 3*(c^4*x^4 -
1)*log(c*x + 1))*log(-c*x + 1)^2)/x^4 + 4*integrate(-1/2*(2*(c*x - 1)*log(
c*x + 1)^3 + (6*c^4*x^4 + 2*c^2*x^2 - 6*(c*x - 1)*log(c*x + 1)^2 - 3*(c^5*
x^5 - c*x)*log(c*x + 1))*log(-c*x + 1))/(c*x^6 - x^5), x)) - 3/4*a*b^2*arc
tanh(c*x)^2/x^4 - 1/4*a^3/x^4
```

3.34.8 Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^5}\,dx = \int \frac{(b\,\text{artanh}\,(cx)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctanh(c*x))^3/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3/x^5, x)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx))^3}{x^5}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x))^3}{x^5}\,dx$$

input
```
int((a + b*atanh(c*x))^3/x^5,x)
```

output
```
int((a + b*atanh(c*x))^3/x^5, x)
```

3.35 $\int (dx)^{5/2}(a + b\text{arctanh}(cx))\,dx$

3.35.1 Optimal result

Integrand size = 16, antiderivative size = 124

$$\int (dx)^{5/2}(a+b\text{arctanh}(cx))\,dx=\frac{4bd^2\sqrt{dx}}{7c^3}+\frac{4b(dx)^{5/2}}{35c}-\frac{2bd^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}+\frac{2(dx)^{7/2}(a+b\text{arctanh}(cx))}{7d}-\frac{2bd^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/2}}$$

output
```
4/35*b*(d*x)^(5/2)/c-2/7*b*d^(5/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/c^(
7/2)+2/7*(d*x)^(7/2)*(a+b*arctanh(c*x))/d-2/7*b*d^(5/2)*arctanh(c^(1/2)*(d
*x)^(1/2)/d^(1/2))/c^(7/2)+4/7*b*d^2*(d*x)^(1/2)/c^3
```

3.35.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.03

$$\int (dx)^{5/2}(a+b\text{arctanh}(cx))\,dx=\frac{(dx)^{5/2}\left(20b\sqrt{c}\sqrt{x}+4bc^{5/2}x^{5/2}+10ac^{7/2}x^{7/2}-10b\arctan\left(\sqrt{c}\sqrt{x}\right)+10bc^{7/2}x^{7/2}\text{arcta}\right.}{35c^{7/2}x^{5/2}}$$

input
```
Integrate[(d*x)^(5/2)*(a + b*ArcTanh[c*x]),x]
```

output
```
((d*x)^(5/2)*(20*b*Sqrt[c]*Sqrt[x] + 4*b*c^(5/2)*x^(5/2) + 10*a*c^(7/2)*x^
(7/2) - 10*b*ArcTan[Sqrt[c]*Sqrt[x]] + 10*b*c^(7/2)*x^(7/2)*ArcTanh[c*x] +
 5*b*Log[1 - Sqrt[c]*Sqrt[x]] - 5*b*Log[1 + Sqrt[c]*Sqrt[x]]))/(35*c^(7/2)
*x^(5/2))
```

3.35.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6464, 262, 262, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2}(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{ 6464}$$

$$\frac{2(dx)^{7/2}(a+b\text{arctanh}(cx))}{7d}-\frac{2bc\int \frac{(dx)^{7/2}}{1-c^2x^2}dx}{7d}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2(dx)^{7/2}(a+b\text{arctanh}(cx))}{7d}-\frac{2bc\left(\frac{d^2\int \frac{(dx)^{3/2}}{1-c^2x^2}dx}{c^2}-\frac{2d(dx)^{5/2}}{5c^2}\right)}{7d}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2(dx)^{7/2}(a+b\text{arctanh}(cx))}{7d}-\frac{2bc\left(\frac{d^2\left(\frac{d^2\int \frac{1}{\sqrt{dx}\left(1-c^2x^2\right)}dx}{c^2}-\frac{2d\sqrt{dx}}{c^2}\right)}{c^2}-\frac{2d(dx)^{5/2}}{5c^2}\right)}{7d}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2(dx)^{7/2}(a+b\text{arctanh}(cx))}{7d}-\frac{2bc\left(\frac{d^2\left(\frac{2d\int \frac{1}{1-c^2x^2}d\sqrt{dx}}{c^2}-\frac{2d\sqrt{dx}}{c^2}\right)}{c^2}-\frac{2d(dx)^{5/2}}{5c^2}\right)}{7d}$$

$$\Big\downarrow \text{ 756}$$

$$\frac{2(dx)^{7/2}(a+b\text{arctanh}(cx))}{7d}-\frac{2bc\left(\frac{d^2\left(\frac{2d\left(\frac{1}{2}d\int\frac{1}{d-cdx}d\sqrt{dx}+\frac{1}{2}d\int\frac{1}{cxd+d}d\sqrt{dx}\right)}{c^2}-\frac{2d\sqrt{dx}}{c^2}\right)}{c^2}-\frac{2d(dx)^{5/2}}{5c^2}\right)}{7d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2(dx)^{7/2}(a+b\text{arctanh}(cx))}{7d}-\frac{2bc\left(\frac{d^2\left(\frac{2d\left(\frac{1}{2}d\int\frac{1}{d-cdx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}\right)}{c^2}-\frac{2d\sqrt{dx}}{c^2}\right)}{c^2}-\frac{2d(dx)^{5/2}}{5c^2}\right)}{7d}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2(dx)^{7/2}(a+b\text{arctanh}(cx))}{7d}-\frac{2bc\left(\frac{d^2\left(\frac{2d\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}\right)}{c^2}-\frac{2d\sqrt{dx}}{c^2}\right)}{c^2}-\frac{2d(dx)^{5/2}}{5c^2}\right)}{7d}$$

input
```
Int[(d*x)^(5/2)*(a + b*ArcTanh[c*x]),x]
```

output
```
(2*(d*x)^(7/2)*(a + b*ArcTanh[c*x]))/(7*d) - (2*b*c*((-2*d*(d*x)^(5/2))/(5
*c^2) + (d^2*((-2*d*Sqrt[d*x])/c^2 + (2*d*((Sqrt[d]*ArcTan[(Sqrt[c]*Sqrt[d
*x])/Sqrt[d]])/(2*Sqrt[c]) + (Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]
)/(2*Sqrt[c])))/c^2))/c^2))/(7*d)
```

3.35. $\int(dx)^{5/2}(a+b\text{arctanh}(cx))\,dx$

3.35.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.35.4 Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

method	result	siz
derivativedivides	$\frac{\frac{2a(dx)^{\frac{7}{2}}}{7}+\frac{2b(dx)^{\frac{7}{2}}\operatorname{arctanh}(cx)}{7}+\frac{4bd(dx)^{\frac{5}{2}}}{35c}+\frac{4b\,d^{3}\sqrt{dx}}{7c^{3}}-\frac{2b\,d^{4}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7c^{3}\sqrt{cd}}-\frac{2b\,d^{4}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7c^{3}\sqrt{cd}}}{d}$	10
default	$\frac{\frac{2a(dx)^{\frac{7}{2}}}{7}+\frac{2b(dx)^{\frac{7}{2}}\operatorname{arctanh}(cx)}{7}+\frac{4bd(dx)^{\frac{5}{2}}}{35c}+\frac{4b\,d^{3}\sqrt{dx}}{7c^{3}}-\frac{2b\,d^{4}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7c^{3}\sqrt{cd}}-\frac{2b\,d^{4}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7c^{3}\sqrt{cd}}}{d}$	10
parts	$\frac{2a(dx)^{\frac{7}{2}}}{7d}+\frac{2b(dx)^{\frac{7}{2}}\operatorname{arctanh}(cx)}{7d}+\frac{4b(dx)^{\frac{5}{2}}}{35c}+\frac{4b\,d^{2}\sqrt{dx}}{7c^{3}}-\frac{2b\,d^{3}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7c^{3}\sqrt{cd}}-\frac{2b\,d^{3}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7c^{3}\sqrt{cd}}$	10

input
```
int((d*x)^(5/2)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/7*a*(d*x)^(7/2)+1/7*b*(d*x)^(7/2)*arctanh(c*x)+2/35*b*d/c*(d*x)^(5/
2)+2/7*b*d^3/c^3*(d*x)^(1/2)-1/7*b*d^4/c^3/(c*d)^(1/2)*arctanh(c*(d*x)^(1/
2)/(c*d)^(1/2))-1/7*b*d^4/c^3/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2)
))
```

3.35.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.39

$$\int (dx)^{5/2}(a+b\operatorname{arctanh}(cx))\,dx = \left[-\frac{10\,bd^{2}\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}c\sqrt{\frac{d}{c}}}{d}\right)-5\,bd^{2}\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\sqrt{dx}c\sqrt{\frac{d}{c}}+d}{cx-1}\right)-\left(5\,bc^{3}d^{2}x^{3}\log\left(-\frac{c}{c}\right.\right.}{35\,c^{3}}\right.$$

input
```
integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
[-1/35*(10*b*d^2*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) - 5*b*d^2*sqrt(
d/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) - (5*b*c^3*d^2*x
^3*log(-(c*x + 1)/(c*x - 1)) + 10*a*c^3*d^2*x^3 + 4*b*c^2*d^2*x^2 + 20*b*d
^2)*sqrt(d*x))/c^3, 1/35*(10*b*d^2*sqrt(-d/c)*arctan(sqrt(d*x)*c*sqrt(-d/c
)/d) + 5*b*d^2*sqrt(-d/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*x
+ 1)) + (5*b*c^3*d^2*x^3*log(-(c*x + 1)/(c*x - 1)) + 10*a*c^3*d^2*x^3 + 4*
b*c^2*d^2*x^2 + 20*b*d^2)*sqrt(d*x))/c^3]
```

3.35.6 Sympy [F]

$$\int (dx)^{5/2}(a+b\text{arctanh}(cx))\,dx=\int (dx)^{\frac{5}{2}}\left(a+b\,\text{atanh}\left(cx\right)\right)\,dx$$

input
```
integrate((d*x)**(5/2)*(a+b*atanh(c*x)),x)
```

output
```
Integral((d*x)**(5/2)*(a + b*atanh(c*x)), x)
```

3.35.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.08

$$\int (dx)^{5/2}(a+b\text{arctanh}(cx))\,dx=\frac{10\,(dx)^{\frac{7}{2}}\,a+\left(10\,(dx)^{\frac{7}{2}}\,\text{artanh}\,(cx)-\frac{\left(\frac{10\,d^5\,\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}c^4}-\frac{5\,d^5\,\log\left(\frac{\sqrt{dx}c-\sqrt{cd}}{\sqrt{dx}c+\sqrt{cd}}\right)}{\sqrt{cd}c^4}-\frac{4\left((dx)^{\frac{5}{2}}\,c^2d^2+5\,\sqrt{dx}\right.}{c^4}\right.}{d}\right.}{35\,d}$$

input
```
integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/35*(10*(d*x)^(7/2)*a + (10*(d*x)^(7/2)*arctanh(c*x) - (10*d^5*arctan(sqr
t(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c^4) - 5*d^5*log((sqrt(d*x)*c - sqrt(c*d))/
(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*c^4) - 4*((d*x)^(5/2)*c^2*d^2 + 5*sq
rt(d*x)*d^4)/c^4)*c/d)*b)/d
```

3.35.8 Giac [F]

$$\int (dx)^{5/2}(a+b\text{arctanh}(cx))\,dx=\int (dx)^{\frac{5}{2}}\left(b\,\text{artanh}\left(cx\right)+a\right)dx$$

input
```
integrate((d*x)^(5/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^(5/2)*(b*arctanh(c*x) + a), x)
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2}(a+b\text{arctanh}(cx))\,dx=\int (a+b\,\text{atanh}(c\,x))\,\left(d\,x\right)^{5/2}\,dx$$

input
```
int((a + b*atanh(c*x))*(d*x)^(5/2),x)
```

output
```
int((a + b*atanh(c*x))*(d*x)^(5/2), x)
```

3.36 $\int (dx)^{3/2}(a + b\text{arctanh}(cx))\,dx$

3.36.1 Optimal result

Integrand size = 16, antiderivative size = 106

$$\int (dx)^{3/2}(a+b\text{arctanh}(cx))\,dx=\frac{4b(dx)^{3/2}}{15c}+\frac{2bd^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}+\frac{2(dx)^{5/2}(a+b\text{arctanh}(cx))}{5d}-\frac{2bd^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/2}}$$

output
```
4/15*b*(d*x)^(3/2)/c+2/5*b*d^(3/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/c^(
5/2)+2/5*(d*x)^(5/2)*(a+b*arctanh(c*x))/d-2/5*b*d^(3/2)*arctanh(c^(1/2)*(d
*x)^(1/2)/d^(1/2))/c^(5/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08

$$\int (dx)^{3/2}(a+b\text{arctanh}(cx))\,dx=\frac{(dx)^{3/2}\left(4bc^{3/2}x^{3/2}+6ac^{5/2}x^{5/2}+6b\arctan\left(\sqrt{c}\sqrt{x}\right)+6bc^{5/2}x^{5/2}\text{arctanh}(cx)+3b\log\right.}{15c^{5/2}x^{3/2}}$$

input
```
Integrate[(d*x)^(3/2)*(a + b*ArcTanh[c*x]),x]
```

output
```
((d*x)^(3/2)*(4*b*c^(3/2)*x^(3/2) + 6*a*c^(5/2)*x^(5/2) + 6*b*ArcTan[Sqrt[
c]*Sqrt[x]] + 6*b*c^(5/2)*x^(5/2)*ArcTanh[c*x] + 3*b*Log[1 - Sqrt[c]*Sqrt[
x]] - 3*b*Log[1 + Sqrt[c]*Sqrt[x]]))/(15*c^(5/2)*x^(3/2))
```

3.36.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6464, 262, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{ 6464}$$

$$\frac{2(dx)^{5/2}(a+b\text{arctanh}(cx))}{5d}-\frac{2bc\int \frac{(dx)^{5/2}}{1-c^2x^2}dx}{5d}$$

$$\Big\downarrow \text{ 262}$$

$$\frac{2(dx)^{5/2}(a+b\text{arctanh}(cx))}{5d}-\frac{2bc\left(\frac{d^2\int \frac{\sqrt{dx}}{1-c^2x^2}dx}{c^2}-\frac{2d(dx)^{3/2}}{3c^2}\right)}{5d}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2(dx)^{5/2}(a+b\text{arctanh}(cx))}{5d}-\frac{2bc\left(\frac{2d\int \frac{d^3x}{d^2-c^2d^2x^2}d\sqrt{dx}}{c^2}-\frac{2d(dx)^{3/2}}{3c^2}\right)}{5d}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2(dx)^{5/2}(a+b\text{arctanh}(cx))}{5d}-\frac{2bc\left(\frac{2d^3\int \frac{dx}{d^2-c^2d^2x^2}d\sqrt{dx}}{c^2}-\frac{2d(dx)^{3/2}}{3c^2}\right)}{5d}$$

$$\Big\downarrow \text{ 827}$$

$$\frac{2(dx)^{5/2}(a+b\text{arctanh}(cx))}{5d}-\frac{2bc\left(\frac{2d^3\left(\frac{\int \frac{1}{d-cdx}d\sqrt{dx}}{2c}-\frac{\int \frac{1}{cxd+d}d\sqrt{dx}}{2c}\right)}{c^2}-\frac{2d(dx)^{3/2}}{3c^2}\right)}{5d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2(dx)^{5/2}(a+b\text{arctanh}(cx))}{5d}-\frac{2bc\left(\frac{2d^3\left(\frac{\int \frac{1}{d-cdx}d\sqrt{dx}}{2c}-\frac{\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}\right)}{c^2}-\frac{2d(dx)^{3/2}}{3c^2}\right)}{5d}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(dx)^{5/2}(a+b\text{arctanh}(cx))}{5d}-\frac{2bc\left(\frac{2d^3\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}\right)}{c^2}-\frac{2d(dx)^{3/2}}{3c^2}\right)}{5d}$$

input
```
Int[(d*x)^(3/2)*(a + b*ArcTanh[c*x]),x]
```

output
```
(2*(d*x)^(5/2)*(a + b*ArcTanh[c*x]))/(5*d) - (2*b*c*((-2*d*(d*x)^(3/2))/(3
*c^2) + (2*d^3*(-1/2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]/(c^(3/2)*Sqrt[d])
 + ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]/(2*c^(3/2)*Sqrt[d])))/c^2))/(5*d)
```

3.36.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.36.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+\frac{2(dx)^{\frac{5}{2}}b\,\mathrm{arctanh}(cx)}{5}+\frac{4bd(dx)^{\frac{3}{2}}}{15c}-\frac{2b\,d^{3}\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5c^{2}\sqrt{cd}}+\frac{2b\,d^{3}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5c^{2}\sqrt{cd}}}{d}$	93
default	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+\frac{2(dx)^{\frac{5}{2}}b\,\mathrm{arctanh}(cx)}{5}+\frac{4bd(dx)^{\frac{3}{2}}}{15c}-\frac{2b\,d^{3}\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5c^{2}\sqrt{cd}}+\frac{2b\,d^{3}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5c^{2}\sqrt{cd}}}{d}$	93
parts	$\frac{2a(dx)^{\frac{5}{2}}}{5d}+\frac{2b(dx)^{\frac{5}{2}}\,\mathrm{arctanh}(cx)}{5d}+\frac{4b(dx)^{\frac{3}{2}}}{15c}+\frac{2b\,d^{2}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5c^{2}\sqrt{cd}}-\frac{2b\,d^{2}\,\mathrm{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5c^{2}\sqrt{cd}}$	93

input
```
int((d*x)^(3/2)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/d*(1/5*(d*x)^(5/2)*a+1/5*(d*x)^(5/2)*b*arctanh(c*x)+2/15*b*d*(d*x)^(3/2)
/c-1/5*b*d^3/c^2/(c*d)^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))+1/5*b*d^3/
c^2/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2)))
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.41

$$\int (dx)^{3/2}(a+b\text{arctanh}(cx))\,dx = \left[\frac{6\,bd\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}c\sqrt{\frac{d}{c}}}{d}\right)+3\,bd\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\sqrt{dx}c\sqrt{\frac{d}{c}}+d}{cx-1}\right)+\left(3\,bc^2dx^2\log\left(-\frac{cx+1}{cx-1}\right)+\right.}{15\,c^2}\right.$$

input
```
integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
[1/15*(6*b*d*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) + 3*b*d*sqrt(d/c)*l
og((c*d*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) + (3*b*c^2*d*x^2*log(-
(c*x + 1)/(c*x - 1)) + 6*a*c^2*d*x^2 + 4*b*c*d*x)*sqrt(d*x))/c^2, 1/15*(6*
b*d*sqrt(-d/c)*arctan(sqrt(d*x)*c*sqrt(-d/c)/d) + 3*b*d*sqrt(-d/c)*log((c*
d*x + 2*sqrt(d*x)*c*sqrt(-d/c) - d)/(c*x + 1)) + (3*b*c^2*d*x^2*log(-(c*x
+ 1)/(c*x - 1)) + 6*a*c^2*d*x^2 + 4*b*c*d*x)*sqrt(d*x))/c^2]
```

3.36.6 Sympy [F]

$$\int (dx)^{3/2}(a+b\text{arctanh}(cx))\,dx = \int (dx)^{\frac{3}{2}}\left(a+b\,\text{atanh}\,(cx)\right)\,dx$$

input
```
integrate((d*x)**(3/2)*(a+b*atanh(c*x)),x)
```

output
```
Integral((d*x)**(3/2)*(a + b*atanh(c*x)), x)
```

3.36.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.11

$$\int (dx)^{3/2}(a+b\text{arctanh}(cx))\,dx = \frac{6\,(dx)^{\frac{5}{2}}\,a+\left(6\,(dx)^{\frac{5}{2}}\,\text{artanh}\,(cx)+\frac{\left(\frac{4\,(dx)^{\frac{3}{2}}d^2}{c^2}+\frac{6\,d^4\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}c^3}+\frac{3\,d^4\log\left(\frac{\sqrt{dx}c-\sqrt{cd}}{\sqrt{dx}c+\sqrt{cd}}\right)}{\sqrt{cd}c^3}\right)c}{d}\right)b}{15\,d}$$

input
```
integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/15*(6*(d*x)^(5/2)*a + (6*(d*x)^(5/2)*arctanh(c*x) + (4*(d*x)^(3/2)*d^2/c
^2 + 6*d^4*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c^3) + 3*d^4*log((sqrt
(d*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*c^3))*c/d)*b)/d
```

3.36.8 Giac [F]

$$\int (dx)^{3/2}(a+b\text{arctanh}(cx))\,dx=\int (dx)^{\frac{3}{2}}\,(b\,\text{artanh}\,(cx)+a)\,dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)*(b*arctanh(c*x) + a), x)
```

3.36.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}(a+b\text{arctanh}(cx))\,dx=\int (a+b\,\text{atanh}(c\,x))\,(d\,x)^{3/2}\,dx$$

input
```
int((a + b*atanh(c*x))*(d*x)^(3/2),x)
```

output
```
int((a + b*atanh(c*x))*(d*x)^(3/2), x)
```

3.37 $\int \sqrt{dx}(a+b\text{arctanh}(cx))\,dx$

3.37.1 Optimal result

Integrand size = 16, antiderivative size = 106

$$\int \sqrt{dx}(a+b\text{arctanh}(cx))\,dx=\frac{4b\sqrt{dx}}{3c}-\frac{2b\sqrt{d}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}+\frac{2(dx)^{3/2}(a+b\text{arctanh}(cx))}{3d}-\frac{2b\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/2}}$$

output
```
2/3*(d*x)^(3/2)*(a+b*arctanh(c*x))/d-2/3*b*arctan(c^(1/2)*(d*x)^(1/2)/d^(1
/2))*d^(1/2)/c^(3/2)-2/3*b*arctanh(c^(1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/c^
(3/2)+4/3*b*(d*x)^(1/2)/c
```

3.37.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.08

$$\int \sqrt{dx}(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{\sqrt{dx}\left(4b\sqrt{c}\sqrt{x}+2ac^{3/2}x^{3/2}-2b\arctan\left(\sqrt{c}\sqrt{x}\right)+2bc^{3/2}x^{3/2}\text{arctanh}(cx)+b\log\left(1-\sqrt{c}\sqrt{x}\right)-b\log\left(1-\right.\right.}{3c^{3/2}\sqrt{x}}$$

input
```
Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x]),x]
```

output
```
(Sqrt[d*x]*(4*b*Sqrt[c]*Sqrt[x] + 2*a*c^(3/2)*x^(3/2) - 2*b*ArcTan[Sqrt[c]
*Sqrt[x]] + 2*b*c^(3/2)*x^(3/2)*ArcTanh[c*x] + b*Log[1 - Sqrt[c]*Sqrt[x]]
- b*Log[1 + Sqrt[c]*Sqrt[x]]))/(3*c^(3/2)*Sqrt[x])
```

3.37.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6464, 262, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6464}$$

$$\frac{2(dx)^{3/2}(a+b\text{arctanh}(cx))}{3d}-\frac{2bc\int \frac{(dx)^{3/2}}{1-c^2x^2}dx}{3d}$$

$$\Big\downarrow \text{262}$$

$$\frac{2(dx)^{3/2}(a+b\text{arctanh}(cx))}{3d}-\frac{2bc\left(\frac{d^2\int \frac{1}{\sqrt{dx}(1-c^2x^2)}dx}{c^2}-\frac{2d\sqrt{dx}}{c^2}\right)}{3d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2(dx)^{3/2}(a+b\text{arctanh}(cx))}{3d}-\frac{2bc\left(\frac{2d\int \frac{1}{1-c^2x^2}d\sqrt{dx}}{c^2}-\frac{2d\sqrt{dx}}{c^2}\right)}{3d}$$

$$\Big\downarrow \text{756}$$

$$\frac{2(dx)^{3/2}(a+b\text{arctanh}(cx))}{3d}-\frac{2bc\left(\frac{2d\left(\frac{1}{2}d\int \frac{1}{d-cdx}d\sqrt{dx}+\frac{1}{2}d\int \frac{1}{cxd+d}d\sqrt{dx}\right)}{c^2}-\frac{2d\sqrt{dx}}{c^2}\right)}{3d}$$

$$\Big\downarrow \text{218}$$

$$\frac{2(dx)^{3/2}(a+b\text{arctanh}(cx))}{3d}-\frac{2bc\left(\frac{2d\left(\frac{1}{2}d\int \frac{1}{d-cdx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}\right)}{c^2}-\frac{2d\sqrt{dx}}{c^2}\right)}{3d}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(dx)^{3/2}(a+b\text{arctanh}(cx))}{3d}-\frac{2bc\left(\frac{2d\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}\right)}{c^2}-\frac{2d\sqrt{dx}}{c^2}\right)}{3d}$$

input
```
Int[Sqrt[d*x]*(a + b*ArcTanh[c*x]),x]
```

output
```
(2*(d*x)^(3/2)*(a + b*ArcTanh[c*x]))/(3*d) - (2*b*c*((-2*d*Sqrt[d*x])/c^2
+ (2*d*((Sqrt[d]*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[c]) + (Sqrt[
d]*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[c])))/c^2))/(3*d)
```

3.37.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.37.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.84

method	result	size
parts	$\frac{2a(dx)^{\frac{3}{2}}}{3d}+\frac{2b(dx)^{\frac{3}{2}}\operatorname{arctanh}(cx)}{3d}+\frac{4b\sqrt{dx}}{3c}-\frac{2bd\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3c\sqrt{cd}}-\frac{2bd\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3c\sqrt{cd}}$	89
derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+\frac{2(dx)^{\frac{3}{2}}b\operatorname{arctanh}(cx)}{3}+\frac{4bd\sqrt{dx}}{3c}-\frac{2b\,d^{2}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3c\sqrt{cd}}-\frac{2b\,d^{2}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3c\sqrt{cd}}}{d}$	93
default	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+\frac{2(dx)^{\frac{3}{2}}b\operatorname{arctanh}(cx)}{3}+\frac{4bd\sqrt{dx}}{3c}-\frac{2b\,d^{2}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3c\sqrt{cd}}-\frac{2b\,d^{2}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3c\sqrt{cd}}}{d}$	93

input
```
int((d*x)^(1/2)*(a+b*arctanh(c*x)),x,method=_RETURNVERBOSE)
```

output
```
2/3*a*(d*x)^(3/2)/d+2/3*b/d*(d*x)^(3/2)*arctanh(c*x)+4/3*b*(d*x)^(1/2)/c-2
/3*b*d/c/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2))-2/3*b*d/c/(c*d)^(1/
2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))
```

3.37.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.10

$$\int \sqrt{dx}(a+b\operatorname{arctanh}(cx))\,dx$$

$$=\left[-\frac{2\,b\sqrt{\frac{d}{c}}\arctan\left(\frac{\sqrt{dx}c\sqrt{\frac{d}{c}}}{d}\right)-b\sqrt{\frac{d}{c}}\log\left(\frac{cdx-2\,\sqrt{dx}c\sqrt{\frac{d}{c}}+d}{cx-1}\right)-\left(bcx\log\left(-\frac{cx+1}{cx-1}\right)+2\,acx+4\,b\right)\sqrt{dx}}{3\,c},\ \frac{2\,b\sqrt{-\cdots}}{\cdots}\right.$$

input
```
integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
[-1/3*(2*b*sqrt(d/c)*arctan(sqrt(d*x)*c*sqrt(d/c)/d) - b*sqrt(d/c)*log((c*
d*x - 2*sqrt(d*x)*c*sqrt(d/c) + d)/(c*x - 1)) - (b*c*x*log(-(c*x + 1)/(c*x
 - 1)) + 2*a*c*x + 4*b)*sqrt(d*x))/c, 1/3*(2*b*sqrt(-d/c)*arctan(sqrt(d*x)
*c*sqrt(-d/c)/d) + b*sqrt(-d/c)*log((c*d*x - 2*sqrt(d*x)*c*sqrt(-d/c) - d)
/(c*x + 1)) + (b*c*x*log(-(c*x + 1)/(c*x - 1)) + 2*a*c*x + 4*b)*sqrt(d*x))
/c]
```

3.37.6 Sympy [F]

$$\int \sqrt{dx}(a+b\text{arctanh}(cx))\,dx=\int \sqrt{dx}(a+b\,\text{atanh}\,(cx))\,dx$$

input
```
integrate((d*x)**(1/2)*(a+b*atanh(c*x)),x)
```

output
```
Integral(sqrt(d*x)*(a + b*atanh(c*x)), x)
```

3.37.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.12

$$\int \sqrt{dx}(a+b\text{arctanh}(cx))\,dx$$
$$=\frac{2\,(dx)^{\frac{3}{2}}\,a+\left(2\,(dx)^{\frac{3}{2}}\,\text{artanh}\,(cx)-\frac{\left(\frac{2\,d^3\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}c^2}-\frac{d^3\log\left(\frac{\sqrt{dx}c-\sqrt{cd}}{\sqrt{dx}c+\sqrt{cd}}\right)}{\sqrt{cd}c^2}-\frac{4\sqrt{dx}d^2}{c^2}\right)c}{d}\right)b}{3\,d}$$

input
```
integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/3*(2*(d*x)^(3/2)*a + (2*(d*x)^(3/2)*arctanh(c*x) - (2*d^3*arctan(sqrt(d*
x)*c/sqrt(c*d))/(sqrt(c*d)*c^2) - d^3*log((sqrt(d*x)*c - sqrt(c*d))/(sqrt(
d*x)*c + sqrt(c*d)))/(sqrt(c*d)*c^2) - 4*sqrt(d*x)*d^2/c^2)*c/d)*b)/d
```

3.37.8 Giac [F]

$$\int \sqrt{dx}(a+b\operatorname{arctanh}(cx))\,dx = \int \sqrt{dx}(b\operatorname{artanh}(cx)+a)\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*(b*arctanh(c*x) + a), x)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a+b\operatorname{arctanh}(cx))\,dx = \int (a+b\operatorname{atanh}(c\,x))\,\sqrt{d\,x}\,dx$$

input
```
int((a + b*atanh(c*x))*(d*x)^(1/2),x)
```

output
```
int((a + b*atanh(c*x))*(d*x)^(1/2), x)
```

3.38 $\int \frac{a+b\,\text{arctanh}(cx)}{\sqrt{dx}}\,dx$

3.38.1 Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{a+b\,\text{arctanh}(cx)}{\sqrt{dx}}\,dx=\frac{2b\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}+\frac{2\sqrt{dx}(a+b\,\text{arctanh}(cx))}{d}-\frac{2b\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{c}\sqrt{d}}$$

output
```
2*b*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/c^(1/2)/d^(1/2)-2*b*arctanh(c^(1/2
)*(d*x)^(1/2)/d^(1/2))/c^(1/2)/d^(1/2)+2*(a+b*arctanh(c*x))*(d*x)^(1/2)/d
```

3.38.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.15

$$\int \frac{a+b\,\text{arctanh}(cx)}{\sqrt{dx}}\,dx=\frac{\sqrt{x}\left(2a\sqrt{c}\sqrt{x}+2b\arctan\left(\sqrt{c}\sqrt{x}\right)+2b\sqrt{c}\sqrt{x}\,\text{arctanh}(cx)+b\log\left(1-\sqrt{c}\sqrt{x}\right)-b\log\left(1+\sqrt{c}\sqrt{x}\right)\right)}{\sqrt{c}\sqrt{dx}}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/Sqrt[d*x],x]
```

output
```
(Sqrt[x]*(2*a*Sqrt[c]*Sqrt[x] + 2*b*ArcTan[Sqrt[c]*Sqrt[x]] + 2*b*Sqrt[c]*
Sqrt[x]*ArcTanh[c*x] + b*Log[1 - Sqrt[c]*Sqrt[x]] - b*Log[1 + Sqrt[c]*Sqrt
[x]]))/(Sqrt[c]*Sqrt[d*x])
```

3.38.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6464, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{6464}$$

$$\frac{2\sqrt{dx}(a+b\text{arctanh}(cx))}{d}-\frac{2bc\int \frac{\sqrt{dx}}{1-c^2x^2}dx}{d}$$

$$\Big\downarrow \text{266}$$

$$\frac{2\sqrt{dx}(a+b\text{arctanh}(cx))}{d}-\frac{4bc\int \frac{d^3x}{d^2-c^2d^2x^2}d\sqrt{dx}}{d^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{dx}(a+b\text{arctanh}(cx))}{d}-4bc\int \frac{dx}{d^2-c^2d^2x^2}d\sqrt{dx}$$

$$\Big\downarrow \text{827}$$

$$\frac{2\sqrt{dx}(a+b\text{arctanh}(cx))}{d}-4bc\left(\frac{\int \frac{1}{d-cdx}d\sqrt{dx}}{2c}-\frac{\int \frac{1}{cxd+d}d\sqrt{dx}}{2c}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{2\sqrt{dx}(a+b\text{arctanh}(cx))}{d}-4bc\left(\frac{\int \frac{1}{d-cdx}d\sqrt{dx}}{2c}-\frac{\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{2\sqrt{dx}(a+b\text{arctanh}(cx))}{d}-4bc\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}\right)$$

input
```
Int[(a + b*ArcTanh[c*x])/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x]*(a + b*ArcTanh[c*x]))/d - 4*b*c*(-1/2*ArcTan[(Sqrt[c]*Sqrt[d*
x])/Sqrt[d]]/(c^(3/2)*Sqrt[d]) + ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]/(2*c
^(3/2)*Sqrt[d]))
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.38.4 Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{2\sqrt{dx}\,a+2\sqrt{dx}\,b\,\operatorname{arctanh}(cx)-\frac{2bd\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{2bd\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}}{d}$	68
default	$\frac{2\sqrt{dx}\,a+2\sqrt{dx}\,b\,\operatorname{arctanh}(cx)-\frac{2bd\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{2bd\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}}{d}$	68
parts	$\frac{2a\sqrt{dx}}{d}+\frac{2b\sqrt{dx}\,\operatorname{arctanh}(cx)}{d}+\frac{2b\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}-\frac{2b\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}$	70

input
```
int((a+b*arctanh(c*x))/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*((d*x)^(1/2)*a+(d*x)^(1/2)*b*arctanh(c*x)-b*d/(c*d)^(1/2)*arctanh(c*(d
*x)^(1/2)/(c*d)^(1/2))+b*d/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2)))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.48

$$\int \frac{a+b\operatorname{arctanh}(cx)}{\sqrt{dx}}\,dx$$
$$=\left[-\frac{2\sqrt{cd}b\arctan\left(\frac{\sqrt{cd}\sqrt{dx}}{cdx}\right)-\sqrt{cd}b\log\left(\frac{cdx-2\sqrt{cd}\sqrt{dx}+d}{cx-1}\right)-\left(bc\log\left(-\frac{cx+1}{cx-1}\right)+2\,ac\right)\sqrt{dx}}{cd},\ \frac{2\sqrt{-cd}b\arctan}{}\right.$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
[-(2*sqrt(c*d)*b*arctan(sqrt(c*d)*sqrt(d*x)/(c*d*x)) - sqrt(c*d)*b*log((c*
d*x - 2*sqrt(c*d)*sqrt(d*x) + d)/(c*x - 1)) - (b*c*log(-(c*x + 1)/(c*x - 1
)) + 2*a*c)*sqrt(d*x))/(c*d), (2*sqrt(-c*d)*b*arctan(sqrt(-c*d)*sqrt(d*x)/
(c*d*x)) - sqrt(-c*d)*b*log((c*d*x - 2*sqrt(-c*d)*sqrt(d*x) - d)/(c*x + 1)
) + (b*c*log(-(c*x + 1)/(c*x - 1)) + 2*a*c)*sqrt(d*x))/(c*d)]
```

3.38.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx)}{\sqrt{dx}}\,dx=\int \frac{a+b\,\text{atanh}\,(cx)}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*atanh(c*x))/(d*x)**(1/2),x)
```

output
```
Integral((a + b*atanh(c*x))/sqrt(d*x), x)
```

3.38.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.21

$$\int \frac{a+b\text{arctanh}(cx)}{\sqrt{dx}}\,dx = \frac{\left(2\sqrt{dx}\,\text{artanh}\,(cx)+\frac{\left(\frac{2\,d^2\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}c}+\frac{d^2\log\left(\frac{\sqrt{dx}c-\sqrt{cd}}{\sqrt{dx}c+\sqrt{cd}}\right)}{\sqrt{cd}c}\right)c}{d}\right)b+2\sqrt{dx}a}{d}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
((2*sqrt(d*x)*arctanh(c*x) + (2*d^2*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*
d)*c) + d^2*log((sqrt(d*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt
(c*d)*c))*c/d)*b + 2*sqrt(d*x)*a)/d
```

3.38.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

$$\int \frac{a+b\text{arctanh}(cx)}{\sqrt{dx}}\,dx = \frac{\left(2\,cd\left(\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}c}+\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}c}\right)+\sqrt{dx}\log\left(-\frac{cx+1}{cx-1}\right)\right)b+2\sqrt{dx}a}{d}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(1/2),x, algorithm="giac")
```

output
```
((2*c*d*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*c) + arctan(sqrt(d*x)*c/
sqrt(-c*d))/(sqrt(-c*d)*c)) + sqrt(d*x)*log(-(c*x + 1)/(c*x - 1)))*b + 2*s
qrt(d*x)*a)/d
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{\sqrt{dx}}\,dx = \int \frac{a+b\,\text{atanh}(c\,x)}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*atanh(c*x))/(d*x)^(1/2),x)
```

output
```
int((a + b*atanh(c*x))/(d*x)^(1/2), x)
```

3.39 $\int \frac{a+b\text{arctanh}(cx)}{(dx)^{3/2}}\,dx$

3.39.1 Optimal result

Integrand size = 16, antiderivative size = 85

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{3/2}}\,dx=\frac{2b\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2(a+b\text{arctanh}(cx))}{d\sqrt{dx}}+\frac{2b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

output
```
2*b*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))*c^(1/2)/d^(3/2)+2*b*arctanh(c^(1/2
)*(d*x)^(1/2)/d^(1/2))*c^(1/2)/d^(3/2)-2*(a+b*arctanh(c*x))/d/(d*x)^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{3/2}}\,dx=\frac{x\left(-2a+2b\sqrt{c}\sqrt{x}\arctan\left(\sqrt{c}\sqrt{x}\right)-2b\text{arctanh}(cx)-b\sqrt{c}\sqrt{x}\log\left(1-\sqrt{c}\sqrt{x}\right)+\right.}{(dx)^{3/2}}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(d*x)^(3/2),x]
```

output
```
(x*(-2*a + 2*b*Sqrt[c]*Sqrt[x]*ArcTan[Sqrt[c]*Sqrt[x]] - 2*b*ArcTanh[c*x]
- b*Sqrt[c]*Sqrt[x]*Log[1 - Sqrt[c]*Sqrt[x]] + b*Sqrt[c]*Sqrt[x]*Log[1 + S
qrt[c]*Sqrt[x]]))/(d*x)^(3/2)
```

3.39.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6464, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \ 6464$$

$$\frac{2bc\int \frac{1}{\sqrt{dx}(1-c^2x^2)}dx}{d}-\frac{2(a+b\text{arctanh}(cx))}{d\sqrt{dx}}$$

$$\Big\downarrow \ 266$$

$$\frac{4bc\int \frac{1}{1-c^2x^2}d\sqrt{dx}}{d^2}-\frac{2(a+b\text{arctanh}(cx))}{d\sqrt{dx}}$$

$$\Big\downarrow \ 756$$

$$\frac{4bc\left(\frac{1}{2}d\int \frac{1}{d-cdx}d\sqrt{dx}+\frac{1}{2}d\int \frac{1}{cxd+d}d\sqrt{dx}\right)}{d^2}-\frac{2(a+b\text{arctanh}(cx))}{d\sqrt{dx}}$$

$$\Big\downarrow \ 218$$

$$\frac{4bc\left(\frac{1}{2}d\int \frac{1}{d-cdx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}\right)}{d^2}-\frac{2(a+b\text{arctanh}(cx))}{d\sqrt{dx}}$$

$$\Big\downarrow \ 221$$

$$\frac{4bc\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}\right)}{d^2}-\frac{2(a+b\text{arctanh}(cx))}{d\sqrt{dx}}$$

input
```
Int[(a + b*ArcTanh[c*x])/(d*x)^(3/2),x]
```

output
```
(-2*(a + b*ArcTanh[c*x]))/(d*Sqrt[d*x]) + (4*b*c*((Sqrt[d]*ArcTan[(Sqrt[c]
*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[c]) + (Sqrt[d]*ArcTanh[(Sqrt[c]*Sqrt[d*x])/S
qrt[d]])/(2*Sqrt[c])))/d^2
```

3.39.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.39.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{2a}{\sqrt{dx}}-\frac{2b\,\operatorname{arctanh}(cx)}{\sqrt{dx}}+\frac{2bc\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{2bc\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}}{d}$	69
default	$\frac{-\frac{2a}{\sqrt{dx}}-\frac{2b\,\operatorname{arctanh}(cx)}{\sqrt{dx}}+\frac{2bc\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{2bc\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{\sqrt{cd}}}{d}$	69
parts	$-\frac{2a}{\sqrt{dx}\,d}-\frac{2b\,\operatorname{arctanh}(cx)}{d\sqrt{dx}}+\frac{2bc\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{d\sqrt{cd}}+\frac{2bc\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{d\sqrt{cd}}$	78

input
```
int((a+b*arctanh(c*x))/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-a/(d*x)^(1/2)-b/(d*x)^(1/2)*arctanh(c*x)+b*c/(c*d)^(1/2)*arctanh(c*(
d*x)^(1/2)/(c*d)^(1/2))+b*c/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2)))
```

3.39.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.60

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{3/2}}\,dx = \left[-\frac{2\,bdx\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)-bdx\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\,\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)+\sqrt{dx}\left(b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)}{d^2x}, -\frac{2\,bdx\sqrt{-\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{c}{d}}}{cx}\right)-bdx\sqrt{-\frac{c}{d}}\log\left(\frac{cx+2\,\sqrt{dx}\sqrt{-\frac{c}{d}}-1}{cx+1}\right)+\sqrt{dx}\left(b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)}{d^2x}\right]$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
[-(2*b*d*x*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) - b*d*x*sqrt(c/d)*l
og((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) + sqrt(d*x)*(b*log(-(c*x +
 1)/(c*x - 1)) + 2*a))/(d^2*x), -(2*b*d*x*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt
(-c/d)/(c*x)) - b*d*x*sqrt(-c/d)*log((c*x + 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c
*x + 1)) + sqrt(d*x)*(b*log(-(c*x + 1)/(c*x - 1)) + 2*a))/(d^2*x)]
```

3.39.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{3/2}}\,dx = \int \frac{a+b\,\text{atanh}\,(cx)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atanh(c*x))/(d*x)**(3/2),x)
```

output
```
Integral((a + b*atanh(c*x))/(d*x)**(3/2), x)
```

3.39.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{3/2}}\,dx=\frac{b\left(\frac{\left(\frac{2\,d\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}}-\frac{d\log\left(\frac{\sqrt{dx}c-\sqrt{cd}}{\sqrt{dx}c+\sqrt{cd}}\right)}{\sqrt{cd}}\right)c}{d}-\frac{2\,\text{artanh}(cx)}{\sqrt{dx}}\right)-\frac{2\,a}{\sqrt{dx}}}{d}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
(b*((2*d*arctan(sqrt(d*x)*c/sqrt(c*d))/sqrt(c*d) - d*log((sqrt(d*x)*c - sq
rt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/sqrt(c*d))*c/d - 2*arctanh(c*x)/sqrt(d
*x)) - 2*a/sqrt(d*x))/d
```

3.39.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{3/2}}\,dx=\frac{2\,bcd\left(\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}d}-\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}d}\right)-\frac{b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}}-\frac{2\,a}{\sqrt{dx}}}{d}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(3/2),x, algorithm="giac")
```

output
```
(2*b*c*d*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d) - arctan(sqrt(d*x)*c
/sqrt(-c*d))/(sqrt(-c*d)*d)) - b*log(-(c*d*x + d)/(c*d*x - d))/sqrt(d*x) -
 2*a/sqrt(d*x))/d
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{3/2}}\,dx=\int \frac{a+b\,\text{atanh}(c\,x)}{(d\,x)^{3/2}}\,dx$$

input
```
int((a + b*atanh(c*x))/(d*x)^(3/2),x)
```

output
```
int((a + b*atanh(c*x))/(d*x)^(3/2), x)
```

3.40 $\int \frac{a+b\text{arctanh}(cx)}{(dx)^{5/2}}\,dx$

3.40.1 Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{5/2}}\,dx = -\frac{4bc}{3d^2\sqrt{dx}}-\frac{2bc^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} -\frac{2(a+b\text{arctanh}(cx))}{3d(dx)^{3/2}}+\frac{2bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}$$

output
```
-2/3*b*c^(3/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(5/2)-2/3*(a+b*arctan
h(c*x))/d/(d*x)^(3/2)+2/3*b*c^(3/2)*arctanh(c^(1/2)*(d*x)^(1/2)/d^(1/2))/d
^(5/2)-4/3*b*c/d^2/(d*x)^(1/2)
```

3.40.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{5/2}}\,dx = -\frac{x\left(2a+4bcx+2bc^{3/2}x^{3/2}\arctan\left(\sqrt{c}\sqrt{x}\right)+2b\text{arctanh}(cx)+bc^{3/2}x^{3/2}\log\left(1-\sqrt{c}\sqrt{x}\right)-bc^{3/2}x^{3/2}\log\left(1-\right.\right.}{3(dx)^{5/2}}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(d*x)^(5/2),x]
```

output
```
-1/3*(x*(2*a + 4*b*c*x + 2*b*c^(3/2)*x^(3/2)*ArcTan[Sqrt[c]*Sqrt[x]] + 2*b
*ArcTanh[c*x] + b*c^(3/2)*x^(3/2)*Log[1 - Sqrt[c]*Sqrt[x]] - b*c^(3/2)*x^(
3/2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(d*x)^(5/2)
```

3.40.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6464, 264, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{5/2}}\,dx$$

$$\Big\downarrow \ 6464$$

$$\frac{2bc\int \frac{1}{(dx)^{3/2}(1-c^2x^2)}dx}{3d}-\frac{2(a+b\text{arctanh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 264$$

$$\frac{2bc\left(\frac{c^2\int \frac{\sqrt{dx}}{1-c^2x^2}dx}{d^2}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\text{arctanh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 266$$

$$\frac{2bc\left(\frac{2c^2\int \frac{d^3x}{d^2-c^2d^2x^2}d\sqrt{dx}}{d^3}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\text{arctanh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{2bc\left(\frac{2c^2\int \frac{dx}{d^2-c^2d^2x^2}d\sqrt{dx}}{d}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\text{arctanh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \ 827$$

$$\frac{2bc\left(\frac{2c^2\left(\frac{\int \frac{1}{d-cdx}d\sqrt{dx}}{2c}-\frac{\int \frac{1}{cxd+d}d\sqrt{dx}}{2c}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\text{arctanh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{2bc\left(\frac{2c^2\left(\frac{\int \frac{1}{d-cdx}d\sqrt{dx}}{2c}-\frac{\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\text{arctanh}(cx))}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{2bc\left(\frac{2c^2\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{3d}-\frac{2(a+b\text{arctanh}(cx))}{3d(dx)^{3/2}}$$

input
```
Int[(a + b*ArcTanh[c*x])/(d*x)^(5/2),x]
```

output
```
(-2*(a + b*ArcTanh[c*x]))/(3*d*(d*x)^(3/2)) + (2*b*c*(-2/(d*Sqrt[d*x]) + (
2*c^2*(-1/2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]/(c^(3/2)*Sqrt[d]) + ArcTan
h[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]/(2*c^(3/2)*Sqrt[d])))/d))/(3*d)
```

3.40.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.40.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}-\frac{2b\,\operatorname{arctanh}(cx)}{3(dx)^{\frac{3}{2}}}+\frac{2b\,c^{2}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3d\sqrt{cd}}-\frac{4bc}{3d\sqrt{dx}}-\frac{2b\,c^{2}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3d\sqrt{cd}}}{d}$	93
default	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}-\frac{2b\,\operatorname{arctanh}(cx)}{3(dx)^{\frac{3}{2}}}+\frac{2b\,c^{2}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3d\sqrt{cd}}-\frac{4bc}{3d\sqrt{dx}}-\frac{2b\,c^{2}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3d\sqrt{cd}}}{d}$	93
parts	$-\frac{2a}{3(dx)^{\frac{3}{2}}d}-\frac{2b\,\operatorname{arctanh}(cx)}{3d(dx)^{\frac{3}{2}}}-\frac{4bc}{3d^{2}\sqrt{dx}}-\frac{2b\,c^{2}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3d^{2}\sqrt{cd}}+\frac{2b\,c^{2}\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{3d^{2}\sqrt{cd}}$	94

input
```
int((a+b*arctanh(c*x))/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/3*a/(d*x)^(3/2)-1/3*b/(d*x)^(3/2)*arctanh(c*x)+1/3*b/d*c^2/(c*d)^(
1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))-2/3*b/d*c/(d*x)^(1/2)-1/3*b/d*c^2/
(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2)))
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.27

$$\int \frac{a+b\operatorname{arctanh}(cx)}{(dx)^{5/2}}\,dx = \left[\frac{2\,bcdx^2\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)+bcdx^2\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)-\left(4\,bcx+b\log\left(-\right.\right.}{3\,d^3x^2}\right.$$

$$\left.-\frac{2\,bcdx^2\sqrt{-\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{c}{d}}}{cx}\right)-bcdx^2\sqrt{-\frac{c}{d}}\log\left(\frac{cx-2\sqrt{dx}\sqrt{-\frac{c}{d}}-1}{cx+1}\right)+\left(4\,bcx+b\log\left(-\frac{cx+1}{cx-1}\right)+2\,a\right)\sqrt{dx}}{3\,d^3x^2}\right]$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(2*b*c*d*x^2*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) + b*c*d*x^2*
sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) - (4*b*c*x + b*
log(-(c*x + 1)/(c*x - 1)) + 2*a)*sqrt(d*x))/(d^3*x^2), -1/3*(2*b*c*d*x^2*s
qrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - b*c*d*x^2*sqrt(-c/d)*log((c
*x - 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) + (4*b*c*x + b*log(-(c*x + 1)/
(c*x - 1)) + 2*a)*sqrt(d*x))/(d^3*x^2)]
```

3.40.6 Sympy [F]

$$\int \frac{a+b\operatorname{arctanh}(cx)}{(dx)^{5/2}}\,dx = \int \frac{a+b\,\operatorname{atanh}(cx)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*atanh(c*x))/(d*x)**(5/2),x)
```

output
```
Integral((a + b*atanh(c*x))/(d*x)**(5/2), x)
```

3.40.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{5/2}}\,dx = -\frac{b\left(\frac{\left(\frac{2\,c\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{c\log\left(\frac{\sqrt{dx}c-\sqrt{cd}}{\sqrt{dx}c+\sqrt{cd}}\right)}{\sqrt{cd}}+\frac{4}{\sqrt{dx}}\right)c}{d}+\frac{2\,\text{artanh}(cx)}{(dx)^{\frac{3}{2}}}\right)+\frac{2\,a}{(dx)^{\frac{3}{2}}}}{3\,d}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*(b*((2*c*arctan(sqrt(d*x)*c/sqrt(c*d))/sqrt(c*d) + c*log((sqrt(d*x)*c
 - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/sqrt(c*d) + 4/sqrt(d*x))*c/d + 2*
arctanh(c*x)/(d*x)^(3/2)) + 2*a/(d*x)^(3/2))/d
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{5/2}}\,dx = -\frac{\frac{2\,bc^2\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}d}+\frac{2\,bc^2\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}d}+\frac{b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}dx}+\frac{2\,(2\,bcdx+ad)}{\sqrt{dx}d^2x}}{3\,d}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(5/2),x, algorithm="giac")
```

output
```
-1/3*(2*b*c^2*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d) + 2*b*c^2*arctan
(sqrt(d*x)*c/sqrt(-c*d))/(sqrt(-c*d)*d) + b*log(-(c*d*x + d)/(c*d*x - d))/
(sqrt(d*x)*d*x) + 2*(2*b*c*d*x + a*d)/(sqrt(d*x)*d^2*x))/d
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{5/2}}\,dx = \int \frac{a+b\,\text{atanh}(c\,x)}{(d\,x)^{5/2}}\,dx$$

input
```
int((a + b*atanh(c*x))/(d*x)^(5/2),x)
```

output
```
int((a + b*atanh(c*x))/(d*x)^(5/2), x)
```

3.41 $\int \frac{a+b\text{arctanh}(cx)}{(dx)^{7/2}}\,dx$

3.41.1 Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{7/2}}\,dx = -\frac{4bc}{15d^2(dx)^{3/2}}+\frac{2bc^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} -\frac{2(a+b\text{arctanh}(cx))}{5d(dx)^{5/2}}+\frac{2bc^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}}$$

output
```
-4/15*b*c/d^2/(d*x)^(3/2)+2/5*b*c^(5/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2)
)/d^(7/2)-2/5*(a+b*arctanh(c*x))/d/(d*x)^(5/2)+2/5*b*c^(5/2)*arctanh(c^(1/
2)*(d*x)^(1/2)/d^(1/2))/d^(7/2)
```

3.41.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.01

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{7/2}}\,dx = \frac{x\left(-6a-4bcx+6bc^{5/2}x^{5/2}\arctan\left(\sqrt{c}\sqrt{x}\right)-6b\text{arctanh}(cx)-3bc^{5/2}x^{5/2}\log\left(1-\right.\right.}{15(dx)^{7/2}}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(d*x)^(7/2),x]
```

output
```
(x*(-6*a - 4*b*c*x + 6*b*c^(5/2)*x^(5/2)*ArcTan[Sqrt[c]*Sqrt[x]] - 6*b*Arc
Tanh[c*x] - 3*b*c^(5/2)*x^(5/2)*Log[1 - Sqrt[c]*Sqrt[x]] + 3*b*c^(5/2)*x^(
5/2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(15*(d*x)^(7/2))
```

3.41.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6464, 264, 266, 756, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 6464}$$

$$\frac{2bc\int \frac{1}{(dx)^{5/2}(1-c^2x^2)}dx}{5d}-\frac{2(a+b\text{arctanh}(cx))}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{2bc\left(\frac{c^2\int \frac{1}{\sqrt{dx}(1-c^2x^2)}dx}{d^2}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2(a+b\text{arctanh}(cx))}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{ 266}$$

$$\frac{2bc\left(\frac{2c^2\int \frac{1}{1-c^2x^2}d\sqrt{dx}}{d^3}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2(a+b\text{arctanh}(cx))}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{ 756}$$

$$\frac{2bc\left(\frac{2c^2\left(\frac{1}{2}d\int \frac{1}{d-cdx}d\sqrt{dx}+\frac{1}{2}d\int \frac{1}{cxd+d}d\sqrt{dx}\right)}{d^3}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2(a+b\text{arctanh}(cx))}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2bc\left(\frac{2c^2\left(\frac{1}{2}d\int \frac{1}{d-cdx}d\sqrt{dx}+\frac{\sqrt{d}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}\right)}{d^3}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2(a+b\text{arctanh}(cx))}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2bc\left(\frac{2c^2\left(\frac{\sqrt{d}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}+\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{c}}\right)}{d^3}-\frac{2}{3d(dx)^{3/2}}\right)}{5d}-\frac{2(a+b\text{arctanh}(cx))}{5d(dx)^{5/2}}$$

input
```
Int[(a + b*ArcTanh[c*x])/(d*x)^(7/2),x]
```

output
```
(-2*(a + b*ArcTanh[c*x]))/(5*d*(d*x)^(5/2)) + (2*b*c*(-2/(3*d*(d*x)^(3/2))
 + (2*c^2*((Sqrt[d]*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[c]) + (Sq
rt[d]*ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[c])))/d^3))/(5*d)
```

3.41.3.1 Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.41.4 Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{-\frac{2a}{5(dx)^{\frac{5}{2}}}-\frac{2b\,\operatorname{arctanh}(cx)}{5(dx)^{\frac{5}{2}}}+\frac{2b\,c^3\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5d^2\sqrt{cd}}+\frac{2b\,c^3\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5d^2\sqrt{cd}}-\frac{4bc}{15d(dx)^{\frac{3}{2}}}}{d}$	93
default	$\frac{-\frac{2a}{5(dx)^{\frac{5}{2}}}-\frac{2b\,\operatorname{arctanh}(cx)}{5(dx)^{\frac{5}{2}}}+\frac{2b\,c^3\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5d^2\sqrt{cd}}+\frac{2b\,c^3\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5d^2\sqrt{cd}}-\frac{4bc}{15d(dx)^{\frac{3}{2}}}}{d}$	93
parts	$-\frac{2a}{5(dx)^{\frac{5}{2}}d}-\frac{2b\,\operatorname{arctanh}(cx)}{5d(dx)^{\frac{5}{2}}}+\frac{2b\,c^3\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5d^3\sqrt{cd}}-\frac{4bc}{15d^2(dx)^{\frac{3}{2}}}+\frac{2b\,c^3\,\operatorname{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{5d^3\sqrt{cd}}$	94

input
```
int((a+b*arctanh(c*x))/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/5*a/(d*x)^(5/2)-1/5*b/(d*x)^(5/2)*arctanh(c*x)+1/5*b/d^2*c^3/(c*d)
^(1/2)*arctanh(c*(d*x)^(1/2)/(c*d)^(1/2))+1/5*b/d^2*c^3/(c*d)^(1/2)*arctan
(c*(d*x)^(1/2)/(c*d)^(1/2))-2/15*b/d*c/(d*x)^(3/2))
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.36

$$\int \frac{a+b\operatorname{arctanh}(cx)}{(dx)^{7/2}}\,dx = \left[-\frac{6\,bc^2dx^3\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)-3\,bc^2dx^3\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)+\left(4\,bcx+3\right.}{15\,d^4x^3}\right.$$

$$-\frac{6\,bc^2dx^3\sqrt{-\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{c}{d}}}{cx}\right)-3\,bc^2dx^3\sqrt{-\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{-\frac{c}{d}}-1}{cx+1}\right)+\left(4\,bcx+3\,b\log\left(-\frac{cx+1}{cx-1}\right)+6\,a\right)}{15\,d^4x^3}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
[-1/15*(6*b*c^2*d*x^3*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) - 3*b*c^
2*d*x^3*sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) + (4*b*
c*x + 3*b*log(-(c*x + 1)/(c*x - 1)) + 6*a)*sqrt(d*x))/(d^4*x^3), -1/15*(6*
b*c^2*d*x^3*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x)) - 3*b*c^2*d*x^3*
sqrt(-c/d)*log((c*x + 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x + 1)) + (4*b*c*x +
3*b*log(-(c*x + 1)/(c*x - 1)) + 6*a)*sqrt(d*x))/(d^4*x^3)]
```

3.41.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{7/2}}\,dx = \int \frac{a+b\,\text{atanh}\,(cx)}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*atanh(c*x))/(d*x)**(7/2),x)
```

output
```
Integral((a + b*atanh(c*x))/(d*x)**(7/2), x)
```

3.41.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{7/2}}\,dx = \frac{b\left(\frac{\left(\frac{6\,c^2\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}d}-\frac{3\,c^2\log\left(\frac{\sqrt{dx}c-\sqrt{cd}}{\sqrt{dx}c+\sqrt{cd}}\right)}{\sqrt{cd}d}-\frac{4}{(dx)^{\frac{3}{2}}}\right)c}{d}-\frac{6\,\text{artanh}(cx)}{(dx)^{\frac{5}{2}}}\right)-\frac{6\,a}{(dx)^{\frac{5}{2}}}}{15\,d}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
1/15*(b*((6*c^2*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d) - 3*c^2*log((s
qrt(d*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*d) - 4/(d*x)
^(3/2))*c/d - 6*arctanh(c*x)/(d*x)^(5/2)) - 6*a/(d*x)^(5/2))/d
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{7/2}}\,dx = \frac{6\,bc^3\left(\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}d^2}-\frac{\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}d^2}\right)-\frac{3\,b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}d^2x^2}-\frac{2\,(2\,bcdx+3\,ad)}{\sqrt{dx}d^3x^2}}{15\,d}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(7/2),x, algorithm="giac")
```

output
```
1/15*(6*b*c^3*(arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^2) - arctan(sqrt
(d*x)*c/sqrt(-c*d))/(sqrt(-c*d)*d^2)) - 3*b*log(-(c*d*x + d)/(c*d*x - d))/
(sqrt(d*x)*d^2*x^2) - 2*(2*b*c*d*x + 3*a*d)/(sqrt(d*x)*d^3*x^2))/d
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{7/2}}\,dx = \int \frac{a+b\,\text{atanh}(c\,x)}{(d\,x)^{7/2}}\,dx$$

input
```
int((a + b*atanh(c*x))/(d*x)^(7/2),x)
```

output
```
int((a + b*atanh(c*x))/(d*x)^(7/2), x)
```

3.42 $\int \frac{a+b\text{arctanh}(cx)}{(dx)^{9/2}}\,dx$

3.42.1 Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{9/2}}\,dx = -\frac{4bc}{35d^2(dx)^{5/2}}-\frac{4bc^3}{7d^4\sqrt{dx}} -\frac{2bc^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{2(a+b\text{arctanh}(cx))}{7d(dx)^{7/2}}+\frac{2bc^{7/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}$$

output
```
-4/35*b*c/d^2/(d*x)^(5/2)-2/7*b*c^(7/2)*arctan(c^(1/2)*(d*x)^(1/2)/d^(1/2)
)/d^(9/2)-2/7*(a+b*arctanh(c*x))/d/(d*x)^(7/2)+2/7*b*c^(7/2)*arctanh(c^(1/
2)*(d*x)^(1/2)/d^(1/2))/d^(9/2)-4/7*b*c^3/d^4/(d*x)^(1/2)
```

3.42.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{9/2}}\,dx = -\frac{\sqrt{dx}\left(10a+4bcx+20bc^3x^3+10bc^{7/2}x^{7/2}\arctan\left(\sqrt{c}\sqrt{x}\right)+10b\text{arctanh}(cx)+5bc^{7/2}x^{7/2}\log\left(1-\sqrt{c}\sqrt{x}\right)\right.}{35d^5x^4}$$

input
```
Integrate[(a + b*ArcTanh[c*x])/(d*x)^(9/2),x]
```

output
```
-1/35*(Sqrt[d*x]*(10*a + 4*b*c*x + 20*b*c^3*x^3 + 10*b*c^(7/2)*x^(7/2)*Arc
Tan[Sqrt[c]*Sqrt[x]] + 10*b*ArcTanh[c*x] + 5*b*c^(7/2)*x^(7/2)*Log[1 - Sqr
t[c]*Sqrt[x]] - 5*b*c^(7/2)*x^(7/2)*Log[1 + Sqrt[c]*Sqrt[x]]))/(d^5*x^4)
```

3.42.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6464, 264, 264, 266, 27, 827, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{9/2}}dx$$

$$\Big\downarrow \text{6464}$$

$$\frac{2bc\int \frac{1}{(dx)^{7/2}(1-c^2x^2)}dx}{7d}-\frac{2(a+b\text{arctanh}(cx))}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2bc\left(\frac{c^2\int \frac{1}{(dx)^{3/2}\left(1-c^2x^2\right)}dx}{d^2}-\frac{2}{5d(dx)^{5/2}}\right)}{7d}-\frac{2(a+b\text{arctanh}(cx))}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{264}$$

$$\frac{2bc\left(\frac{c^2\left(\frac{c^2\int \frac{\sqrt{dx}}{1-c^2x^2}dx}{d^2}-\frac{2}{d\sqrt{dx}}\right)}{d^2}-\frac{2}{5d(dx)^{5/2}}\right)}{7d}-\frac{2(a+b\text{arctanh}(cx))}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{266}$$

$$\frac{2bc\left(\frac{c^2\left(\frac{2c^2\int \frac{d^3x}{d^2-c^2d^2x^2}d\sqrt{dx}}{d^3}-\frac{2}{d\sqrt{dx}}\right)}{d^2}-\frac{2}{5d(dx)^{5/2}}\right)}{7d}-\frac{2(a+b\text{arctanh}(cx))}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2bc\left(\frac{c^2\left(\frac{2c^2\int \frac{dx}{\frac{d^2-c^2d^2x^2}{d}}d\sqrt{dx}}{d}-\frac{2}{d\sqrt{dx}}\right)}{d^2}-\frac{2}{5d(dx)^{5/2}}\right)}{7d}-\frac{2(a+b\text{arctanh}(cx))}{7d(dx)^{7/2}}$$

$\Big\downarrow$ 827

$$\frac{2bc\left(\frac{c^2\left(\frac{2c^2\left(\frac{\int \frac{1}{d-cdx}d\sqrt{dx}}{2c}-\frac{\int \frac{1}{cxd+d}d\sqrt{dx}}{2c}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{d^2}-\frac{2}{5d(dx)^{5/2}}\right)}{7d}-\frac{2(a+b\text{arctanh}(cx))}{7d(dx)^{7/2}}$$

$\Big\downarrow$ 218

$$\frac{2bc\left(\frac{c^2\left(\frac{2c^2\left(\frac{\int \frac{1}{d-cdx}d\sqrt{dx}}{2c}-\frac{\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{d^2}-\frac{2}{5d(dx)^{5/2}}\right)}{7d}-\frac{2(a+b\text{arctanh}(cx))}{7d(dx)^{7/2}}$$

$\Big\downarrow$ 221

$$\frac{2bc\left(\frac{c^2\left(\frac{2c^2\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/2}\sqrt{d}}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{d^2}-\frac{2}{5d(dx)^{5/2}}\right)}{7d}-\frac{2(a+b\text{arctanh}(cx))}{7d(dx)^{7/2}}$$

input
```
Int[(a + b*ArcTanh[c*x])/(d*x)^(9/2),x]
```

3.42. $\int \frac{a+b\text{arctanh}(cx)}{(dx)^{9/2}}\,dx$

output
```
(-2*(a + b*ArcTanh[c*x]))/(7*d*(d*x)^(7/2)) + (2*b*c*(-2/(5*d*(d*x)^(5/2))
 + (c^2*(-2/(d*Sqrt[d*x]) + (2*c^2*(-1/2*ArcTan[(Sqrt[c]*Sqrt[d*x])/Sqrt[d
]]/(c^(3/2)*Sqrt[d]) + ArcTanh[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]]/(2*c^(3/2)*Sqr
t[d])))/d))/d^2))/(7*d)
```

3.42.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.42.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{-\frac{2a}{7(dx)^{\frac{7}{2}}}-\frac{2b\,\text{arctanh}(cx)}{7(dx)^{\frac{7}{2}}}-\frac{4bc}{35d(dx)^{\frac{5}{2}}}-\frac{4b\,c^{3}}{7d^{3}\sqrt{dx}}+\frac{2b\,c^{4}\,\text{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7d^{3}\sqrt{cd}}-\frac{2b\,c^{4}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7d^{3}\sqrt{cd}}}{d}$	107
default	$\frac{-\frac{2a}{7(dx)^{\frac{7}{2}}}-\frac{2b\,\text{arctanh}(cx)}{7(dx)^{\frac{7}{2}}}-\frac{4bc}{35d(dx)^{\frac{5}{2}}}-\frac{4b\,c^{3}}{7d^{3}\sqrt{dx}}+\frac{2b\,c^{4}\,\text{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7d^{3}\sqrt{cd}}-\frac{2b\,c^{4}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7d^{3}\sqrt{cd}}}{d}$	107
parts	$-\frac{2a}{7(dx)^{\frac{7}{2}}d}-\frac{2b\,\text{arctanh}(cx)}{7d(dx)^{\frac{7}{2}}}-\frac{2b\,c^{4}\arctan\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7d^{4}\sqrt{cd}}-\frac{4bc}{35d^{2}(dx)^{\frac{5}{2}}}-\frac{4b\,c^{3}}{7d^{4}\sqrt{dx}}+\frac{2b\,c^{4}\,\text{arctanh}\left(\frac{c\sqrt{dx}}{\sqrt{cd}}\right)}{7d^{4}\sqrt{cd}}$	108

input
```
int((a+b*arctanh(c*x))/(d*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-1/7*a/(d*x)^(7/2)-1/7*b/(d*x)^(7/2)*arctanh(c*x)-2/35*b/d*c/(d*x)^(5
/2)-2/7*b/d^3*c^3/(d*x)^(1/2)+1/7*b/d^3*c^4/(c*d)^(1/2)*arctanh(c*(d*x)^(1
/2)/(c*d)^(1/2))-1/7*b/d^3*c^4/(c*d)^(1/2)*arctan(c*(d*x)^(1/2)/(c*d)^(1/2
)))
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.18

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{9/2}}\,dx = \left[\frac{10\,bc^3dx^4\sqrt{\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{\frac{c}{d}}}{cx}\right)+5\,bc^3dx^4\sqrt{\frac{c}{d}}\log\left(\frac{cx+2\sqrt{dx}\sqrt{\frac{c}{d}}+1}{cx-1}\right)-\left(20\,bc^3x^3+\right.}{35\,d^5x^4}\right.$$

$$-\frac{10\,bc^3dx^4\sqrt{-\frac{c}{d}}\arctan\left(\frac{\sqrt{dx}\sqrt{-\frac{c}{d}}}{cx}\right)-5\,bc^3dx^4\sqrt{-\frac{c}{d}}\log\left(\frac{cx-2\sqrt{dx}\sqrt{-\frac{c}{d}}-1}{cx+1}\right)+\left(20\,bc^3x^3+4\,bcx+5\,b\log\left(-\frac{c}{d}\right.\right.}{35\,d^5x^4}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/35*(10*b*c^3*d*x^4*sqrt(c/d)*arctan(sqrt(d*x)*sqrt(c/d)/(c*x)) + 5*b*c^
3*d*x^4*sqrt(c/d)*log((c*x + 2*sqrt(d*x)*sqrt(c/d) + 1)/(c*x - 1)) - (20*b
*c^3*x^3 + 4*b*c*x + 5*b*log(-(c*x + 1)/(c*x - 1)) + 10*a)*sqrt(d*x))/(d^5
*x^4), -1/35*(10*b*c^3*d*x^4*sqrt(-c/d)*arctan(sqrt(d*x)*sqrt(-c/d)/(c*x))
 - 5*b*c^3*d*x^4*sqrt(-c/d)*log((c*x - 2*sqrt(d*x)*sqrt(-c/d) - 1)/(c*x +
1)) + (20*b*c^3*x^3 + 4*b*c*x + 5*b*log(-(c*x + 1)/(c*x - 1)) + 10*a)*sqrt
(d*x))/(d^5*x^4)]
```

3.42.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{9/2}}\,dx=\int \frac{a+b\,\text{atanh}\,(cx)}{(dx)^{\frac{9}{2}}}\,dx$$

input
```
integrate((a+b*atanh(c*x))/(d*x)**(9/2),x)
```

output
```
Integral((a + b*atanh(c*x))/(d*x)**(9/2), x)
```

3.42.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{9/2}}\,dx = -\frac{b\left(\frac{\left(\frac{10\,c^3\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}d^2}+\frac{5\,c^3\log\left(\frac{\sqrt{dx}c-\sqrt{cd}}{\sqrt{dx}c+\sqrt{cd}}\right)}{\sqrt{cd}d^2}+\frac{4\left(5\,c^2d^2x^2+d^2\right)}{(dx)^{\frac{5}{2}}d^2}\right)c}{d}+\frac{10\,\text{artanh}(cx)}{(dx)^{\frac{7}{2}}}\right)+\frac{10\,a}{(dx)^{\frac{7}{2}}}}{35\,d}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="maxima")
```

output
```
-1/35*(b*((10*c^3*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^2) + 5*c^3*lo
g((sqrt(d*x)*c - sqrt(c*d))/(sqrt(d*x)*c + sqrt(c*d)))/(sqrt(c*d)*d^2) + 4
*(5*c^2*d^2*x^2 + d^2)/((d*x)^(5/2)*d^2))*c/d + 10*arctanh(c*x)/(d*x)^(7/2
)) + 10*a/(d*x)^(7/2))/d
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{9/2}}\,dx = -\frac{\frac{10\,bc^4\arctan\left(\frac{\sqrt{dx}c}{\sqrt{cd}}\right)}{\sqrt{cd}d^3}+\frac{10\,bc^4\arctan\left(\frac{\sqrt{dx}c}{\sqrt{-cd}}\right)}{\sqrt{-cd}d^3}+\frac{5\,b\log\left(-\frac{cdx+d}{cdx-d}\right)}{\sqrt{dx}d^3x^3}+\frac{2\,(10\,bc^3d^3x^3+2\,bcd^3x+5\,ad^3)}{\sqrt{dx}d^6x^3}}{35\,d}$$

input
```
integrate((a+b*arctanh(c*x))/(d*x)^(9/2),x, algorithm="giac")
```

output
```
-1/35*(10*b*c^4*arctan(sqrt(d*x)*c/sqrt(c*d))/(sqrt(c*d)*d^3) + 10*b*c^4*a
rctan(sqrt(d*x)*c/sqrt(-c*d))/(sqrt(-c*d)*d^3) + 5*b*log(-(c*d*x + d)/(c*d
*x - d))/(sqrt(d*x)*d^3*x^3) + 2*(10*b*c^3*d^3*x^3 + 2*b*c*d^3*x + 5*a*d^3
)/(sqrt(d*x)*d^6*x^3))/d
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx)}{(dx)^{9/2}}\,dx = \int \frac{a+b\,\text{atanh}(c\,x)}{(d\,x)^{9/2}}\,dx$$

input
```
int((a + b*atanh(c*x))/(d*x)^(9/2),x)
```

output
```
int((a + b*atanh(c*x))/(d*x)^(9/2), x)
```

3.43 $\int (dx)^m(a+b\,\mathrm{arctanh}(cx))^3\,dx$

3.43.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m(a+b\,\mathrm{arctanh}(cx))^3\,dx=\mathrm{Int}\left((dx)^m(a+b\,\mathrm{arctanh}(cx))^3,x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

3.43.2 Mathematica [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\,\mathrm{arctanh}(cx))^3\,dx=\int (dx)^m(a+b\,\mathrm{arctanh}(cx))^3\,dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^3, x]
```

3.43.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m(a+b\text{arctanh}(cx))^3\,dx$$
$$\Big\downarrow \text{6468}$$
$$\int (dx)^m(a+b\text{arctanh}(cx))^3dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x])^3,x]
```

output
```
$Aborted
```

3.43.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.43.4 Maple [N/A] (verified)

Not integrable

Time = 1.56 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m\,(a+b\,\text{arctanh}\,(cx))^3\,dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x))^3,x)
```

3.43.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

$$\int (dx)^m (a + b\text{arctanh}(cx))^3 \, dx = \int (b\,\text{artanh}\,(cx) + a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x)^3 + 3*a*b^2*arctanh(c*x)^2 + 3*a^2*b*arctanh(c*
x) + a^3)*(d*x)^m, x)
```

3.43.6 Sympy [N/A]

Not integrable

Time = 4.91 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m (a + b\text{arctanh}(cx))^3 \, dx = \int (dx)^m (a + b\,\text{atanh}\,(cx))^3 \, dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x))**3,x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c*x))**3, x)
```

3.43.7 Maxima [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 379, normalized size of antiderivative = 23.69

$$\int (dx)^m (a + b\text{arctanh}(cx))^3 \, dx = \int (b\,\text{artanh}\,(cx) + a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="maxima")
```

output
```
-1/8*b^3*d^m*x*x^m*log(-c*x + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1))
 + integrate(1/8*((b^3*c*d^m*(m + 1)*x - b^3*d^m*(m + 1))*x^m*log(c*x + 1)
^3 + 6*(a*b^2*c*d^m*(m + 1)*x - a*b^2*d^m*(m + 1))*x^m*log(c*x + 1)^2 + 12
*(a^2*b*c*d^m*(m + 1)*x - a^2*b*d^m*(m + 1))*x^m*log(c*x + 1) + 3*((b^3*c*
d^m*(m + 1)*x - b^3*d^m*(m + 1))*x^m*log(c*x + 1) - (2*a*b^2*d^m*(m + 1) -
 (2*a*b^2*c*d^m*(m + 1) + b^3*c*d^m)*x)*x^m)*log(-c*x + 1)^2 - 3*((b^3*c*d
^m*(m + 1)*x - b^3*d^m*(m + 1))*x^m*log(c*x + 1)^2 + 4*(a*b^2*c*d^m*(m + 1
)*x - a*b^2*d^m*(m + 1))*x^m*log(c*x + 1) + 4*(a^2*b*c*d^m*(m + 1)*x - a^2
*b*d^m*(m + 1))*x^m)*log(-c*x + 1))/(c*(m + 1)*x - m - 1), x)
```

3.43.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\text{arctanh}(cx))^3\,dx=\int (b\,\text{artanh}\,(cx)+a)^3(dx)^m\,\,dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^3*(d*x)^m, x)
```

3.43.9 Mupad [N/A]

Not integrable

Time = 3.89 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\text{arctanh}(cx))^3\,dx=\int (a+b\,\text{atanh}(c\,x))^3\,(d\,x)^m\,dx$$

input
```
int((a + b*atanh(c*x))^3*(d*x)^m,x)
```

output
```
int((a + b*atanh(c*x))^3*(d*x)^m, x)
```

3.44 $\int (dx)^m (a + b\text{arctanh}(cx))^2\, dx$

3.44.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m (a + b\text{arctanh}(cx))^2\, dx = \text{Int}\big((dx)^m (a + b\text{arctanh}(cx))^2, x\big)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c*x))^2,x)
```

3.44.2 Mathematica [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m (a + b\text{arctanh}(cx))^2\, dx = \int (dx)^m (a + b\text{arctanh}(cx))^2\, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^2, x]
```

3.44.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m(a+b\text{arctanh}(cx))^2 \, dx$$

$$\Big\downarrow \text{ 6468}$$

$$\int (dx)^m(a+b\text{arctanh}(cx))^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x])^2,x]
```

output
```
$Aborted
```

3.44.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.44.4 Maple [N/A] (verified)

Not integrable

Time = 1.57 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b \,\text{arctanh}\left(cx\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x))^2,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x))^2,x)
```

3.44.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int (dx)^m(a+b\text{arctanh}(cx))^2\,dx=\int (b\,\text{artanh}\,(cx)+a)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2)*(d*x)^m, x)
```

3.44.6 Sympy [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m(a+b\text{arctanh}(cx))^2\,dx=\int (dx)^m\,(a+b\,\text{atanh}\,(cx))^2\,dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x))**2,x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c*x))**2, x)
```

3.44.7 Maxima [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 218, normalized size of antiderivative = 13.62

$$\int (dx)^m(a+b\text{arctanh}(cx))^2\,dx=\int (b\,\text{artanh}\,(cx)+a)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*d^m*x*x^m*log(-c*x + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1))
- integrate(-1/4*((b^2*c*d^m*(m + 1)*x - b^2*d^m*(m + 1))*x^m*log(c*x + 1)
^2 + 4*(a*b*c*d^m*(m + 1)*x - a*b*d^m*(m + 1))*x^m*log(c*x + 1) - 2*((b^2*
c*d^m*(m + 1)*x - b^2*d^m*(m + 1))*x^m*log(c*x + 1) - (2*a*b*d^m*(m + 1) -
 (2*a*b*c*d^m*(m + 1) + b^2*c*d^m)*x)*x^m)*log(-c*x + 1))/(c*(m + 1)*x - m
 - 1), x)
```

3.44.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\text{arctanh}(cx))^2\,dx=\int (b\,\text{artanh}\,(cx)+a)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^2*(d*x)^m, x)
```

3.44.9 Mupad [N/A]

Not integrable

Time = 3.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\text{arctanh}(cx))^2\,dx=\int \left(a+b\,\text{atanh}(c\,x)\right)^2\,{\left(d\,x\right)}^m\,dx$$

input
```
int((a + b*atanh(c*x))^2*(d*x)^m,x)
```

output
```
int((a + b*atanh(c*x))^2*(d*x)^m, x)
```

3.45 $\int (dx)^m(a+b\text{arctanh}(cx))\,dx$

3.45.1 Optimal result

Integrand size = 14, antiderivative size = 72

$$\int (dx)^m(a+b\text{arctanh}(cx))\,dx=\frac{(dx)^{1+m}(a+b\text{arctanh}(cx))}{d(1+m)}-\frac{bc(dx)^{2+m}\,\text{Hypergeometric2F1}\left(1,\frac{2+m}{2},\frac{4+m}{2},c^2x^2\right)}{d^2(1+m)(2+m)}$$

output
```
(d*x)^(1+m)*(a+b*arctanh(c*x))/d/(1+m)-b*c*(d*x)^(2+m)*hypergeom([1, 1+1/2
*m],[2+1/2*m],c^2*x^2)/d^2/(1+m)/(2+m)
```

3.45.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int (dx)^m(a+b\text{arctanh}(cx))\,dx=-\frac{x(dx)^m\left(-((2+m)(a+b\text{arctanh}(cx)))+bcx\,\text{Hypergeometric2F1}\left(1,1+\frac{m}{2},2+\frac{m}{2},c^2x^2\right)\right)}{(1+m)(2+m)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x]),x]
```

output
```
-((x*(d*x)^m*(-((2 + m)*(a + b*ArcTanh[c*x])) + b*c*x*Hypergeometric2F1[1,
 1 + m/2, 2 + m/2, c^2*x^2]))/((1 + m)*(2 + m)))
```

3.45.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6464, 278}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m(a+b\text{arctanh}(cx))\,dx$$

$$\Big\downarrow \text{6464}$$

$$\frac{(dx)^{m+1}(a+b\text{arctanh}(cx))}{d(m+1)}-\frac{bc\int \frac{(dx)^{m+1}}{1-c^2x^2}dx}{d(m+1)}$$

$$\Big\downarrow \text{278}$$

$$\frac{(dx)^{m+1}(a+b\text{arctanh}(cx))}{d(m+1)}-\frac{bc(dx)^{m+2}\,\text{Hypergeometric2F1}\left(1,\frac{m+2}{2},\frac{m+4}{2},c^2x^2\right)}{d^2(m+1)(m+2)}$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcTanh[c*x]))/(d*(1 + m)) - (b*c*(d*x)^(2 + m)*Hype
rgeometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d^2*(1 + m)*(2 + m))
```

3.45.3.1 Defintions of rubi rules used

rule 278
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*((
c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/2, (m + 1)/2 + 1, (
-b)*(x^2/a)], x] /; FreeQ[{a, b, c, m, p}, x] &&  !IGtQ[p, 0] && (ILtQ[p, 0
] || GtQ[a, 0])
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.45.4 Maple [F]

$$\int (dx)^m (a + b \operatorname{arctanh}(cx)) \, dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x)),x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x)),x)
```

3.45.5 Fricas [F]

$$\int (dx)^m (a + b\text{arctanh}(cx)) \, dx = \int (b \operatorname{artanh}(cx) + a)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)*(d*x)^m, x)
```

3.45.6 Sympy [F]

$$\int (dx)^m (a + b\text{arctanh}(cx)) \, dx = \int (dx)^m (a + b \operatorname{atanh}(cx)) \, dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x)),x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c*x)), x)
```

3.45.7 Maxima [F]

$$\int (dx)^m(a + b\text{arctanh}(cx))\,dx = \int (b\,\text{artanh}\,(cx) + a)(dx)^m\;dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
1/2*(2*c*d^m*integrate(x*x^m/(c^2*(m + 1)*x^2 - m - 1), x) + (d^m*x*x^m*lo
g(c*x + 1) - d^m*x*x^m*log(-c*x + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(d*(m +
 1))
```

3.45.8 Giac [F]

$$\int (dx)^m(a + b\text{arctanh}(cx))\,dx = \int (b\,\text{artanh}\,(cx) + a)(dx)^m\;dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)*(d*x)^m, x)
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m(a + b\text{arctanh}(cx))\,dx = \int (a + b\,\text{atanh}(c\,x))\;(d\,x)^m\,dx$$

input
```
int((a + b*atanh(c*x))*(d*x)^m,x)
```

output
```
int((a + b*atanh(c*x))*(d*x)^m, x)
```

3.46 $\int \frac{(dx)^m}{a+b\,\mathbf{arctanh}(cx)}\,dx$

3.46.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{a+b\operatorname{arctanh}(cx)}\,dx=\operatorname{Int}\left(\frac{(dx)^m}{a+b\operatorname{arctanh}(cx)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctanh(c*x)),x)
```

3.46.2 Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\operatorname{arctanh}(cx)}\,dx=\int \frac{(dx)^m}{a+b\operatorname{arctanh}(cx)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x]), x]
```

3.46.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx)} \, dx$$

$$\Big\downarrow \text{ 6468}$$

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTanh[c*x]),x]
```

output
```
$Aborted
```

3.46.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.46.4 Maple [N/A] (verified)

Not integrable

Time = 1.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b \,\operatorname{arctanh}\left(c x \right)}d x$$

input
```
int((d*x)^m/(a+b*arctanh(c*x)),x)
```

output
```
int((d*x)^m/(a+b*arctanh(c*x)),x)
```

3.46.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx)}\,dx = \int \frac{(dx)^m}{b\,\text{artanh}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctanh(c*x) + a), x)
```

3.46.6 Sympy [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx)}\,dx = \int \frac{(dx)^m}{a+b\,\text{atanh}\,(cx)}\,dx$$

input
```
integrate((d*x)**m/(a+b*atanh(c*x)),x)
```

output
```
Integral((d*x)**m/(a + b*atanh(c*x)), x)
```

3.46.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx)}\,dx = \int \frac{(dx)^m}{b\,\text{artanh}\,(cx)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x) + a), x)
```

3.46.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\text{arctanh}(cx)}\,dx = \int \frac{(dx)^m}{b\,\text{artanh}\,(cx) + a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x) + a), x)
```

3.46.9 Mupad [N/A]

Not integrable

Time = 3.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{a + b\text{arctanh}(cx)}\,dx = \int \frac{(d\,x)^m}{a + b\,\text{atanh}\,(c\,x)}\,dx$$

input
```
int((d*x)^m/(a + b*atanh(c*x)),x)
```

output
```
int((d*x)^m/(a + b*atanh(c*x)), x)
```

3.47 $\int \frac{(dx)^m}{(a+b\,\mathrm{arctanh}(cx))^2}\,dx$

3.47.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(dx)^m}{(a+b\,\mathrm{arctanh}(cx))^2}\,dx=\mathrm{Int}\left(\frac{(dx)^m}{(a+b\,\mathrm{arctanh}(cx))^2},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctanh(c*x))^2,x)
```

3.47.2 Mathematica [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{(a+b\,\mathrm{arctanh}(cx))^2}\,dx=\int \frac{(dx)^m}{(a+b\,\mathrm{arctanh}(cx))^2}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x])^2, x]
```

3.47.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx))^2} \, dx$$

$$\Big\downarrow \; 6468$$

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx))^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTanh[c*x])^2,x]
```

output
```
$Aborted
```

3.47.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.47.4 Maple [N/A] (verified)

Not integrable

Time = 1.63 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b \operatorname{arctanh}(cx))^2}dx$$

input
```
int((d*x)^m/(a+b*arctanh(c*x))^2,x)
```

output
```
int((d*x)^m/(a+b*arctanh(c*x))^2,x)
```

3.47.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\,\text{artanh}\,(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arctanh(c*x)^2 + 2*a*b*arctanh(c*x) + a^2), x)
```

3.47.6 Sympy [N/A]

Not integrable

Time = 8.43 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx))^2}\,dx = \int \frac{(dx)^m}{(a+b\,\text{atanh}\,(cx))^2}\,dx$$

input
```
integrate((d*x)**m/(a+b*atanh(c*x))**2,x)
```

output
```
Integral((d*x)**m/(a + b*atanh(c*x))**2, x)
```

3.47.7 Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 116, normalized size of antiderivative = 7.25

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\,\text{artanh}\,(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="maxima")
```

output
```
2*(c^2*d^m*x^2 - d^m)*x^m/(b^2*c*log(c*x + 1) - b^2*c*log(-c*x + 1) + 2*a*
b*c) + integrate(-2*(c^2*d^m*(m + 2)*x^2 - d^m*m)*x^m/(b^2*c*x*log(c*x + 1
) - b^2*c*x*log(-c*x + 1) + 2*a*b*c*x), x)
```

3.47.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx))^2}\,dx = \int \frac{(dx)^m}{(b\,\text{artanh}\,(cx)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x) + a)^2, x)
```

3.47.9 Mupad [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx))^2}\,dx = \int \frac{(d\,x)^m}{(a+b\,\text{atanh}\,(c\,x))^2}\,dx$$

input
```
int((d*x)^m/(a + b*atanh(c*x))^2,x)
```

output
```
int((d*x)^m/(a + b*atanh(c*x))^2, x)
```

3.48 $\int (a + b\mathbf{arctanh}(cx))^p \, dx$

3.48.1 Optimal result

Integrand size = 10, antiderivative size = 10

$$\int (a + b\text{arctanh}(cx))^p \, dx = \text{Int}((a + b\text{arctanh}(cx))^p, x)$$

output
```
Unintegrable((a+b*arctanh(c*x))^p,x)
```

3.48.2 Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a + b\text{arctanh}(cx))^p \, dx = \int (a + b\text{arctanh}(cx))^p \, dx$$

input
```
Integrate[(a + b*ArcTanh[c*x])^p,x]
```

output
```
Integrate[(a + b*ArcTanh[c*x])^p, x]
```

3.48.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6444}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\text{arctanh}(cx))^p \, dx$$

$$\Big\downarrow \text{ 6444}$$

$$\int (a + b\text{arctanh}(cx))^p dx$$

input
```
Int[(a + b*ArcTanh[c*x])^p,x]
```

output
```
$Aborted
```

3.48.3.1 Defintions of rubi rules used

rule 6444
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrab
le[(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.48.4 Maple [N/A] (verified)

Not integrable

Time = 0.42 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a + b \operatorname{arctanh}(cx))^p \, dx$$

input
```
int((a+b*arctanh(c*x))^p,x)
```

output
```
int((a+b*arctanh(c*x))^p,x)
```

3.48.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\text{arctanh}(cx))^p\,dx=\int (b\,\text{artanh}\,(cx)+a)^p\,dx$$

input
```
integrate((a+b*arctanh(c*x))^p,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x) + a)^p, x)
```

3.48.6 Sympy [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int (a+b\text{arctanh}(cx))^p\,dx=\int (a+b\,\text{atanh}\,(cx))^p\ dx$$

input
```
integrate((a+b*atanh(c*x))**p,x)
```

output
```
Integral((a + b*atanh(c*x))**p, x)
```

3.48.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a+b\text{arctanh}(cx))^p\,dx=\int (b\,\text{artanh}\,(cx)+a)^p\,dx$$

input
```
integrate((a+b*arctanh(c*x))^p,x, algorithm="maxima")
```

output
```
integrate((b*arctanh(c*x) + a)^p, x)
```

3.48.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a + b\text{arctanh}(cx))^p \, dx = \int (b \,\text{artanh}\,(cx) + a)^p \, dx$$

input
```
integrate((a+b*arctanh(c*x))^p,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x) + a)^p, x)
```

3.48.9 Mupad [N/A]

Not integrable

Time = 3.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int (a + b\text{arctanh}(cx))^p \, dx = \int (a + b \,\text{atanh}(c\,x))^p \, dx$$

input
```
int((a + b*atanh(c*x))^p,x)
```

output
```
int((a + b*atanh(c*x))^p, x)
```

3.49 $\int (dx)^m(a+b\text{arctanh}(cx))^p\,dx$

3.49.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (dx)^m(a+b\text{arctanh}(cx))^p\,dx=\text{Int}\left((dx)^m(a+b\text{arctanh}(cx))^p,x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c*x))^p,x)
```

3.49.2 Mathematica [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\text{arctanh}(cx))^p\,dx=\int (dx)^m(a+b\text{arctanh}(cx))^p\,dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^p,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x])^p, x]
```

3.49.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m (a + b\text{arctanh}(cx))^p \, dx$$
$$\Big\downarrow \; 6468$$
$$\int (dx)^m (a + b\text{arctanh}(cx))^p dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x])^p,x]
```

output
```
$Aborted
```

3.49.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.49.4 Maple [N/A] (verified)

Not integrable

Time = 0.84 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx)^m (a + b \operatorname{arctanh}(cx))^p \, dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x))^p,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x))^p,x)
```

3.49.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m (a + b\text{arctanh}(cx))^p \, dx = \int (dx)^m (b \operatorname{artanh}(cx) + a)^p \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="fricas")
```

output
```
integral((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

3.49.6 Sympy [N/A]

Not integrable

Time = 130.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (dx)^m (a + b\text{arctanh}(cx))^p \, dx = \int (dx)^m (a + b \operatorname{atanh}(cx))^p \, dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x))**p,x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c*x))**p, x)
```

3.49.7 Maxima [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m (a + b\text{arctanh}(cx))^p \, dx = \int (dx)^m (b \operatorname{artanh}(cx) + a)^p \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="maxima")
```

output
```
integrate((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

3.49.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\text{arctanh}(cx))^p\,dx=\int (dx)^m\,(b\,\text{artanh}\,(cx)+a)^p\,dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x))^p,x, algorithm="giac")
```

output
```
integrate((d*x)^m*(b*arctanh(c*x) + a)^p, x)
```

3.49.9 Mupad [N/A]

Not integrable

Time = 3.60 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (dx)^m(a+b\text{arctanh}(cx))^p\,dx=\int (a+b\,\text{atanh}(c\,x))^p\,(d\,x)^m\,dx$$

input
```
int((a + b*atanh(c*x))^p*(d*x)^m,x)
```

output
```
int((a + b*atanh(c*x))^p*(d*x)^m, x)
```

3.50 $\int x^7(a + b\text{arctanh}(cx^2))\,dx$

3.50.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^7\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{bx^2}{8c^3}+\frac{bx^6}{24c}-\frac{b\text{arctanh}(cx^2)}{8c^4}+\frac{1}{8}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)$$

output
```
1/8*b*x^2/c^3+1/24*b*x^6/c-1/8*b*arctanh(c*x^2)/c^4+1/8*x^8*(a+b*arctanh(c
*x^2))
```

3.50.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int x^7\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{bx^2}{8c^3}+\frac{bx^6}{24c}+\frac{ax^8}{8}+\frac{1}{8}bx^8\text{arctanh}\left(cx^2\right)+\frac{b\log\left(1-cx^2\right)}{16c^4}-\frac{b\log\left(1+cx^2\right)}{16c^4}$$

input
```
Integrate[x^7*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(b*x^2)/(8*c^3) + (b*x^6)/(24*c) + (a*x^8)/8 + (b*x^8*ArcTanh[c*x^2])/8 +
(b*Log[1 - c*x^2])/(16*c^4) - (b*Log[1 + c*x^2])/(16*c^4)
```

3.50.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 807, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a + b\text{arctanh}(cx^2))\, dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{8}x^8(a + b\text{arctanh}(cx^2)) - \frac{1}{4}bc\int \frac{x^9}{1-c^2x^4}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{8}x^8(a + b\text{arctanh}(cx^2)) - \frac{1}{8}bc\int \frac{x^8}{1-c^2x^4}dx^2$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{8}x^8(a + b\text{arctanh}(cx^2)) - \frac{1}{8}bc\int \left(-\frac{x^4}{c^2} + \frac{1}{c^4(1-c^2x^4)} - \frac{1}{c^4}\right)dx^2$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{8}x^8(a + b\text{arctanh}(cx^2)) - \frac{1}{8}bc\left(\frac{\text{arctanh}(cx^2)}{c^5} - \frac{x^2}{c^4} - \frac{x^6}{3c^2}\right)$$

input
```
Int[x^7*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(x^8*(a + b*ArcTanh[c*x^2]))/8 - (b*c*(-(x^2/c^4) - x^6/(3*c^2) + ArcTanh[
c*x^2]/c^5))/8
```

3.50.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.50.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$-\frac{-3 b \,\operatorname{arctanh}\left(c \,x^{2}\right) x^{8} c^{4}-3 a \,c^{4} x^{8}-b \,c^{3} x^{6}-3 b c \,x^{2}+3 b \,\operatorname{arctanh}\left(c \,x^{2}\right)}{24 c^{4}}$	56
default	$\frac{a \,x^{8}}{8}+\frac{b \,x^{8} \operatorname{arctanh}\left(c \,x^{2}\right)}{8}+\frac{b \,x^{6}}{24 c}+\frac{b \,x^{2}}{8 c^{3}}-\frac{b \ln \left(c \,x^{2}+1\right)}{16 c^{4}}+\frac{b \ln \left(c \,x^{2}-1\right)}{16 c^{4}}$	66
parts	$\frac{a \,x^{8}}{8}+\frac{b \,x^{8} \operatorname{arctanh}\left(c \,x^{2}\right)}{8}+\frac{b \,x^{6}}{24 c}+\frac{b \,x^{2}}{8 c^{3}}-\frac{b \ln \left(c \,x^{2}+1\right)}{16 c^{4}}+\frac{b \ln \left(c \,x^{2}-1\right)}{16 c^{4}}$	66
risch	$\frac{x^{8} b \ln \left(c \,x^{2}+1\right)}{16}-\frac{x^{8} b \ln \left(-c \,x^{2}+1\right)}{16}+\frac{a \,x^{8}}{8}+\frac{b \,x^{6}}{24 c}+\frac{b \,x^{2}}{8 c^{3}}+\frac{b \ln \left(c \,x^{2}-1\right)}{16 c^{4}}-\frac{b \ln \left(c \,x^{2}+1\right)}{16 c^{4}}$	83

input
```
int(x^7*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-3*b*arctanh(c*x^2)*x^8*c^4-3*a*c^4*x^8-b*c^3*x^6-3*b*c*x^2+3*b*arc
tanh(c*x^2))/c^4
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int x^7\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{6\,ac^4x^8+2\,bc^3x^6+6\,bcx^2+3\,\left(bc^4x^8-b\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{48\,c^4}$$

input
```
integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
1/48*(6*a*c^4*x^8 + 2*b*c^3*x^6 + 6*b*c*x^2 + 3*(b*c^4*x^8 - b)*log(-(c*x^
2 + 1)/(c*x^2 - 1)))/c^4
```

3.50.6 Sympy [A] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.07

$$\int x^7\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\begin{cases}\frac{ax^8}{8}+\frac{bx^8\,\text{atanh}\left(cx^2\right)}{8}+\frac{bx^6}{24c}+\frac{bx^2}{8c^3}-\frac{b\,\text{atanh}\left(cx^2\right)}{8c^4} & \text{for}\ c\neq 0\\ \frac{ax^8}{8} & \text{otherwise}\end{cases}$$

input
```
integrate(x**7*(a+b*atanh(c*x**2)),x)
```

output
```
Piecewise((a*x**8/8 + b*x**8*atanh(c*x**2)/8 + b*x**6/(24*c) + b*x**2/(8*c
**3) - b*atanh(c*x**2)/(8*c**4), Ne(c, 0)), (a*x**8/8, True))
```

3.50.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\begin{aligned}\int x^7\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx\\ &=\frac{1}{8}\,ax^8\\ &\quad+\frac{1}{48}\,\left(6\,x^8\,\text{artanh}\left(cx^2\right)+c\left(\frac{2\,\left(c^2x^6+3\,x^2\right)}{c^4}-\frac{3\,\log\left(cx^2+1\right)}{c^5}+\frac{3\,\log\left(cx^2-1\right)}{c^5}\right)\right)b\end{aligned}$$

input
```
integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
1/8*a*x^8 + 1/48*(6*x^8*arctanh(c*x^2) + c*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*lo
g(c*x^2 + 1)/c^5 + 3*log(c*x^2 - 1)/c^5))*b
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int x^7\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{1}{16}\,bx^8\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{8}\,ax^8+\frac{bx^6}{24\,c}+\frac{bx^2}{8\,c^3}-\frac{b\log\left(cx^2+1\right)}{16\,c^4}+\frac{b\log\left(cx^2-1\right)}{16\,c^4}$$

input
```
integrate(x^7*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
1/16*b*x^8*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/8*a*x^8 + 1/24*b*x^6/c + 1/8*
b*x^2/c^3 - 1/16*b*log(c*x^2 + 1)/c^4 + 1/16*b*log(c*x^2 - 1)/c^4
```

3.50.9 Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\int x^7\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{a\,x^8}{8}+\frac{b\,x^2}{8\,c^3}+\frac{b\,x^6}{24\,c}+\frac{b\,x^8\,\ln\left(c\,x^2+1\right)}{16}-\frac{b\,x^8\,\ln\left(1-c\,x^2\right)}{16}+\frac{b\,\text{atan}(c\,x^2\,1\text{i})\,1\text{i}}{8\,c^4}$$

input
```
int(x^7*(a + b*atanh(c*x^2)),x)
```

output
```
(a*x^8)/8 + (b*x^2)/(8*c^3) + (b*x^6)/(24*c) + (b*atan(c*x^2*1i)*1i)/(8*c^
4) + (b*x^8*log(c*x^2 + 1))/16 - (b*x^8*log(1 - c*x^2))/16
```

3.51 $\int x^5(a + b\text{arctanh}(cx^2))\, dx$

3.51.1 Optimal result

Integrand size = 14, antiderivative size = 48

$$\int x^5\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{bx^4}{12c}+\frac{1}{6}x^6\left(a+b\text{arctanh}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{12c^3}$$

output
```
1/12*b*x^4/c+1/6*x^6*(a+b*arctanh(c*x^2))+1/12*b*ln(-c^2*x^4+1)/c^3
```

3.51.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int x^5\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{bx^4}{12c}+\frac{ax^6}{6}+\frac{1}{6}bx^6\text{arctanh}\left(cx^2\right)+\frac{b\log\left(1-c^2x^4\right)}{12c^3}$$

input
```
Integrate[x^5*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(b*x^4)/(12*c) + (a*x^6)/6 + (b*x^6*ArcTanh[c*x^2])/6 + (b*Log[1 - c^2*x^4
])/(12*c^3)
```

3.51.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a+b\text{arctanh}(cx^2))\, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{6}x^6 (a+b\text{arctanh}(cx^2))-\frac{1}{3}bc\int \frac{x^7}{1-c^2x^4}dx$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{6}x^6 (a+b\text{arctanh}(cx^2))-\frac{1}{12}bc\int \frac{x^4}{1-c^2x^4}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{6}x^6 (a+b\text{arctanh}(cx^2))-\frac{1}{12}bc\int \left(-\frac{1}{c^2}-\frac{1}{c^2\,(c^2x^4-1)}\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6 (a+b\text{arctanh}(cx^2))-\frac{1}{12}bc\left(-\frac{x^4}{c^2}-\frac{\log\left(1-c^2x^4\right)}{c^4}\right)$$

input
```
Int[x^5*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(x^6*(a + b*ArcTanh[c*x^2]))/6 - (b*c*(-(x^4/c^2) - Log[1 - c^2*x^4]/c^4))
/12
```

3.51.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.51.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6}\operatorname{arctanh}\left(c\,x^{2}\right)}{6}+\frac{b\,x^{4}}{12c}+\frac{b\ln\left(c^{2}x^{4}-1\right)}{12c^{3}}$	45
parts	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6}\operatorname{arctanh}\left(c\,x^{2}\right)}{6}+\frac{b\,x^{4}}{12c}+\frac{b\ln\left(c^{2}x^{4}-1\right)}{12c^{3}}$	45
parallelrisch	$\frac{2b\operatorname{arctanh}\left(c\,x^{2}\right)x^{6}c^{3}+2a\,c^{3}x^{6}+b\,c^{2}x^{4}+2b\ln\left(c\,x^{2}-1\right)+2b\operatorname{arctanh}\left(c\,x^{2}\right)}{12c^{3}}$	59
risch	$\frac{b\,x^{6}\ln\left(c\,x^{2}+1\right)}{12}-\frac{b\,x^{6}\ln\left(-c\,x^{2}+1\right)}{12}+\frac{a\,x^{6}}{6}+\frac{b\,x^{4}}{12c}+\frac{b\ln\left(c^{2}x^{4}-1\right)}{12c^{3}}$	62

input
```
int(x^5*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/6*a*x^6+1/6*b*x^6*arctanh(c*x^2)+1/12*b*x^4/c+1/12*b/c^3*ln(c^2*x^4-1)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int x^5(a+b\text{arctanh}(cx^2))\,dx=\frac{bc^3x^6\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ac^3x^6+bc^2x^4+b\log\left(c^2x^4-1\right)}{12\,c^3}$$

input
```
integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
1/12*(b*c^3*x^6*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^3*x^6 + b*c^2*x^4 +
b*log(c^2*x^4 - 1))/c^3
```

3.51.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 85 vs. 2(39) = 78.

Time = 4.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.77

$$\int x^5(a+b\text{arctanh}(cx^2))\,dx$$
$$=\begin{cases}\frac{ax^6}{6}+\frac{bx^6\,\text{atanh}\,(cx^2)}{6}+\frac{bx^4}{12c}+\frac{b\log\left(x-\sqrt{-\frac{1}{c}}\right)}{6c^3}+\frac{b\log\left(x+\sqrt{-\frac{1}{c}}\right)}{6c^3}-\frac{b\,\text{atanh}\,(cx^2)}{6c^3} & \text{for } c\neq 0\\ \frac{ax^6}{6} & \text{otherwise}\end{cases}$$

input
```
integrate(x**5*(a+b*atanh(c*x**2)),x)
```

output
```
Piecewise((a*x**6/6 + b*x**6*atanh(c*x**2)/6 + b*x**4/(12*c) + b*log(x - s
qrt(-1/c))/(6*c**3) + b*log(x + sqrt(-1/c))/(6*c**3) - b*atanh(c*x**2)/(6*
c**3), Ne(c, 0)), (a*x**6/6, True))
```

3.51.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int x^5(a+b\text{arctanh}(cx^2))\,dx=\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\,\text{artanh}\,(cx^2)+\left(\frac{x^4}{c^2}+\frac{\log\left(c^2x^4-1\right)}{c^4}\right)c\right)b$$

input
```
integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/12*(2*x^6*arctanh(c*x^2) + (x^4/c^2 + log(c^2*x^4 - 1)/c^4)*
c)*b
```

3.51.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int x^5\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{1}{12}\,bx^6\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{6}\,ax^6+\frac{bx^4}{12\,c}+\frac{b\log\left(c^2x^4-1\right)}{12\,c^3}$$

input
```
integrate(x^5*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
1/12*b*x^6*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/6*a*x^6 + 1/12*b*x^4/c + 1/12
*b*log(c^2*x^4 - 1)/c^3
```

3.51.9 Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int x^5\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{a\,x^6}{6}+\frac{b\,\ln\left(c^2\,x^4-1\right)}{12\,c^3}+\frac{b\,x^4}{12\,c}+\frac{b\,x^6\,\ln\left(c\,x^2+1\right)}{12}-\frac{b\,x^6\,\ln\left(1-c\,x^2\right)}{12}$$

input
```
int(x^5*(a + b*atanh(c*x^2)),x)
```

output
```
(a*x^6)/6 + (b*log(c^2*x^4 - 1))/(12*c^3) + (b*x^4)/(12*c) + (b*x^6*log(c*
x^2 + 1))/12 - (b*x^6*log(1 - c*x^2))/12
```

3.52 $\int x^3(a + b\mathbf{arctanh}(cx^2))\,dx$

3.52.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^3(a + b\text{arctanh}(cx^2))\,dx = \frac{bx^2}{4c} - \frac{b\text{arctanh}(cx^2)}{4c^2} + \frac{1}{4}x^4(a + b\text{arctanh}(cx^2))$$

output
```
1/4*b*x^2/c-1/4*b*arctanh(c*x^2)/c^2+1/4*x^4*(a+b*arctanh(c*x^2))
```

3.52.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int x^3(a + b\text{arctanh}(cx^2))\,dx = \frac{bx^2}{4c} + \frac{ax^4}{4} + \frac{1}{4}bx^4\text{arctanh}(cx^2) + \frac{b\log(1 - cx^2)}{8c^2} - \frac{b\log(1 + cx^2)}{8c^2}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(b*x^2)/(4*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*x^2])/4 + (b*Log[1 - c*x^2])/
(8*c^2) - (b*Log[1 + c*x^2])/(8*c^2)
```

3.52.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 807, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\text{arctanh}(cx^2))\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^2))-\frac{1}{2}bc\int \frac{x^5}{1-c^2x^4}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^2))-\frac{1}{4}bc\int \frac{x^4}{1-c^2x^4}dx^2$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^2))-\frac{1}{4}bc\left(\frac{\int \frac{1}{1-c^2x^4}dx^2}{c^2}-\frac{x^2}{c^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^2))-\frac{1}{4}bc\left(\frac{\text{arctanh}(cx^2)}{c^3}-\frac{x^2}{c^2}\right)$$

input
```
Int[x^3*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(x^4*(a + b*ArcTanh[c*x^2]))/4 - (b*c*(-(x^2/c^2) + ArcTanh[c*x^2]/c^3))/4
```

3.52.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.52.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

method	result	size
parallelrisch	$-\frac{-\operatorname{arctanh}\left(c\,x^{2}\right) b\,c^{2} x^{4}-a\,c^{2} x^{4}-b c\,x^{2}+b \,\operatorname{arctanh}\left(c\,x^{2}\right)}{4 c^{2}}$	46
default	$\frac{a\,x^{4}}{4}+\frac{b\,x^{4} \operatorname{arctanh}\left(c\,x^{2}\right)}{4}+\frac{b\,x^{2}}{4 c}-\frac{b \ln \left(c\,x^{2}+1\right)}{8 c^{2}}+\frac{b \ln \left(c\,x^{2}-1\right)}{8 c^{2}}$	57
parts	$\frac{a\,x^{4}}{4}+\frac{b\,x^{4} \operatorname{arctanh}\left(c\,x^{2}\right)}{4}+\frac{b\,x^{2}}{4 c}-\frac{b \ln \left(c\,x^{2}+1\right)}{8 c^{2}}+\frac{b \ln \left(c\,x^{2}-1\right)}{8 c^{2}}$	57
risch	$\frac{b\,x^{4} \ln \left(c\,x^{2}+1\right)}{8}-\frac{b\,x^{4} \ln \left(-c\,x^{2}+1\right)}{8}+\frac{a\,x^{4}}{4}+\frac{b\,x^{2}}{4 c}+\frac{b \ln \left(c\,x^{2}-1\right)}{8 c^{2}}-\frac{b \ln \left(c\,x^{2}+1\right)}{8 c^{2}}+\frac{b^{2}}{16 a\,c^{2}}$	85

input
```
int(x^3*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-arctanh(c*x^2)*b*c^2*x^4-a*c^2*x^4-b*c*x^2+b*arctanh(c*x^2))/c^2
```

3.52.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int x^3(a+b\operatorname{arctanh}(cx^2))\,dx=\frac{2\,ac^2x^4+2\,bcx^2+(bc^2x^4-b)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{8\,c^2}$$

input
```
integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
1/8*(2*a*c^2*x^4 + 2*b*c*x^2 + (b*c^2*x^4 - b)*log(-(c*x^2 + 1)/(c*x^2 - 1
)))/c^2
```

3.52.6 Sympy [A] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int x^3(a+b\operatorname{arctanh}(cx^2))\,dx=\begin{cases}\frac{ax^4}{4}+\frac{bx^4\operatorname{atanh}(cx^2)}{4}+\frac{bx^2}{4c}-\frac{b\operatorname{atanh}(cx^2)}{4c^2} & \text{for } c\neq 0\\ \frac{ax^4}{4} & \text{otherwise}\end{cases}$$

input
```
integrate(x**3*(a+b*atanh(c*x**2)),x)
```

output
```
Piecewise((a*x**4/4 + b*x**4*atanh(c*x**2)/4 + b*x**2/(4*c) - b*atanh(c*x*
*2)/(4*c**2), Ne(c, 0)), (a*x**4/4, True))
```

3.52.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int x^3(a+b\operatorname{arctanh}(cx^2))\,dx$$
$$=\frac{1}{4}\,ax^4+\frac{1}{8}\left(2\,x^4\operatorname{artanh}\left(cx^2\right)+c\left(\frac{2\,x^2}{c^2}-\frac{\log\left(cx^2+1\right)}{c^3}+\frac{\log\left(cx^2-1\right)}{c^3}\right)\right)b$$

input
```
integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/8*(2*x^4*arctanh(c*x^2) + c*(2*x^2/c^2 - log(c*x^2 + 1)/c^3
+ log(c*x^2 - 1)/c^3))*b
```

3.52.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(37) = 74.

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.21

$$\int x^3\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)\,dx = \frac{1}{2}\,c\left(\frac{(cx^2+1)b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{\left(\frac{(cx^2+1)^2c^3}{(cx^2-1)^2}-\frac{2\,(cx^2+1)c^3}{cx^2-1}+c^3\right)(cx^2-1)}+\frac{\frac{2\,(cx^2+1)a}{cx^2-1}+\frac{(cx^2+1)b}{cx^2-1}-b}{\frac{(cx^2+1)^2c^3}{(cx^2-1)^2}-\frac{2\,(cx^2+1)c^3}{cx^2-1}+c^3}\right)$$

input
```
integrate(x^3*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
1/2*c*((c*x^2 + 1)*b*log(-(c*x^2 + 1)/(c*x^2 - 1))/(((c*x^2 + 1)^2*c^3/(c*
x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) + c^3)*(c*x^2 - 1)) + (2*(c*x^2
 + 1)*a/(c*x^2 - 1) + (c*x^2 + 1)*b/(c*x^2 - 1) - b)/((c*x^2 + 1)^2*c^3/(c
*x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) + c^3))
```

3.52.9 Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int x^3\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)\,dx = \frac{a\,x^4}{4}+\frac{b\,x^2}{4\,c}+\frac{b\,x^4\,\ln\left(c\,x^2+1\right)}{8}-\frac{b\,x^4\,\ln\left(1-c\,x^2\right)}{8}+\frac{b\,\mathrm{atan}(c\,x^2\,1\mathrm{i})\,1\mathrm{i}}{4\,c^2}$$

input
```
int(x^3*(a + b*atanh(c*x^2)),x)
```

output
```
(a*x^4)/4 + (b*x^2)/(4*c) + (b*atan(c*x^2*1i)*1i)/(4*c^2) + (b*x^4*log(c*x
^2 + 1))/8 - (b*x^4*log(1 - c*x^2))/8
```

3.53 $\int x(a + b\text{arctanh}(cx^2))\, dx$

3.53.1 Optimal result

Integrand size = 12, antiderivative size = 37

$$\int x(a + b\text{arctanh}(cx^2))\, dx = \frac{1}{2}x^2(a + b\text{arctanh}(cx^2)) + \frac{b\log(1 - c^2x^4)}{4c}$$

output
```
1/2*x^2*(a+b*arctanh(c*x^2))+1/4*b*ln(-c^2*x^4+1)/c
```

3.53.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x(a + b\text{arctanh}(cx^2))\, dx = \frac{ax^2}{2} + \frac{1}{2}bx^2\text{arctanh}(cx^2) + \frac{b\log(1 - c^2x^4)}{4c}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(a*x^2)/2 + (b*x^2*ArcTanh[c*x^2])/2 + (b*Log[1 - c^2*x^4])/(4*c)
```

3.53.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6452, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}x^2\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)-bc\int \frac{x^3}{1-c^2x^4}dx$$

$$\Big\downarrow \text{ 792}$$

$$\frac{1}{2}x^2\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)+\frac{b\log\left(1-c^2x^4\right)}{4c}$$

input
```
Int[x*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(x^2*(a + b*ArcTanh[c*x^2]))/2 + (b*Log[1 - c^2*x^4])/(4*c)
```

3.53.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.53.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\,x^{2}\operatorname{arctanh}\left(c\,x^{2}\right)}{2}+\frac{b\ln\left(-c^{2}x^{4}+1\right)}{4c}$	37
derivativedivides	$\frac{ac\,x^{2}+b\left(c\,x^{2}\operatorname{arctanh}\left(c\,x^{2}\right)+\frac{\ln\left(-c^{2}x^{4}+1\right)}{2}\right)}{2c}$	40
default	$\frac{ac\,x^{2}+b\left(c\,x^{2}\operatorname{arctanh}\left(c\,x^{2}\right)+\frac{\ln\left(-c^{2}x^{4}+1\right)}{2}\right)}{2c}$	40
parallelrisch	$\frac{b\ \operatorname{arctanh}\left(c\,x^{2}\right)x^{2}c+ac\,x^{2}+b\ln\left(c\,x^{2}-1\right)+b\ \operatorname{arctanh}\left(c\,x^{2}\right)}{2c}$	43
risch	$\frac{b\,x^{2}\ln\left(c\,x^{2}+1\right)}{4}-\frac{b\,x^{2}\ln\left(-c\,x^{2}+1\right)}{4}+\frac{a\,x^{2}}{2}+\frac{b\ln\left(c^{2}x^{4}-1\right)}{4c}$	53

input
```
int(x*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/2*b*x^2*arctanh(c*x^2)+1/4*b*ln(-c^2*x^4+1)/c
```

3.53.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int x(a+b\operatorname{arctanh}(cx^2))\ dx=\frac{bcx^2\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,acx^2+b\log\left(c^2x^4-1\right)}{4\,c}$$

input
```
integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
1/4*(b*c*x^2*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x^2 + b*log(c^2*x^4 - 1
))/c
```

3.53.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(29) = 58$.

Time = 2.59 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.92

$$\int x\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)\,dx = \begin{cases} \frac{ax^2}{2}+\frac{bx^2\operatorname{atanh}\left(cx^2\right)}{2}+\frac{b\log\left(x-\sqrt{-\frac{1}{c}}\right)}{2c}+\frac{b\log\left(x+\sqrt{-\frac{1}{c}}\right)}{2c}-\frac{b\operatorname{atanh}\left(cx^2\right)}{2c} & \text{for } c\neq 0 \\ \frac{ax^2}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(x*(a+b*atanh(c*x**2)),x)
```

output
```
Piecewise((a*x**2/2 + b*x**2*atanh(c*x**2)/2 + b*log(x - sqrt(-1/c))/(2*c)
 + b*log(x + sqrt(-1/c))/(2*c) - b*atanh(c*x**2)/(2*c), Ne(c, 0)), (a*x**2
/2, True))
```

3.53.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)\,dx = \frac{1}{2}\,ax^2+\frac{\left(2\,cx^2\operatorname{artanh}\left(cx^2\right)+\log\left(-c^2x^4+1\right)\right)b}{4\,c}$$

input
```
integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/4*(2*c*x^2*arctanh(c*x^2) + log(-c^2*x^4 + 1))*b/c
```

3.53.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(33) = 66$.

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 5.08

$$\int x\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)\,dx$$
$$=\frac{1}{2}\,ax^2+\frac{1}{2}\,bc\left(\frac{\log\left(\frac{|-cx^2-1|}{|cx^2-1|}\right)}{c^2}-\frac{\log\left(\left|-\frac{cx^2+1}{cx^2-1}+1\right|\right)}{c^2}+\frac{\log\left(-\frac{\frac{c\left(\frac{cx^2+1}{cx^2-1}+1\right)}{\frac{(cx^2+1)c}{cx^2-1}-c}+1}{\frac{c\left(\frac{cx^2+1}{cx^2-1}+1\right)}{\frac{(cx^2+1)c}{cx^2-1}-c}-1}\right)}{c^2\left(\frac{cx^2+1}{cx^2-1}-1\right)}\right)$$

input
```
integrate(x*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
1/2*a*x^2 + 1/2*b*c*(log(abs(-c*x^2 - 1)/abs(c*x^2 - 1))/c^2 - log(abs(-(c
*x^2 + 1)/(c*x^2 - 1) + 1))/c^2 + log(-(c*((c*x^2 + 1)/(c*x^2 - 1) + 1)/((
c*x^2 + 1)*c/(c*x^2 - 1) - c) + 1)/(c*((c*x^2 + 1)/(c*x^2 - 1) + 1)/((c*x^
2 + 1)*c/(c*x^2 - 1) - c) - 1))/(c^2*((c*x^2 + 1)/(c*x^2 - 1) - 1)))
```

3.53.9 Mupad [B] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int x\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)\,dx=\frac{a\,x^2}{2}+\frac{b\,\ln\left(c^2\,x^4-1\right)}{4\,c}+\frac{b\,x^2\,\ln\left(c\,x^2+1\right)}{4}-\frac{b\,x^2\,\ln\left(1-c\,x^2\right)}{4}$$

input
```
int(x*(a + b*atanh(c*x^2)),x)
```

output
```
(a*x^2)/2 + (b*log(c^2*x^4 - 1))/(4*c) + (b*x^2*log(c*x^2 + 1))/4 - (b*x^2
*log(1 - c*x^2))/4
```

3.54 $\int \frac{a+b\,\mathrm{arctanh}(cx^2)}{x}\,dx$

3.54.1 Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \frac{a+b\,\mathrm{arctanh}(cx^2)}{x}\,dx = a\log(x)-\frac{1}{4}b\,\mathrm{PolyLog}\left(2,-cx^2\right)+\frac{1}{4}b\,\mathrm{PolyLog}\left(2,cx^2\right)$$

output
```
a*ln(x)-1/4*b*polylog(2,-c*x^2)+1/4*b*polylog(2,c*x^2)
```

3.54.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{a+b\,\mathrm{arctanh}(cx^2)}{x}\,dx = a\log(x)+\frac{1}{4}b\left(-\mathrm{PolyLog}\left(2,-cx^2\right)+\mathrm{PolyLog}\left(2,cx^2\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/x,x]
```

output
```
a*Log[x] + (b*(-PolyLog[2, -(c*x^2)] + PolyLog[2, c*x^2]))/4
```

3.54.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6450, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^2)}{x} \, dx$$

$$\Big\downarrow \text{6450}$$

$$\frac{1}{2}\int \frac{a+b\text{arctanh}(cx^2)}{x^2}dx^2$$

$$\Big\downarrow \text{6446}$$

$$\frac{1}{2}\left(a\log\left(x^2\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,-cx^2\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,cx^2\right)\right)$$

input
```
Int[(a + b*ArcTanh[c*x^2])/x,x]
```

output
```
(a*Log[x^2] - (b*PolyLog[2, -(c*x^2)])/2 + (b*PolyLog[2, c*x^2])/2)/2
```

3.54.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.54.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 4.20

method	result
default	$a\ln(x)+b\Big(\ln(x)\operatorname{arctanh}(c\,x^2)-2c\Big(-\frac{\ln(x)(\ln(1-x\sqrt{c})+\ln(1+x\sqrt{c}))}{4c}-\frac{\operatorname{dilog}(1-x\sqrt{c})+\operatorname{dilog}(1+x\sqrt{c})}{4c}+\frac{\ln(x}{}$
parts	$a\ln(x)+b\Big(\ln(x)\operatorname{arctanh}(c\,x^2)-2c\Big(-\frac{\ln(x)(\ln(1-x\sqrt{c})+\ln(1+x\sqrt{c}))}{4c}-\frac{\operatorname{dilog}(1-x\sqrt{c})+\operatorname{dilog}(1+x\sqrt{c})}{4c}+\frac{\ln(x}{}$
risch	$a\ln(x)-\frac{\ln(-c\,x^2+1)\ln(x)b}{2}+\frac{b\ln(x)\ln(1-x\sqrt{c})}{2}+\frac{b\ln(x)\ln(1+x\sqrt{c})}{2}+\frac{b\operatorname{dilog}(1-x\sqrt{c})}{2}+\frac{b\operatorname{dilog}(1+x\sqrt{c})}{2}-\frac{\ln(1+x}{}$

input
```
int((a+b*arctanh(c*x^2))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(ln(x)*arctanh(c*x^2)-2*c*(-1/4*ln(x)*(ln(1-x*c^(1/2))+ln(1+x*c^
(1/2)))/c-1/4*(dilog(1-x*c^(1/2))+dilog(1+x*c^(1/2)))/c+1/4*ln(x)*(ln(1+x*
(-c)^(1/2))+ln(1-x*(-c)^(1/2)))/c+1/4*(dilog(1+x*(-c)^(1/2))+dilog(1-x*(-c
)^(1/2)))/c))
```

3.54.5 Fricas [F]

$$\int \frac{a+b\text{arctanh}(cx^2)}{x}\,dx=\int \frac{b\,\text{artanh}\left(cx^2\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))/x,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x^2) + a)/x, x)
```

3.54.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx^2)}{x}\,dx=\int \frac{a+b\,\text{atanh}\left(cx^2\right)}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))/x,x)
```

output
```
Integral((a + b*atanh(c*x**2))/x, x)
```

3.54.7 Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx^2)}{x}\,dx = \int \frac{b\,\text{artanh}\left(cx^2\right) + a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))/x,x, algorithm="maxima")
```

output
```
1/2*b*integrate((log(c*x^2 + 1) - log(-c*x^2 + 1))/x, x) + a*log(x)
```

3.54.8 Giac [F]

$$\int \frac{a + b\text{arctanh}(cx^2)}{x}\,dx = \int \frac{b\,\text{artanh}\left(cx^2\right) + a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)/x, x)
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^2)}{x}\,dx = \int \frac{a + b\,\text{atanh}(c\,x^2)}{x}\,dx$$

input
```
int((a + b*atanh(c*x^2))/x,x)
```

output
```
int((a + b*atanh(c*x^2))/x, x)
```

3.55 $\int \frac{a+b\text{arctanh}(cx^2)}{x^3}\,dx$

3.55.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^3}\,dx = -\frac{a+b\text{arctanh}(cx^2)}{2x^2} + bc\log(x) - \frac{1}{4}bc\log\left(1-c^2x^4\right)$$

output
```
1/2*(-a-b*arctanh(c*x^2))/x^2+b*c*ln(x)-1/4*b*c*ln(-c^2*x^4+1)
```

3.55.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^3}\,dx = -\frac{a}{2x^2} - \frac{b\text{arctanh}(cx^2)}{2x^2} + bc\log(x) - \frac{1}{4}bc\log\left(1-c^2x^4\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*ArcTanh[c*x^2])/(2*x^2) + b*c*Log[x] - (b*c*Log[1 - c^2*x^
4])/4
```

3.55.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6452, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^3}dx$$

$$\Big\downarrow \text{6452}$$

$$bc\int \frac{1}{x(1-c^2x^4)}dx-\frac{a+b\text{arctanh}(cx^2)}{2x^2}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{4}bc\int \frac{1}{x^4(1-c^2x^4)}dx^4-\frac{a+b\text{arctanh}(cx^2)}{2x^2}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{4}bc\left(c^2\int \frac{1}{1-c^2x^4}dx^4+\int \frac{1}{x^4}dx^4\right)-\frac{a+b\text{arctanh}(cx^2)}{2x^2}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{4}bc\left(c^2\int \frac{1}{1-c^2x^4}dx^4+\log(x^4)\right)-\frac{a+b\text{arctanh}(cx^2)}{2x^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{4}bc(\log(x^4)-\log(1-c^2x^4))-\frac{a+b\text{arctanh}(cx^2)}{2x^2}$$

input
```
Int[(a + b*ArcTanh[c*x^2])/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c*x^2])/x^2 + (b*c*(Log[x^4] - Log[1 - c^2*x^4]))/4
```

3.55.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.55.4 Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

method	result	size
default	$-\frac{a}{2x^2}+b\left(-\frac{\operatorname{arctanh}(c\,x^2)}{2x^2}+c\left(-\frac{\ln(c\,x^2-1)}{4}-\frac{\ln(c\,x^2+1)}{4}+\ln(x)\right)\right)$	47
parts	$-\frac{a}{2x^2}+b\left(-\frac{\operatorname{arctanh}(c\,x^2)}{2x^2}+c\left(-\frac{\ln(c\,x^2-1)}{4}-\frac{\ln(c\,x^2+1)}{4}+\ln(x)\right)\right)$	47
parallelrisch	$\frac{2bc\ln(x)x^2-\ln(c\,x^2-1)x^2bc-b\operatorname{arctanh}(c\,x^2)x^2c-b\operatorname{arctanh}(c\,x^2)-a}{2x^2}$	56
risch	$-\frac{b\ln(c\,x^2+1)}{4x^2}+\frac{4bc\ln(x)x^2-bc\ln(c^2x^4-1)x^2+b\ln(-c\,x^2+1)-2a}{4x^2}$	62

input
```
int((a+b*arctanh(c*x^2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+b*(-1/2/x^2*arctanh(c*x^2)+c*(-1/4*ln(c*x^2-1)-1/4*ln(c*x^2+1)+
ln(x)))
```

3.55.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{x^3}\,dx = -\frac{bcx^2\log\left(c^2x^4-1\right)-4\,bcx^2\log\left(x\right)+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{4\,x^2}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="fricas")
```

output
```
-1/4*(b*c*x^2*log(c^2*x^4 - 1) - 4*b*c*x^2*log(x) + b*log(-(c*x^2 + 1)/(c*
x^2 - 1)) + 2*a)/x^2
```

3.55.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(39) = 78.

Time = 4.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{x^3}\,dx$$
$$=\begin{cases} -\frac{a}{2x^2}+bc\log\left(x\right)-\frac{bc\log\left(x-\sqrt{-\frac{1}{c}}\right)}{2}-\frac{bc\log\left(x+\sqrt{-\frac{1}{c}}\right)}{2}+\frac{bc\operatorname{atanh}\left(cx^2\right)}{2}-\frac{b\operatorname{atanh}\left(cx^2\right)}{2x^2} & \text{for } c\neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atanh(c*x**2))/x**3,x)
```

output
```
Piecewise((-a/(2*x**2) + b*c*log(x) - b*c*log(x - sqrt(-1/c))/2 - b*c*log(
x + sqrt(-1/c))/2 + b*c*atanh(c*x**2)/2 - b*atanh(c*x**2)/(2*x**2), Ne(c,
0)), (-a/(2*x**2), True))
```

3.55.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^3}\,dx = -\frac{1}{4}\left(c\left(\log\left(c^2x^4-1\right)-\log\left(x^4\right)\right)+\frac{2\,\text{artanh}\left(cx^2\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="maxima")
```

output
```
-1/4*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*b - 1/2*a/x^
2
```

3.55.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^3}\,dx = -\frac{1}{4}\,bc\log\left(c^2x^4-1\right)+bc\log\left(x\right)-\frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{4\,x^2}-\frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^3,x, algorithm="giac")
```

output
```
-1/4*b*c*log(c^2*x^4 - 1) + b*c*log(x) - 1/4*b*log(-(c*x^2 + 1)/(c*x^2 - 1
))/x^2 - 1/2*a/x^2
```

3.55.9 Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^3}\,dx = b\,c\,\ln\left(x\right)-\frac{a}{2\,x^2}-\frac{b\,c\,\ln\left(c^2\,x^4-1\right)}{4}-\frac{b\,\ln\left(c\,x^2+1\right)}{4\,x^2}+\frac{b\,\ln\left(1-c\,x^2\right)}{4\,x^2}$$

input
```
int((a + b*atanh(c*x^2))/x^3,x)
```

output
```
b*c*log(x) - a/(2*x^2) - (b*c*log(c^2*x^4 - 1))/4 - (b*log(c*x^2 + 1))/(4*
x^2) + (b*log(1 - c*x^2))/(4*x^2)
```

3.56 $\int \frac{a+b\text{arctanh}(cx^2)}{x^5}\,dx$

3.56.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^5}\,dx=-\frac{bc}{4x^2}+\frac{1}{4}bc^2\text{arctanh}(cx^2)-\frac{a+b\text{arctanh}(cx^2)}{4x^4}$$

output
```
-1/4*b*c/x^2+1/4*b*c^2*arctanh(c*x^2)+1/4*(-a-b*arctanh(c*x^2))/x^4
```

3.56.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^5}\,dx=-\frac{a}{4x^4}-\frac{bc}{4x^2}-\frac{b\text{arctanh}(cx^2)}{4x^4}-\frac{1}{8}bc^2\log\left(1-cx^2\right)+\frac{1}{8}bc^2\log\left(1+cx^2\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/x^5,x]
```

output
```
-1/4*a/x^4 - (b*c)/(4*x^2) - (b*ArcTanh[c*x^2])/(4*x^4) - (b*c^2*Log[1 - c
*x^2])/8 + (b*c^2*Log[1 + c*x^2])/8
```

3.56.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6452, 807, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^5}dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}bc\int \frac{1}{x^3(1-c^2x^4)}dx-\frac{a+b\text{arctanh}(cx^2)}{4x^4}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{4}bc\int \frac{1}{x^4(1-c^2x^4)}dx^2-\frac{a+b\text{arctanh}(cx^2)}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{4}bc\left(c^2\int \frac{1}{1-c^2x^4}dx^2-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx^2)}{4x^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}bc\left(c\text{arctanh}(cx^2)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx^2)}{4x^4}$$

input
```
Int[(a + b*ArcTanh[c*x^2])/x^5,x]
```

output
```
-1/4*(a + b*ArcTanh[c*x^2])/x^4 + (b*c*(-x^(-2) + c*ArcTanh[c*x^2]))/4
```

3.56.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.56.4 Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

method	result	size
parallelrisch	$-\frac{-\operatorname{arctanh}\left(c\,x^{2}\right) b\,c^{2} x^{4}+a\,c^{2} x^{4}+b c\,x^{2}+b \,\operatorname{arctanh}\left(c\,x^{2}\right)+a}{4 x^{4}}$	45
default	$-\frac{a}{4 x^{4}}-\frac{b \,\operatorname{arctanh}\left(c\,x^{2}\right)}{4 x^{4}}-\frac{b\,c^{2} \ln \left(c\,x^{2}-1\right)}{8}+\frac{b\,c^{2} \ln \left(c\,x^{2}+1\right)}{8}-\frac{b c}{4 x^{2}}$	55
parts	$-\frac{a}{4 x^{4}}-\frac{b \,\operatorname{arctanh}\left(c\,x^{2}\right)}{4 x^{4}}-\frac{b\,c^{2} \ln \left(c\,x^{2}-1\right)}{8}+\frac{b\,c^{2} \ln \left(c\,x^{2}+1\right)}{8}-\frac{b c}{4 x^{2}}$	55
risch	$-\frac{b \ln \left(c\,x^{2}+1\right)}{8 x^{4}}+\frac{b\,c^{2} \ln \left(c\,x^{2}+1\right) x^{4}-b\,c^{2} \ln \left(c\,x^{2}-1\right) x^{4}-2 b c\,x^{2}+b \ln \left(-c\,x^{2}+1\right)-2 a}{8 x^{4}}$	76

input
```
int((a+b*arctanh(c*x^2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-arctanh(c*x^2)*b*c^2*x^4+a*c^2*x^4+b*c*x^2+b*arctanh(c*x^2)+a)/x^4
```

3.56.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^5}\,dx = -\frac{2\,bcx^2 - (bc^2x^4 - b)\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,a}{8\,x^4}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="fricas")
```

output
```
-1/8*(2*b*c*x^2 - (b*c^2*x^4 - b)*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^4
```

3.56.6 Sympy [A] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^5}\,dx = -\frac{a}{4x^4} + \frac{bc^2\,\text{atanh}\,(cx^2)}{4} - \frac{bc}{4x^2} - \frac{b\,\text{atanh}\,(cx^2)}{4x^4}$$

input
```
integrate((a+b*atanh(c*x**2))/x**5,x)
```

output
```
-a/(4*x**4) + b*c**2*atanh(c*x**2)/4 - b*c/(4*x**2) - b*atanh(c*x**2)/(4*x
**4)
```

3.56.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^5}\,dx = \frac{1}{8}\left(\left(c\log\left(cx^2+1\right) - c\log\left(cx^2-1\right) - \frac{2}{x^2}\right)c - \frac{2\,\text{artanh}\,(cx^2)}{x^4}\right)b - \frac{a}{4\,x^4}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="maxima")
```

output
```
1/8*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^
4)*b - 1/4*a/x^4
```

3.56.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.63

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{x^5}\,dx = \frac{1}{8}\,bc^2\log\left(cx^2+1\right) - \frac{1}{8}\,bc^2\log\left(cx^2-1\right) - \frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{8\,x^4} - \frac{bcx^2+a}{4\,x^4}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^5,x, algorithm="giac")
```

output
```
1/8*b*c^2*log(c*x^2 + 1) - 1/8*b*c^2*log(c*x^2 - 1) - 1/8*b*log(-(c*x^2 +
1)/(c*x^2 - 1))/x^4 - 1/4*(b*c*x^2 + a)/x^4
```

3.56.9 Mupad [B] (verification not implemented)

Time = 3.71 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{x^5}\,dx = \frac{b\,c^2\operatorname{atanh}(c\,x^2)}{4} - \frac{\frac{a}{4}+\frac{b\,\ln(c\,x^2+1)}{8}-\frac{b\,\ln(1-c\,x^2)}{8}+\frac{b\,c\,x^2}{4}}{x^4}$$

input
```
int((a + b*atanh(c*x^2))/x^5,x)
```

output
```
(b*c^2*atanh(c*x^2))/4 - (a/4 + (b*log(c*x^2 + 1))/8 - (b*log(1 - c*x^2))/
8 + (b*c*x^2)/4)/x^4
```

3.57 $\int \frac{a+b\,\text{arctanh}(cx^2)}{x^7}\,dx$

3.57.1 Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^7}\,dx = -\frac{bc}{12x^4} - \frac{a+b\text{arctanh}(cx^2)}{6x^6} + \frac{1}{3}bc^3\log(x) - \frac{1}{12}bc^3\log\left(1-c^2x^4\right)$$

output
```
-1/12*b*c/x^4+1/6*(-a-b*arctanh(c*x^2))/x^6+1/3*b*c^3*ln(x)-1/12*b*c^3*ln(
-c^2*x^4+1)
```

3.57.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^7}\,dx = -\frac{a}{6x^6} - \frac{bc}{12x^4} - \frac{b\text{arctanh}(cx^2)}{6x^6} + \frac{1}{3}bc^3\log(x) - \frac{1}{12}bc^3\log\left(1-c^2x^4\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/x^7,x]
```

output
```
-1/6*a/x^6 - (b*c)/(12*x^4) - (b*ArcTanh[c*x^2])/(6*x^6) + (b*c^3*Log[x])/
3 - (b*c^3*Log[1 - c^2*x^4])/12
```

3.57.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^7}dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}bc\int \frac{1}{x^5(1-c^2x^4)}dx-\frac{a+b\text{arctanh}(cx^2)}{6x^6}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{12}bc\int \frac{1}{x^8(1-c^2x^4)}dx^4-\frac{a+b\text{arctanh}(cx^2)}{6x^6}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{12}bc\int \left(-\frac{c^4}{c^2x^4-1}+\frac{c^2}{x^4}+\frac{1}{x^8}\right)dx^4-\frac{a+b\text{arctanh}(cx^2)}{6x^6}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}bc\left(c^2\log(x^4)-c^2\log(1-c^2x^4)-\frac{1}{x^4}\right)-\frac{a+b\text{arctanh}(cx^2)}{6x^6}$$

input
```
Int[(a + b*ArcTanh[c*x^2])/x^7,x]
```

output
```
-1/6*(a + b*ArcTanh[c*x^2])/x^6 + (b*c*(-x^(-4) + c^2*Log[x^4] - c^2*Log[1
 - c^2*x^4]))/12
```

3.57.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.57.4 Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{a}{6x^{6}}+b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{6x^{6}}+\frac{c\left(-\frac{c^{2}\ln\left(c\,x^{2}-1\right)}{4}-\frac{c^{2}\ln\left(c\,x^{2}+1\right)}{4}-\frac{1}{4x^{4}}+c^{2}\ln\left(x\right)\right)}{3}\right)$	63
parts	$-\frac{a}{6x^{6}}+b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{6x^{6}}+\frac{c\left(-\frac{c^{2}\ln\left(c\,x^{2}-1\right)}{4}-\frac{c^{2}\ln\left(c\,x^{2}+1\right)}{4}-\frac{1}{4x^{4}}+c^{2}\ln\left(x\right)\right)}{3}\right)$	63
risch	$-\frac{b\ln\left(c\,x^{2}+1\right)}{12x^{6}}+\frac{4b\,c^{3}\ln\left(x\right)x^{6}-b\,c^{3}\ln\left(c^{2}x^{4}-1\right)x^{6}-bc\,x^{2}+b\ln\left(-c\,x^{2}+1\right)-2a}{12x^{6}}$	73
parallelrisch	$\frac{4b\,c^{3}\ln\left(x\right)x^{6}-2\ln\left(c\,x^{2}-1\right)x^{6}b\,c^{3}-2b\,\operatorname{arctanh}\left(c\,x^{2}\right)x^{6}c^{3}-b\,c^{3}x^{6}-bc\,x^{2}-2b\,\operatorname{arctanh}\left(c\,x^{2}\right)-2a}{12x^{6}}$	78

input
```
int((a+b*arctanh(c*x^2))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6+b*(-1/6/x^6*arctanh(c*x^2)+1/3*c*(-1/4*c^2*ln(c*x^2-1)-1/4*c^2*
ln(c*x^2+1)-1/4/x^4+c^2*ln(x)))
```

3.57.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^7}\,dx = -\frac{bc^3x^6\log\left(c^2x^4-1\right)-4\,bc^3x^6\log\left(x\right)+bcx^2+b\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,a}{12\,x^6}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="fricas")
```

output
```
-1/12*(b*c^3*x^6*log(c^2*x^4 - 1) - 4*b*c^3*x^6*log(x) + b*c*x^2 + b*log(-
(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x^6
```

3.57.6 Sympy [A] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.73

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^7}\,dx = \begin{cases} -\frac{a}{6x^6}+\frac{bc^3\log\left(x\right)}{3}-\frac{bc^3\log\left(x-\sqrt{-\frac{1}{c}}\right)}{6}-\frac{bc^3\log\left(x+\sqrt{-\frac{1}{c}}\right)}{6}+\frac{bc^3\,\text{atanh}\left(cx^2\right)}{6}-\frac{bc}{12x^4}-\frac{b\,\text{atanh}\left(cx^2\right)}{6x^6} & \text{for}\ c\neq 0 \\ -\frac{a}{6x^6} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atanh(c*x**2))/x**7,x)
```

output
```
Piecewise((-a/(6*x**6) + b*c**3*log(x)/3 - b*c**3*log(x - sqrt(-1/c))/6 -
b*c**3*log(x + sqrt(-1/c))/6 + b*c**3*atanh(c*x**2)/6 - b*c/(12*x**4) - b*
atanh(c*x**2)/(6*x**6), Ne(c, 0)), (-a/(6*x**6), True))
```

3.57.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^7}\,dx = -\frac{1}{12}\left(\left(c^2\log\left(c^2x^4-1\right) - c^2\log\left(x^4\right) + \frac{1}{x^4}\right)c + \frac{2\,\text{artanh}\left(cx^2\right)}{x^6}\right)b - \frac{a}{6\,x^6}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="maxima")
```

output
```
-1/12*((c^2*log(c^2*x^4 - 1) - c^2*log(x^4) + 1/x^4)*c + 2*arctanh(c*x^2)/
x^6)*b - 1/6*a/x^6
```

3.57.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^7}\,dx = -\frac{1}{12}\,bc^3\log\left(c^2x^4-1\right) + \frac{1}{3}\,bc^3\log\left(x\right) - \frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{12\,x^6} - \frac{bcx^2+2\,a}{12\,x^6}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^7,x, algorithm="giac")
```

output
```
-1/12*b*c^3*log(c^2*x^4 - 1) + 1/3*b*c^3*log(x) - 1/12*b*log(-(c*x^2 + 1)/
(c*x^2 - 1))/x^6 - 1/12*(b*c*x^2 + 2*a)/x^6
```

3.57.9 Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^7}\,dx = \frac{b\,c^3\,\ln\left(x\right)}{3} - \frac{b\,c^3\,\ln\left(c^2\,x^4-1\right)}{12} - \frac{a}{6\,x^6} - \frac{b\,c}{12\,x^4} - \frac{b\,\ln\left(c\,x^2+1\right)}{12\,x^6} + \frac{b\,\ln\left(1-c\,x^2\right)}{12\,x^6}$$

input
```
int((a + b*atanh(c*x^2))/x^7,x)
```

output
```
(b*c^3*log(x))/3 - (b*c^3*log(c^2*x^4 - 1))/12 - a/(6*x^6) - (b*c)/(12*x^4
) - (b*log(c*x^2 + 1))/(12*x^6) + (b*log(1 - c*x^2))/(12*x^6)
```

3.58 $\int x^4(a+b\text{arctanh}(cx^2))\,dx$

3.58.1 Optimal result

Integrand size = 14, antiderivative size = 65

$$\int x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{2bx^3}{15c}+\frac{b\arctan\left(\sqrt{c}x\right)}{5c^{5/2}}-\frac{b\text{arctanh}\left(\sqrt{c}x\right)}{5c^{5/2}}+\frac{1}{5}x^5\left(a+b\text{arctanh}\left(cx^2\right)\right)$$

output
```
2/15*b*x^3/c+1/5*b*arctan(x*c^(1/2))/c^(5/2)+1/5*x^5*(a+b*arctanh(c*x^2))-
1/5*b*arctanh(x*c^(1/2))/c^(5/2)
```

3.58.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.43

$$\int x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{2bx^3}{15c}+\frac{ax^5}{5}+\frac{b\arctan\left(\sqrt{c}x\right)}{5c^{5/2}}+\frac{1}{5}bx^5\text{arctanh}\left(cx^2\right)+\frac{b\log\left(1-\sqrt{c}x\right)}{10c^{5/2}}-\frac{b\log\left(1+\sqrt{c}x\right)}{10c^{5/2}}$$

input
```
Integrate[x^4*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(2*b*x^3)/(15*c) + (a*x^5)/5 + (b*ArcTan[Sqrt[c]*x])/(5*c^(5/2)) + (b*x^5*
ArcTanh[c*x^2])/5 + (b*Log[1 - Sqrt[c]*x])/(10*c^(5/2)) - (b*Log[1 + Sqrt[
c]*x])/(10*c^(5/2))
```

3.58.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6452, 843, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(a+b\text{arctanh}\left(cx^2\right)\right) \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{5} x^5 \left(a+b\text{arctanh}\left(cx^2\right)\right)-\frac{2}{5} bc \int \frac{x^6}{1-c^2 x^4}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{5} x^5 \left(a+b\text{arctanh}\left(cx^2\right)\right)-\frac{2}{5} bc \left(\frac{\int \frac{x^2}{1-c^2x^4}dx}{c^2}-\frac{x^3}{3c^2}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{1}{5} x^5 \left(a+b\text{arctanh}\left(cx^2\right)\right)-\frac{2}{5} bc \left(\frac{\frac{\int \frac{1}{1-cx^2}dx}{2c}-\frac{\int \frac{1}{cx^2+1}dx}{2c}}{c^2}-\frac{x^3}{3c^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{5} x^5 \left(a+b\text{arctanh}\left(cx^2\right)\right)-\frac{2}{5} bc \left(\frac{\frac{\int \frac{1}{1-cx^2}dx}{2c}-\frac{\arctan\left(\sqrt{c}x\right)}{2c^{3/2}}}{c^2}-\frac{x^3}{3c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{5} x^5 \left(a+b\text{arctanh}\left(cx^2\right)\right)-\frac{2}{5} bc \left(\frac{\frac{\text{arctanh}\left(\sqrt{c}x\right)}{2c^{3/2}}-\frac{\arctan\left(\sqrt{c}x\right)}{2c^{3/2}}}{c^2}-\frac{x^3}{3c^2}\right)$$

input
```
Int[x^4*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(-2*b*c*(-1/3*x^3/c^2 + (-1/2*ArcTan[Sqrt[c]*x]/c^(3/2) + ArcTanh[Sqrt[c]*
x]/(2*c^(3/2)))/c^2))/5 + (x^5*(a + b*ArcTanh[c*x^2]))/5
```

3.58.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.58.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

method	result
default	$\frac{a\,x^{5}}{5}+\frac{b\,x^{5}\operatorname{arctanh}\left(c\,x^{2}\right)}{5}+\frac{2 b\,x^{3}}{15 c}-\frac{b\operatorname{arctanh}\left(x\sqrt{c}\right)}{5 c^{\frac{5}{2}}}+\frac{b\arctan\left(x\sqrt{c}\right)}{5 c^{\frac{5}{2}}}$
parts	$\frac{a\,x^{5}}{5}+\frac{b\,x^{5}\operatorname{arctanh}\left(c\,x^{2}\right)}{5}+\frac{2 b\,x^{3}}{15 c}-\frac{b\operatorname{arctanh}\left(x\sqrt{c}\right)}{5 c^{\frac{5}{2}}}+\frac{b\arctan\left(x\sqrt{c}\right)}{5 c^{\frac{5}{2}}}$
risch	$\frac{b\,x^{5}\ln\left(c\,x^{2}+1\right)}{10}-\frac{b\,x^{5}\ln\left(-c\,x^{2}+1\right)}{10}+\frac{a\,x^{5}}{5}+\frac{2 b\,x^{3}}{15 c}+\frac{b\ln\left(1-x\sqrt{c}\right)}{10 c^{\frac{5}{2}}}-\frac{b\ln\left(1+x\sqrt{c}\right)}{10 c^{\frac{5}{2}}}+\frac{\sqrt{-c}\,\ln\left(-\sqrt{-c}\,c-c^{2}x\right)b}{10 c^{3}}-\frac{\sqrt{-c}\,\ln$

input
```
int(x^4*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+1/5*b*x^5*arctanh(c*x^2)+2/15*b*x^3/c-1/5*b*arctanh(x*c^(1/2))/c
^(5/2)+1/5*b*arctan(x*c^(1/2))/c^(5/2)
```

3.58.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(49) = 98$.

Time = 0.28 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.03

$$\int x^4(a + b\text{arctanh}(cx^2))\,dx$$
$$= \left[\frac{3\,bc^3x^5\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 6\,ac^3x^5 + 4\,bc^2x^3 + 6\,b\sqrt{c}\arctan\left(\sqrt{c}x\right) + 3\,b\sqrt{c}\log\left(\frac{cx^2-2\sqrt{c}x+1}{cx^2-1}\right)}{30\,c^3}, \frac{3\,bc^3x^5\log\left(-\right.}{}\right.$$

input
```
integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
[1/30*(3*b*c^3*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c^3*x^5 + 4*b*c^2*x
^3 + 6*b*sqrt(c)*arctan(sqrt(c)*x) + 3*b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x
+ 1)/(c*x^2 - 1)))/c^3, 1/30*(3*b*c^3*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) +
6*a*c^3*x^5 + 4*b*c^2*x^3 + 6*b*sqrt(-c)*arctan(sqrt(-c)*x) - 3*b*sqrt(-c)
*log((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)))/c^3]
```

3.58.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(58) = 116$.

Time = 3.67 (sec) , antiderivative size = 944, normalized size of antiderivative = 14.52

$$\int x^4(a + b\text{arctanh}(cx^2))\,dx = \text{Too large to display}$$

input
```
integrate(x**4*(a+b*atanh(c*x**2)),x)
```

output
```
Piecewise((12*a*c**3*x**5*sqrt(-1/c)*sqrt(1/c)/(15*c**4*(-1/c)**(3/2)*sqrt
(1/c) - 15*c**4*sqrt(-1/c)*(1/c)**(3/2) + 90*c**3*sqrt(-1/c)*sqrt(1/c)) +
12*b*c**3*x**5*sqrt(-1/c)*sqrt(1/c)*atanh(c*x**2)/(15*c**4*(-1/c)**(3/2)*s
qrt(1/c) - 15*c**4*sqrt(-1/c)*(1/c)**(3/2) + 90*c**3*sqrt(-1/c)*sqrt(1/c))
 + 8*b*c**2*x**3*sqrt(-1/c)*sqrt(1/c)/(15*c**4*(-1/c)**(3/2)*sqrt(1/c) - 1
5*c**4*sqrt(-1/c)*(1/c)**(3/2) + 90*c**3*sqrt(-1/c)*sqrt(1/c)) - 3*b*c*(-1
/c)**(3/2)*log(x + sqrt(-1/c))/(15*c**4*(-1/c)**(3/2)*sqrt(1/c) - 15*c**4*
sqrt(-1/c)*(1/c)**(3/2) + 90*c**3*sqrt(-1/c)*sqrt(1/c)) + 3*b*c*(1/c)**(3/
2)*log(x + sqrt(-1/c))/(15*c**4*(-1/c)**(3/2)*sqrt(1/c) - 15*c**4*sqrt(-1/
c)*(1/c)**(3/2) + 90*c**3*sqrt(-1/c)*sqrt(1/c)) - 6*b*sqrt(-1/c)*log(x - s
qrt(-1/c))/(15*c**4*(-1/c)**(3/2)*sqrt(1/c) - 15*c**4*sqrt(-1/c)*(1/c)**(3
/2) + 90*c**3*sqrt(-1/c)*sqrt(1/c)) - 9*b*sqrt(-1/c)*log(x + sqrt(-1/c))/(
15*c**4*(-1/c)**(3/2)*sqrt(1/c) - 15*c**4*sqrt(-1/c)*(1/c)**(3/2) + 90*c**
3*sqrt(-1/c)*sqrt(1/c)) + 12*b*sqrt(-1/c)*log(x - sqrt(1/c))/(15*c**4*(-1/
c)**(3/2)*sqrt(1/c) - 15*c**4*sqrt(-1/c)*(1/c)**(3/2) + 90*c**3*sqrt(-1/c)
*sqrt(1/c)) + 12*b*sqrt(-1/c)*atanh(c*x**2)/(15*c**4*(-1/c)**(3/2)*sqrt(1/
c) - 15*c**4*sqrt(-1/c)*(1/c)**(3/2) + 90*c**3*sqrt(-1/c)*sqrt(1/c)) + 6*b
*sqrt(1/c)*log(x - sqrt(-1/c))/(15*c**4*(-1/c)**(3/2)*sqrt(1/c) - 15*c**4*
sqrt(-1/c)*(1/c)**(3/2) + 90*c**3*sqrt(-1/c)*sqrt(1/c)) - 9*b*sqrt(1/c)*lo
g(x + sqrt(-1/c))/(15*c**4*(-1/c)**(3/2)*sqrt(1/c) - 15*c**4*sqrt(-1/c)...
```

3.58.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int x^4 (a + b\text{arctanh}(cx^2)) \, dx$$
$$= \frac{1}{5} a x^5 + \frac{1}{30} \left(6 x^5 \operatorname{artanh}(cx^2) + c \left(\frac{4 x^3}{c^2} + \frac{6 \arctan(\sqrt{c}x)}{c^{\frac{7}{2}}} + \frac{3 \log\left(\frac{cx - \sqrt{c}}{cx + \sqrt{c}}\right)}{c^{\frac{7}{2}}} \right) \right) b$$

input
```
integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/30*(6*x^5*arctanh(c*x^2) + c*(4*x^3/c^2 + 6*arctan(sqrt(c)*x
)/c^(7/2) + 3*log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(7/2)))*b
```

3.58.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.12

$$\int x^4\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)\,dx=\frac{1}{10}\,bx^5\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{5}\,ax^5+\frac{2\,bx^3}{15\,c}+\frac{b\arctan\left(\sqrt{c}x\right)}{5\,c^{\frac{5}{2}}}+\frac{b\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{5\,\sqrt{-c}c^2}$$

input
```
integrate(x^4*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
1/10*b*x^5*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/5*a*x^5 + 2/15*b*x^3/c + 1/5*
b*arctan(sqrt(c)*x)/c^(5/2) + 1/5*b*arctan(c*x/sqrt(-c))/(sqrt(-c)*c^2)
```

3.58.9 Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int x^4\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)\,dx=\frac{a\,x^5}{5}+\frac{2\,b\,x^3}{15\,c}+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,x\right)}{5\,c^{5/2}}+\frac{b\,x^5\,\ln\left(c\,x^2+1\right)}{10}-\frac{b\,x^5\,\ln\left(1-c\,x^2\right)}{10}+\frac{b\,\mathrm{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5\,c^{5/2}}$$

input
```
int(x^4*(a + b*atanh(c*x^2)),x)
```

output
```
(a*x^5)/5 + (2*b*x^3)/(15*c) + (b*atan(c^(1/2)*x))/(5*c^(5/2)) + (b*atan(c
^(1/2)*x*1i)*1i)/(5*c^(5/2)) + (b*x^5*log(c*x^2 + 1))/10 - (b*x^5*log(1 -
c*x^2))/10
```

3.59 $\int x^2(a+b\text{arctanh}(cx^2))\,dx$

3.59.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{2bx}{3c}-\frac{b\arctan\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{b\text{arctanh}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{1}{3}x^3\left(a+b\text{arctanh}\left(cx^2\right)\right)$$

output
```
2/3*b*x/c-1/3*b*arctan(x*c^(1/2))/c^(3/2)+1/3*x^3*(a+b*arctanh(c*x^2))-1/3
*b*arctanh(x*c^(1/2))/c^(3/2)
```

3.59.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{2bx}{3c}+\frac{ax^3}{3}-\frac{b\arctan\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{1}{3}bx^3\text{arctanh}\left(cx^2\right)+\frac{b\log\left(1-\sqrt{c}x\right)}{6c^{3/2}}-\frac{b\log\left(1+\sqrt{c}x\right)}{6c^{3/2}}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(2*b*x)/(3*c) + (a*x^3)/3 - (b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) + (b*x^3*Arc
Tanh[c*x^2])/3 + (b*Log[1 - Sqrt[c]*x])/(6*c^(3/2)) - (b*Log[1 + Sqrt[c]*x
])/(6*c^(3/2))
```

3.59.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6452, 843, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + b\text{arctanh}(cx^2))\ dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3 (a + b\text{arctanh}(cx^2)) - \frac{2}{3}bc\int \frac{x^4}{1-c^2x^4}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{3}x^3 (a + b\text{arctanh}(cx^2)) - \frac{2}{3}bc\left(\frac{\int \frac{1}{1-c^2x^4}dx}{c^2} - \frac{x}{c^2}\right)$$

$$\Big\downarrow \text{756}$$

$$\frac{1}{3}x^3 (a + b\text{arctanh}(cx^2)) - \frac{2}{3}bc\left(\frac{\frac{1}{2}\int \frac{1}{1-cx^2}dx + \frac{1}{2}\int \frac{1}{cx^2+1}dx}{c^2} - \frac{x}{c^2}\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{3}x^3 (a + b\text{arctanh}(cx^2)) - \frac{2}{3}bc\left(\frac{\frac{1}{2}\int \frac{1}{1-cx^2}dx + \frac{\arctan(\sqrt{c}x)}{2\sqrt{c}}}{c^2} - \frac{x}{c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}x^3 (a + b\text{arctanh}(cx^2)) - \frac{2}{3}bc\left(\frac{\frac{\arctan(\sqrt{c}x)}{2\sqrt{c}} + \frac{\text{arctanh}(\sqrt{c}x)}{2\sqrt{c}}}{c^2} - \frac{x}{c^2}\right)$$

input
```
Int[x^2*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(-2*b*c*(-(x/c^2) + (ArcTan[Sqrt[c]*x]/(2*Sqrt[c]) + ArcTanh[Sqrt[c]*x]/(2
*Sqrt[c]))/c^2))/3 + (x^3*(a + b*ArcTanh[c*x^2]))/3
```

3.59.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.59.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result
default	$\frac{a\,x^{3}}{3}+\frac{b\,x^{3}\operatorname{arctanh}\left(c\,x^{2}\right)}{3}+\frac{2bx}{3c}-\frac{b\operatorname{arctanh}\left(x\sqrt{c}\right)}{3c^{\frac{3}{2}}}-\frac{b\arctan\left(x\sqrt{c}\right)}{3c^{\frac{3}{2}}}$
parts	$\frac{a\,x^{3}}{3}+\frac{b\,x^{3}\operatorname{arctanh}\left(c\,x^{2}\right)}{3}+\frac{2bx}{3c}-\frac{b\operatorname{arctanh}\left(x\sqrt{c}\right)}{3c^{\frac{3}{2}}}-\frac{b\arctan\left(x\sqrt{c}\right)}{3c^{\frac{3}{2}}}$
risch	$\frac{b\,x^{3}\ln\left(c\,x^{2}+1\right)}{6}-\frac{b\,x^{3}\ln\left(-c\,x^{2}+1\right)}{6}+\frac{a\,x^{3}}{3}-\frac{b\ln\left(1+x\sqrt{c}\right)}{6c^{\frac{3}{2}}}+\frac{b\ln\left(x\sqrt{c}-1\right)}{6c^{\frac{3}{2}}}+\frac{2bx}{3c}-\frac{\sqrt{-c}\,\ln\left(x\sqrt{-c}-1\right)b}{6c^{2}}+\frac{\sqrt{-c}\,\ln(1+x}{6c^{2}}$

input
```
int(x^2*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/3*b*x^3*arctanh(c*x^2)+2/3*b*x/c-1/3*b*arctanh(x*c^(1/2))/c^(3
/2)-1/3*b*arctan(x*c^(1/2))/c^(3/2)
```

3.59.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.95

$$\int x^2(a+b\text{arctanh}(cx^2))\,dx = \left[\frac{bc^2x^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+2\,ac^2x^3+4\,bcx-2\,b\sqrt{c}\arctan\left(\sqrt{c}x\right)+b\sqrt{c}\log\left(\frac{cx^2-2\,\sqrt{c}x+1}{cx^2-1}\right)}{6\,c^2},\ \frac{bc^2x^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{}\right.$$

input
```
integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
[1/6*(b*c^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^3 + 4*b*c*x - 2*
b*sqrt(c)*arctan(sqrt(c)*x) + b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x
^2 - 1)))/c^2, 1/6*(b*c^2*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c^2*x^3
+ 4*b*c*x + 2*b*sqrt(-c)*arctan(sqrt(-c)*x) - b*sqrt(-c)*log((c*x^2 + 2*sq
rt(-c)*x - 1)/(c*x^2 + 1)))/c^2]
```

3.59.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 670 vs. $2(56) = 112$.

Time = 2.82 (sec) , antiderivative size = 670, normalized size of antiderivative = 10.63

$$\int x^2(a+b\text{arctanh}(cx^2))\,dx = \begin{cases} \frac{4ac^2x^3\sqrt{-\frac{1}{c}}}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}}+\frac{4ac^2x^3\sqrt{\frac{1}{c}}}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}}+\frac{4bc^2x^3\sqrt{-\frac{1}{c}}\,\text{atanh}\,(cx^2)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}}+\frac{4bc^2x^3\sqrt{\frac{1}{c}}\,\text{atanh}\,(cx^2)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}}-\frac{bc^2\left(-\frac{1}{c}\right)^{\frac{3}{2}}\sqrt{\frac{1}{c}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{12c^2\sqrt{-\frac{1}{c}}+12c^2\sqrt{\frac{1}{c}}}+ \\ \frac{ax^3}{3} \end{cases}$$

input
```
integrate(x**2*(a+b*atanh(c*x**2)),x)
```

output
```
Piecewise((4*a*c**2*x**3*sqrt(-1/c)/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c
)) + 4*a*c**2*x**3*sqrt(1/c)/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 4*
b*c**2*x**3*sqrt(-1/c)*atanh(c*x**2)/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/
c)) + 4*b*c**2*x**3*sqrt(1/c)*atanh(c*x**2)/(12*c**2*sqrt(-1/c) + 12*c**2*
sqrt(1/c)) - b*c**2*(-1/c)**(3/2)*sqrt(1/c)*log(x + sqrt(-1/c))/(12*c**2*s
qrt(-1/c) + 12*c**2*sqrt(1/c)) + b*c**2*sqrt(-1/c)*(1/c)**(3/2)*log(x + sq
rt(-1/c))/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 8*b*c*x*sqrt(-1/c)/(1
2*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 8*b*c*x*sqrt(1/c)/(12*c**2*sqrt(-
1/c) + 12*c**2*sqrt(1/c)) - 6*b*c*sqrt(-1/c)*sqrt(1/c)*log(x + sqrt(-1/c))
/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 4*b*c*sqrt(-1/c)*sqrt(1/c)*log
(x - sqrt(1/c))/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 4*b*c*sqrt(-1/c
)*sqrt(1/c)*atanh(c*x**2)/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) - 4*b*l
og(x - sqrt(-1/c))/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 4*b*log(x -
sqrt(1/c))/(12*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)) + 4*b*atanh(c*x**2)/(1
2*c**2*sqrt(-1/c) + 12*c**2*sqrt(1/c)), Ne(c, 0)), (a*x**3/3, True))
```

3.59.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int x^2(a+b\text{arctanh}(cx^2))\,dx$$
$$=\frac{1}{3}ax^3+\frac{1}{6}\left(2\,x^3\,\text{artanh}\left(cx^2\right)+c\left(\frac{4\,x}{c^2}-\frac{2\,\arctan\left(\sqrt{c}x\right)}{c^{\frac{5}{2}}}+\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)\right)b$$

input
```
integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x^2) + c*(4*x/c^2 - 2*arctan(sqrt(c)*x)/c
^(5/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(5/2)))*b
```

3.59.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

$$\int x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=-\frac{1}{3}\,bc^5\left(\frac{\arctan\left(\sqrt{c}x\right)}{c^{\frac{13}{2}}}-\frac{\arctan\left(\frac{cx}{\sqrt{-c}}\right)}{\sqrt{-c}c^6}\right)+\frac{1}{6}\,bx^3\log\left(-\frac{cx^2+1}{cx^2-1}\right)+\frac{1}{3}\,ax^3+\frac{2\,bx}{3\,c}$$

input
```
integrate(x^2*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
-1/3*b*c^5*(arctan(sqrt(c)*x)/c^(13/2) - arctan(c*x/sqrt(-c))/(sqrt(-c)*c^
6)) + 1/6*b*x^3*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/3*a*x^3 + 2/3*b*x/c
```

3.59.9 Mupad [B] (verification not implemented)

Time = 3.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11

$$\int x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\frac{a\,x^3}{3}-\frac{b\,\text{atan}\left(\sqrt{c}\,x\right)}{3\,c^{3/2}}+\frac{2\,b\,x}{3\,c}+\frac{b\,x^3\,\ln\left(c\,x^2+1\right)}{6}-\frac{b\,x^3\,\ln\left(1-c\,x^2\right)}{6}+\frac{b\,\text{atan}\left(\sqrt{c}\,x\,1\text{i}\right)\,1\text{i}}{3\,c^{3/2}}$$

input
```
int(x^2*(a + b*atanh(c*x^2)),x)
```

output
```
(a*x^3)/3 - (b*atan(c^(1/2)*x))/(3*c^(3/2)) + (b*atan(c^(1/2)*x*1i)*1i)/(3
*c^(3/2)) + (2*b*x)/(3*c) + (b*x^3*log(c*x^2 + 1))/6 - (b*x^3*log(1 - c*x^
2))/6
```

3.60 $\int (a + b\text{arctanh}(cx^2)) \, dx$

3.60.1 Optimal result

Integrand size = 10, antiderivative size = 44

$$\int (a + b\text{arctanh}(cx^2)) \, dx = ax + \frac{b\arctan(\sqrt{c}x)}{\sqrt{c}} - \frac{b\text{arctanh}(\sqrt{c}x)}{\sqrt{c}} + bx\text{arctanh}(cx^2)$$

output
```
a*x+b*x*arctanh(c*x^2)+b*arctan(x*c^(1/2))/c^(1/2)-b*arctanh(x*c^(1/2))/c^
(1/2)
```

3.60.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int (a + b\text{arctanh}(cx^2)) \, dx = ax + bx\text{arctanh}(cx^2) + \frac{b(2\arctan(\sqrt{c}x) + \log(1 - \sqrt{c}x) - \log(1 + \sqrt{c}x))}{2\sqrt{c}}$$

input
```
Integrate[a + b*ArcTanh[c*x^2],x]
```

output
```
a*x + b*x*ArcTanh[c*x^2] + (b*(2*ArcTan[Sqrt[c]*x] + Log[1 - Sqrt[c]*x] -
Log[1 + Sqrt[c]*x]))/(2*Sqrt[c])
```

3.60.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx$$

$$\Big\downarrow \ 2009$$

$$ax+\frac{b\arctan\left(\sqrt{c}x\right)}{\sqrt{c}}+bx\text{arctanh}\left(cx^2\right)-\frac{b\text{arctanh}\left(\sqrt{c}x\right)}{\sqrt{c}}$$

input
```
Int[a + b*ArcTanh[c*x^2],x]
```

output
```
a*x + (b*ArcTan[Sqrt[c]*x])/Sqrt[c] - (b*ArcTanh[Sqrt[c]*x])/Sqrt[c] + b*x
*ArcTanh[c*x^2]
```

3.60.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.60.4 Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84

method	result	s
default	$ax+bx\ \text{arctanh}\left(c\,x^2\right)+\frac{b\arctan\left(x\sqrt{c}\right)}{\sqrt{c}}-\frac{b\ \text{arctanh}\left(x\sqrt{c}\right)}{\sqrt{c}}$	3
parts	$ax+bx\ \text{arctanh}\left(c\,x^2\right)+\frac{b\arctan\left(x\sqrt{c}\right)}{\sqrt{c}}-\frac{b\ \text{arctanh}\left(x\sqrt{c}\right)}{\sqrt{c}}$	3
risch	$ax+\frac{bx\ln\left(c\,x^2+1\right)}{2}-\frac{bx\ln\left(-c\,x^2+1\right)}{2}+\frac{b\sqrt{-c}\,\ln\left(cx+\sqrt{-c}\right)}{2c}-\frac{b\sqrt{-c}\,\ln\left(-cx+\sqrt{-c}\right)}{2c}+\frac{b\ln\left(1-x\sqrt{c}\right)}{2\sqrt{c}}-\frac{b\ln\left(1+x\sqrt{c}\right)}{2\sqrt{c}}$	1

input
```
int(a+b*arctanh(c*x^2),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arctanh(c*x^2)+b*arctan(x*c^(1/2))/c^(1/2)-b*arctanh(x*c^(1/2))/c^
(1/2)
```

3.60.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.64

$$\int (a + b\text{arctanh}(cx^2))\,dx = \left[\frac{bcx\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,acx + 2\,b\sqrt{c}\arctan\left(\sqrt{c}x\right) + b\sqrt{c}\log\left(\frac{cx^2-2\,\sqrt{c}x+1}{cx^2-1}\right)}{2\,c}, \frac{bcx\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,acx + 2\right.$$

input
```
integrate(a+b*arctanh(c*x^2),x, algorithm="fricas")
```

output
```
[1/2*(b*c*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x + 2*b*sqrt(c)*arctan(s
qrt(c)*x) + b*sqrt(c)*log((c*x^2 - 2*sqrt(c)*x + 1)/(c*x^2 - 1)))/c, 1/2*(
b*c*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*c*x + 2*b*sqrt(-c)*arctan(sqrt(-
c)*x) - b*sqrt(-c)*log((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)))/c]
```

3.60.6 Sympy [A] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 702, normalized size of antiderivative = 15.95

$$\int (a + b\text{arctanh}(cx^2))\,dx = ax + b\left(\begin{cases} \frac{4cx\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}\,\text{atanh}\,(cx^2)}{c^2(-\frac{1}{c})^{\frac{3}{2}}\sqrt{\frac{1}{c}} - c^2\sqrt{-\frac{1}{c}}(\frac{1}{c})^{\frac{3}{2}} + 6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}} - \frac{c(-\frac{1}{c})^{\frac{3}{2}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{c^2(-\frac{1}{c})^{\frac{3}{2}}\sqrt{\frac{1}{c}} - c^2\sqrt{-\frac{1}{c}}(\frac{1}{c})^{\frac{3}{2}} + 6c\sqrt{-\frac{1}{c}}\sqrt{\frac{1}{c}}} + \frac{c(\frac{1}{c})^{\frac{3}{2}}\log\left(x+\sqrt{-\frac{1}{c}}\right)}{c^2(-\frac{1}{c})^{\frac{3}{2}}\sqrt{\frac{1}{c}} - c^2\sqrt{-\frac{1}{c}}(\frac{1}{c})^{\frac{3}{2}} + 6c\sqrt{-\frac{1}{c}} \\ 0 \end{cases}\right.$$

input
```
integrate(a+b*atanh(c*x**2),x)
```

output
```
a*x + b*Piecewise((4*c*x*sqrt(-1/c)*sqrt(1/c)*atanh(c*x**2)/(c**2*(-1/c)**
(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c))
 - c*(-1/c)**(3/2)*log(x + sqrt(-1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) - c**
2*sqrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c)) + c*(1/c)**(3/2)*log
(x + sqrt(-1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(3
/2) + 6*c*sqrt(-1/c)*sqrt(1/c)) - 2*sqrt(-1/c)*log(x - sqrt(-1/c))/(c**2*(
-1/c)**(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqr
t(1/c)) - 3*sqrt(-1/c)*log(x + sqrt(-1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) -
 c**2*sqrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c)) + 4*sqrt(-1/c)*l
og(x - sqrt(1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(
3/2) + 6*c*sqrt(-1/c)*sqrt(1/c)) + 4*sqrt(-1/c)*atanh(c*x**2)/(c**2*(-1/c)
**(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c
)) + 2*sqrt(1/c)*log(x - sqrt(-1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) - c**2*
sqrt(-1/c)*(1/c)**(3/2) + 6*c*sqrt(-1/c)*sqrt(1/c)) - 3*sqrt(1/c)*log(x +
sqrt(-1/c))/(c**2*(-1/c)**(3/2)*sqrt(1/c) - c**2*sqrt(-1/c)*(1/c)**(3/2) +
 6*c*sqrt(-1/c)*sqrt(1/c)), Ne(c, 0)), (0, True))
```

3.60.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \left(a + b\text{arctanh}(cx^2)\right) dx = \frac{1}{2}\left(c\left(\frac{2 \arctan\left(\sqrt{c}x\right)}{c^{\frac{3}{2}}} + \frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) + 2\,x\,\text{artanh}\left(cx^2\right)\right)b + ax$$

input
```
integrate(a+b*arctanh(c*x^2),x, algorithm="maxima")
```

output
```
1/2*(c*(2*arctan(sqrt(c)*x)/c^(3/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))
/c^(3/2)) + 2*x*arctanh(c*x^2))*b + a*x
```

3.60.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(36) = 72$.

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.89

$$\int (a + b\text{arctanh}(cx^2))\, dx$$
$$= \frac{1}{2}\left(c\left(\frac{2\sqrt{|c|}\arctan\left(x\sqrt{|c|}\right)}{c^2} - \frac{\sqrt{|c|}\log\left(\left|x + \frac{1}{\sqrt{|c|}}\right|\right)}{c^2} + \frac{\sqrt{|c|}\log\left(\left|x - \frac{1}{\sqrt{|c|}}\right|\right)}{c^2}\right) + x\log\left(-\frac{cx^2+1}{cx^2-1}\right)\right)$$
$$+ ax$$

input
```
integrate(a+b*arctanh(c*x^2),x, algorithm="giac")
```

output
```
1/2*(c*(2*sqrt(abs(c))*arctan(x*sqrt(abs(c)))/c^2 - sqrt(abs(c))*log(abs(x
 + 1/sqrt(abs(c))))/c^2 + sqrt(abs(c))*log(abs(x - 1/sqrt(abs(c))))/c^2) +
 x*log(-(c*x^2 + 1)/(c*x^2 - 1)))*b + a*x
```

3.60.9 Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int (a + b\text{arctanh}(cx^2))\, dx = a\,x + \frac{b\,\text{atan}\left(\sqrt{c}\,x\right)}{\sqrt{c}} + \frac{b\,x\,\ln\left(c\,x^2+1\right)}{2}$$
$$- \frac{b\,x\,\ln\left(1-c\,x^2\right)}{2} + \frac{b\,\text{atan}\left(\sqrt{c}\,x\,1\text{i}\right)\,1\text{i}}{\sqrt{c}}$$

input
```
int(a + b*atanh(c*x^2),x)
```

output
```
a*x + (b*atan(c^(1/2)*x))/c^(1/2) + (b*atan(c^(1/2)*x*1i)*1i)/c^(1/2) + (b
*x*log(c*x^2 + 1))/2 - (b*x*log(1 - c*x^2))/2
```

3.61 $\int \frac{a+b\text{arctanh}(cx^2)}{x^2} dx$

3.61.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^2} dx = b\sqrt{c}\arctan\left(\sqrt{c}x\right)+b\sqrt{c}\text{arctanh}\left(\sqrt{c}x\right)-\frac{a+b\text{arctanh}(cx^2)}{x}$$

output
```
(-a-b*arctanh(c*x^2))/x+b*arctan(x*c^(1/2))*c^(1/2)+b*arctanh(x*c^(1/2))*c
^(1/2)
```

3.61.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^2} dx = -\frac{a}{x}+b\sqrt{c}\arctan\left(\sqrt{c}x\right)-\frac{b\text{arctanh}(cx^2)}{x} -\frac{1}{2}b\sqrt{c}\log\left(1-\sqrt{c}x\right)+\frac{1}{2}b\sqrt{c}\log\left(1+\sqrt{c}x\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/x^2,x]
```

output
```
-(a/x) + b*Sqrt[c]*ArcTan[Sqrt[c]*x] - (b*ArcTanh[c*x^2])/x - (b*Sqrt[c]*L
og[1 - Sqrt[c]*x])/2 + (b*Sqrt[c]*Log[1 + Sqrt[c]*x])/2
```

3.61.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^2}dx$$

$$\Big\downarrow \text{ 6452}$$

$$2bc\int \frac{1}{1-c^2x^4}dx-\frac{a+b\text{arctanh}(cx^2)}{x}$$

$$\Big\downarrow \text{ 756}$$

$$2bc\left(\frac{1}{2}\int \frac{1}{1-cx^2}dx+\frac{1}{2}\int \frac{1}{cx^2+1}dx\right)-\frac{a+b\text{arctanh}(cx^2)}{x}$$

$$\Big\downarrow \text{ 216}$$

$$2bc\left(\frac{1}{2}\int \frac{1}{1-cx^2}dx+\frac{\arctan\left(\sqrt{c}x\right)}{2\sqrt{c}}\right)-\frac{a+b\text{arctanh}(cx^2)}{x}$$

$$\Big\downarrow \text{ 219}$$

$$2bc\left(\frac{\arctan\left(\sqrt{c}x\right)}{2\sqrt{c}}+\frac{\text{arctanh}\left(\sqrt{c}x\right)}{2\sqrt{c}}\right)-\frac{a+b\text{arctanh}(cx^2)}{x}$$

input
```
Int[(a + b*ArcTanh[c*x^2])/x^2,x]
```

output
```
2*b*c*(ArcTan[Sqrt[c]*x]/(2*Sqrt[c]) + ArcTanh[Sqrt[c]*x]/(2*Sqrt[c])) - (
a + b*ArcTanh[c*x^2])/x
```

3.61.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.61.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.91

method	result	size
default	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^{2}\right)}{x}+b\,\mathrm{arctanh}\left(x\sqrt{c}\right)\sqrt{c}+b\arctan\left(x\sqrt{c}\right)\sqrt{c}$	42
parts	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^{2}\right)}{x}+b\,\mathrm{arctanh}\left(x\sqrt{c}\right)\sqrt{c}+b\arctan\left(x\sqrt{c}\right)\sqrt{c}$	42
risch	$-\frac{b\ln\left(c\,x^{2}+1\right)}{2x}+b\arctan\left(x\sqrt{c}\right)\sqrt{c}-\frac{a}{x}+\frac{b\ln\left(-c\,x^{2}+1\right)}{2x}+b\,\mathrm{arctanh}\left(x\sqrt{c}\right)\sqrt{c}$	59

input
```
int((a+b*arctanh(c*x^2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-b/x*arctanh(c*x^2)+b*arctanh(x*c^(1/2))*c^(1/2)+b*arctan(x*c^(1/2))*c
^(1/2)
```

3.61.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(38) = 76$.

Time = 0.28 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.41

$$\int \frac{a + b\operatorname{arctanh}(cx^2)}{x^2}\,dx = \left[\frac{2\,b\sqrt{c}x\arctan\left(\sqrt{c}x\right) + b\sqrt{c}x\log\left(\frac{cx^2+2\,\sqrt{c}x+1}{cx^2-1}\right) - b\log\left(-\frac{cx^2+1}{cx^2-1}\right) - 2\,a}{2\,x}, -\frac{2\,b\sqrt{-c}x\arctan\left(\sqrt{-c}x\right) - b\sqrt{-c}x\log\left(\frac{cx^2+2\,\sqrt{-c}x-1}{cx^2+1}\right) + b\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,a}{2\,x}\right]$$

input
```
integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="fricas")
```

output
```
[1/2*(2*b*sqrt(c)*x*arctan(sqrt(c)*x) + b*sqrt(c)*x*log((c*x^2 + 2*sqrt(c)
*x + 1)/(c*x^2 - 1)) - b*log(-(c*x^2 + 1)/(c*x^2 - 1)) - 2*a)/x, -1/2*(2*b
*sqrt(-c)*x*arctan(sqrt(-c)*x) - b*sqrt(-c)*x*log((c*x^2 + 2*sqrt(-c)*x -
1)/(c*x^2 + 1)) + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)/x]
```

3.61.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1374 vs. $2(42) = 84$.

Time = 3.86 (sec) , antiderivative size = 1374, normalized size of antiderivative = 29.87

$$\int \frac{a + b\operatorname{arctanh}(cx^2)}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*atanh(c*x**2))/x**2,x)
```

output
```
Piecewise((-a/x, Eq(c, 0)), (-(a - oo*b)/x, Eq(c, -1/x**2)), (-(a + oo*b)/
x, Eq(c, x**(-2))), (-a*c*x**4*sqrt(-1/c)/(c*x**5*sqrt(-1/c) + c*x**5*sqrt
(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - a*c*x**4*sqrt(1/c)/(c*x**5*sqrt(
-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) + a*sqrt(-1/c)/
(c**2*x**5*sqrt(-1/c) + c**2*x**5*sqrt(1/c) - x*sqrt(-1/c) - x*sqrt(1/c))
+ a*sqrt(1/c)/(c**2*x**5*sqrt(-1/c) + c**2*x**5*sqrt(1/c) - x*sqrt(-1/c) -
 x*sqrt(1/c)) + b*c**2*x**5*sqrt(-1/c)*sqrt(1/c)*log(x + sqrt(-1/c))/(c*x*
*5*sqrt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - b*c**
2*x**5*sqrt(-1/c)*sqrt(1/c)*log(x - sqrt(1/c))/(c*x**5*sqrt(-1/c) + c*x**5
*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - b*c**2*x**5*sqrt(-1/c)*sqrt
(1/c)*atanh(c*x**2)/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c
 - x*sqrt(1/c)/c) + b*c*x**5*log(x - sqrt(-1/c))/(c*x**5*sqrt(-1/c) + c*x*
*5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - b*c*x**5*log(x - sqrt(1/c
))/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c)
 - b*c*x**5*atanh(c*x**2)/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-
1/c)/c - x*sqrt(1/c)/c) - b*c*x**4*sqrt(-1/c)*atanh(c*x**2)/(c*x**5*sqrt(-
1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) - b*c*x**4*sqrt(
1/c)*atanh(c*x**2)/(c*x**5*sqrt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c
- x*sqrt(1/c)/c) - b*x*sqrt(-1/c)*sqrt(1/c)*log(x + sqrt(-1/c))/(c*x**5*sq
rt(-1/c) + c*x**5*sqrt(1/c) - x*sqrt(-1/c)/c - x*sqrt(1/c)/c) + b*x*sqr...
```

3.61.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{x^2}\,dx = \frac{1}{2}\left(c\left(\frac{2\arctan\left(\sqrt{c}x\right)}{\sqrt{c}}-\frac{\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{2\operatorname{artanh}\left(cx^2\right)}{x}\right)b-\frac{a}{x}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="maxima")
```

output
```
1/2*(c*(2*arctan(sqrt(c)*x)/sqrt(c) - log((c*x - sqrt(c))/(c*x + sqrt(c)))
/sqrt(c)) - 2*arctanh(c*x^2)/x)*b - a/x
```

3.61.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(38) = 76.

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^2}\,dx = \frac{1}{2}\,bc\left(\frac{2\,\arctan\left(x\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}-\frac{\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}\right) - \frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{2\,x}-\frac{a}{x}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^2,x, algorithm="giac")
```

output
```
1/2*b*c*(2*arctan(x*sqrt(abs(c)))/sqrt(abs(c)) + log(abs(x + 1/sqrt(abs(c)
)))/sqrt(abs(c)) - log(abs(x - 1/sqrt(abs(c))))/sqrt(abs(c))) - 1/2*b*log(
-(c*x^2 + 1)/(c*x^2 - 1))/x - a/x
```

3.61.9 Mupad [B] (verification not implemented)

Time = 3.41 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^2}\,dx = b\,\sqrt{c}\,\text{atan}\left(\sqrt{c}\,x\right)-\frac{a}{x}-\frac{b\,\ln\left(c\,x^2+1\right)}{2\,x}+\frac{b\,\ln\left(1-c\,x^2\right)}{2\,x}-b\,\sqrt{c}\,\text{atan}\left(\sqrt{c}\,x\,1\text{i}\right)\,1\text{i}$$

input
```
int((a + b*atanh(c*x^2))/x^2,x)
```

output
```
b*c^(1/2)*atan(c^(1/2)*x) - a/x - b*c^(1/2)*atan(c^(1/2)*x*1i)*1i - (b*log
(c*x^2 + 1))/(2*x) + (b*log(1 - c*x^2))/(2*x)
```

3.62 $\int \frac{a+b\operatorname{arctanh}(cx^2)}{x^4}\,dx$

3.62.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{x^4}\,dx = -\frac{2bc}{3x}-\frac{1}{3}bc^{3/2}\arctan\left(\sqrt{c}x\right)+\frac{1}{3}bc^{3/2}\operatorname{arctanh}\left(\sqrt{c}x\right)-\frac{a+b\operatorname{arctanh}(cx^2)}{3x^3}$$

output
```
-2/3*b*c/x-1/3*b*c^(3/2)*arctan(x*c^(1/2))+1/3*(-a-b*arctanh(c*x^2))/x^3+1
/3*b*c^(3/2)*arctanh(x*c^(1/2))
```

3.62.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{x^4}\,dx = -\frac{a}{3x^3}-\frac{2bc}{3x}-\frac{1}{3}bc^{3/2}\arctan\left(\sqrt{c}x\right)-\frac{b\operatorname{arctanh}(cx^2)}{3x^3}-\frac{1}{6}bc^{3/2}\log\left(1-\sqrt{c}x\right)+\frac{1}{6}bc^{3/2}\log\left(1+\sqrt{c}x\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/x^4,x]
```

output
```
-1/3*a/x^3 - (2*b*c)/(3*x) - (b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 - (b*ArcTanh[
c*x^2])/(3*x^3) - (b*c^(3/2)*Log[1 - Sqrt[c]*x])/6 + (b*c^(3/2)*Log[1 + Sq
rt[c]*x])/6
```

3.62.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6452, 847, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^4}dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{2}{3}bc\int \frac{1}{x^2(1-c^2x^4)}dx-\frac{a+b\text{arctanh}(cx^2)}{3x^3}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{2}{3}bc\left(c^2\int \frac{x^2}{1-c^2x^4}dx-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx^2)}{3x^3}$$

$$\Big\downarrow \text{ 827}$$

$$\frac{2}{3}bc\left(c^2\left(\frac{\int \frac{1}{1-cx^2}dx}{2c}-\frac{\int \frac{1}{cx^2+1}dx}{2c}\right)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx^2)}{3x^3}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2}{3}bc\left(c^2\left(\frac{\int \frac{1}{1-cx^2}dx}{2c}-\frac{\arctan(\sqrt{c}x)}{2c^{3/2}}\right)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx^2)}{3x^3}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2}{3}bc\left(c^2\left(\frac{\text{arctanh}(\sqrt{c}x)}{2c^{3/2}}-\frac{\arctan(\sqrt{c}x)}{2c^{3/2}}\right)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}(cx^2)}{3x^3}$$

input
```
Int[(a + b*ArcTanh[c*x^2])/x^4,x]
```

output
```
(2*b*c*(-x^(-1) + c^2*(-1/2*ArcTan[Sqrt[c]*x]/c^(3/2) + ArcTanh[Sqrt[c]*x]
/(2*c^(3/2)))))/3 - (a + b*ArcTanh[c*x^2])/(3*x^3)
```

3.62.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.62.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result
default	$-\frac{a}{3x^{3}}-\frac{b\,\operatorname{arctanh}\left(c\,x^{2}\right)}{3x^{3}}-\frac{2bc}{3x}+\frac{b\,c^{\frac{3}{2}}\,\operatorname{arctanh}\left(x\sqrt{c}\right)}{3}-\frac{b\,c^{\frac{3}{2}}\,\arctan\left(x\sqrt{c}\right)}{3}$
parts	$-\frac{a}{3x^{3}}-\frac{b\,\operatorname{arctanh}\left(c\,x^{2}\right)}{3x^{3}}-\frac{2bc}{3x}+\frac{b\,c^{\frac{3}{2}}\,\operatorname{arctanh}\left(x\sqrt{c}\right)}{3}-\frac{b\,c^{\frac{3}{2}}\,\arctan\left(x\sqrt{c}\right)}{3}$
risch	$-\frac{b\ln\left(c\,x^{2}+1\right)}{6x^{3}}-\frac{-c^{\frac{3}{2}}b\ln\left(-c^{\frac{11}{2}}-c^{6}x\right)x^{3}+c^{\frac{3}{2}}b\ln\left(c^{\frac{11}{2}}-c^{6}x\right)x^{3}-bc\sqrt{-c}\,\ln\left(c^{4}\sqrt{-c}-c^{5}x\right)x^{3}+bc\sqrt{-c}\,\ln\left(-c^{4}\sqrt{-c}-c^{5}x\right)x^{3}+4bc\,x}{6x^{3}}$

input
```
int((a+b*arctanh(c*x^2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3-1/3*b/x^3*arctanh(c*x^2)-2/3*b*c/x+1/3*b*c^(3/2)*arctanh(x*c^(1
/2))-1/3*b*c^(3/2)*arctan(x*c^(1/2))
```

3.62.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(47) = 94$.

Time = 0.27 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.87

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^4}\,dx = \left[-\frac{2\,bc^{\frac{3}{2}}x^3\arctan\left(\sqrt{c}x\right) - bc^{\frac{3}{2}}x^3\log\left(\frac{cx^2+2\sqrt{c}x+1}{cx^2-1}\right) + 4\,bcx^2 + b\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,a}{6\,x^3}, -\frac{2\,b\sqrt{-c}cx^3\arctan\left(\sqrt{-c}x\right) - b\sqrt{-c}cx^3\log\left(\frac{cx^2-2\sqrt{-c}x-1}{cx^2+1}\right) + 4\,bcx^2 + b\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,a}{6\,x^3}\right]$$

input
```
integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="fricas")
```

output
```
[-1/6*(2*b*c^(3/2)*x^3*arctan(sqrt(c)*x) - b*c^(3/2)*x^3*log((c*x^2 + 2*sq
rt(c)*x + 1)/(c*x^2 - 1)) + 4*b*c*x^2 + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) +
2*a)/x^3, -1/6*(2*b*sqrt(-c)*c*x^3*arctan(sqrt(-c)*x) - b*sqrt(-c)*c*x^3*l
og((c*x^2 - 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + 4*b*c*x^2 + b*log(-(c*x^2 + 1
)/(c*x^2 - 1)) + 2*a)/x^3]
```

3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1904 vs. $2(60) = 120$.

Time = 5.12 (sec) , antiderivative size = 1904, normalized size of antiderivative = 30.22

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*atanh(c*x**2))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*b)/(3*x**3), Eq(c, -1/x**2)),
 (-(a + oo*b)/(3*x**3), Eq(c, x**(-2))), (-a*c*x**4*sqrt(-1/c)/(3*c*x**7*s
qrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c)
 + a*c*x**4*sqrt(1/c)/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*s
qrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) + a*sqrt(-1/c)/(3*c**2*x**7*sqrt(-1/c) -
 3*c**2*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c) + 3*x**3*sqrt(1/c)) - a*sqrt(1/
c)/(3*c**2*x**7*sqrt(-1/c) - 3*c**2*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c) + 3
*x**3*sqrt(1/c)) + b*c**3*x**7*sqrt(-1/c)*sqrt(1/c)*log(x + sqrt(-1/c))/(3
*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqr
t(1/c)/c) - b*c**3*x**7*sqrt(-1/c)*sqrt(1/c)*log(x - sqrt(1/c))/(3*c*x**7*
sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c
) - b*c**3*x**7*sqrt(-1/c)*sqrt(1/c)*atanh(c*x**2)/(3*c*x**7*sqrt(-1/c) -
3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) - b*c**2*x*
*7*log(x - sqrt(-1/c))/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*
sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) + b*c**2*x**7*log(x - sqrt(1/c))/(3*c*x
**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/
c)/c) + b*c**2*x**7*atanh(c*x**2)/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c
) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sqrt(1/c)/c) - 2*b*c**2*x**6*sqrt(-1/c)/(
3*c*x**7*sqrt(-1/c) - 3*c*x**7*sqrt(1/c) - 3*x**3*sqrt(-1/c)/c + 3*x**3*sq
rt(1/c)/c) + 2*b*c**2*x**6*sqrt(1/c)/(3*c*x**7*sqrt(-1/c) - 3*c*x**7*sq...
```

3.62.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^4} \, dx$$
$$= -\frac{1}{6}\left(\left(2\sqrt{c}\arctan\left(\sqrt{c}x\right) + \sqrt{c}\log\left(\frac{cx - \sqrt{c}}{cx + \sqrt{c}}\right) + \frac{4}{x}\right)c + \frac{2\ \text{artanh}\left(cx^2\right)}{x^3}\right)b - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="maxima")
```

output
```
-1/6*((2*sqrt(c)*arctan(sqrt(c)*x) + sqrt(c)*log((c*x - sqrt(c))/(c*x + sq
rt(c))) + 4/x)*c + 2*arctanh(c*x^2)/x^3)*b - 1/3*a/x^3
```

3.62.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(47) = 94$.

Time = 0.32 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.51

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^4}\,dx = -\frac{bc^3\arctan\left(x\sqrt{|c|}\right)}{3\,|c|^{\frac{3}{2}}} + \frac{bc^3\log\left(\left|x + \frac{1}{\sqrt{|c|}}\right|\right)}{6\,|c|^{\frac{3}{2}}} - \frac{bc^3\log\left(\left|x - \frac{1}{\sqrt{|c|}}\right|\right)}{6\,|c|^{\frac{3}{2}}} - \frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{6\,x^3} - \frac{2\,bcx^2 + a}{3\,x^3}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^4,x, algorithm="giac")
```

output
```
-1/3*b*c^3*arctan(x*sqrt(abs(c)))/abs(c)^(3/2) + 1/6*b*c^3*log(abs(x + 1/s
qrt(abs(c))))/abs(c)^(3/2) - 1/6*b*c^3*log(abs(x - 1/sqrt(abs(c))))/abs(c)
^(3/2) - 1/6*b*log(-(c*x^2 + 1)/(c*x^2 - 1))/x^3 - 1/3*(2*b*c*x^2 + a)/x^3
```

3.62.9 Mupad [B] (verification not implemented)

Time = 3.48 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^4}\,dx = \frac{b\,\ln\left(1 - c\,x^2\right)}{6\,x^3} - \frac{b\,c^{3/2}\,\text{atan}\left(\sqrt{c}\,x\right)}{3} - \frac{b\,\ln\left(c\,x^2 + 1\right)}{6\,x^3} - \frac{2\,b\,c\,x^2 + a}{3\,x^3} - \frac{b\,c^{3/2}\,\text{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{3}$$

input
```
int((a + b*atanh(c*x^2))/x^4,x)
```

output
```
(b*log(1 - c*x^2))/(6*x^3) - (b*c^(3/2)*atan(c^(1/2)*x))/3 - (b*c^(3/2)*at
an(c^(1/2)*x*1i)*1i)/3 - (b*log(c*x^2 + 1))/(6*x^3) - (a + 2*b*c*x^2)/(3*x
^3)
```

3.63 $\int \frac{a+b\text{arctanh}(cx^2)}{x^6} dx$

3.63.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^6} dx = -\frac{2bc}{15x^3} + \frac{1}{5}bc^{5/2}\arctan\left(\sqrt{c}x\right) + \frac{1}{5}bc^{5/2}\text{arctanh}\left(\sqrt{c}x\right) - \frac{a+b\text{arctanh}(cx^2)}{5x^5}$$

output
```
-2/15*b*c/x^3+1/5*b*c^(5/2)*arctan(x*c^(1/2))+1/5*(-a-b*arctanh(c*x^2))/x^
5+1/5*b*c^(5/2)*arctanh(x*c^(1/2))
```

3.63.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^6} dx = -\frac{a}{5x^5} - \frac{2bc}{15x^3} + \frac{1}{5}bc^{5/2}\arctan\left(\sqrt{c}x\right) - \frac{b\text{arctanh}(cx^2)}{5x^5} - \frac{1}{10}bc^{5/2}\log\left(1-\sqrt{c}x\right) + \frac{1}{10}bc^{5/2}\log\left(1+\sqrt{c}x\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/x^6,x]
```

output
```
-1/5*a/x^5 - (2*b*c)/(15*x^3) + (b*c^(5/2)*ArcTan[Sqrt[c]*x])/5 - (b*ArcTa
nh[c*x^2])/(5*x^5) - (b*c^(5/2)*Log[1 - Sqrt[c]*x])/10 + (b*c^(5/2)*Log[1
+ Sqrt[c]*x])/10
```

3.63.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6452, 847, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(cx^2\right)}{x^6} \, dx$$
$$\Big\downarrow \text{6452}$$
$$\frac{2}{5} b c \int \frac{1}{x^4 \left(1-c^2 x^4\right)}dx-\frac{a+b\text{arctanh}\left(cx^2\right)}{5 x^5}$$
$$\Big\downarrow \text{847}$$
$$\frac{2}{5} b c \left(c^2 \int \frac{1}{1-c^2 x^4}dx-\frac{1}{3 x^3}\right)-\frac{a+b\text{arctanh}\left(cx^2\right)}{5 x^5}$$
$$\Big\downarrow \text{756}$$
$$\frac{2}{5} b c \left(c^2 \left(\frac{1}{2} \int \frac{1}{1-c x^2}dx+\frac{1}{2} \int \frac{1}{c x^2+1}dx\right)-\frac{1}{3 x^3}\right)-\frac{a+b\text{arctanh}\left(cx^2\right)}{5 x^5}$$
$$\Big\downarrow \text{216}$$
$$\frac{2}{5} b c \left(c^2 \left(\frac{1}{2} \int \frac{1}{1-c x^2}dx+\frac{\arctan\left(\sqrt{c} x\right)}{2 \sqrt{c}}\right)-\frac{1}{3 x^3}\right)-\frac{a+b\text{arctanh}\left(cx^2\right)}{5 x^5}$$
$$\Big\downarrow \text{219}$$
$$\frac{2}{5} b c \left(c^2 \left(\frac{\arctan\left(\sqrt{c} x\right)}{2 \sqrt{c}}+\frac{\text{arctanh}\left(\sqrt{c} x\right)}{2 \sqrt{c}}\right)-\frac{1}{3 x^3}\right)-\frac{a+b\text{arctanh}\left(cx^2\right)}{5 x^5}$$

input
```
Int[(a + b*ArcTanh[c*x^2])/x^6,x]
```

output
```
(2*b*c*(-1/3*1/x^3 + c^2*(ArcTan[Sqrt[c]*x]/(2*Sqrt[c]) + ArcTanh[Sqrt[c]*
x]/(2*Sqrt[c]))))/5 - (a + b*ArcTanh[c*x^2])/(5*x^5)
```

3.63.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.63.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

method	result	size
default	$-\frac{a}{5x^5}-\frac{b\,\operatorname{arctanh}\left(c\,x^2\right)}{5x^5}+\frac{b\,c^{\frac{5}{2}}\operatorname{arctanh}\left(x\sqrt{c}\right)}{5}+\frac{b\,c^{\frac{5}{2}}\arctan\left(x\sqrt{c}\right)}{5}-\frac{2bc}{15x^3}$	51
parts	$-\frac{a}{5x^5}-\frac{b\,\operatorname{arctanh}\left(c\,x^2\right)}{5x^5}+\frac{b\,c^{\frac{5}{2}}\operatorname{arctanh}\left(x\sqrt{c}\right)}{5}+\frac{b\,c^{\frac{5}{2}}\arctan\left(x\sqrt{c}\right)}{5}-\frac{2bc}{15x^3}$	51
risch	$-\frac{b\ln\left(c\,x^2+1\right)}{10x^5}-\frac{a}{5x^5}+\frac{b\ln\left(-c\,x^2+1\right)}{10x^5}+\frac{b\,c^{\frac{5}{2}}\operatorname{arctanh}\left(x\sqrt{c}\right)}{5}-\frac{2bc}{15x^3}+\frac{b\,c^{\frac{5}{2}}\arctan\left(x\sqrt{c}\right)}{5}$	68

input
```
int((a+b*arctanh(c*x^2))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5-1/5*b/x^5*arctanh(c*x^2)+1/5*b*c^(5/2)*arctanh(x*c^(1/2))+1/5*b
*c^(5/2)*arctan(x*c^(1/2))-2/15*b*c/x^3
```

3.63.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(47) = 94$.

Time = 0.28 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.97

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^6}\,dx = \left[\frac{6\,bc^{\frac{5}{2}}x^5\arctan\left(\sqrt{c}x\right) + 3\,bc^{\frac{5}{2}}x^5\log\left(\frac{cx^2+2\,\sqrt{c}x+1}{cx^2-1}\right) - 4\,bcx^2 - 3\,b\log\left(-\frac{cx^2+1}{cx^2-1}\right) - 6\,a}{30\,x^5}, -\frac{6\,b\sqrt{-c}c^2x^5\arctan\left(\sqrt{-c}x\right) - 3\,b\sqrt{-c}c^2x^5\log\left(\frac{cx^2+2\,\sqrt{-c}x-1}{cx^2+1}\right) + 4\,bcx^2 + 3\,b\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 6\,a}{30\,x^5}\right]$$

input
```
integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="fricas")
```

output
```
[1/30*(6*b*c^(5/2)*x^5*arctan(sqrt(c)*x) + 3*b*c^(5/2)*x^5*log((c*x^2 + 2*
sqrt(c)*x + 1)/(c*x^2 - 1)) - 4*b*c*x^2 - 3*b*log(-(c*x^2 + 1)/(c*x^2 - 1)
) - 6*a)/x^5, -1/30*(6*b*sqrt(-c)*c^2*x^5*arctan(sqrt(-c)*x) - 3*b*sqrt(-c
)*c^2*x^5*log((c*x^2 + 2*sqrt(-c)*x - 1)/(c*x^2 + 1)) + 4*b*c*x^2 + 3*b*lo
g(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a)/x^5]
```

3.63.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1948 vs. $2(61) = 122$.

Time = 6.91 (sec) , antiderivative size = 1948, normalized size of antiderivative = 30.92

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^6}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*atanh(c*x**2))/x**6,x)
```

output
```
Piecewise((-a/(5*x**5), Eq(c, 0)), (-(a - oo*b)/(5*x**5), Eq(c, -1/x**2)),
 (-(a + oo*b)/(5*x**5), Eq(c, x**(-2))), (-3*a*c*x**4*sqrt(-1/c)/(15*c*x**
9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1
/c)/c) - 3*a*c*x**4*sqrt(1/c)/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c)
- 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) + 3*a*sqrt(-1/c)/(15*c**2*x*
*9*sqrt(-1/c) + 15*c**2*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c) - 15*x**5*sqrt
(1/c)) + 3*a*sqrt(1/c)/(15*c**2*x**9*sqrt(-1/c) + 15*c**2*x**9*sqrt(1/c) -
 15*x**5*sqrt(-1/c) - 15*x**5*sqrt(1/c)) + 3*b*c**4*x**9*sqrt(-1/c)*sqrt(1
/c)*log(x + sqrt(-1/c))/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x
**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) - 3*b*c**4*x**9*sqrt(-1/c)*sqrt(1/
c)*log(x - sqrt(1/c))/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**
5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) - 3*b*c**4*x**9*sqrt(-1/c)*sqrt(1/c)
*atanh(c*x**2)/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(
-1/c)/c - 15*x**5*sqrt(1/c)/c) + 3*b*c**3*x**9*log(x - sqrt(-1/c))/(15*c*x
**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt
(1/c)/c) - 3*b*c**3*x**9*log(x - sqrt(1/c))/(15*c*x**9*sqrt(-1/c) + 15*c*x
**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)/c) - 3*b*c**3*x**
9*atanh(c*x**2)/(15*c*x**9*sqrt(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt
(-1/c)/c - 15*x**5*sqrt(1/c)/c) - 2*b*c**2*x**6*sqrt(-1/c)/(15*c*x**9*sqrt
(-1/c) + 15*c*x**9*sqrt(1/c) - 15*x**5*sqrt(-1/c)/c - 15*x**5*sqrt(1/c)...
```

3.63.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \frac{a + b\text{arctanh}(cx^2)}{x^6}\,dx$$
$$= \frac{1}{30}\left(\left(6\,c^{\frac{3}{2}}\arctan\left(\sqrt{c}x\right) - 3\,c^{\frac{3}{2}}\log\left(\frac{cx-\sqrt{c}}{cx+\sqrt{c}}\right) - \frac{4}{x^3}\right)c - \frac{6\,\text{artanh}\left(cx^2\right)}{x^5}\right)b - \frac{a}{5\,x^5}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="maxima")
```

output
```
1/30*((6*c^(3/2)*arctan(sqrt(c)*x) - 3*c^(3/2)*log((c*x - sqrt(c))/(c*x +
sqrt(c))) - 4/x^3)*c - 6*arctanh(c*x^2)/x^5)*b - 1/5*a/x^5
```

3.63.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.44

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^6}\,dx = \frac{1}{10}\,bc^3\left(\frac{2\,\arctan\left(x\sqrt{|c|}\right)}{\sqrt{|c|}}+\frac{\log\left(\left|x+\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}-\frac{\log\left(\left|x-\frac{1}{\sqrt{|c|}}\right|\right)}{\sqrt{|c|}}\right) - \frac{b\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{10\,x^5}-\frac{2\,bcx^2+3\,a}{15\,x^5}$$

input
```
integrate((a+b*arctanh(c*x^2))/x^6,x, algorithm="giac")
```

output
```
1/10*b*c^3*(2*arctan(x*sqrt(abs(c)))/sqrt(abs(c)) + log(abs(x + 1/sqrt(abs
(c))))/sqrt(abs(c)) - log(abs(x - 1/sqrt(abs(c))))/sqrt(abs(c))) - 1/10*b*
log(-(c*x^2 + 1)/(c*x^2 - 1))/x^5 - 1/15*(2*b*c*x^2 + 3*a)/x^5
```

3.63.9 Mupad [B] (verification not implemented)

Time = 3.79 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.13

$$\int \frac{a+b\text{arctanh}(cx^2)}{x^6}\,dx = \frac{b\,c^{5/2}\,\mathrm{atan}\left(\sqrt{c}\,x\right)}{5}-\frac{\frac{2\,b\,c\,x^2}{3}+a}{5\,x^5}-\frac{b\,\ln\left(c\,x^2+1\right)}{10\,x^5}+\frac{b\,\ln\left(1-c\,x^2\right)}{10\,x^5}-\frac{b\,c^{5/2}\,\mathrm{atan}\left(\sqrt{c}\,x\,1\mathrm{i}\right)\,1\mathrm{i}}{5}$$

input
```
int((a + b*atanh(c*x^2))/x^6,x)
```

output
```
(b*c^(5/2)*atan(c^(1/2)*x))/5 - (a + (2*b*c*x^2)/3)/(5*x^5) - (b*c^(5/2)*a
tan(c^(1/2)*x*1i)*1i)/5 - (b*log(c*x^2 + 1))/(10*x^5) + (b*log(1 - c*x^2))
/(10*x^5)
```

3.64 $\int x^7(a+b\text{arctanh}(cx^2))^2\,dx$

3.64.1 Optimal result

Integrand size = 16, antiderivative size = 125

$$\int x^7\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\frac{abx^2}{4c^3}+\frac{b^2x^4}{24c^2}+\frac{b^2x^2\text{arctanh}(cx^2)}{4c^3}+\frac{bx^6(a+b\text{arctanh}(cx^2))}{12c}-\frac{(a+b\text{arctanh}(cx^2))^2}{8c^4}+\frac{1}{8}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)^2+\frac{b^2\log\left(1-c^2x^4\right)}{6c^4}$$

output
```
1/4*a*b*x^2/c^3+1/24*b^2*x^4/c^2+1/4*b^2*x^2*arctanh(c*x^2)/c^3+1/12*b*x^6
*(a+b*arctanh(c*x^2))/c-1/8*(a+b*arctanh(c*x^2))^2/c^4+1/8*x^8*(a+b*arctan
h(c*x^2))^2+1/6*b^2*ln(-c^2*x^4+1)/c^4
```

3.64.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.17

$$\int x^7\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\frac{6abcx^2+b^2c^2x^4+2abc^3x^6+3a^2c^4x^8+2bcx^2(3ac^3x^6+b(3+c^2x^4))\,\text{arctanh}(cx^2)+3b^2(-1+c^4x^8)\,\text{arcta}}{24c^4}$$

input
```
Integrate[x^7*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
(6*a*b*c*x^2 + b^2*c^2*x^4 + 2*a*b*c^3*x^6 + 3*a^2*c^4*x^8 + 2*b*c*x^2*(3*
a*c^3*x^6 + b*(3 + c^2*x^4))*ArcTanh[c*x^2] + 3*b^2*(-1 + c^4*x^8)*ArcTanh
[c*x^2]^2 + b*(3*a + 4*b)*Log[1 - c*x^2] - 3*a*b*Log[1 + c*x^2] + 4*b^2*Lo
g[1 + c*x^2])/(24*c^4)
```

3.64.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6454, 6452, 6542, 6452, 243, 49, 2009, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+b\text{arctanh}\left(cx^2\right)\right)^2 \, dx$$

$$\Big\downarrow \text{ 6454}$$

$$\frac{1}{2}\int x^6 \left(a+b\text{arctanh}\left(cx^2\right)\right)^2 dx^2$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{1}{2}bc\int \frac{x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2x^4}dx^2\right)$$

$$\Big\downarrow \text{ 6542}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\int \frac{x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2x^4}dx^2}{c^2}-\frac{\int x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\int \frac{x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2x^4}dx^2}{c^2}-\frac{\frac{1}{3}x^6\left(a+b\text{arctanh}\left(cx^2\right)\right)-\frac{1}{3}bc\int \frac{x^6}{1-c^2x^4}dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\int \frac{x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2x^4}dx^2}{c^2}-\frac{\frac{1}{3}x^6\left(a+b\text{arctanh}\left(cx^2\right)\right)-\frac{1}{6}bc\int \frac{x^4}{1-c^2x^4}dx^4}{c^2}\right)\right)$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\int\frac{x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2x^4}dx^2}{c^2}-\frac{\frac{1}{3}x^6\left(a+b\text{arctanh}\left(cx^2\right)\right)-\frac{1}{6}bc\int\left(-\frac{1}{c^2}-\frac{1}{c^2(c^2x\right.}}{c^2}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\int\frac{x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2x^4}dx^2}{c^2}-\frac{\frac{1}{3}x^6\left(a+b\text{arctanh}\left(cx^2\right)\right)-\frac{1}{6}bc\left(-\frac{x^4}{c^2}-\frac{\log(1-c}{c^4}\right.}{c^2}\right.\right.$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{\int\frac{a+b\text{arctanh}\left(cx^2\right)}{1-c^2x^4}dx^2}{c^2}-\frac{\int\left(a+b\text{arctanh}\left(cx^2\right)\right)dx^2}{c^2}}{c^2}-\frac{\frac{1}{3}x^6\left(a+b\text{arctanh}\left(cx^2\right)\right)-}{c^2}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{\int\frac{a+b\text{arctanh}\left(cx^2\right)}{1-c^2x^4}dx^2}{c^2}-\frac{ax^2+bx^2\text{arctanh}\left(cx^2\right)+\frac{b\log\left(1-c^2x^4\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^6\left(a+b\text{arctan}\right.}{}\right.\right.$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{2}\left(\frac{1}{4}x^8\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2bc^3}-\frac{ax^2+bx^2\text{arctanh}\left(cx^2\right)+\frac{b\log\left(1-c^2x^4\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^6\left(a+b\text{arctanh}\right.}{}\right.\right.$$

input
```
Int[x^7*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
((x^8*(a + b*ArcTanh[c*x^2])^2)/4 - (b*c*(-(((x^6*(a + b*ArcTanh[c*x^2]))/
3 - (b*c*(-(x^4/c^2) - Log[1 - c^2*x^4]/c^4))/6)/c^2) + ((a + b*ArcTanh[c*
x^2])^2/(2*b*c^3) - (a*x^2 + b*x^2*ArcTanh[c*x^2] + (b*Log[1 - c^2*x^4])/(
2*c))/c^2)/c^2))/2)/2
```

3.64. $\int x^7\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx$

3.64.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.64.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.30

method	result
parallelrisch	$\frac{3b^2 \operatorname{arctanh}(c x^2)^2 x^8 c^4+6ab \operatorname{arctanh}(c x^2) x^8 c^4+3a^2 c^4 x^8+2b^2 \operatorname{arctanh}(c x^2) x^6 c^3+2ab c^3 x^6+b^2 c^2 x^4+6b^2 \operatorname{arctanh}(c x^2) x^2 c+}{24c^4}$
risch	$\frac{b^2(c^4x^8-1)\ln(c x^2+1)^2}{32c^4}+\frac{b(-3x^8 b\ln(-c x^2+1)c^4+6a c^4x^8+2b c^3x^6+6bc x^2+3b\ln(-c x^2+1))\ln(c x^2+1)}{48c^4}+\frac{b^2x^8\ln(-c}{32}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^7*(a+b*arctanh(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/24*(3*b^2*arctanh(c*x^2)^2*x^8*c^4+6*a*b*arctanh(c*x^2)*x^8*c^4+3*a^2*c^
4*x^8+2*b^2*arctanh(c*x^2)*x^6*c^3+2*a*b*c^3*x^6+b^2*c^2*x^4+6*b^2*arctanh
(c*x^2)*x^2*c+6*a*b*c*x^2-3*b^2*arctanh(c*x^2)^2+8*ln(c*x^2-1)*b^2-6*arcta
nh(c*x^2)*a*b+8*arctanh(c*x^2)*b^2+b^2)/c^4
```

3.64.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.41

$$\int x^7 \left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2 dx$$
$$=\frac{12\,a^2c^4x^8+8\,abc^3x^6+4\,b^2c^2x^4+24\,abcx^2+3\,(b^2c^4x^8-b^2)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2-4\,(3\,ab-4\,b^2)\log\left(cx^2+1\right)}{96\,c^4}$$

input
```
integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

output
```
1/96*(12*a^2*c^4*x^8 + 8*a*b*c^3*x^6 + 4*b^2*c^2*x^4 + 24*a*b*c*x^2 + 3*(b
^2*c^4*x^8 - b^2)*log(-(c*x^2 + 1)/(c*x^2 - 1))^2 - 4*(3*a*b - 4*b^2)*log(
c*x^2 + 1) + 4*(3*a*b + 4*b^2)*log(c*x^2 - 1) + 4*(3*a*b*c^4*x^8 + b^2*c^3
*x^6 + 3*b^2*c*x^2)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^4
```

3.64.6 Sympy [A] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.65

$$\int x^7 \left(a + b\text{arctanh}\left(cx^2\right)\right)^2 dx$$
$$= \begin{cases} \frac{a^2x^8}{8} + \frac{abx^8 \operatorname{atanh}\left(cx^2\right)}{4} + \frac{abx^6}{12c} + \frac{abx^2}{4c^3} - \frac{ab \operatorname{atanh}\left(cx^2\right)}{4c^4} + \frac{b^2x^8 \operatorname{atanh}^2\left(cx^2\right)}{8} + \frac{b^2x^6 \operatorname{atanh}\left(cx^2\right)}{12c} + \frac{b^2x^4}{24c^2} + \frac{b^2x^2 \operatorname{atanh}\left(cx^2\right)}{4c^3} \\ \frac{a^2x^8}{8} \end{cases}$$

input
```
integrate(x**7*(a+b*atanh(c*x**2))**2,x)
```

output
```
Piecewise((a**2*x**8/8 + a*b*x**8*atanh(c*x**2)/4 + a*b*x**6/(12*c) + a*b*
x**2/(4*c**3) - a*b*atanh(c*x**2)/(4*c**4) + b**2*x**8*atanh(c*x**2)**2/8
+ b**2*x**6*atanh(c*x**2)/(12*c) + b**2*x**4/(24*c**2) + b**2*x**2*atanh(c
*x**2)/(4*c**3) + b**2*log(x - sqrt(-1/c))/(3*c**4) + b**2*log(x + sqrt(-1
/c))/(3*c**4) - b**2*atanh(c*x**2)**2/(8*c**4) - b**2*atanh(c*x**2)/(3*c**
4), Ne(c, 0)), (a**2*x**8/8, True))
```

3.64.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.74

$$\int x^7 \left(a + b\text{arctanh}\left(cx^2\right)\right)^2 dx = \frac{1}{8} b^2x^8 \operatorname{artanh}\left(cx^2\right)^2 + \frac{1}{8} a^2x^8$$
$$+ \frac{1}{24} \left(6 x^8 \operatorname{artanh}\left(cx^2\right) + c\left(\frac{2\left(c^2x^6 + 3x^2\right)}{c^4} - \frac{3 \log\left(cx^2 + 1\right)}{c^5} + \frac{3 \log\left(cx^2 - 1\right)}{c^5}\right)\right)ab$$
$$+ \frac{1}{96} \left(4c\left(\frac{2\left(c^2x^6 + 3x^2\right)}{c^4} - \frac{3 \log\left(cx^2 + 1\right)}{c^5} + \frac{3 \log\left(cx^2 - 1\right)}{c^5}\right) \operatorname{artanh}\left(cx^2\right) + \frac{4c^2x^4 - 2\left(3 \log\left(cx^2 - \right.\right.}{}\right.$$

input
```
integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*x^8*arctanh(c*x^2)^2 + 1/8*a^2*x^8 + 1/24*(6*x^8*arctanh(c*x^2) +
c*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log(c*x^2 + 1)/c^5 + 3*log(c*x^2 - 1)/c^5))
*a*b + 1/96*(4*c*(2*(c^2*x^6 + 3*x^2)/c^4 - 3*log(c*x^2 + 1)/c^5 + 3*log(c
*x^2 - 1)/c^5)*arctanh(c*x^2) + (4*c^2*x^4 - 2*(3*log(c*x^2 - 1) - 8)*log(
c*x^2 + 1) + 3*log(c*x^2 + 1)^2 + 3*log(c*x^2 - 1)^2 + 16*log(c*x^2 - 1))/
c^4)*b^2
```

3.64.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\int x^7 (a + b\text{arctanh}(cx^2))^2 \, dx = \frac{1}{8} a^2 x^8 + \frac{abx^6}{12c} + \frac{b^2 x^4}{24c^2} + \frac{1}{32} \left(b^2 x^8 - \frac{b^2}{c^4}\right) \log\left(-\frac{cx^2+1}{cx^2-1}\right)^2 + \frac{1}{24} \left(3abx^8 + \frac{b^2 x^6}{c} + \frac{3b^2 x^2}{c^3}\right) \log\left(-\frac{cx^2+1}{cx^2-1}\right) + \frac{abx^2}{4c^3} - \frac{(3ab - 4b^2) \log(cx^2+1)}{24c^4} + \frac{(3ab + 4b^2) \log(cx^2-1)}{24c^4}$$

input
```
integrate(x^7*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

output
```
1/8*a^2*x^8 + 1/12*a*b*x^6/c + 1/24*b^2*x^4/c^2 + 1/32*(b^2*x^8 - b^2/c^4)
*log(-(c*x^2 + 1)/(c*x^2 - 1))^2 + 1/24*(3*a*b*x^8 + b^2*x^6/c + 3*b^2*x^2
/c^3)*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 1/4*a*b*x^2/c^3 - 1/24*(3*a*b - 4*b^
2)*log(c*x^2 + 1)/c^4 + 1/24*(3*a*b + 4*b^2)*log(c*x^2 - 1)/c^4
```

3.64.9 Mupad [B] (verification not implemented)

Time = 4.22 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.68

$$\int x^7 (a + b\text{arctanh}(cx^2))^2 \, dx = \frac{a^2 x^8}{8} + \frac{b^2 \ln(c x^2 - 1)}{6c^4} + \frac{b^2 \ln(c x^2 + 1)}{6c^4} - \frac{b^2 \ln(c x^2+1)^2}{32c^4} - \frac{b^2 \ln(1 - c x^2)^2}{32c^4} + \frac{b^2 x^4}{24c^2} + \frac{b^2 x^8 \ln(c x^2+1)^2}{32} + \frac{b^2 x^8 \ln(1 - c x^2)^2}{32} + \frac{b^2 x^2 \ln(c x^2+1)}{8c^3} - \frac{b^2 x^2 \ln(1 - c x^2)}{8c^3} + \frac{b^2 x^6 \ln(c x^2+1)}{24c} - \frac{b^2 x^6 \ln(1 - c x^2)}{24c} + \frac{a b \ln(c x^2 - 1)}{8c^4} - \frac{a b \ln(c x^2 + 1)}{8c^4} + \frac{a b x^8 \ln(c x^2+1)}{8} - \frac{a b x^8 \ln(1 - c x^2)}{8} + \frac{b^2 \ln(c x^2+1) \ln(1 - c x^2)}{16c^4} + \frac{a b x^2}{4c^3} + \frac{a b x^6}{12c} - \frac{b^2 x^8 \ln(c x^2+1) \ln(1 - c x^2)}{16}$$

input
```
int(x^7*(a + b*atanh(c*x^2))^2,x)
```

output
```
(a^2*x^8)/8 + (b^2*log(c*x^2 - 1))/(6*c^4) + (b^2*log(c*x^2 + 1))/(6*c^4)
- (b^2*log(c*x^2 + 1)^2)/(32*c^4) - (b^2*log(1 - c*x^2)^2)/(32*c^4) + (b^2
*x^4)/(24*c^2) + (b^2*x^8*log(c*x^2 + 1)^2)/32 + (b^2*x^8*log(1 - c*x^2)^2
)/32 + (b^2*x^2*log(c*x^2 + 1))/(8*c^3) - (b^2*x^2*log(1 - c*x^2))/(8*c^3)
 + (b^2*x^6*log(c*x^2 + 1))/(24*c) - (b^2*x^6*log(1 - c*x^2))/(24*c) + (a*
b*log(c*x^2 - 1))/(8*c^4) - (a*b*log(c*x^2 + 1))/(8*c^4) + (a*b*x^8*log(c*
x^2 + 1))/8 - (a*b*x^8*log(1 - c*x^2))/8 + (b^2*log(c*x^2 + 1)*log(1 - c*x
^2))/(16*c^4) + (a*b*x^2)/(4*c^3) + (a*b*x^6)/(12*c) - (b^2*x^8*log(c*x^2
+ 1)*log(1 - c*x^2))/16
```

3.64. $\int x^7 (a + b\text{arctanh}(cx^2))^2 \, dx$

3.65 $\int x^5(a+b\text{arctanh}(cx^2))^2\,dx$

3.65.1 Optimal result

Integrand size = 16, antiderivative size = 146

$$\begin{aligned}\int x^5\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=&\frac{b^2x^2}{6c^2}-\frac{b^2\text{arctanh}(cx^2)}{6c^3}+\frac{bx^4(a+b\text{arctanh}(cx^2))}{6c}\\&+\frac{(a+b\text{arctanh}(cx^2))^2}{6c^3}+\frac{1}{6}x^6\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\\&-\frac{b(a+b\text{arctanh}(cx^2))\log\left(\frac{2}{1-cx^2}\right)}{3c^3}\\&-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{6c^3}\end{aligned}$$

output
```
1/6*b^2*x^2/c^2-1/6*b^2*arctanh(c*x^2)/c^3+1/6*b*x^4*(a+b*arctanh(c*x^2))/
c+1/6*(a+b*arctanh(c*x^2))^2/c^3+1/6*x^6*(a+b*arctanh(c*x^2))^2-1/3*b*(a+b
*arctanh(c*x^2))*ln(2/(-c*x^2+1))/c^3-1/6*b^2*polylog(2,1-2/(-c*x^2+1))/c^
3
```

3.65.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int x^5\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx$$

$$=\frac{b^2cx^2+abc^2x^4+a^2c^3x^6+b^2(-1+c^3x^6)\,\text{arctanh}(cx^2)^2+b\text{arctanh}(cx^2)\left(-b+bc^2x^4+2ac^3x^6-2b\log\left(\right.\right.}{6c^3}$$

input
```
Integrate[x^5*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
(b^2*c*x^2 + a*b*c^2*x^4 + a^2*c^3*x^6 + b^2*(-1 + c^3*x^6)*ArcTanh[c*x^2]
^2 + b*ArcTanh[c*x^2]*(-b + b*c^2*x^4 + 2*a*c^3*x^6 - 2*b*Log[1 + E^(-2*Ar
cTanh[c*x^2])]) + a*b*Log[-1 + c^2*x^4] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*
x^2])])/(6*c^3)
```

3.65.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6454, 6452, 6542, 6452, 262, 219, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b\text{arctanh}\left(cx^2\right)\right)^2 \, dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{2} \int x^4 \left(a+b\text{arctanh}\left(cx^2\right)\right)^2 dx^2$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2} \left(\frac{1}{3} x^6 \left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{2}{3} bc \int \frac{x^6 \left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2 x^4}dx^2\right)$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{2} \left(\frac{1}{3} x^6 \left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{2}{3} bc \left(\frac{\int \frac{x^2 \left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2 x^4}dx^2}{c^2}-\frac{\int x^2 \left(a+b\text{arctanh}\left(cx^2\right)\right)dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2} \left(\frac{1}{3} x^6 \left(a+b\text{arctanh}\left(cx^2\right)\right)^2-\frac{2}{3} bc \left(\frac{\int \frac{x^2 \left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2 x^4}dx^2}{c^2}-\frac{\frac{1}{2} x^4 \left(a+b\text{arctanh}\left(cx^2\right)\right)-\frac{1}{2} bc \int \frac{x^4}{1-c^2 x^4}dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\text{arctanh}(cx^2)\right)^2-\frac{2}{3}bc\left(\frac{\int\frac{x^2\left(a+b\text{arctanh}(cx^2)\right)}{1-c^2x^4}dx^2}{c^2}-\frac{\frac{1}{2}x^4\left(a+b\text{arctanh}(cx^2)\right)-\frac{1}{2}bc\left(\frac{\int\frac{1}{1-c^2x^4}dx^2}{c^2}-\right.}{c^2}\right.\right.$$

$\downarrow$ 219

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\text{arctanh}(cx^2)\right)^2-\frac{2}{3}bc\left(\frac{\int\frac{x^2\left(a+b\text{arctanh}(cx^2)\right)}{1-c^2x^4}dx^2}{c^2}-\frac{\frac{1}{2}x^4\left(a+b\text{arctanh}(cx^2)\right)-\frac{1}{2}bc\left(\frac{\text{arctanh}(cx^2)}{c^3}\right.}{c^2}\right.\right.$$

$\downarrow$ 6546

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\text{arctanh}(cx^2)\right)^2-\frac{2}{3}bc\left(\frac{\frac{\int\frac{a+b\text{arctanh}(cx^2)}{1-cx^2}dx^2}{c}-\frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^4\left(a+b\text{arctanh}(cx^2)\right)-\frac{1}{2}b}{c^2}\right.\right.$$

$\downarrow$ 6470

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\text{arctanh}(cx^2)\right)^2-\frac{2}{3}bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}(cx^2)\right)}{c}-b\int\frac{\log\left(\frac{2}{1-cx^2}\right)}{1-c^2x^4}dx^2}{c}-\frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^4\left(a\right.}{}\right.\right.$$

$\downarrow$ 2849

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\text{arctanh}(cx^2)\right)^2-\frac{2}{3}bc\left(\frac{\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx^2}\right)}{1-\frac{2}{1-cx^2}}d\frac{1}{1-cx^2}}{c}+\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}(cx^2)\right)}{c}}{c}-\frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x}{}\right.\right.$$

$\downarrow$ 2752

$$\frac{1}{2}\left(\frac{1}{3}x^6\left(a+b\text{arctanh}(cx^2)\right)^2-\frac{2}{3}bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}(cx^2)\right)}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{2c}}{c}-\frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^4}{}\right.\right.$$

input
```
Int[x^5*(a + b*ArcTanh[c*x^2])^2,x]
```

3.65. $\int x^5\left(a+b\text{arctanh}(cx^2)\right)^2\,dx$

output
```
((x^6*(a + b*ArcTanh[c*x^2])^2)/3 - (2*b*c*(-(((x^4*(a + b*ArcTanh[c*x^2])
)/2 - (b*c*(-(x^2/c^2) + ArcTanh[c*x^2]/c^3))/2)/c^2) + (-1/2*(a + b*ArcTa
nh[c*x^2])^2/(b*c^2) + (((a + b*ArcTanh[c*x^2])*Log[2/(1 - c*x^2)])/c + (b
*PolyLog[2, 1 - 2/(1 - c*x^2)])/(2*c))/c)/c^2))/3)/2
```

3.65.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.65.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(132) = 264$.

Time = 1.35 (sec) , antiderivative size = 380, normalized size of antiderivative = 2.60

method	result
risch	$\frac{b^{2} x^{4} \ln\left(c\,x^{2}+1\right)}{12 c}+\frac{b^{2} x^{6} \ln\left(c\,x^{2}+1\right)^{2}}{24}-\frac{b^{2} \ln\left(c\,x^{2}+1\right)}{12 c^{3}}+\frac{b^{2} \ln\left(c\,x^{2}+1\right)^{2}}{24 c^{3}}-\frac{b^{2} \operatorname{dilog}\left(\frac{c\,x^{2}}{2}+\frac{1}{2}\right)}{6 c^{3}}-\frac{2 b^{2} \ln\left(c\,x^{2}-1\right)}{9 c^{3}}-\frac{b^{2} x^{4} \ln(-}{12}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^5*(a+b*arctanh(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*b^2/c*x^4*ln(c*x^2+1)+1/24*b^2*x^6*ln(c*x^2+1)^2-1/12*b^2/c^3*ln(c*x^
2+1)+1/24/c^3*b^2*ln(c*x^2+1)^2-1/6*b^2/c^3*dilog(1/2*c*x^2+1/2)-2/9*b^2/c
^3*ln(c*x^2-1)-1/12*b^2/c*x^4*ln(-c*x^2+1)-1/6*a*b*x^6*ln(-c*x^2+1)+1/6*a*
b/c^3*ln(c*x^2-1)-17/108/c^3*b^2+1/24*b^2*x^6*ln(-c*x^2+1)^2+11/36/c^3*b^2
*ln(-c*x^2+1)-1/24/c^3*b^2*ln(-c*x^2+1)^2+1/6/c*a*b*x^4+1/6*b^2*x^2/c^2+1/
6*a^2*x^6+1/6*b*a*x^6*ln(c*x^2+1)+1/6*b*a/c^3*ln(c*x^2+1)-1/12*b^2*ln(-c*x
^2+1)*ln(c*x^2+1)*x^6-1/12*b^2/c^3*ln(-c*x^2+1)*ln(c*x^2+1)+1/6*b^2/c^3*ln
(1/2-1/2*c*x^2)*ln(c*x^2+1)-1/6*b^2/c^3*ln(1/2-1/2*c*x^2)*ln(1/2*c*x^2+1/2
)
```

3.65.5 Fricas [F]

$$\int x^5 (a + b\text{arctanh}(cx^2))^2 \, dx = \int (b \,\text{artanh}\,(cx^2) + a)^2 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^5*arctanh(c*x^2)^2 + 2*a*b*x^5*arctanh(c*x^2) + a^2*x^5, x)
```

3.65.6 Sympy [F]

$$\int x^5 (a + b\text{arctanh}(cx^2))^2 \, dx = \int x^5 (a + b \,\text{atanh}\,(cx^2))^2 \, dx$$

input
```
integrate(x**5*(a+b*atanh(c*x**2))**2,x)
```

output
```
Integral(x**5*(a + b*atanh(c*x**2))**2, x)
```

3.65.7 Maxima [F]

$$\int x^5 (a + b\text{arctanh}(cx^2))^2 \, dx = \int (b \,\text{artanh}\,(cx^2) + a)^2 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/6*a^2*x^6 + 1/6*(2*x^6*arctanh(c*x^2) + (x^4/c^2 + log(c^2*x^4 - 1)/c^4)
*c)*a*b + 1/432*(18*x^6*log(-c*x^2 + 1)^2 - 2*c^4*(2*(c^2*x^6 + 3*x^2)/c^6
 - 3*log(c*x^2 + 1)/c^7 + 3*log(c*x^2 - 1)/c^7) + 3*c^3*(x^4/c^4 + log(c^2
*x^4 - 1)/c^6) + 1296*c^3*integrate(1/9*x^7*log(c*x^2 + 1)/(c^4*x^4 - c^2)
, x) - 9*c^2*(2*x^2/c^4 - log(c*x^2 + 1)/c^5 + log(c*x^2 - 1)/c^5) - 6*c*(
(2*c^2*x^6 + 3*c*x^4 + 6*x^2)/c^3 + 6*log(c*x^2 - 1)/c^4)*log(-c*x^2 + 1)
+ 648*c*integrate(1/9*x^3*log(c*x^2 + 1)/(c^4*x^4 - c^2), x) + 6*(3*c^3*x^
6*log(c*x^2 + 1)^2 + (2*c^3*x^6 - 3*c^2*x^4 + 6*c*x^2 - 6*(c^3*x^6 + 1)*lo
g(c*x^2 + 1))*log(-c*x^2 + 1))/c^3 + (4*c^3*x^6 + 15*c^2*x^4 + 66*c*x^2 +
18*log(c*x^2 - 1)^2 + 66*log(c*x^2 - 1))/c^3 - 18*log(9*c^4*x^4 - 9*c^2)/c
^3 + 648*integrate(1/9*x*log(c*x^2 + 1)/(c^4*x^4 - c^2), x))*b^2
```

3.65.8 Giac [F]

$$\int x^5 (a + b\text{arctanh}(cx^2))^2 \, dx = \int (b \,\text{artanh}\,(cx^2) + a)^2 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2*x^5, x)
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int x^5 (a + b\text{arctanh}(cx^2))^2 \, dx = \int x^5 \left(a + b \,\text{atanh}(c\, x^2)\right)^2 \, dx$$

input
```
int(x^5*(a + b*atanh(c*x^2))^2,x)
```

output
```
int(x^5*(a + b*atanh(c*x^2))^2, x)
```

3.66 $\int x^3 (a + b\text{arctanh}(cx^2))^2 \, dx$

3.66.1 Optimal result

Integrand size = 16, antiderivative size = 91

$$\int x^3 (a + b\text{arctanh}(cx^2))^2 \, dx = \frac{abx^2}{2c} + \frac{b^2x^2\text{arctanh}(cx^2)}{2c} - \frac{(a + b\text{arctanh}(cx^2))^2}{4c^2} + \frac{1}{4}x^4 (a + b\text{arctanh}(cx^2))^2 + \frac{b^2 \log(1 - c^2x^4)}{4c^2}$$

output
```
1/2*a*b*x^2/c+1/2*b^2*x^2*arctanh(c*x^2)/c-1/4*(a+b*arctanh(c*x^2))^2/c^2+
1/4*x^4*(a+b*arctanh(c*x^2))^2+1/4*b^2*ln(-c^2*x^4+1)/c^2
```

3.66.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int x^3 (a + b\text{arctanh}(cx^2))^2 \, dx = \frac{2abcx^2 + a^2c^2x^4 + 2bcx^2(b + acx^2)\,\text{arctanh}(cx^2) + b^2(-1 + c^2x^4)\,\text{arctanh}(cx^2)^2 + b(a + b)\log(1 - cx^2) -}{4c^2}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
(2*a*b*c*x^2 + a^2*c^2*x^4 + 2*b*c*x^2*(b + a*c*x^2)*ArcTanh[c*x^2] + b^2*
(-1 + c^2*x^4)*ArcTanh[c*x^2]^2 + b*(a + b)*Log[1 - c*x^2] - a*b*Log[1 + c
*x^2] + b^2*Log[1 + c*x^2])/(4*c^2)
```

3.66.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6454, 6452, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\text{arctanh}\left(cx^2\right)\right)^2 dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{2}\int x^2 \left(a+b\text{arctanh}\left(cx^2\right)\right)^2 dx^2$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-bc\int \frac{x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2x^4}dx^2\right)$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-bc\left(\frac{\int \frac{a+b\text{arctanh}\left(cx^2\right)}{1-c^2x^4}dx^2}{c^2}-\frac{\int \left(a+b\text{arctanh}\left(cx^2\right)\right)dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-bc\left(\frac{\int \frac{a+b\text{arctanh}\left(cx^2\right)}{1-c^2x^4}dx^2}{c^2}-\frac{ax^2+bx^2\text{arctanh}\left(cx^2\right)+\frac{b\log\left(1-c^2x^4\right)}{2c}}{c^2}\right)\right)$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-bc\left(\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2bc^3}-\frac{ax^2+bx^2\text{arctanh}\left(cx^2\right)+\frac{b\log\left(1-c^2x^4\right)}{2c}}{c^2}\right)\right)$$

input
```
Int[x^3*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
((x^4*(a + b*ArcTanh[c*x^2])^2)/2 - b*c*((a + b*ArcTanh[c*x^2])^2/(2*b*c^3
) - (a*x^2 + b*x^2*ArcTanh[c*x^2] + (b*Log[1 - c^2*x^4])/(2*c))/c^2))/2
```

3.66.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.66.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

method	result
parallelrisch	$\frac{b^2 \operatorname{arctanh}(c x^2)^2 x^4 c^2 + 2x^4 \operatorname{arctanh}(c x^2) ab c^2 + a^2 c^2 x^4 + 2b^2 \operatorname{arctanh}(c x^2) x^2 c + 2abc x^2 - b^2 \operatorname{arctanh}(c x^2)^2 + 2\ln(c x^2 - 1) b^2 -}{4c^2}$
risch	$\frac{b^2 (c^2 x^4 - 1) \ln(c x^2 + 1)^2}{16c^2} + \frac{b(-2b x^4 \ln(-c x^2 + 1) a c^2 + 4a^2 c^2 x^4 + 4abc x^2 + 2b \ln(-c x^2 + 1) a + b^2) \ln(c x^2 + 1)}{16a c^2} + \frac{b^2 x^4 \ln(-c}{16}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^3*(a+b*arctanh(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*arctanh(c*x^2)^2*x^4*c^2+2*x^4*arctanh(c*x^2)*a*b*c^2+a^2*c^2*x^4
+2*b^2*arctanh(c*x^2)*x^2*c+2*a*b*c*x^2-b^2*arctanh(c*x^2)^2+2*ln(c*x^2-1)
*b^2-2*arctanh(c*x^2)*a*b+2*arctanh(c*x^2)*b^2)/c^2
```

3.66.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.52

$$\int x^3\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx$$
$$=\frac{4\,a^2c^2x^4+8\,abcx^2+\left(b^2c^2x^4-b^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)^2-4\left(ab-b^2\right)\log\left(cx^2+1\right)+4\left(ab+b^2\right)\log\left(cx^2-1\right)+\cdots}{16\,c^2}$$

input
```
integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

output
```
1/16*(4*a^2*c^2*x^4 + 8*a*b*c*x^2 + (b^2*c^2*x^4 - b^2)*log(-(c*x^2 + 1)/(
c*x^2 - 1))^2 - 4*(a*b - b^2)*log(c*x^2 + 1) + 4*(a*b + b^2)*log(c*x^2 - 1
) + 4*(a*b*c^2*x^4 + b^2*c*x^2)*log(-(c*x^2 + 1)/(c*x^2 - 1)))/c^2
```

3.66.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(78) = 156.

Time = 4.01 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.79

$$\int x^3\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx$$
$$=\begin{cases}\frac{a^2x^4}{4}+\frac{abx^4\,\text{atanh}\left(cx^2\right)}{2}+\frac{abx^2}{2c}-\frac{ab\,\text{atanh}\left(cx^2\right)}{2c^2}+\frac{b^2x^4\,\text{atanh}^2\left(cx^2\right)}{4}+\frac{b^2x^2\,\text{atanh}\left(cx^2\right)}{2c}+\frac{b^2\log\left(x-\sqrt{-\frac{1}{c}}\right)}{2c^2}+\frac{b^2\log\left(x+\sqrt{-\cdots}\right)}{2c^2}\cdots\\ \frac{a^2x^4}{4}\end{cases}$$

input
```
integrate(x**3*(a+b*atanh(c*x**2))**2,x)
```

output
```
Piecewise((a**2*x**4/4 + a*b*x**4*atanh(c*x**2)/2 + a*b*x**2/(2*c) - a*b*a
tanh(c*x**2)/(2*c**2) + b**2*x**4*atanh(c*x**2)**2/4 + b**2*x**2*atanh(c*x
**2)/(2*c) + b**2*log(x - sqrt(-1/c))/(2*c**2) + b**2*log(x + sqrt(-1/c))/
(2*c**2) - b**2*atanh(c*x**2)**2/(4*c**2) - b**2*atanh(c*x**2)/(2*c**2), N
e(c, 0)), (a**2*x**4/4, True))
```

3.66.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(81) = 162$.

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.04

$$\int x^3 \left(a + b\text{arctanh}(cx^2)\right)^2 \, dx = \frac{1}{4} b^2 x^4 \operatorname{artanh}\left(cx^2\right)^2 + \frac{1}{4} a^2 x^4 + \frac{1}{4} \left(2 x^4 \operatorname{artanh}\left(cx^2\right) + c \left(\frac{2 x^2}{c^2} - \frac{\log\left(cx^2+1\right)}{c^3} + \frac{\log\left(cx^2-1\right)}{c^3}\right)\right) ab + \frac{1}{16} \left(4 c \left(\frac{2 x^2}{c^2} - \frac{\log\left(cx^2+1\right)}{c^3} + \frac{\log\left(cx^2-1\right)}{c^3}\right) \operatorname{artanh}\left(cx^2\right) - \frac{2 \left(\log\left(cx^2-1\right) - 2\right) \log\left(cx^2+1\right) - \log$$

input
```
integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctanh(c*x^2)^2 + 1/4*a^2*x^4 + 1/4*(2*x^4*arctanh(c*x^2) + c
*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3))*a*b + 1/16*(4*c*(2
*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3)*arctanh(c*x^2) - (2*(l
og(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - log(c*x^2 - 1)^2 -
4*log(c*x^2 - 1))/c^2)*b^2
```

3.66.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(81) = 162$.

Time = 0.31 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.97

$$\int x^3 \left(a + b\text{arctanh}(cx^2)\right)^2 \, dx = \frac{1}{4} \left(\frac{(cx^2+1)b^2 \log\left(-\frac{cx^2+1}{cx^2-1}\right)^2}{\left(\frac{(cx^2+1)^2 c^3}{(cx^2-1)^2} - \frac{2(cx^2+1)c^3}{cx^2-1} + c^3\right)(cx^2-1)} + \frac{2\left(\frac{2(cx^2+1)ab}{cx^2-1} + \frac{(cx^2+1)b^2}{cx^2-1} - b^2\right)\log\left(-\frac{cx^2+1}{cx^2-1}\right)}{\frac{(cx^2+1)^2 c^3}{(cx^2-1)^2} - \frac{2(cx^2+1)c^3}{cx^2-1} + c^3} + \frac{4\left(\frac{(cx^2+1)a^2}{cx^2-1}\right.}{\frac{(cx^2+1)^2 c^3}{(cx^2-1)^2} -} \right.$$

input
```
integrate(x^3*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

output
```
1/4*((c*x^2 + 1)*b^2*log(-(c*x^2 + 1)/(c*x^2 - 1))^2/(((c*x^2 + 1)^2*c^3/(
c*x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 - 1) + c^3)*(c*x^2 - 1)) + 2*(2*(c
*x^2 + 1)*a*b/(c*x^2 - 1) + (c*x^2 + 1)*b^2/(c*x^2 - 1) - b^2)*log(-(c*x^2
 + 1)/(c*x^2 - 1))/((c*x^2 + 1)^2*c^3/(c*x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c
*x^2 - 1) + c^3) + 4*((c*x^2 + 1)*a^2/(c*x^2 - 1) + (c*x^2 + 1)*a*b/(c*x^2
 - 1) - a*b)/((c*x^2 + 1)^2*c^3/(c*x^2 - 1)^2 - 2*(c*x^2 + 1)*c^3/(c*x^2 -
 1) + c^3) - 2*b^2*log(-(c*x^2 + 1)/(c*x^2 - 1) + 1)/c^3 + 2*b^2*log(-(c*x
^2 + 1)/(c*x^2 - 1))/c^3)*c
```

3.66.9 Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.02

$$\begin{aligned}\int x^3\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2\,dx=&\ \frac{a^2\,x^4}{4}+\frac{b^2\,\ln\left(c\,x^2-1\right)}{4\,c^2}+\frac{b^2\,\ln\left(c\,x^2+1\right)}{4\,c^2}-\frac{b^2\ln\left(c\,x^2+1\right)^2}{16\,c^2}\\&-\frac{b^2\ln\left(1-c\,x^2\right)^2}{16\,c^2}+\frac{b^2\,x^4\ln\left(c\,x^2+1\right)^2}{16}+\frac{b^2\,x^4\ln\left(1-c\,x^2\right)^2}{16}\\&+\frac{b^2\,x^2\,\ln\left(c\,x^2+1\right)}{4\,c}-\frac{b^2\,x^2\,\ln\left(1-c\,x^2\right)}{4\,c}\\&+\frac{a\,b\,\ln\left(c\,x^2-1\right)}{4\,c^2}-\frac{a\,b\,\ln\left(c\,x^2+1\right)}{4\,c^2}+\frac{a\,b\,x^4\,\ln\left(c\,x^2+1\right)}{4}\\&-\frac{a\,b\,x^4\,\ln\left(1-c\,x^2\right)}{4}+\frac{b^2\,\ln\left(c\,x^2+1\right)\,\ln\left(1-c\,x^2\right)}{8\,c^2}\\&+\frac{a\,b\,x^2}{2\,c}-\frac{b^2\,x^4\,\ln\left(c\,x^2+1\right)\,\ln\left(1-c\,x^2\right)}{8}\end{aligned}$$

input
```
int(x^3*(a + b*atanh(c*x^2))^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*log(c*x^2 - 1))/(4*c^2) + (b^2*log(c*x^2 + 1))/(4*c^2)
- (b^2*log(c*x^2 + 1)^2)/(16*c^2) - (b^2*log(1 - c*x^2)^2)/(16*c^2) + (b^2
*x^4*log(c*x^2 + 1)^2)/16 + (b^2*x^4*log(1 - c*x^2)^2)/16 + (b^2*x^2*log(c
*x^2 + 1))/(4*c) - (b^2*x^2*log(1 - c*x^2))/(4*c) + (a*b*log(c*x^2 - 1))/(
4*c^2) - (a*b*log(c*x^2 + 1))/(4*c^2) + (a*b*x^4*log(c*x^2 + 1))/4 - (a*b*
x^4*log(1 - c*x^2))/4 + (b^2*log(c*x^2 + 1)*log(1 - c*x^2))/(8*c^2) + (a*b
*x^2)/(2*c) - (b^2*x^4*log(c*x^2 + 1)*log(1 - c*x^2))/8
```

3.67 $\int x(a+b\operatorname{arctanh}(cx^2))^2\,dx$

3.67.1 Optimal result

Integrand size = 14, antiderivative size = 94

$$\int x\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2\,dx=\frac{\left(a+b\operatorname{arctanh}(cx^2)\right)^2}{2c}+\frac{1}{2}x^2\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2-\frac{b(a+b\operatorname{arctanh}(cx^2))\log\left(\frac{2}{1-cx^2}\right)}{c}-\frac{b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{2c}$$

output
```
1/2*(a+b*arctanh(c*x^2))^2/c+1/2*x^2*(a+b*arctanh(c*x^2))^2-b*(a+b*arctanh
(c*x^2))*ln(2/(-c*x^2+1))/c-1/2*b^2*polylog(2,1-2/(-c*x^2+1))/c
```

3.67.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int x\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2\,dx$$
$$=\frac{b^2(-1+cx^2)\operatorname{arctanh}(cx^2)^2+2b\operatorname{arctanh}(cx^2)\left(acx^2-b\log\left(1+e^{-2\operatorname{arctanh}(cx^2)}\right)\right)+a(acx^2+b\log(1-c}{2c}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
(b^2*(-1 + c*x^2)*ArcTanh[c*x^2]^2 + 2*b*ArcTanh[c*x^2]*(a*c*x^2 - b*Log[1
 + E^(-2*ArcTanh[c*x^2])]) + a*(a*c*x^2 + b*Log[1 - c^2*x^4]) + b^2*PolyLo
g[2, -E^(-2*ArcTanh[c*x^2])])/(2*c)
```

3.67.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6454, 6436, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{2}\int \left(a+b\text{arctanh}\left(cx^2\right)\right)^2dx^2$$

$$\Big\downarrow \text{6436}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-2bc\int \frac{x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2x^4}dx^2\right)$$

$$\Big\downarrow \text{6546}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-2bc\left(\frac{\int \frac{a+b\text{arctanh}\left(cx^2\right)}{1-cx^2}dx^2}{c}-\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{6470}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)}{c}-b\int \frac{\log\left(\frac{2}{1-cx^2}\right)}{1-c^2x^4}dx^2}{c}-\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-2bc\left(\frac{\frac{b\int \frac{\log\left(\frac{2}{1-cx^2}\right)}{1-\frac{2}{1-cx^2}}d\frac{1}{1-cx^2}}{c}+\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)}{c}}{c}-\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}\left(x^2\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{c}+\frac{b\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{2c}}{c}-\frac{\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2}{2bc^2}\right)\right)$$

input
```
Int[x*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
(x^2*(a + b*ArcTanh[c*x^2])^2 - 2*b*c*(-1/2*(a + b*ArcTanh[c*x^2])^2/(b*c^
2) + (((a + b*ArcTanh[c*x^2])*Log[2/(1 - c*x^2)])/c + (b*PolyLog[2, 1 - 2/
(1 - c*x^2)])/(2*c))/c))/2
```

3.67.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.67.4 Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.43

method	result
derivativedivides	$\frac{a^2cx^2+b^2\left(\operatorname{arctanh}(cx^2)^2(cx^2-1)+2\operatorname{arctanh}(cx^2)^2-2\operatorname{arctanh}(cx^2)\ln\left(1+\frac{(cx^2+1)^2}{-c^2x^4+1}\right)-\operatorname{polylog}\left(2,-\frac{(cx^2+1)^2}{-c^2x^4+1}\right)\right.}{2c}$
default	$\frac{a^2cx^2+b^2\left(\operatorname{arctanh}(cx^2)^2(cx^2-1)+2\operatorname{arctanh}(cx^2)^2-2\operatorname{arctanh}(cx^2)\ln\left(1+\frac{(cx^2+1)^2}{-c^2x^4+1}\right)-\operatorname{polylog}\left(2,-\frac{(cx^2+1)^2}{-c^2x^4+1}\right)\right.}{2c}$
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\operatorname{arctanh}(cx^2)^2(cx^2-1)+2\operatorname{arctanh}(cx^2)^2-2\operatorname{arctanh}(cx^2)\ln\left(1+\frac{(cx^2+1)^2}{-c^2x^4+1}\right)-\operatorname{polylog}\left(2,-\frac{(cx^2+1)^2}{-c^2x^4+1}\right)\right.}{2c}$
risch	$\frac{b^2\ln\left(\frac{1}{2}-\frac{cx^2}{2}\right)\ln(cx^2+1)}{2c}-\frac{b^2\ln\left(\frac{1}{2}-\frac{cx^2}{2}\right)\ln\left(\frac{cx^2}{2}+\frac{1}{2}\right)}{2c}+\frac{ba\ln(cx^2+1)x^2}{2}+\frac{ba\ln(cx^2+1)}{2c}-\frac{\ln(-cx^2+1)^2b^2}{8c}$

input
```
int(x*(a+b*arctanh(c*x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/c*(a^2*c*x^2+b^2*(arctanh(c*x^2)^2*(c*x^2-1)+2*arctanh(c*x^2)^2-2*arct
anh(c*x^2)*ln(1+(c*x^2+1)^2/(-c^2*x^4+1))-polylog(2,-(c*x^2+1)^2/(-c^2*x^4
+1)))+2*a*b*c*x^2*arctanh(c*x^2)+a*b*ln(-c^2*x^4+1))
```

3.67.5 Fricas [F]

$$\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int \left(b\,\text{artanh}\left(cx^2\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x*arctanh(c*x^2)^2 + 2*a*b*x*arctanh(c*x^2) + a^2*x, x)
```

3.67.6 Sympy [F]

$$\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int x\left(a+b\,\text{atanh}\left(cx^2\right)\right)^2\,dx$$

input
```
integrate(x*(a+b*atanh(c*x**2))**2,x)
```

output
```
Integral(x*(a + b*atanh(c*x**2))**2, x)
```

3.67.7 Maxima [F]

$$\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int \left(b\,\text{artanh}\left(cx^2\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 + 1/8*(x^2*log(-c*x^2 + 1)^2 - c^2*(2*x^2/c^2 - log(c*x^2 + 1)
/c^3 + log(c*x^2 - 1)/c^3) - 2*c*(x^2/c + log(c*x^2 - 1)/c^2)*log(-c*x^2 +
 1) + 12*c*integrate(x^3*log(c*x^2 + 1)/(c^2*x^4 - 1), x) + (c*x^2*log(c*x
^2 + 1)^2 + 2*(c*x^2 - (c*x^2 + 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/c + (2
*c*x^2 + log(c*x^2 - 1)^2 + 2*log(c*x^2 - 1))/c - log(c^2*x^4 - 1)/c + 4*i
ntegrate(x*log(c*x^2 + 1)/(c^2*x^4 - 1), x))*b^2 + 1/2*(2*c*x^2*arctanh(c*
x^2) + log(-c^2*x^4 + 1))*a*b/c
```

3.67.8 Giac [F]

$$\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2*x, x)
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int x\,\left(a+b\,\text{atanh}\left(c\,x^2\right)\right)^2\,dx$$

input
```
int(x*(a + b*atanh(c*x^2))^2,x)
```

output
```
int(x*(a + b*atanh(c*x^2))^2, x)
```

3.68 $\int \frac{(a+b\,\mathrm{arctanh}(cx^2))^2}{x}\,dx$

3.68.1 Optimal result

Integrand size = 16, antiderivative size = 137

$$\begin{aligned}\int \frac{(a+b\,\mathrm{arctanh}(cx^2))^2}{x}\,dx &= (a+b\,\mathrm{arctanh}(cx^2))^2\,\mathrm{arctanh}\left(1-\frac{2}{1-cx^2}\right)\\ &\quad-\frac{1}{2}b(a+b\,\mathrm{arctanh}(cx^2))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\\ &\quad+\frac{1}{2}b(a+b\,\mathrm{arctanh}(cx^2))\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-cx^2}\right)\\ &\quad+\frac{1}{4}b^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-cx^2}\right)\\ &\quad-\frac{1}{4}b^2\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-cx^2}\right)\end{aligned}$$

output
```
-(a+b*arctanh(c*x^2))^2*arctanh(-1+2/(-c*x^2+1))-1/2*b*(a+b*arctanh(c*x^2)
)*polylog(2,1-2/(-c*x^2+1))+1/2*b*(a+b*arctanh(c*x^2))*polylog(2,-1+2/(-c*
x^2+1))+1/4*b^2*polylog(3,1-2/(-c*x^2+1))-1/4*b^2*polylog(3,-1+2/(-c*x^2+1
))
```

3.68.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.32

$$\int \frac{(a+b\operatorname{arctanh}(cx^2))^2}{x}\,dx = a^2\log(x)+\frac{1}{2}ab\left(-\operatorname{PolyLog}\left(2,-cx^2\right)+\operatorname{PolyLog}\left(2,cx^2\right)\right) +\frac{1}{2}b^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\operatorname{arctanh}\left(cx^2\right)^3 -\operatorname{arctanh}\left(cx^2\right)^2\log\left(1+e^{-2\operatorname{arctanh}(cx^2)}\right) +\operatorname{arctanh}\left(cx^2\right)^2\log\left(1-e^{2\operatorname{arctanh}(cx^2)}\right) +\operatorname{arctanh}\left(cx^2\right)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(cx^2)}\right) +\operatorname{arctanh}\left(cx^2\right)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(cx^2)}\right) +\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}(cx^2)}\right) -\frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(cx^2)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^2/x,x]
```

output
```
a^2*Log[x] + (a*b*(-PolyLog[2, -(c*x^2)] + PolyLog[2, c*x^2]))/2 + (b^2*((
I/24)*Pi^3 - (2*ArcTanh[c*x^2]^3)/3 - ArcTanh[c*x^2]^2*Log[1 + E^(-2*ArcTa
nh[c*x^2])] + ArcTanh[c*x^2]^2*Log[1 - E^(2*ArcTanh[c*x^2])] + ArcTanh[c*x
^2]*PolyLog[2, -E^(-2*ArcTanh[c*x^2])] + ArcTanh[c*x^2]*PolyLog[2, E^(2*Ar
cTanh[c*x^2])] + PolyLog[3, -E^(-2*ArcTanh[c*x^2])]/2 - PolyLog[3, E^(2*Ar
cTanh[c*x^2])]/2))/2
```

3.68.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6450, 6448, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{x}\,dx$$

$$\Big\downarrow \text{ 6450}$$

$$\frac{1}{2}\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{x^2}dx^2$$

$$\Big\downarrow \text{ 6448}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}(cx^2)\right)^2-4bc\int \frac{\left(a+b\text{arctanh}(cx^2)\right)\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)}{1-c^2x^4}dx^2\right)$$

$$\Big\downarrow \text{ 6614}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}(cx^2)\right)^2-4bc\left(\frac{1}{2}\int \frac{\left(a+b\text{arctanh}(cx^2)\right)\log\left(2-\frac{2}{1-cx^2}\right)}{1-c^2x^4}dx^2-\frac{1}{2}\int \frac{(\cdots}{\cdots}\right.\right.$$

$$\Big\downarrow \text{ 6620}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}(cx^2)\right)^2-4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}(cx^2)\right)}{2c}-\frac{1}{2}b\cdots\right.\right.\right.$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}(cx^2)\right)^2-4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}(cx^2)\right)}{2c}-\frac{b\,\text{Po}\cdots}{\cdots}\right.\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x^2])^2/x,x]
```

output
```
(2*(a + b*ArcTanh[c*x^2])^2*ArcTanh[1 - 2/(1 - c*x^2)] - 4*b*c*((((a + b*A
rcTanh[c*x^2])*PolyLog[2, 1 - 2/(1 - c*x^2)])/(2*c) - (b*PolyLog[3, 1 - 2/
(1 - c*x^2)])/(4*c))/2 + (-1/2*((a + b*ArcTanh[c*x^2])*PolyLog[2, -1 + 2/(
1 - c*x^2)])/c + (b*PolyLog[3, -1 + 2/(1 - c*x^2)])/(4*c))/2))/2
```

3.68. $\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{x}\,dx$

3.68.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.68.4 Maple [F]

$$\int \frac{(a + b\,\mathrm{arctanh}\,(c\,x^2))^2}{x}dx$$

input
```
int((a+b*arctanh(c*x^2))^2/x,x)
```

output
```
int((a+b*arctanh(c*x^2))^2/x,x)
```

3.68.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x}\,dx = \int \frac{\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x, x)
```

3.68.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x}\,dx = \int \frac{\left(a+b\,\text{atanh}\left(cx^2\right)\right)^2}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))**2/x,x)
```

output
```
Integral((a + b*atanh(c*x**2))**2/x, x)
```

3.68.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x}\,dx = \int \frac{\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + integrate(1/4*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/x + a*
b*(log(c*x^2 + 1) - log(-c*x^2 + 1))/x, x)
```

3.68.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x} \, dx = \int \frac{(b\,\text{artanh}\,(cx^2) + a)^2}{x} \, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2/x, x)
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x} \, dx = \int \frac{(a + b\,\text{atanh}(c\,x^2))^2}{x} \, dx$$

input
```
int((a + b*atanh(c*x^2))^2/x,x)
```

output
```
int((a + b*atanh(c*x^2))^2/x, x)
```

3.69 $\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{x^3}\,dx$

3.69.1 Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^3}\,dx = \frac{1}{2}c\left(a+b\text{arctanh}\left(cx^2\right)\right)^2 - \frac{(a+b\text{arctanh}(cx^2))^2}{2x^2} + bc\left(a+b\text{arctanh}\left(cx^2\right)\right)\log\left(2-\frac{2}{1+cx^2}\right) - \frac{1}{2}b^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx^2}\right)$$

output
```
1/2*c*(a+b*arctanh(c*x^2))^2-1/2*(a+b*arctanh(c*x^2))^2/x^2+b*c*(a+b*arcta
nh(c*x^2))*ln(2-2/(c*x^2+1))-1/2*b^2*c*polylog(2,-1+2/(c*x^2+1))
```

3.69.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^3}\,dx = -\frac{a^2}{2x^2} + abc\left(-\frac{\text{arctanh}(cx^2)}{cx^2} + \log\left(cx^2\right) - \frac{1}{2}\log\left(1-c^2x^4\right)\right) + \frac{1}{2}b^2c\left(\text{arctanh}\left(cx^2\right)\left(\text{arctanh}\left(cx^2\right) - \frac{\text{arctanh}(cx^2)}{cx^2} + 2\log\left(1-e^{-2\text{arctanh}(cx^2)}\right)\right) - \text{PolyLog}\left(2,e^{-2\text{arctanh}(cx^2)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^2/x^3,x]
```

output
```
-1/2*a^2/x^2 + a*b*c*(-(ArcTanh[c*x^2]/(c*x^2)) + Log[c*x^2] - Log[1 - c^2
*x^4]/2) + (b^2*c*(ArcTanh[c*x^2]*(ArcTanh[c*x^2] - ArcTanh[c*x^2]/(c*x^2)
 + 2*Log[1 - E^(-2*ArcTanh[c*x^2])]) - PolyLog[2, E^(-2*ArcTanh[c*x^2])]))
/2
```

3.69.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6454, 6452, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^3}dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{2}\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^4}dx^2$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}\left(2bc\int \frac{a+b\text{arctanh}(cx^2)}{x^2(1-c^2x^4)}dx^2-\frac{(a+b\text{arctanh}(cx^2))^2}{x^2}\right)$$

$$\Big\downarrow \text{6550}$$

$$\frac{1}{2}\left(2bc\left(\int \frac{a+b\text{arctanh}(cx^2)}{x^2(cx^2+1)}dx^2+\frac{(a+b\text{arctanh}(cx^2))^2}{2b}\right)-\frac{(a+b\text{arctanh}(cx^2))^2}{x^2}\right)$$

$$\Big\downarrow \text{6494}$$

$$\frac{1}{2}\left(2bc\left(-bc\int \frac{\log\left(2-\frac{2}{cx^2+1}\right)}{1-c^2x^4}dx^2+\frac{(a+b\text{arctanh}(cx^2))^2}{2b}+\log\left(2-\frac{2}{cx^2+1}\right)(a+b\text{arctanh}(cx^2))\right)-\frac{(a+$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{2}\left(2bc\left(\frac{(a+b\text{arctanh}(cx^2))^2}{2b}+\log\left(2-\frac{2}{cx^2+1}\right)(a+b\text{arctanh}(cx^2))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx^2+1}-1\right)\right)-\frac{(a$$

input
```
Int[(a + b*ArcTanh[c*x^2])^2/x^3,x]
```

output
```
(-((a + b*ArcTanh[c*x^2])^2/x^2) + 2*b*c*((a + b*ArcTanh[c*x^2])^2/(2*b) +
 (a + b*ArcTanh[c*x^2])*Log[2 - 2/(1 + c*x^2)] - (b*PolyLog[2, -1 + 2/(1 +
 c*x^2)])/2))/2
```

3.69.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.69.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.03 (sec) , antiderivative size = 820, normalized size of antiderivative = 9.43

method	result	size
default	Expression too large to display	820
parts	Expression too large to display	820

input
```
int((a+b*arctanh(c*x^2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+b^2*(-1/2/x^2*arctanh(c*x^2)^2+2*c*(-1/4*arctanh(c*x^2)*ln(c*
x^2-1)-1/4*arctanh(c*x^2)*ln(c*x^2+1)+arctanh(c*x^2)*ln(x)-1/2*c*(Sum(1/4*
(ln(x-_alpha)*ln(c*x^2-1)-2*c*(1/4/_alpha/c*ln(x-_alpha)^2-1/2*_alpha*ln(x
-_alpha)*ln(1/2*(x+_alpha)/_alpha)-1/2*_alpha*dilog(1/2*(x+_alpha)/_alpha)
))/c,_alpha=RootOf(_Z^2*c-1))+Sum(-1/4*(ln(x-_alpha)*ln(c*x^2-1)-2*c*(1/2*
ln(x-_alpha)*(ln((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1)-x+_alpha)/RootOf(
_Z^2*c+2*_Z*_alpha*c-2,index=1))+ln((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2
)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)))/c+1/2*(dilog((RootOf(
_Z^2*c+2*_Z*_alpha*c-2,index=1)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,in
dex=1))+dilog((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)-x+_alpha)/RootOf(_Z^
2*c+2*_Z*_alpha*c-2,index=2)))/c))/c,_alpha=RootOf(_Z^2*c+1))+Sum(1/4*(ln(
x-_alpha)*ln(c*x^2+1)-2*c*(1/2*ln(x-_alpha)*(ln((RootOf(_Z^2*c+2*_Z*_alpha
*c+2,index=1)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=1))+ln((RootOf
(_Z^2*c+2*_Z*_alpha*c+2,index=2)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c+2,i
ndex=2)))/c+1/2*(dilog((RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=1)-x+_alpha)/R
ootOf(_Z^2*c+2*_Z*_alpha*c+2,index=1))+dilog((RootOf(_Z^2*c+2*_Z*_alpha*c+
2,index=2)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=2)))/c))/c,_alpha
=RootOf(_Z^2*c-1))+Sum(-1/4*(ln(x-_alpha)*ln(c*x^2+1)-2*c*(1/4/_alpha/c*ln
(x-_alpha)^2+1/2*_alpha*ln(x-_alpha)*ln(1/2*(x+_alpha)/_alpha)+1/2*_alpha*
dilog(1/2*(x+_alpha)/_alpha)))/c,_alpha=RootOf(_Z^2*c+1))-ln(x)*(ln(1-x...
```

3.69.5 Fricas [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^3} \, dx = \int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^3} \, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^3, x)
```

3.69.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^3} \, dx = \int \frac{(a + b \operatorname{atanh}(cx^2))^2}{x^3} \, dx$$

input
```
integrate((a+b*atanh(c*x**2))**2/x**3,x)
```

output
```
Integral((a + b*atanh(c*x**2))**2/x**3, x)
```

3.69.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^3} \, dx = \int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^3} \, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*a*b - 1/8*b^
2*(log(-c*x^2 + 1)^2/x^2 + 2*integrate(-((c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*
(c*x^2 - (c*x^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^5 - x^3), x)) -
 1/2*a^2/x^2
```

3.69.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^3}\,dx = \int \frac{(b\,\text{artanh}\,(cx^2) + a)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2/x^3, x)
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^3}\,dx = \int \frac{(a + b\,\text{atanh}(c\,x^2))^2}{x^3}\,dx$$

input
```
int((a + b*atanh(c*x^2))^2/x^3,x)
```

output
```
int((a + b*atanh(c*x^2))^2/x^3, x)
```

3.70 $\int \frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{x^5}\,dx$

3.70.1 Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^5}\,dx = -\frac{bc(a+b\text{arctanh}(cx^2))}{2x^2}+\frac{1}{4}c^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2 - \frac{(a+b\text{arctanh}(cx^2))^2}{4x^4}+b^2c^2\log(x)-\frac{1}{4}b^2c^2\log\left(1-c^2x^4\right)$$

output
```
-1/2*b*c*(a+b*arctanh(c*x^2))/x^2+1/4*c^2*(a+b*arctanh(c*x^2))^2-1/4*(a+b*
arctanh(c*x^2))^2/x^4+b^2*c^2*ln(x)-1/4*b^2*c^2*ln(-c^2*x^4+1)
```

3.70.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^5}\,dx = \frac{1}{4}\left(-\frac{a^2}{x^4}-\frac{2abc}{x^2}-\frac{2b(a+bcx^2)\,\text{arctanh}(cx^2)}{x^4} + \frac{b^2(-1+c^2x^4)\,\text{arctanh}(cx^2)^2}{x^4}+4b^2c^2\log(x) - b(a+b)c^2\log\left(1-cx^2\right)+(a-b)bc^2\log\left(1+cx^2\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^2/x^5,x]
```

output
```
(-(a^2/x^4) - (2*a*b*c)/x^2 - (2*b*(a + b*c*x^2)*ArcTanh[c*x^2])/x^4 + (b^
2*(-1 + c^2*x^4)*ArcTanh[c*x^2]^2)/x^4 + 4*b^2*c^2*Log[x] - b*(a + b)*c^2*
Log[1 - c*x^2] + (a - b)*b*c^2*Log[1 + c*x^2])/4
```

3.70.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6454, 6452, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{x^5}dx$$

$$\Big\downarrow \text{ 6454}$$

$$\frac{1}{2}\int \frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{x^6}dx^2$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}\left(bc\int \frac{a+b\text{arctanh}\left(cx^2\right)}{x^4\left(1-c^2x^4\right)}dx^2-\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2x^4}\right)$$

$$\Big\downarrow \text{ 6544}$$

$$\frac{1}{2}\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(cx^2\right)}{1-c^2x^4}dx^2+\int \frac{a+b\text{arctanh}\left(cx^2\right)}{x^4}dx^2\right)-\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2x^4}\right)$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(cx^2\right)}{1-c^2x^4}dx^2+bc\int \frac{1}{x^2\left(1-c^2x^4\right)}dx^2-\frac{a+b\text{arctanh}\left(cx^2\right)}{x^2}\right)-\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2x^4}\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{2}\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(cx^2\right)}{1-c^2x^4}dx^2+\frac{1}{2}bc\int \frac{1}{x^2\left(1-c^2x^4\right)}dx^4-\frac{a+b\text{arctanh}\left(cx^2\right)}{x^2}\right)-\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2x^4}\right)$$

$\Big\downarrow$ 47

$$\frac{1}{2}\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx^2)}{1-c^2x^4}dx^2+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^4}dx^4+\int\frac{1}{x^2}dx^4\right)-\frac{a+b\text{arctanh}(cx^2)}{x^2}\right)-\frac{(a+b\text{arcta}}{2x}\right.$$

$\Big\downarrow$ 14

$$\frac{1}{2}\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx^2)}{1-c^2x^4}dx^2+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^4}dx^4+\log\left(x^4\right)\right)-\frac{a+b\text{arctanh}(cx^2)}{x^2}\right)-\frac{(a+b\text{arctan}}{2x^4}\right.$$

$\Big\downarrow$ 16

$$\frac{1}{2}\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx^2)}{1-c^2x^4}dx^2-\frac{a+b\text{arctanh}(cx^2)}{x^2}+\frac{1}{2}bc\left(\log\left(x^4\right)-\log\left(1-c^2x^4\right)\right)\right)-\frac{(a+b\text{arctanh}(cx^2}{2x^4}\right.$$

$\Big\downarrow$ 6510

$$\frac{1}{2}\left(bc\left(\frac{c(a+b\text{arctanh}(cx^2))^2}{2b}-\frac{a+b\text{arctanh}(cx^2)}{x^2}+\frac{1}{2}bc\left(\log\left(x^4\right)-\log\left(1-c^2x^4\right)\right)\right)-\frac{(a+b\text{arctanh}(cx^2))^2}{2x^4}\right)$$

input
```
Int[(a + b*ArcTanh[c*x^2])^2/x^5,x]
```

output
```
(-1/2*(a + b*ArcTanh[c*x^2])^2/x^4 + b*c*(-((a + b*ArcTanh[c*x^2])/x^2) +
(c*(a + b*ArcTanh[c*x^2])^2)/(2*b) + (b*c*(Log[x^4] - Log[1 - c^2*x^4]))/2
))/2
```

3.70.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.70.4 Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.73

method	result
parallelrisch	$\frac{b^{2} \operatorname{arctanh}\left(c\,x^{2}\right)^{2} x^{4} c^{2}+4 b^{2} c^{2} \ln \left(x\right) x^{4}-2 \ln \left(c\,x^{2}-1\right) b^{2} c^{2} x^{4}+2 x^{4} \operatorname{arctanh}\left(c\,x^{2}\right) a b\,c^{2}-2 \operatorname{arctanh}\left(c\,x^{2}\right) x^{4} b^{2} c^{2}-a^{2} c^{2} x^{4}-2 b^{2} \operatorname{arc}}{4 x^{4}}$
risch	$\frac{b^{2}\left(c^{2} x^{4}-1\right) \ln \left(c\,x^{2}+1\right)^{2}}{16 x^{4}}-\frac{b\left(b\,c^{2} \ln \left(-c\,x^{2}+1\right) x^{4}+2 b c\,x^{2}-b \ln \left(-c\,x^{2}+1\right)+2 a\right) \ln \left(c\,x^{2}+1\right)}{8 x^{4}}+\frac{b^{2} c^{2} x^{4} \ln \left(-c\,x^{2}+1\right)^{2}+16 b^{2} c^{2}}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int((a+b*arctanh(c*x^2))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*arctanh(c*x^2)^2*x^4*c^2+4*b^2*c^2*ln(x)*x^4-2*ln(c*x^2-1)*b^2*c^
2*x^4+2*x^4*arctanh(c*x^2)*a*b*c^2-2*arctanh(c*x^2)*x^4*b^2*c^2-a^2*c^2*x^
4-2*b^2*arctanh(c*x^2)*x^2*c-2*a*b*c*x^2-b^2*arctanh(c*x^2)^2-2*arctanh(c*
x^2)*a*b-a^2)/x^4
```

3.70.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.72

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^5}\,dx = \frac{16\,b^2c^2x^4\log{(x)} + 4\,(ab - b^2)c^2x^4\log{(cx^2+1)} - 4\,(ab + b^2)c^2x^4\log{(cx^2-1)} - 8\,abcx^2 + (b^2c^2x^4 - b^2)\,\mathrm{l}}{16\,x^4}$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="fricas")
```

output
```
1/16*(16*b^2*c^2*x^4*log(x) + 4*(a*b - b^2)*c^2*x^4*log(c*x^2 + 1) - 4*(a*
b + b^2)*c^2*x^4*log(c*x^2 - 1) - 8*a*b*c*x^2 + (b^2*c^2*x^4 - b^2)*log(-(
c*x^2 + 1)/(c*x^2 - 1))^2 - 4*a^2 - 4*(b^2*c*x^2 + a*b)*log(-(c*x^2 + 1)/(
c*x^2 - 1)))/x^4
```

3.70.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(80) = 160$.

Time = 5.76 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.99

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^5}\,dx = \begin{cases} -\frac{a^2}{4x^4} + \frac{abc^2\operatorname{atanh}{(cx^2)}}{2} - \frac{abc}{2x^2} - \frac{ab\operatorname{atanh}{(cx^2)}}{2x^4} + b^2c^2\log{(x)} - \frac{b^2c^2\log{\left(x - \sqrt{-\frac{1}{c}}\right)}}{2} - \frac{b^2c^2\log{\left(x + \sqrt{-\frac{1}{c}}\right)}}{2} + \frac{b^2c^2\operatorname{atanh}^2{(}}{4} \\ -\frac{a^2}{4x^4} \end{cases}$$

input
```
integrate((a+b*atanh(c*x**2))**2/x**5,x)
```

output
```
Piecewise((-a**2/(4*x**4) + a*b*c**2*atanh(c*x**2)/2 - a*b*c/(2*x**2) - a*
b*atanh(c*x**2)/(2*x**4) + b**2*c**2*log(x) - b**2*c**2*log(x - sqrt(-1/c)
)/2 - b**2*c**2*log(x + sqrt(-1/c))/2 + b**2*c**2*atanh(c*x**2)**2/4 + b**
2*c**2*atanh(c*x**2)/2 - b**2*c*atanh(c*x**2)/(2*x**2) - b**2*atanh(c*x**2
)**2/(4*x**4), Ne(c, 0)), (-a**2/(4*x**4), True))
```

3.70.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(80) = 160.

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.99

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^5}\, dx$$
$$= \frac{1}{4}\left(\left(c\log\left(cx^2+1\right) - c\log\left(cx^2-1\right) - \frac{2}{x^2}\right)c - \frac{2\,\text{artanh}\left(cx^2\right)}{x^4}\right)ab$$
$$+ \frac{1}{16}\left(\left(2\left(\log\left(cx^2-1\right)-2\right)\log\left(cx^2+1\right) - \log\left(cx^2+1\right)^2 - \log\left(cx^2-1\right)^2 - 4\log\left(cx^2-1\right) + 16\log(\right.\right.$$
$$- \frac{b^2\,\text{artanh}\left(cx^2\right)^2}{4\,x^4} - \frac{a^2}{4\,x^4}$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="maxima")
```

output
```
1/4*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^
4)*a*b + 1/16*((2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 -
 log(c*x^2 - 1)^2 - 4*log(c*x^2 - 1) + 16*log(x))*c^2 + 4*(c*log(c*x^2 + 1
) - c*log(c*x^2 - 1) - 2/x^2)*c*arctanh(c*x^2))*b^2 - 1/4*b^2*arctanh(c*x^
2)^2/x^4 - 1/4*a^2/x^4
```

3.70.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^5}\, dx = \int \frac{\left(b\,\text{artanh}\left(cx^2\right) + a\right)^2}{x^5}\, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2/x^5, x)
```

3.70.9 Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.16

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^5}\,dx = \frac{b^2\,c^2\ln{(c\,x^2+1)}^2}{16} - \frac{b^2\,c^2\,\ln{(c\,x^2-1)}}{4} - \frac{b^2\,c^2\,\ln{(c\,x^2+1)}}{4} - \frac{a^2}{4\,x^4} + \frac{b^2\,c^2\ln{(1-c\,x^2)}^2}{16} - \frac{b^2\ln{(c\,x^2+1)}^2}{16\,x^4} - \frac{b^2\ln{(1-c\,x^2)}^2}{16\,x^4} + b^2\,c^2\,\ln{(x)} - \frac{a\,b\,c^2\,\ln{(c\,x^2-1)}}{4} + \frac{a\,b\,c^2\,\ln{(c\,x^2+1)}}{4} - \frac{a\,b\,c}{2\,x^2} - \frac{a\,b\,\ln{(c\,x^2+1)}}{4\,x^4} + \frac{a\,b\,\ln{(1-c\,x^2)}}{4\,x^4} - \frac{b^2\,c^2\,\ln{(c\,x^2+1)}\,\ln{(1-c\,x^2)}}{8} - \frac{b^2\,c\,\ln{(c\,x^2+1)}}{4\,x^2} + \frac{b^2\,c\,\ln{(1-c\,x^2)}}{4\,x^2} + \frac{b^2\,\ln{(c\,x^2+1)}\,\ln{(1-c\,x^2)}}{8\,x^4}$$

input
```
int((a + b*atanh(c*x^2))^2/x^5,x)
```

output
```
(b^2*c^2*log(c*x^2 + 1)^2)/16 - (b^2*c^2*log(c*x^2 - 1))/4 - (b^2*c^2*log(
c*x^2 + 1))/4 - a^2/(4*x^4) + (b^2*c^2*log(1 - c*x^2)^2)/16 - (b^2*log(c*x
^2 + 1)^2)/(16*x^4) - (b^2*log(1 - c*x^2)^2)/(16*x^4) + b^2*c^2*log(x) - (
a*b*c^2*log(c*x^2 - 1))/4 + (a*b*c^2*log(c*x^2 + 1))/4 - (a*b*c)/(2*x^2) -
 (a*b*log(c*x^2 + 1))/(4*x^4) + (a*b*log(1 - c*x^2))/(4*x^4) - (b^2*c^2*lo
g(c*x^2 + 1)*log(1 - c*x^2))/8 - (b^2*c*log(c*x^2 + 1))/(4*x^2) + (b^2*c*l
og(1 - c*x^2))/(4*x^2) + (b^2*log(c*x^2 + 1)*log(1 - c*x^2))/(8*x^4)
```

3.71 $\int x^4 (a + b\text{arctanh}(cx^2))^2 \, dx$

3.71.1 Optimal result

Integrand size = 16, antiderivative size = 1173

$$\int x^4 (a + b\text{arctanh}(cx^2))^2 \, dx = \text{Too large to display}$$

output
```
1/5*I*b^2*arctan(x*c^(1/2))^2/c^(5/2)+1/5*I*b^2*polylog(2,1-2/(1-I*x*c^(1/
2)))/c^(5/2)+1/5*I*b^2*polylog(2,1-2/(1+I*x*c^(1/2)))/c^(5/2)+1/20*x^5*(2*
a-b*ln(-c*x^2+1))^2+2/5*a*b*arctan(x*c^(1/2))/c^(5/2)-1/15*b^2*x^3*ln(-c*x
^2+1)/c-1/5*b^2*arctan(x*c^(1/2))*ln(-c*x^2+1)/c^(5/2)+1/15*b*x^3*(2*a-b*l
n(-c*x^2+1))/c-1/5*b*arctanh(x*c^(1/2))*(2*a-b*ln(-c*x^2+1))/c^(5/2)+2/15*
b^2*x^3*ln(c*x^2+1)/c+1/5*a*b*x^5*ln(c*x^2+1)+1/5*b^2*arctan(x*c^(1/2))*ln
(c*x^2+1)/c^(5/2)-1/5*b^2*arctanh(x*c^(1/2))*ln(c*x^2+1)/c^(5/2)-1/10*b^2*
x^5*ln(-c*x^2+1)*ln(c*x^2+1)+2/5*b^2*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2))
)/c^(5/2)-2/5*b^2*arctan(x*c^(1/2))*ln(2/(1-I*x*c^(1/2)))/c^(5/2)+1/5*b^2*
arctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))/c^(5/2)+2/5*b^2*
arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))/c^(5/2)-2/5*b^2*arctanh(x*c^(1/2))
*ln(2/(1+x*c^(1/2)))/c^(5/2)+1/5*b^2*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1
/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))/c^(5/2)+1/5*b^2*arctanh(x
*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2))
)/c^(5/2)+1/5*b^2*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2))
)/c^(5/2)+2/15*a*b*x^3/c-1/10*I*b^2*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x
*c^(1/2)))/c^(5/2)-1/10*I*b^2*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1
/2)))/c^(5/2)-4/15*b^2*arctan(x*c^(1/2))/c^(5/2)-4/15*b^2*arctanh(x*c^(1/2
))/c^(5/2)-1/5*b^2*arctanh(x*c^(1/2))^2/c^(5/2)+1/25*b^2*x^5*ln(-c*x^2+1)+
1/25*b*x^5*(2*a-b*ln(-c*x^2+1))+1/20*b^2*x^5*ln(c*x^2+1)^2+1/5*b^2*poly...
```

3.71.2 Mathematica [F]

$$\int x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx$$

input
```
Integrate[x^4*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
Integrate[x^4*(a + b*ArcTanh[c*x^2])^2, x]
```

3.71.3 Rubi [A] (verified)

Time = 2.50 (sec) , antiderivative size = 1173, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6456, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx$$

$$\Big\downarrow \text{6456}$$

$$\int \left(\frac{1}{4}x^4\left(2a-b\log\left(1-cx^2\right)\right)^2-\frac{1}{2}bx^4\log\left(cx^2+1\right)\left(b\log\left(1-cx^2\right)-2a\right)+\frac{1}{4}b^2x^4\log^2\left(cx^2+1\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{20}(2a-b\log(1-cx^2))^2x^5+\frac{1}{20}b^2\log^2(cx^2+1)x^5-\frac{2}{25}abx^5+\frac{1}{25}b^2\log(1-cx^2)x^5+\frac{1}{25}b(2a-b\log(1-cx^2))x^5+\frac{1}{5}ab\log(cx^2+1)x^5-\frac{1}{10}b^2\log(1-cx^2)\log(cx^2+1)x^5-\frac{b^2\log(1-cx^2)x^3}{15c}+\frac{b(2a-b\log(1-cx^2))x^3}{15c}+\frac{2b^2\log(cx^2+1)x^3}{15c}+\frac{2abx^3}{15c}+\frac{8b^2x}{15c^2}+\frac{ib^2\arctan(\sqrt{c}x)^2}{5c^{5/2}}-\frac{b^2\text{arctanh}(\sqrt{c}x)^2}{5c^{5/2}}-\frac{4b^2\arctan(\sqrt{c}x)}{15c^{5/2}}+\frac{2ab\arctan(\sqrt{c}x)}{5c^{5/2}}-\frac{4b^2\text{arctanh}(\sqrt{c}x)}{15c^{5/2}}+\frac{2b^2\text{arctanh}(\sqrt{c}x)\log\left(\frac{2}{1-\sqrt{c}x}\right)}{5c^{5/2}}-\frac{2b^2\arctan(\sqrt{c}x)\log\left(\frac{2}{1-i\sqrt{c}x}\right)}{5c^{5/2}}+\frac{b^2\arctan(\sqrt{c}x)\log\left(\frac{(1+i)(1-\sqrt{c}x)}{1-i\sqrt{c}x}\right)}{5c^{5/2}}+\frac{2b^2\arctan(\sqrt{c}x)\log\left(\frac{2}{i\sqrt{c}x+1}\right)}{5c^{5/2}}-\frac{2b^2\text{arctanh}(\sqrt{c}x)\log\left(\frac{2}{\sqrt{c}x+1}\right)}{5c^{5/2}}+\frac{b^2\text{arctanh}(\sqrt{c}x)\log\left(-\frac{2\sqrt{c}(1-\sqrt{-c}x)}{(\sqrt{-c}-\sqrt{c})(\sqrt{c}x+1)}\right)}{5c^{5/2}}+\frac{b^2\text{arctanh}(\sqrt{c}x)\log\left(\frac{2\sqrt{c}(\sqrt{-c}x+1)}{(\sqrt{-c}+\sqrt{c})(\sqrt{c}x+1)}\right)}{5c^{5/2}}+\frac{b^2\arctan(\sqrt{c}x)\log\left(\frac{(1-i)(\sqrt{c}x+1)}{1-i\sqrt{c}x}\right)}{5c^{5/2}}-\frac{b^2\arctan(\sqrt{c}x)\log(1-cx^2)}{5c^{5/2}}-\frac{b\text{arctanh}(\sqrt{c}x)(2a-b\log(1-cx^2))}{5c^{5/2}}+\frac{b^2\arctan(\sqrt{c}x)\log(cx^2+1)}{5c^{5/2}}-\frac{b^2\text{arctanh}(\sqrt{c}x)\log(cx^2+1)}{5c^{5/2}}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-\sqrt{c}x}\right)}{5c^{5/2}}+\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1-i\sqrt{c}x}\right)}{5c^{5/2}}-\frac{ib^2\text{PolyLog}\left(2,1-\frac{(1+i)(1-\sqrt{c}x)}{1-i\sqrt{c}x}\right)}{10c^{5/2}}+\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{i\sqrt{c}x+1}\right)}{5c^{5/2}}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{\sqrt{c}x+1}\right)}{5c^{5/2}}-\frac{b^2\text{PolyLog}\left(2,\frac{2\sqrt{c}(1-\sqrt{-c}x)}{(\sqrt{-c}-\sqrt{c})(\sqrt{c}x+1)}+1\right)}{10c^{5/2}}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2\sqrt{c}(\sqrt{-c}x+1)}{(\sqrt{-c}+\sqrt{c})(\sqrt{c}x+1)}\right)}{10c^{5/2}}-\frac{ib^2\text{PolyLog}\left(2,1-\frac{(1-i)(\sqrt{c}x+1)}{1-i\sqrt{c}x}\right)}{10c^{5/2}}$$

input
```
Int[x^4*(a + b*ArcTanh[c*x^2])^2,x]
```

3.71. $\int x^4(a+b\text{arctanh}(cx^2))^2\,dx$

output
```
(8*b^2*x)/(15*c^2) + (2*a*b*x^3)/(15*c) - (2*a*b*x^5)/25 + (2*a*b*ArcTan[S
qrt[c]*x])/(5*c^(5/2)) - (4*b^2*ArcTan[Sqrt[c]*x])/(15*c^(5/2)) + ((I/5)*b
^2*ArcTan[Sqrt[c]*x]^2)/c^(5/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(15*c^(5/2))
- (b^2*ArcTanh[Sqrt[c]*x]^2)/(5*c^(5/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2
/(1 - Sqrt[c]*x)])/(5*c^(5/2)) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqr
t[c]*x)])/(5*c^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x
))/(1 - I*Sqrt[c]*x)])/(5*c^(5/2)) + (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I
*Sqrt[c]*x)])/(5*c^(5/2)) - (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x
)])/(5*c^(5/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x)
)/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/(5*c^(5/2)) + (b^2*ArcTanh[Sqrt
[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]
*x))])/(5*c^(5/2)) + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/
(1 - I*Sqrt[c]*x)])/(5*c^(5/2)) - (b^2*x^3*Log[1 - c*x^2])/(15*c) + (b^2*x
^5*Log[1 - c*x^2])/25 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/(5*c^(5/2))
 + (b*x^3*(2*a - b*Log[1 - c*x^2]))/(15*c) + (b*x^5*(2*a - b*Log[1 - c*x^2
]))/25 - (b*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(5*c^(5/2)) + (x^
5*(2*a - b*Log[1 - c*x^2])^2)/20 + (2*b^2*x^3*Log[1 + c*x^2])/(15*c) + (a*
b*x^5*Log[1 + c*x^2])/5 + (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2
)) - (b^2*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/(5*c^(5/2)) - (b^2*x^5*Log[1
- c*x^2]*Log[1 + c*x^2])/10 + (b^2*x^5*Log[1 + c*x^2]^2)/20 + (b^2*Poly...
```

3.71.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6456
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

3.71.4 Maple [F]

$$\int x^4 \left(a + b \operatorname{arctanh}\left(c x^2\right)\right)^2 dx$$

input
```
int(x^4*(a+b*arctanh(c*x^2))^2,x)
```

output
```
int(x^4*(a+b*arctanh(c*x^2))^2,x)
```

3.71.5 Fricas [F]

$$\int x^4 \left(a + b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2 x^4 \, dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^4*arctanh(c*x^2)^2 + 2*a*b*x^4*arctanh(c*x^2) + a^2*x^4, x)
```

3.71.6 Sympy [F]

$$\int x^4 \left(a + b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int x^4 \left(a + b \operatorname{atanh}\left(cx^2\right)\right)^2 \, dx$$

input
```
integrate(x**4*(a+b*atanh(c*x**2))**2,x)
```

output
```
Integral(x**4*(a + b*atanh(c*x**2))**2, x)
```

3.71.7 Maxima [F]

$$\int x^4 \left(a + b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^2 x^4 \, dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*x^5 + 1/15*(6*x^5*arctanh(c*x^2) + c*(4*x^3/c^2 + 6*arctan(sqrt(c)
*x)/c^(7/2) + 3*log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(7/2)))*a*b + 1/20*
(x^5*log(-c*x^2 + 1)^2 - 5*integrate(-1/5*(5*(c*x^6 - x^4)*log(c*x^2 + 1)^
2 - 2*(2*c*x^6 + 5*(c*x^6 - x^4)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 -
 1), x))*b^2
```

3.71.8 Giac [F]

$$\int x^4 \left(a + b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int \left(b\,\text{artanh}\left(cx^2\right) + a\right)^2 x^4 \, dx$$

input
```
integrate(x^4*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2*x^4, x)
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a + b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int x^4 \left(a + b\,\text{atanh}\left(c\,x^2\right)\right)^2 dx$$

input
```
int(x^4*(a + b*atanh(c*x^2))^2,x)
```

output
```
int(x^4*(a + b*atanh(c*x^2))^2, x)
```

3.72 $\int x^2(a+b\operatorname{arctanh}(cx^2))^2\,dx$

3.72.1 Optimal result

Integrand size = 16, antiderivative size = 1129

$$\int x^2\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2\,dx=\text{Too large to display}$$

output
```
1/12*x^3*(2*a-b*ln(-c*x^2+1))^2-1/3*b^2*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^
(1/2))/(1-I*x*c^(1/2)))/c^(3/2)-1/3*I*b^2*arctan(x*c^(1/2))^2/c^(3/2)-1/3*
I*b^2*polylog(2,1-2/(1-I*x*c^(1/2)))/c^(3/2)-1/3*I*b^2*polylog(2,1-2/(1+I*
x*c^(1/2)))/c^(3/2)-2/3*a*b*arctan(x*c^(1/2))/c^(3/2)-2/3*b^2*x*ln(-c*x^2+
1)/c+1/3*b^2*arctan(x*c^(1/2))*ln(-c*x^2+1)/c^(3/2)-1/3*b*arctanh(x*c^(1/2
))*(2*a-b*ln(-c*x^2+1))/c^(3/2)+2/3*b^2*x*ln(c*x^2+1)/c+1/3*a*b*x^3*ln(c*x
^2+1)-1/3*b^2*arctan(x*c^(1/2))*ln(c*x^2+1)/c^(3/2)-1/3*b^2*arctanh(x*c^(1
/2))*ln(c*x^2+1)/c^(3/2)-1/6*b^2*x^3*ln(-c*x^2+1)*ln(c*x^2+1)+2/3*b^2*arct
anh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))/c^(3/2)+2/3*b^2*arctan(x*c^(1/2))*ln(2/
(1-I*x*c^(1/2)))/c^(3/2)-1/3*b^2*arctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/
(1-I*x*c^(1/2)))/c^(3/2)-2/3*b^2*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))/c
^(3/2)-2/3*b^2*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))/c^(3/2)+1/3*b^2*arct
anh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^
(1/2)))/c^(3/2)+1/3*b^2*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/(
(-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(3/2)+1/6*I*b^2*polylog(2,1-(1+I)*(1-
x*c^(1/2))/(1-I*x*c^(1/2)))/c^(3/2)+1/6*I*b^2*polylog(2,1+(-1+I)*(1+x*c^(1
/2))/(1-I*x*c^(1/2)))/c^(3/2)+4/3*a*b*x/c-1/6*b^2*polylog(2,1-2*(1+x*(-c)^
(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(3/2)-2/9*a*b*x^3+4/3
*b^2*arctan(x*c^(1/2))/c^(3/2)-4/3*b^2*arctanh(x*c^(1/2))/c^(3/2)-1/3*b^2*
arctanh(x*c^(1/2))^2/c^(3/2)+1/9*b^2*x^3*ln(-c*x^2+1)+1/9*b*x^3*(2*a-b*...
```

3.72.2 Mathematica [F]

$$\int x^2 \left(a + b \text{arctanh}\left(cx^2\right)\right)^2 dx = \int x^2 \left(a + b \text{arctanh}\left(cx^2\right)\right)^2 dx$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
Integrate[x^2*(a + b*ArcTanh[c*x^2])^2, x]
```

3.72.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 1129, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6456, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a + b \text{arctanh}\left(cx^2\right)\right)^2 dx$$

$$\Big\downarrow \text{ 6456}$$

$$\int \left(\frac{1}{4}x^2 \left(2a - b \log\left(1 - cx^2\right)\right)^2 - \frac{1}{2}bx^2 \log\left(cx^2 + 1\right) \left(b \log\left(1 - cx^2\right) - 2a\right) + \frac{1}{4}b^2x^2 \log^2\left(cx^2 + 1\right)\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{12}\left(2a-b\log\left(1-cx^2\right)\right)^2x^3+\frac{1}{12}b^2\log^2\left(cx^2+1\right)x^3-\frac{2}{9}abx^3+\frac{1}{9}b^2\log\left(1-cx^2\right)x^3+\frac{1}{9}b\left(2a-b\log\left(1-cx^2\right)\right)x^3+\frac{1}{3}ab\log\left(cx^2+1\right)x^3-\frac{1}{6}b^2\log\left(1-cx^2\right)\log\left(cx^2+1\right)x^3-\frac{2b^2\log\left(1-cx^2\right)x}{3c}+\frac{2b^2\log\left(cx^2+1\right)x}{3c}+\frac{4abx}{3c}-\frac{ib^2\arctan\left(\sqrt{c}x\right)^2}{3c^{3/2}}-\frac{b^2\text{arctanh}\left(\sqrt{c}x\right)^2}{3c^{3/2}}+\frac{4b^2\arctan\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{2ab\arctan\left(\sqrt{c}x\right)}{3c^{3/2}}-\frac{4b^2\text{arctanh}\left(\sqrt{c}x\right)}{3c^{3/2}}+\frac{2b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt{c}x}\right)}{3c^{3/2}}+\frac{2b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{1-i\sqrt{c}x}\right)}{3c^{3/2}}-\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1+i)\left(1-\sqrt{c}x\right)}{1-i\sqrt{c}x}\right)}{3c^{3/2}}-\frac{2b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{i\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{2b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{\sqrt{c}x+1}\right)}{3c^{3/2}}+\frac{b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(-\frac{2\sqrt{c}\left(1-\sqrt{-c}x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)}{3c^{3/2}}+\frac{b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2\sqrt{c}\left(\sqrt{-c}x+1\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)}{3c^{3/2}}-\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1-i)\left(\sqrt{c}x+1\right)}{1-i\sqrt{c}x}\right)}{3c^{3/2}}+\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(1-cx^2\right)}{3c^{3/2}}-\frac{b\text{arctanh}\left(\sqrt{c}x\right)\left(2a-b\log\left(1-cx^2\right)\right)}{3c^{3/2}}-\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(cx^2+1\right)}{3c^{3/2}}-\frac{b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(cx^2+1\right)}{3c^{3/2}}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-\sqrt{c}x}\right)}{3c^{3/2}}-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1-i\sqrt{c}x}\right)}{3c^{3/2}}+\frac{ib^2\text{PolyLog}\left(2,1-\frac{(1+i)\left(1-\sqrt{c}x\right)}{1-i\sqrt{c}x}\right)}{6c^{3/2}}-\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{i\sqrt{c}x+1}\right)}{3c^{3/2}}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{\sqrt{c}x+1}\right)}{3c^{3/2}}-\frac{b^2\text{PolyLog}\left(2,\frac{2\sqrt{c}\left(1-\sqrt{-c}x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(\sqrt{c}x+1\right)}+1\right)}{6c^{3/2}}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2\sqrt{c}\left(\sqrt{-c}x+1\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)}{6c^{3/2}}+\frac{ib^2\text{PolyLog}\left(2,1-\frac{(1-i)\left(\sqrt{c}x+1\right)}{1-i\sqrt{c}x}\right)}{6c^{3/2}}$$

input
```
Int[x^2*(a + b*ArcTanh[c*x^2])^2,x]
```

3.72. $\int x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx$

output
```
(4*a*b*x)/(3*c) - (2*a*b*x^3)/9 - (2*a*b*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) +
(4*b^2*ArcTan[Sqrt[c]*x])/(3*c^(3/2)) - ((I/3)*b^2*ArcTan[Sqrt[c]*x]^2)/c^
(3/2) - (4*b^2*ArcTanh[Sqrt[c]*x])/(3*c^(3/2)) - (b^2*ArcTanh[Sqrt[c]*x]^2
)/(3*c^(3/2)) + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/(3*c^(3/
2)) + (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/(3*c^(3/2)) - (b^
2*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c
^(3/2)) - (2*b^2*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/(3*c^(3/2)) -
 (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/(3*c^(3/2)) + (b^2*ArcT
anh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1
+ Sqrt[c]*x))])/(3*c^(3/2)) + (b^2*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 +
Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/(3*c^(3/2)) - (b^2*A
rcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/(3*c^(3
/2)) - (2*b^2*x*Log[1 - c*x^2])/(3*c) + (b^2*x^3*Log[1 - c*x^2])/9 + (b^2*
ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/(3*c^(3/2)) + (b*x^3*(2*a - b*Log[1 - c*
x^2]))/9 - (b*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/(3*c^(3/2)) + (
x^3*(2*a - b*Log[1 - c*x^2])^2)/12 + (2*b^2*x*Log[1 + c*x^2])/(3*c) + (a*b
*x^3*Log[1 + c*x^2])/3 - (b^2*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)
) - (b^2*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/(3*c^(3/2)) - (b^2*x^3*Log[1 -
 c*x^2]*Log[1 + c*x^2])/6 + (b^2*x^3*Log[1 + c*x^2]^2)/12 + (b^2*PolyLog[2
, 1 - 2/(1 - Sqrt[c]*x)])/(3*c^(3/2)) - ((I/3)*b^2*PolyLog[2, 1 - 2/(1 ...
```

3.72.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6456
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

3.72.4 Maple [F]

$$\int x^2 \left(a + b \operatorname{arctanh}\left(c x^2\right)\right)^2 dx$$

input
```
int(x^2*(a+b*arctanh(c*x^2))^2,x)
```

output
```
int(x^2*(a+b*arctanh(c*x^2))^2,x)
```

3.72.5 Fricas [F]

$$\int x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctanh(c*x^2)^2 + 2*a*b*x^2*arctanh(c*x^2) + a^2*x^2, x)
```

3.72.6 Sympy [F]

$$\int x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int x^2\left(a+b\operatorname{atanh}\left(cx^2\right)\right)^2\,dx$$

input
```
integrate(x**2*(a+b*atanh(c*x**2))**2,x)
```

output
```
Integral(x**2*(a + b*atanh(c*x**2))**2, x)
```

3.72.7 Maxima [F]

$$\int x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x^2) + c*(4*x/c^2 - 2*arctan(sqrt(c)*x)
/c^(5/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(5/2)))*a*b + 1/12*(x^3*
log(-c*x^2 + 1)^2 - 3*integrate(-1/3*(3*(c*x^4 - x^2)*log(c*x^2 + 1)^2 - 2
*(2*c*x^4 + 3*(c*x^4 - x^2)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1),
x))*b^2
```

3.72.8 Giac [F]

$$\int x^2 (a + b\text{arctanh}(cx^2))^2 \, dx = \int (b \,\text{artanh}\,(cx^2) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2*x^2, x)
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int x^2 (a + b\text{arctanh}(cx^2))^2 \, dx = \int x^2 \, (a + b \,\text{atanh}(c\,x^2))^2 \, dx$$

input
```
int(x^2*(a + b*atanh(c*x^2))^2,x)
```

output
```
int(x^2*(a + b*atanh(c*x^2))^2, x)
```

3.73 $\int (a + b\,\mathrm{arctanh}(cx^2))^2\, dx$

3.73.1 Optimal result

Integrand size = 12, antiderivative size = 958

$$\begin{aligned}
\int \left(a+b\operatorname{arctanh}(cx^2)\right)^2 dx = {} & a^2x+\frac{2ab\arctan\left(\sqrt{c}x\right)}{\sqrt{c}}+\frac{ib^2\arctan\left(\sqrt{c}x\right)^2}{\sqrt{c}} \\
& -\frac{2ab\operatorname{arctanh}\left(\sqrt{c}x\right)}{\sqrt{c}}-\frac{b^2\operatorname{arctanh}\left(\sqrt{c}x\right)^2}{\sqrt{c}} \\
& +\frac{2b^2\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt{c}x}\right)}{\sqrt{c}} \\
& -\frac{2b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{1-i\sqrt{c}x}\right)}{\sqrt{c}} \\
& +\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1+i)(1-\sqrt{c}x)}{1-i\sqrt{c}x}\right)}{\sqrt{c}} \\
& +\frac{2b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{1+i\sqrt{c}x}\right)}{\sqrt{c}} \\
& -\frac{2b^2\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{1+\sqrt{c}x}\right)}{\sqrt{c}} \\
& +\frac{b^2\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(-\frac{2\sqrt{c}(1-\sqrt{-c}x)}{(\sqrt{-c}-\sqrt{c})(1+\sqrt{c}x)}\right)}{\sqrt{c}} \\
& +\frac{b^2\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2\sqrt{c}(1+\sqrt{-c}x)}{(\sqrt{-c}+\sqrt{c})(1+\sqrt{c}x)}\right)}{\sqrt{c}} \\
& +\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1-i)(1+\sqrt{c}x)}{1-i\sqrt{c}x}\right)}{\sqrt{c}} \\
& -abx\log\left(1-cx^2\right)-\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(1-cx^2\right)}{\sqrt{c}} \\
& +\frac{b^2\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(1-cx^2\right)}{\sqrt{c}}+\frac{1}{4}b^2x\log^2\left(1-cx^2\right) \\
& +abx\log\left(1+cx^2\right)+\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(1+cx^2\right)}{\sqrt{c}} \\
& -\frac{b^2\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(1+cx^2\right)}{\sqrt{c}} \\
& -\frac{1}{2}b^2x\log\left(1-cx^2\right)\log\left(1+cx^2\right)+\frac{1}{4}b^2x\log^2\left(1+cx^2\right) \\
& +\frac{b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-\sqrt{c}x}\right)}{\sqrt{c}}+\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1-i\sqrt{c}x}\right)}{\sqrt{c}} \\
& -\frac{ib^2\operatorname{PolyLog}\left(2,1-\frac{(1+i)(1-\sqrt{c}x)}{1-i\sqrt{c}x}\right)}{2\sqrt{c}} \\
& + ib^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+i\sqrt{c}x}\right) \quad + b^2\operatorname{PolyLog}\left(2,1-\frac{2}{1+\sqrt{c}x}\right)
\end{aligned}$$

output
```
-b^2*arctanh(x*c^(1/2))^2/c^(1/2)+b^2*polylog(2,1-2/(1-x*c^(1/2)))/c^(1/2)
+a^2*x+b^2*polylog(2,1-2/(1+x*c^(1/2)))/c^(1/2)-a*b*x*ln(-c*x^2+1)+I*b^2*a
rctan(x*c^(1/2))^2/c^(1/2)+a*b*x*ln(c*x^2+1)-b^2*arctan(x*c^(1/2))*ln(-c*x
^2+1)/c^(1/2)+b^2*arctanh(x*c^(1/2))*ln(-c*x^2+1)/c^(1/2)+b^2*arctan(x*c^(
1/2))*ln(c*x^2+1)/c^(1/2)-b^2*arctanh(x*c^(1/2))*ln(c*x^2+1)/c^(1/2)+b^2*a
rctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))/c^(1/2)+b^2*arcta
nh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(
1/2)))/c^(1/2)+b^2*arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^
(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(1/2)+b^2*arctan(x*c^(1/2))*ln((1-I)*(1+x*
c^(1/2))/(1-I*x*c^(1/2)))/c^(1/2)+I*b^2*polylog(2,1-2/(1-I*x*c^(1/2)))/c^(
1/2)+I*b^2*polylog(2,1-2/(1+I*x*c^(1/2)))/c^(1/2)-1/2*b^2*x*ln(-c*x^2+1)*l
n(c*x^2+1)+2*a*b*arctan(x*c^(1/2))/c^(1/2)-2*a*b*arctanh(x*c^(1/2))/c^(1/2
)+2*b^2*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))/c^(1/2)-2*b^2*arctan(x*c^(1
/2))*ln(2/(1-I*x*c^(1/2)))/c^(1/2)+2*b^2*arctan(x*c^(1/2))*ln(2/(1+I*x*c^(
1/2)))/c^(1/2)-2*b^2*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))/c^(1/2)-1/2*I*
b^2*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))/c^(1/2)-1/2*I*b^2*pol
ylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))/c^(1/2)+1/4*b^2*x*ln(-c*x^2
+1)^2+1/4*b^2*x*ln(c*x^2+1)^2-1/2*b^2*polylog(2,1+2*(1-x*(-c)^(1/2))*c^(1/
2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))/c^(1/2)-1/2*b^2*polylog(2,1-2*(1+x*
(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))/c^(1/2)
```

3.73.2 Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.59

$$\int \left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx$$

$$=\frac{1}{2}x\left(2a^2+4ab\text{arctanh}\left(cx^2\right)+\frac{4ab\left(\arctan\left(\sqrt{cx^2}\right)-\text{arctanh}\left(\sqrt{cx^2}\right)\right)}{\sqrt{cx^2}}\right.$$

$$+\frac{b^2\left(-2i\arctan\left(\sqrt{cx^2}\right)^2+4\arctan\left(\sqrt{cx^2}\right)\text{arctanh}(cx^2)+2\sqrt{cx^2}\text{arctanh}(cx^2)^2+2\arctan\left(\sqrt{cx^2}\right)\text{lo}\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
(x*(2*a^2 + 4*a*b*ArcTanh[c*x^2] + (4*a*b*(ArcTan[Sqrt[c*x^2]] - ArcTanh[S
qrt[c*x^2]]))/Sqrt[c*x^2] + (b^2*((-2*I)*ArcTan[Sqrt[c*x^2]]^2 + 4*ArcTan[
Sqrt[c*x^2]]*ArcTanh[c*x^2] + 2*Sqrt[c*x^2]*ArcTanh[c*x^2]^2 + 2*ArcTan[Sq
rt[c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] + 2*ArcTanh[c*x^2]*Log[1
 - Sqrt[c*x^2]] - Log[2]*Log[1 - Sqrt[c*x^2]] + Log[1 - Sqrt[c*x^2]]^2/2 -
 Log[1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c*x^2])] - 2*ArcTanh[c*x^
2]*Log[1 + Sqrt[c*x^2]] + Log[2]*Log[1 + Sqrt[c*x^2]] + Log[((1 + I) - (1
- I)*Sqrt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] + Log[(-1/2 - I/2)*(I + Sqrt[c*x
^2])]*Log[1 + Sqrt[c*x^2]] - Log[1 + Sqrt[c*x^2]]^2/2 - Log[1 - Sqrt[c*x^2
]]*Log[((1 + I) + (1 - I)*Sqrt[c*x^2])/2] - (I/2)*PolyLog[2, -E^((4*I)*Arc
Tan[Sqrt[c*x^2]])] + PolyLog[2, (1 - Sqrt[c*x^2])/2] - PolyLog[2, (-1/2 -
I/2)*(-1 + Sqrt[c*x^2])] - PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c*x^2])] - P
olyLog[2, (1 + Sqrt[c*x^2])/2] + PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c*x^2])]
 + PolyLog[2, (1/2 + I/2)*(1 + Sqrt[c*x^2])]))/Sqrt[c*x^2]))/2
```

3.73.3 Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 958, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6438, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}(cx^2)\right)^2 dx$$

$$\Big\downarrow \text{6438}$$

$$\int \left(a^2-ab\log\left(1-cx^2\right)+ab\log\left(cx^2+1\right)+\frac{1}{4}b^2\log^2\left(1-cx^2\right)+\frac{1}{4}b^2\log^2\left(cx^2+1\right)-\frac{1}{2}b^2\log\left(1-cx^2\right)\log\left(cx^2\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$xa^2+\frac{2b\arctan\left(\sqrt{c}x\right)a}{\sqrt{c}}-\frac{2b\text{arctanh}\left(\sqrt{c}x\right)a}{\sqrt{c}}-bx\log\left(1-cx^2\right)a+bx\log\left(cx^2+1\right)a+\frac{ib^2\arctan\left(\sqrt{c}x\right)^2}{\sqrt{c}}-\frac{b^2\text{arctanh}\left(\sqrt{c}x\right)^2}{\sqrt{c}}+\frac{1}{4}b^2x\log^2\left(1-cx^2\right)+\frac{1}{4}b^2x\log^2\left(cx^2+1\right)+\frac{2b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt{c}x}\right)}{\sqrt{c}}-\frac{2b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{1-i\sqrt{c}x}\right)}{\sqrt{c}}+\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1+i)(1-\sqrt{c}x)}{1-i\sqrt{c}x}\right)}{\sqrt{c}}+\frac{2b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{i\sqrt{c}x+1}\right)}{\sqrt{c}}-\frac{2b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{\sqrt{c}x+1}\right)}{\sqrt{c}}+\frac{b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(-\frac{2\sqrt{c}(1-\sqrt{-c}x)}{(\sqrt{-c}-\sqrt{c})(\sqrt{c}x+1)}\right)}{\sqrt{c}}+\frac{b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2\sqrt{c}(\sqrt{-c}x+1)}{(\sqrt{-c}+\sqrt{c})(\sqrt{c}x+1)}\right)}{\sqrt{c}}+\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1-i)(\sqrt{c}x+1)}{1-i\sqrt{c}x}\right)}{\sqrt{c}}-\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(1-cx^2\right)}{\sqrt{c}}+\frac{b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(1-cx^2\right)}{\sqrt{c}}+\frac{b^2\arctan\left(\sqrt{c}x\right)\log\left(cx^2+1\right)}{\sqrt{c}}-\frac{b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(cx^2+1\right)}{\sqrt{c}}-\frac{1}{2}b^2x\log\left(1-cx^2\right)\log\left(cx^2+1\right)+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{1-\sqrt{c}x}\right)}{\sqrt{c}}+\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{1-i\sqrt{c}x}\right)}{\sqrt{c}}-\frac{ib^2\text{PolyLog}\left(2,1-\frac{(1+i)(1-\sqrt{c}x)}{1-i\sqrt{c}x}\right)}{2\sqrt{c}}+\frac{ib^2\text{PolyLog}\left(2,1-\frac{2}{i\sqrt{c}x+1}\right)}{\sqrt{c}}+\frac{b^2\text{PolyLog}\left(2,1-\frac{2}{\sqrt{c}x+1}\right)}{\sqrt{c}}-\frac{b^2\text{PolyLog}\left(2,\frac{2\sqrt{c}(1-\sqrt{-c}x)}{(\sqrt{-c}-\sqrt{c})(\sqrt{c}x+1)}+1\right)}{2\sqrt{c}}-\frac{b^2\text{PolyLog}\left(2,1-\frac{2\sqrt{c}(\sqrt{-c}x+1)}{(\sqrt{-c}+\sqrt{c})(\sqrt{c}x+1)}\right)}{2\sqrt{c}}-\frac{ib^2\text{PolyLog}\left(2,1-\frac{(1-i)(\sqrt{c}x+1)}{1-i\sqrt{c}x}\right)}{2\sqrt{c}}$$

input
```
Int[(a + b*ArcTanh[c*x^2])^2,x]
```

3.73. $\int\left(a+b\text{arctanh}(cx^2)\right)^2\,dx$

output
```
a^2*x + (2*a*b*ArcTan[Sqrt[c]*x])/Sqrt[c] + (I*b^2*ArcTan[Sqrt[c]*x]^2)/Sq
rt[c] - (2*a*b*ArcTanh[Sqrt[c]*x])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]^2)/Sq
rt[c] + (2*b^2*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/Sqrt[c] - (2*b^2
*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]
*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (2*b^2*Arc
Tan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/Sqrt[c] - (2*b^2*ArcTanh[Sqrt[c]*
x]*Log[2/(1 + Sqrt[c]*x)])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[
c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/Sqrt[c] + (b
^2*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c
])*(1 + Sqrt[c]*x))])/Sqrt[c] + (b^2*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + S
qrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] - a*b*x*Log[1 - c*x^2] - (b^2*ArcTa
n[Sqrt[c]*x]*Log[1 - c*x^2])/Sqrt[c] + (b^2*ArcTanh[Sqrt[c]*x]*Log[1 - c*x
^2])/Sqrt[c] + (b^2*x*Log[1 - c*x^2]^2)/4 + a*b*x*Log[1 + c*x^2] + (b^2*Ar
cTan[Sqrt[c]*x]*Log[1 + c*x^2])/Sqrt[c] - (b^2*ArcTanh[Sqrt[c]*x]*Log[1 +
c*x^2])/Sqrt[c] - (b^2*x*Log[1 - c*x^2]*Log[1 + c*x^2])/2 + (b^2*x*Log[1 +
 c*x^2]^2)/4 + (b^2*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/Sqrt[c] + (I*b^2*Po
lyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)])/Sqrt[c] - ((I/2)*b^2*PolyLog[2, 1 - ((1
 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/Sqrt[c] + (I*b^2*PolyLog[2, 1 -
 2/(1 + I*Sqrt[c]*x)])/Sqrt[c] + (b^2*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)])/S
qrt[c] - (b^2*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - ...
```

3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6438
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandI
ntegrand[(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p, x], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

3.73.4 Maple [F]

$$\int \left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2dx$$

input
```
int((a+b*arctanh(c*x^2))^2,x)
```

output
```
int((a+b*arctanh(c*x^2))^2,x)
```

3.73.5 Fricas [F]

$$\int \left(a+b\text{arctanh}(cx^2)\right)^2\,dx=\int \left(b\,\text{artanh}\left(cx^2\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2, x)
```

3.73.6 Sympy [F]

$$\int \left(a+b\text{arctanh}(cx^2)\right)^2\,dx=\int \left(a+b\,\text{atanh}\left(cx^2\right)\right)^2\,dx$$

input
```
integrate((a+b*atanh(c*x**2))**2,x)
```

output
```
Integral((a + b*atanh(c*x**2))**2, x)
```

3.73.7 Maxima [F]

$$\int \left(a+b\text{arctanh}(cx^2)\right)^2\,dx=\int \left(b\,\text{artanh}\left(cx^2\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

output
```
(c*(2*arctan(sqrt(c)*x)/c^(3/2) + log((c*x - sqrt(c))/(c*x + sqrt(c)))/c^(
3/2)) + 2*x*arctanh(c*x^2))*a*b + 1/4*(x*log(-c*x^2 + 1)^2 - integrate(-((
c*x^2 - 1)*log(c*x^2 + 1)^2 - 2*(2*c*x^2 + (c*x^2 - 1)*log(c*x^2 + 1))*log
(-c*x^2 + 1))/(c*x^2 - 1), x))*b^2 + a^2*x
```

3.73.8 Giac [F]

$$\int \left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2\,dx=\int \left(b\operatorname{artanh}\left(cx^2\right)+a\right)^2\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2, x)
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2\,dx=\int \left(a+b\operatorname{atanh}\left(c\,x^2\right)\right)^2\,dx$$

input
```
int((a + b*atanh(c*x^2))^2,x)
```

output
```
int((a + b*atanh(c*x^2))^2, x)
```

3.74 $\int \frac{\left(a+b\,\mathrm{arctanh}\left(cx^2\right)\right)^2}{x^2}\,dx$

3.74.1 Optimal result

Integrand size = 16, antiderivative size = 942

$$
\begin{aligned}
\int \frac{(a+b\operatorname{arctanh}(cx^2))^2}{x^2}\,dx ={}& 2ab\sqrt{c}\arctan\left(\sqrt{c}x\right) + ib^2\sqrt{c}\arctan\left(\sqrt{c}x\right)^2 \\
&+ b^2\sqrt{c}\operatorname{arctanh}\left(\sqrt{c}x\right)^2 - 2b^2\sqrt{c}\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt{c}x}\right) \\
&- 2b^2\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{1-i\sqrt{c}x}\right) \\
&+ b^2\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1+i)\left(1-\sqrt{c}x\right)}{1-i\sqrt{c}x}\right) \\
&+ 2b^2\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{1+i\sqrt{c}x}\right) \\
&+ 2b^2\sqrt{c}\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{1+\sqrt{c}x}\right) \\
&- b^2\sqrt{c}\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(-\frac{2\sqrt{c}(1-\sqrt{-c}x)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(1+\sqrt{c}x\right)}\right) \\
&- b^2\sqrt{c}\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2\sqrt{c}(1+\sqrt{-c}x)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(1+\sqrt{c}x\right)}\right) \\
&+ b^2\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1-i)\left(1+\sqrt{c}x\right)}{1-i\sqrt{c}x}\right) \\
&- b^2\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(1-cx^2\right) \\
&+ b\sqrt{c}\operatorname{arctanh}\left(\sqrt{c}x\right)\left(2a-b\log\left(1-cx^2\right)\right) \\
&- \frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x} - \frac{ab\log\left(1+cx^2\right)}{x} \\
&+ b^2\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(1+cx^2\right) \\
&+ b^2\sqrt{c}\operatorname{arctanh}\left(\sqrt{c}x\right)\log\left(1+cx^2\right) \\
&+ \frac{b^2\log\left(1-cx^2\right)\log\left(1+cx^2\right)}{2x} - \frac{b^2\log^2\left(1+cx^2\right)}{4x} \\
&- b^2\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{2}{1-\sqrt{c}x}\right) \\
&+ ib^2\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{2}{1-i\sqrt{c}x}\right) \\
&- \frac{1}{2}ib^2\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{(1+i)\left(1-\sqrt{c}x\right)}{1-i\sqrt{c}x}\right) \\
&+ ib^2\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{2}{1+i\sqrt{c}x}\right) \\
&- b^2\sqrt{c}\operatorname{PolyLog}\left(2,1-\frac{2}{1+\sqrt{c}x}\right) \\
&+ \frac{1}{2}b^2\sqrt{c}\operatorname{PolyLog}\left(2,1+\frac{2\sqrt{c}(1-\sqrt{-c}x)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(1+\sqrt{c}x\right)}\right)
\end{aligned}
$$

output
```
-b^2*polylog(2,1-2/(1-x*c^(1/2)))*c^(1/2)+b^2*arctanh(x*c^(1/2))^2*c^(1/2)
-b^2*polylog(2,1-2/(1+x*c^(1/2)))*c^(1/2)-a*b*ln(c*x^2+1)/x+I*b^2*arctan(x
*c^(1/2))^2*c^(1/2)-b^2*arctan(x*c^(1/2))*ln(-c*x^2+1)*c^(1/2)+b*arctanh(x
*c^(1/2))*(2*a-b*ln(-c*x^2+1))*c^(1/2)+b^2*arctan(x*c^(1/2))*ln(c*x^2+1)*c
^(1/2)+b^2*arctanh(x*c^(1/2))*ln(c*x^2+1)*c^(1/2)+b^2*arctan(x*c^(1/2))*ln
((1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))*c^(1/2)-b^2*arctanh(x*c^(1/2))*ln(-2
*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))*c^(1/2)-b^2*
arctanh(x*c^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x
*c^(1/2)))*c^(1/2)+b^2*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(
1/2)))*c^(1/2)+I*b^2*polylog(2,1-2/(1-I*x*c^(1/2)))*c^(1/2)+I*b^2*polylog(
2,1-2/(1+I*x*c^(1/2)))*c^(1/2)+1/2*b^2*ln(-c*x^2+1)*ln(c*x^2+1)/x+2*a*b*ar
ctan(x*c^(1/2))*c^(1/2)-2*b^2*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))*c^(1/
2)-2*b^2*arctan(x*c^(1/2))*ln(2/(1-I*x*c^(1/2)))*c^(1/2)+2*b^2*arctan(x*c^
(1/2))*ln(2/(1+I*x*c^(1/2)))*c^(1/2)+2*b^2*arctanh(x*c^(1/2))*ln(2/(1+x*c^
(1/2)))*c^(1/2)-1/2*I*b^2*polylog(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))
*c^(1/2)-1/2*I*b^2*polylog(2,1+(-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))*c^(1/
2)-1/4*(2*a-b*ln(-c*x^2+1))^2/x-1/4*b^2*ln(c*x^2+1)^2/x+1/2*b^2*polylog(2,
1+2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))*c^(1/2)+1
/2*b^2*polylog(2,1-2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^
(1/2)))*c^(1/2)
```

3.74.2 Mathematica [A] (verified)

Time = 2.82 (sec) , antiderivative size = 566, normalized size of antiderivative = 0.60

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^2}\,dx$$

$$= \frac{-2a^2 - 4ab\text{arctanh}(cx^2) + 4ab\sqrt{cx^2}\left(\arctan\left(\sqrt{cx^2}\right) + \text{arctanh}\left(\sqrt{cx^2}\right)\right) + b^2\sqrt{cx^2}\left(-2i\arctan\left(\sqrt{cx^2}\right)^2 \cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^2/x^2,x]
```

output
```
(-2*a^2 - 4*a*b*ArcTanh[c*x^2] + 4*a*b*Sqrt[c*x^2]*(ArcTan[Sqrt[c*x^2]] +
ArcTanh[Sqrt[c*x^2]]) + b^2*Sqrt[c*x^2]*((-2*I)*ArcTan[Sqrt[c*x^2]]^2 + 4*
ArcTan[Sqrt[c*x^2]]*ArcTanh[c*x^2] - (2*ArcTanh[c*x^2]^2)/Sqrt[c*x^2] + 2*
ArcTan[Sqrt[c*x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c*x^2]])] - 2*ArcTanh[c*x
^2]*Log[1 - Sqrt[c*x^2]] + Log[2]*Log[1 - Sqrt[c*x^2]] - Log[1 - Sqrt[c*x^
2]]^2/2 + Log[1 - Sqrt[c*x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c*x^2])] + 2*Arc
Tanh[c*x^2]*Log[1 + Sqrt[c*x^2]] - Log[2]*Log[1 + Sqrt[c*x^2]] - Log[((1 +
 I) - (1 - I)*Sqrt[c*x^2])/2]*Log[1 + Sqrt[c*x^2]] - Log[(-1/2 - I/2)*(I +
 Sqrt[c*x^2])]*Log[1 + Sqrt[c*x^2]] + Log[1 + Sqrt[c*x^2]]^2/2 + Log[1 - S
qrt[c*x^2]]*Log[((1 + I) + (1 - I)*Sqrt[c*x^2])/2] - (I/2)*PolyLog[2, -E^(
(4*I)*ArcTan[Sqrt[c*x^2]])] - PolyLog[2, (1 - Sqrt[c*x^2])/2] + PolyLog[2,
 (-1/2 - I/2)*(-1 + Sqrt[c*x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c*x
^2])] + PolyLog[2, (1 + Sqrt[c*x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt
[c*x^2])] - PolyLog[2, (1/2 + I/2)*(1 + Sqrt[c*x^2])]))/(2*x)
```

3.74.3 Rubi [A] (verified)

Time = 1.69 (sec) , antiderivative size = 942, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6456, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{x^2} \, dx$$

$$\Big\downarrow \text{6456}$$

$$\int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^2}-\frac{b\log\left(cx^2+1\right)\left(b\log\left(1-cx^2\right)-2a\right)}{2x^2}+\frac{b^2\log^2\left(cx^2+1\right)}{4x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\sqrt{c}\arctan\left(\sqrt{c}x\right)^2b^2+\sqrt{c}\text{carctanh}\left(\sqrt{c}x\right)^2b^2-\frac{\log^2\left(cx^2+1\right)b^2}{4x}-2\sqrt{c}\text{carctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt{c}x}\right)b^2-2\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{1-i\sqrt{c}x}\right)b^2+\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1+i)\left(1-\sqrt{c}x\right)}{1-i\sqrt{c}x}\right)b^2+2\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{i\sqrt{c}x+1}\right)b^2+2\sqrt{c}\text{carctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{\sqrt{c}x+1}\right)b^2-\sqrt{c}\text{carctanh}\left(\sqrt{c}x\right)\log\left(-\frac{2\sqrt{c}\left(1-\sqrt{-c}x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)b^2-\sqrt{c}\text{carctanh}\left(\sqrt{c}x\right)\log\left(\frac{2\sqrt{c}\left(\sqrt{-c}x+1\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)b^2+\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1-i)\left(\sqrt{c}x+1\right)}{1-i\sqrt{c}x}\right)b^2-\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(1-cx^2\right)b^2+\sqrt{c}\arctan\left(\sqrt{c}x\right)\log\left(cx^2+1\right)b^2+\sqrt{c}\text{carctanh}\left(\sqrt{c}x\right)\log\left(cx^2+1\right)b^2+\frac{\log\left(1-cx^2\right)\log\left(cx^2+1\right)b^2}{2x}-\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{2}{1-\sqrt{c}x}\right)b^2+i\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{2}{1-i\sqrt{c}x}\right)b^2-\frac{1}{2}i\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{(1+i)\left(1-\sqrt{c}x\right)}{1-i\sqrt{c}x}\right)b^2+i\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{2}{i\sqrt{c}x+1}\right)b^2-\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{2}{\sqrt{c}x+1}\right)b^2+\frac{1}{2}\sqrt{c}\,\text{PolyLog}\left(2,\frac{2\sqrt{c}\left(1-\sqrt{-c}x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(\sqrt{c}x+1\right)}+1\right)b^2+\frac{1}{2}\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{2\sqrt{c}\left(\sqrt{-c}x+1\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)b^2-\frac{1}{2}i\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{(1-i)\left(\sqrt{c}x+1\right)}{1-i\sqrt{c}x}\right)b^2+2a\sqrt{c}\arctan\left(\sqrt{c}x\right)b+\sqrt{c}\text{carctanh}\left(\sqrt{c}x\right)\left(2a-b\log\left(1-cx^2\right)\right)b-\frac{a\log\left(cx^2+1\right)b}{x}-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x}$$

input

```
Int[(a + b*ArcTanh[c*x^2])^2/x^2,x]
```

3.74. $\int \frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{x^2}\,dx$

output
```
2*a*b*Sqrt[c]*ArcTan[Sqrt[c]*x] + I*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]^2 + b^2*
Sqrt[c]*ArcTanh[Sqrt[c]*x]^2 - 2*b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[2/(1 -
 Sqrt[c]*x)] - 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)] +
b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]
*x)] + 2*b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)] + 2*b^2*Sq
rt[c]*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTanh[Sqrt
[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c
]*x))] - b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/(
(Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))] + b^2*Sqrt[c]*ArcTan[Sqrt[c]*x]*Log
[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] - b^2*Sqrt[c]*ArcTan[Sqrt[c]
*x]*Log[1 - c*x^2] + b*Sqrt[c]*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2])
 - (2*a - b*Log[1 - c*x^2])^2/(4*x) - (a*b*Log[1 + c*x^2])/x + b^2*Sqrt[c]
*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2] + b^2*Sqrt[c]*ArcTanh[Sqrt[c]*x]*Log[1 +
 c*x^2] + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])/(2*x) - (b^2*Log[1 + c*x^2]^
2)/(4*x) - b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)] + I*b^2*Sqrt[c]*P
olyLog[2, 1 - 2/(1 - I*Sqrt[c]*x)] - (I/2)*b^2*Sqrt[c]*PolyLog[2, 1 - ((1
+ I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - 2/
(1 + I*Sqrt[c]*x)] - b^2*Sqrt[c]*PolyLog[2, 1 - 2/(1 + Sqrt[c]*x)] + (b^2*
Sqrt[c]*PolyLog[2, 1 + (2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*
(1 + Sqrt[c]*x))])/2 + (b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c]*(1 + Sqrt...
```

3.74.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6456
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

3.74.4 Maple [F]

$$\int \frac{(a + b \operatorname{arctanh}(c x^2))^2}{x^2} dx$$

input
```
int((a+b*arctanh(c*x^2))^2/x^2,x)
```

output
```
int((a+b*arctanh(c*x^2))^2/x^2,x)
```

3.74.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx^2)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^2, x)
```

3.74.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^2}\,dx=\int \frac{(a+b\,\text{atanh}\,(cx^2))^2}{x^2}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))**2/x**2,x)
```

output
```
Integral((a + b*atanh(c*x**2))**2/x**2, x)
```

3.74.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx^2)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="maxima")
```

output
```
(c*(2*arctan(sqrt(c)*x)/sqrt(c) - log((c*x - sqrt(c))/(c*x + sqrt(c)))/sqr
t(c)) - 2*arctanh(c*x^2)/x)*a*b - 1/4*b^2*(log(-c*x^2 + 1)^2/x + integrate
(-((c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(2*c*x^2 - (c*x^2 - 1)*log(c*x^2 + 1))
*log(-c*x^2 + 1))/(c*x^4 - x^2), x)) - a^2/x
```

3.74.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx^2) + a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2/x^2, x)
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^2}\,dx = \int \frac{(a + b\,\text{atanh}(c\,x^2))^2}{x^2}\,dx$$

input
```
int((a + b*atanh(c*x^2))^2/x^2,x)
```

output
```
int((a + b*atanh(c*x^2))^2/x^2, x)
```

3.75 $\int \frac{\left(a+b\operatorname{arctanh}(cx^2)\right)^2}{x^4}\,dx$

3.75.1 Optimal result

Integrand size = 16, antiderivative size = 1102

$$\int \frac{\left(a+b\operatorname{arctanh}(cx^2)\right)^2}{x^4}\,dx = \text{Too large to display}$$

output
```
-1/3*I*b^2*c^(3/2)*polylog(2,1-2/(1+I*x*c^(1/2)))+1/6*I*b^2*c^(3/2)*polylo
g(2,1-(1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))+1/6*I*b^2*c^(3/2)*polylog(2,1+(
-1+I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))+1/6*b^2*ln(-c*x^2+1)*ln(c*x^2+1)/x^3-
2/3*b^2*c^(3/2)*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))+2/3*b^2*c^(3/2)*arc
tan(x*c^(1/2))*ln(2/(1-I*x*c^(1/2)))-1/3*b^2*c^(3/2)*arctan(x*c^(1/2))*ln(
(1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))-2/3*b^2*c^(3/2)*arctan(x*c^(1/2))*ln(
2/(1+I*x*c^(1/2)))+2/3*b^2*c^(3/2)*arctanh(x*c^(1/2))*ln(2/(1+x*c^(1/2)))-
1/3*b^2*c^(3/2)*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2))*c^(1/2)/((-c)^(1
/2)-c^(1/2))/(1+x*c^(1/2)))-1/3*b^2*c^(3/2)*arctanh(x*c^(1/2))*ln(2*(1+x*(
-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))-1/3*b^2*c^(3/2)*arc
tan(x*c^(1/2))*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))-2/3*a*b*c^(3/2)*arc
tan(x*c^(1/2))+1/3*b^2*c*ln(-c*x^2+1)/x+1/3*b^2*c^(3/2)*arctan(x*c^(1/2))*
ln(-c*x^2+1)-1/3*b*c*(2*a-b*ln(-c*x^2+1))/x+1/3*b*c^(3/2)*arctanh(x*c^(1/2
))*(2*a-b*ln(-c*x^2+1))-1/3*a*b*ln(c*x^2+1)/x^3-2/3*b^2*c*ln(c*x^2+1)/x-1/
3*b^2*c^(3/2)*arctan(x*c^(1/2))*ln(c*x^2+1)+1/3*b^2*c^(3/2)*arctanh(x*c^(1
/2))*ln(c*x^2+1)-2/3*a*b*c/x-1/3*I*b^2*c^(3/2)*arctan(x*c^(1/2))^2-1/3*I*b
^2*c^(3/2)*polylog(2,1-2/(1-I*x*c^(1/2)))-1/12*(2*a-b*ln(-c*x^2+1))^2/x^3+
4/3*b^2*c^(3/2)*arctan(x*c^(1/2))+4/3*b^2*c^(3/2)*arctanh(x*c^(1/2))+1/3*b
^2*c^(3/2)*arctanh(x*c^(1/2))^2-1/12*b^2*ln(c*x^2+1)^2/x^3-1/3*b^2*c^(3/2)
*polylog(2,1-2/(1-x*c^(1/2)))-1/3*b^2*c^(3/2)*polylog(2,1-2/(1+x*c^(1/2...
```

3.75.2 Mathematica [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^4}\,dx=\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^4}\,dx$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^2/x^4,x]
```

output
```
Integrate[(a + b*ArcTanh[c*x^2])^2/x^4, x]
```

3.75.3 Rubi [A] (verified)

Time = 2.11 (sec) , antiderivative size = 1102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6456, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^4}\,dx$$

$$\Big\downarrow \text{6456}$$

$$\int \left(\frac{(2a-b\log(1-cx^2))^2}{4x^4}-\frac{b\log(cx^2+1)\,(b\log(1-cx^2)-2a)}{2x^4}+\frac{b^2\log^2(cx^2+1)}{4x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{1}{3}ic^{3/2}\arctan\left(\sqrt{c}x\right)^2 b^2+\frac{1}{3}c^{3/2}\text{arctanh}\left(\sqrt{c}x\right)^2 b^2-\frac{\log^2\left(cx^2+1\right)b^2}{12x^3}+\frac{4}{3}c^{3/2}\arctan\left(\sqrt{c}x\right)b^2+\frac{4}{3}c^{3/2}\text{arctanh}\left(\sqrt{c}x\right)b^2-\frac{2}{3}c^{3/2}\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt{c}x}\right)b^2+\frac{2}{3}c^{3/2}\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{1-i\sqrt{c}x}\right)b^2-\frac{1}{3}c^{3/2}\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1+i)\left(1-\sqrt{c}x\right)}{1-i\sqrt{c}x}\right)b^2-\frac{2}{3}c^{3/2}\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{i\sqrt{c}x+1}\right)b^2+\frac{2}{3}c^{3/2}\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{\sqrt{c}x+1}\right)b^2-\frac{1}{3}c^{3/2}\text{arctanh}\left(\sqrt{c}x\right)\log\left(-\frac{2\sqrt{c}\left(1-\sqrt{-c}x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)b^2-\frac{1}{3}c^{3/2}\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2\sqrt{c}\left(\sqrt{-c}x+1\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)b^2-\frac{1}{3}c^{3/2}\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1-i)\left(\sqrt{c}x+1\right)}{1-i\sqrt{c}x}\right)b^2+\frac{1}{3}c^{3/2}\arctan\left(\sqrt{c}x\right)\log\left(1-cx^2\right)b^2+\frac{c\log\left(1-cx^2\right)b^2}{3x}-\frac{1}{3}c^{3/2}\arctan\left(\sqrt{c}x\right)\log\left(cx^2+1\right)b^2+\frac{1}{3}c^{3/2}\text{arctanh}\left(\sqrt{c}x\right)\log\left(cx^2+1\right)b^2+\frac{\log\left(1-cx^2\right)\log\left(cx^2+1\right)b^2}{6x^3}-\frac{2c\log\left(cx^2+1\right)b^2}{3x}-\frac{1}{3}c^{3/2}\text{PolyLog}\left(2,1-\frac{2}{1-\sqrt{c}x}\right)b^2-\frac{1}{3}ic^{3/2}\text{PolyLog}\left(2,1-\frac{2}{1-i\sqrt{c}x}\right)b^2+\frac{1}{6}ic^{3/2}\text{PolyLog}\left(2,1-\frac{(1+i)\left(1-\sqrt{c}x\right)}{1-i\sqrt{c}x}\right)b^2-\frac{1}{3}ic^{3/2}\text{PolyLog}\left(2,1-\frac{2}{i\sqrt{c}x+1}\right)b^2-\frac{1}{3}c^{3/2}\text{PolyLog}\left(2,1-\frac{2}{\sqrt{c}x+1}\right)b^2+\frac{1}{6}c^{3/2}\text{PolyLog}\left(2,\frac{2\sqrt{c}\left(1-\sqrt{-c}x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(\sqrt{c}x+1\right)}+1\right)b^2+\frac{1}{6}c^{3/2}\text{PolyLog}\left(2,1-\frac{2\sqrt{c}\left(\sqrt{-c}x+1\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)b^2+\frac{1}{6}ic^{3/2}\text{PolyLog}\left(2,1-\frac{(1-i)\left(\sqrt{c}x+1\right)}{1-i\sqrt{c}x}\right)b^2-\frac{2}{3}ac^{3/2}\arctan\left(\sqrt{c}x\right)b+\frac{1}{3}c^{3/2}\text{arctanh}\left(\sqrt{c}x\right)\left(2a-b\log\left(1-cx^2\right)\right)b-\frac{c\left(2a-b\log\left(1-cx^2\right)\right)b}{3x}-\frac{a\log\left(cx^2+1\right)b}{3x^3}-\frac{2acb}{3x}-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{12x^3}$$

input
```
Int[(a + b*ArcTanh[c*x^2])^2/x^4,x]
```

3.75. $\int \frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{x^4}\,dx$

output
```
(-2*a*b*c)/(3*x) - (2*a*b*c^(3/2)*ArcTan[Sqrt[c]*x])/3 + (4*b^2*c^(3/2)*Ar
cTan[Sqrt[c]*x])/3 - (I/3)*b^2*c^(3/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(3/2
)*ArcTanh[Sqrt[c]*x])/3 + (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]^2)/3 - (2*b^2*c^
(3/2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 - Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTan
[Sqrt[c]*x]*Log[2/(1 - I*Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*L
og[((1 + I)*(1 - Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/3 - (2*b^2*c^(3/2)*ArcTan
[Sqrt[c]*x]*Log[2/(1 + I*Sqrt[c]*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[Sqrt[c]*x
]*Log[2/(1 + Sqrt[c]*x)])/3 - (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt
[c]*(1 - Sqrt[-c]*x))/((Sqrt[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/3 - (b^2*c^
(3/2)*ArcTanh[Sqrt[c]*x]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqr
t[c])*(1 + Sqrt[c]*x))])/3 - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(
1 + Sqrt[c]*x))/(1 - I*Sqrt[c]*x)])/3 + (b^2*c*Log[1 - c*x^2])/(3*x) + (b^
2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/3 - (b*c*(2*a - b*Log[1 - c*x^
2]))/(3*x) + (b*c^(3/2)*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/3 - (
2*a - b*Log[1 - c*x^2])^2/(12*x^3) - (a*b*Log[1 + c*x^2])/(3*x^3) - (2*b^2
*c*Log[1 + c*x^2])/(3*x) - (b^2*c^(3/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/
3 + (b^2*c^(3/2)*ArcTanh[Sqrt[c]*x]*Log[1 + c*x^2])/3 + (b^2*Log[1 - c*x^2
]*Log[1 + c*x^2])/(6*x^3) - (b^2*Log[1 + c*x^2]^2)/(12*x^3) - (b^2*c^(3/2)
*PolyLog[2, 1 - 2/(1 - Sqrt[c]*x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - 2
/(1 - I*Sqrt[c]*x)] + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(1 - Sq...
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6456
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

3.75.4 Maple [F]

$$\int \frac{(a + b \operatorname{arctanh}(c x^2))^2}{x^4} dx$$

input
```
int((a+b*arctanh(c*x^2))^2/x^4,x)
```

output
```
int((a+b*arctanh(c*x^2))^2/x^4,x)
```

3.75.5 Fricas [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^4} \, dx = \int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^4} \, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^4, x)
```

3.75.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^4} \, dx = \int \frac{(a + b \operatorname{atanh}(cx^2))^2}{x^4} \, dx$$

input
```
integrate((a+b*atanh(c*x**2))**2/x**4,x)
```

output
```
Integral((a + b*atanh(c*x**2))**2/x**4, x)
```

3.75.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^4} \, dx = \int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^4} \, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*((2*sqrt(c)*arctan(sqrt(c)*x) + sqrt(c)*log((c*x - sqrt(c))/(c*x + sq
rt(c))) + 4/x)*c + 2*arctanh(c*x^2)/x^3)*a*b - 1/12*b^2*(log(-c*x^2 + 1)^2
/x^3 + 3*integrate(-1/3*(3*(c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(2*c*x^2 - 3*(
c*x^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^6 - x^4), x)) - 1/3*a^2/x
^3
```

3.75.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^4}\,dx = \int \frac{(b\,\text{artanh}\,(cx^2) + a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2/x^4, x)
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^4}\,dx = \int \frac{(a + b\,\text{atanh}(c\,x^2))^2}{x^4}\,dx$$

input
```
int((a + b*atanh(c*x^2))^2/x^4,x)
```

output
```
int((a + b*atanh(c*x^2))^2/x^4, x)
```

3.76 $\int \frac{(a+b\,\mathrm{arctanh}(cx^2))^2}{x^6}\,dx$

3.76.1 Optimal result

Integrand size = 16, antiderivative size = 1176

$$\int \frac{(a+b\,\mathrm{arctanh}(cx^2))^2}{x^6}\,dx = \text{Too large to display}$$

output
```
1/5*I*b^2*c^(5/2)*polylog(2,1-2/(1-I*x*c^(1/2)))+1/5*I*b^2*c^(5/2)*polylog
(2,1-2/(1+I*x*c^(1/2)))+2/5*a*b*c^(5/2)*arctan(x*c^(1/2))+1/15*b^2*c*ln(-c
*x^2+1)/x^3-1/5*b^2*c^2*ln(-c*x^2+1)/x-1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*l
n(-c*x^2+1)-1/15*b*c*(2*a-b*ln(-c*x^2+1))/x^3-1/5*b*c^2*(2*a-b*ln(-c*x^2+1
))/x+1/5*b*c^(5/2)*arctanh(x*c^(1/2))*(2*a-b*ln(-c*x^2+1))-1/5*a*b*ln(c*x^
2+1)/x^5-2/15*b^2*c*ln(c*x^2+1)/x^3+1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln(c
*x^2+1)+1/5*b^2*c^(5/2)*arctanh(x*c^(1/2))*ln(c*x^2+1)+1/10*b^2*ln(-c*x^2+
1)*ln(c*x^2+1)/x^5-2/5*b^2*c^(5/2)*arctanh(x*c^(1/2))*ln(2/(1-x*c^(1/2)))-
2/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln(2/(1-I*x*c^(1/2)))+1/5*b^2*c^(5/2)*ar
ctan(x*c^(1/2))*ln((1+I)*(1-x*c^(1/2))/(1-I*x*c^(1/2)))+2/5*b^2*c^(5/2)*ar
ctan(x*c^(1/2))*ln(2/(1+I*x*c^(1/2)))+2/5*b^2*c^(5/2)*arctanh(x*c^(1/2))*l
n(2/(1+x*c^(1/2)))-1/5*b^2*c^(5/2)*arctanh(x*c^(1/2))*ln(-2*(1-x*(-c)^(1/2
))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+x*c^(1/2)))-1/5*b^2*c^(5/2)*arctanh(x*c
^(1/2))*ln(2*(1+x*(-c)^(1/2))*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+x*c^(1/2)))+
1/5*b^2*c^(5/2)*arctan(x*c^(1/2))*ln((1-I)*(1+x*c^(1/2))/(1-I*x*c^(1/2)))-
2/15*a*b*c/x^3+2/5*a*b*c^2/x-1/10*I*b^2*c^(5/2)*polylog(2,1-(1+I)*(1-x*c^(
1/2))/(1-I*x*c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(2,1+(-1+I)*(1+x*c^(1/2))
/(1-I*x*c^(1/2)))+1/5*I*b^2*c^(5/2)*arctan(x*c^(1/2))^2-1/20*(2*a-b*ln(-c*
x^2+1))^2/x^5-4/15*b^2*c^(5/2)*arctan(x*c^(1/2))+4/15*b^2*c^(5/2)*arctanh(
x*c^(1/2))+1/5*b^2*c^(5/2)*arctanh(x*c^(1/2))^2-1/20*b^2*ln(c*x^2+1)^2/...
```

3.76.2 Mathematica [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^6}\,dx = \int \frac{(a+b\text{arctanh}(cx^2))^2}{x^6}\,dx$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^2/x^6,x]
```

output
```
Integrate[(a + b*ArcTanh[c*x^2])^2/x^6, x]
```

3.76.3 Rubi [A] (verified)

Time = 2.22 (sec) , antiderivative size = 1176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6456, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^6}\,dx$$

$$\Big\downarrow \text{6456}$$

$$\int \left(\frac{(2a-b\log(1-cx^2))^2}{4x^6}-\frac{b\log(cx^2+1)(b\log(1-cx^2)-2a)}{2x^6}+\frac{b^2\log^2(cx^2+1)}{4x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}ib^2\arctan\left(\sqrt{c}x\right)^2c^{5/2}+\frac{1}{5}b^2\text{arctanh}\left(\sqrt{c}x\right)^2c^{5/2}-\frac{4}{15}b^2\arctan\left(\sqrt{c}x\right)c^{5/2}+\frac{2}{5}ab\arctan\left(\sqrt{c}x\right)c^{5/2}+\frac{4}{15}b^2\text{arctanh}\left(\sqrt{c}x\right)c^{5/2}-\frac{2}{5}b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{1-\sqrt{c}x}\right)c^{5/2}-\frac{2}{5}b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{1-i\sqrt{c}x}\right)c^{5/2}+\frac{1}{5}b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1+i)\left(1-\sqrt{c}x\right)}{1-i\sqrt{c}x}\right)c^{5/2}+\frac{2}{5}b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{2}{i\sqrt{c}x+1}\right)c^{5/2}+\frac{2}{5}b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2}{\sqrt{c}x+1}\right)c^{5/2}-\frac{1}{5}b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(-\frac{2\sqrt{c}\left(1-\sqrt{-c}x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)c^{5/2}-\frac{1}{5}b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(\frac{2\sqrt{c}\left(\sqrt{-c}x+1\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)c^{5/2}+\frac{1}{5}b^2\arctan\left(\sqrt{c}x\right)\log\left(\frac{(1-i)\left(\sqrt{c}x+1\right)}{1-i\sqrt{c}x}\right)c^{5/2}-\frac{1}{5}b^2\arctan\left(\sqrt{c}x\right)\log\left(1-cx^2\right)c^{5/2}+\frac{1}{5}b\text{arctanh}\left(\sqrt{c}x\right)\left(2a-b\log\left(1-cx^2\right)\right)c^{5/2}+\frac{1}{5}b^2\arctan\left(\sqrt{c}x\right)\log\left(cx^2+1\right)c^{5/2}+\frac{1}{5}b^2\text{arctanh}\left(\sqrt{c}x\right)\log\left(cx^2+1\right)c^{5/2}-\frac{1}{5}b^2\text{PolyLog}\left(2,1-\frac{2}{1-\sqrt{c}x}\right)c^{5/2}+\frac{1}{5}ib^2\text{PolyLog}\left(2,1-\frac{2}{1-i\sqrt{c}x}\right)c^{5/2}-\frac{1}{10}ib^2\text{PolyLog}\left(2,1-\frac{(1+i)\left(1-\sqrt{c}x\right)}{1-i\sqrt{c}x}\right)c^{5/2}+\frac{1}{5}ib^2\text{PolyLog}\left(2,1-\frac{2}{i\sqrt{c}x+1}\right)c^{5/2}-\frac{1}{5}b^2\text{PolyLog}\left(2,1-\frac{2}{\sqrt{c}x+1}\right)c^{5/2}+\frac{1}{10}b^2\text{PolyLog}\left(2,\frac{2\sqrt{c}\left(1-\sqrt{-c}x\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(\sqrt{c}x+1\right)}+1\right)c^{5/2}+\frac{1}{10}b^2\text{PolyLog}\left(2,1-\frac{2\sqrt{c}\left(\sqrt{-c}x+1\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(\sqrt{c}x+1\right)}\right)c^{5/2}-\frac{1}{10}ib^2\text{PolyLog}\left(2,1-\frac{(1-i)\left(\sqrt{c}x+1\right)}{1-i\sqrt{c}x}\right)c^{5/2}-\frac{b^2\log\left(1-cx^2\right)c^2}{5x}-\frac{b\left(2a-b\log\left(1-cx^2\right)\right)c^2}{5x}-\frac{8b^2c^2}{15x}+\frac{2abc^2}{5x}+\frac{b^2\log\left(1-cx^2\right)c}{15x^3}-\frac{b\left(2a-b\log\left(1-cx^2\right)\right)c}{15x^3}-\frac{2b^2\log\left(cx^2+1\right)c}{15x^3}-\frac{2abc}{15x^3}-\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{20x^5}-\frac{b^2\log^2\left(cx^2+1\right)}{20x^5}+\frac{b^2\log\left(1-cx^2\right)\log\left(cx^2+1\right)}{10x^5}-\frac{ab\log\left(cx^2+1\right)}{5x^5}$$

input
```
Int[(a + b*ArcTanh[c*x^2])^2/x^6,x]
```

3.76. $\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{x^6}\,dx$

output
```
(-2*a*b*c)/(15*x^3) + (2*a*b*c^2)/(5*x) - (8*b^2*c^2)/(15*x) + (2*a*b*c^(5
/2)*ArcTan[Sqrt[c]*x])/5 - (4*b^2*c^(5/2)*ArcTan[Sqrt[c]*x])/15 + (I/5)*b^
2*c^(5/2)*ArcTan[Sqrt[c]*x]^2 + (4*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x])/15 + (b
^2*c^(5/2)*ArcTanh[Sqrt[c]*x]^2)/5 - (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log
[2/(1 - Sqrt[c]*x)])/5 - (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 - I*Sqr
t[c]*x)])/5 + (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[((1 + I)*(1 - Sqrt[c]*x))
/(1 - I*Sqrt[c]*x)])/5 + (2*b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[2/(1 + I*Sqr
t[c]*x)])/5 + (2*b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[2/(1 + Sqrt[c]*x)])/5
- (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]*x))/((Sqrt
[-c] - Sqrt[c])*(1 + Sqrt[c]*x))])/5 - (b^2*c^(5/2)*ArcTanh[Sqrt[c]*x]*Log
[(2*Sqrt[c]*(1 + Sqrt[-c]*x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]*x))])/5 +
 (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[((1 - I)*(1 + Sqrt[c]*x))/(1 - I*Sqrt[
c]*x)])/5 + (b^2*c*Log[1 - c*x^2])/(15*x^3) - (b^2*c^2*Log[1 - c*x^2])/(5*
x) - (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[1 - c*x^2])/5 - (b*c*(2*a - b*Log[
1 - c*x^2]))/(15*x^3) - (b*c^2*(2*a - b*Log[1 - c*x^2]))/(5*x) + (b*c^(5/2
)*ArcTanh[Sqrt[c]*x]*(2*a - b*Log[1 - c*x^2]))/5 - (2*a - b*Log[1 - c*x^2]
)^2/(20*x^5) - (a*b*Log[1 + c*x^2])/(5*x^5) - (2*b^2*c*Log[1 + c*x^2])/(15
*x^3) + (b^2*c^(5/2)*ArcTan[Sqrt[c]*x]*Log[1 + c*x^2])/5 + (b^2*c^(5/2)*Ar
cTanh[Sqrt[c]*x]*Log[1 + c*x^2])/5 + (b^2*Log[1 - c*x^2]*Log[1 + c*x^2])/(
10*x^5) - (b^2*Log[1 + c*x^2]^2)/(20*x^5) - (b^2*c^(5/2)*PolyLog[2, 1 -...
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6456
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

3.76.4 Maple [F]

$$\int \frac{(a + b \operatorname{arctanh}(c x^2))^2}{x^6} dx$$

input
```
int((a+b*arctanh(c*x^2))^2/x^6,x)
```

output
```
int((a+b*arctanh(c*x^2))^2/x^6,x)
```

3.76.5 Fricas [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^6} \, dx = \int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^6} \, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)/x^6, x)
```

3.76.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^6} \, dx = \int \frac{(a + b \operatorname{atanh}(cx^2))^2}{x^6} \, dx$$

input
```
integrate((a+b*atanh(c*x**2))**2/x**6,x)
```

output
```
Integral((a + b*atanh(c*x**2))**2/x**6, x)
```

3.76.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{x^6} \, dx = \int \frac{(b \operatorname{artanh}(cx^2) + a)^2}{x^6} \, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="maxima")
```

output
```
1/15*((6*c^(3/2)*arctan(sqrt(c)*x) - 3*c^(3/2)*log((c*x - sqrt(c))/(c*x +
sqrt(c))) - 4/x^3)*c - 6*arctanh(c*x^2)/x^5)*a*b - 1/20*b^2*(log(-c*x^2 +
1)^2/x^5 + 5*integrate(-1/5*(5*(c*x^2 - 1)*log(c*x^2 + 1)^2 + 2*(2*c*x^2 -
 5*(c*x^2 - 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^8 - x^6), x)) - 1/5*a
^2/x^5
```

3.76.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{x^6}\,dx=\int \frac{\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2}{x^6}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/x^6,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2/x^6, x)
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{x^6}\,dx=\int \frac{\left(a+b\,\text{atanh}(c\,x^2)\right)^2}{x^6}\,dx$$

input
```
int((a + b*atanh(c*x^2))^2/x^6,x)
```

output
```
int((a + b*atanh(c*x^2))^2/x^6, x)
```

3.77 $\int x^3(a+b\operatorname{arctanh}(cx^2))^3\,dx$

3.77.1 Optimal result

Integrand size = 16, antiderivative size = 141

$$\int x^3\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^3\,dx=\frac{3b(a+b\operatorname{arctanh}(cx^2))^2}{4c^2}+\frac{3bx^2(a+b\operatorname{arctanh}(cx^2))^2}{4c}-\frac{(a+b\operatorname{arctanh}(cx^2))^3}{4c^2}+\frac{1}{4}x^4\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^3-\frac{3b^2(a+b\operatorname{arctanh}(cx^2))\log\left(\frac{2}{1-cx^2}\right)}{2c^2}-\frac{3b^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{4c^2}$$

output
```
3/4*b*(a+b*arctanh(c*x^2))^2/c^2+3/4*b*x^2*(a+b*arctanh(c*x^2))^2/c-1/4*(a
+b*arctanh(c*x^2))^3/c^2+1/4*x^4*(a+b*arctanh(c*x^2))^3-3/2*b^2*(a+b*arcta
nh(c*x^2))*ln(2/(-c*x^2+1))/c^2-3/4*b^3*polylog(2,1-2/(-c*x^2+1))/c^2
```

3.77.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.31

$$\int x^3\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^3\,dx$$

$$=\frac{6b^2(-1+cx^2)\left(a+b+acx^2\right)\operatorname{arctanh}(cx^2)^2+2b^3(-1+c^2x^4)\operatorname{arctanh}(cx^2)^3+6b\operatorname{arctanh}(cx^2)\left(acx^2(2b+\right.}{}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*x^2])^3,x]
```

output
```
(6*b^2*(-1 + c*x^2)*(a + b + a*c*x^2)*ArcTanh[c*x^2]^2 + 2*b^3*(-1 + c^2*x
^4)*ArcTanh[c*x^2]^3 + 6*b*ArcTanh[c*x^2]*(a*c*x^2*(2*b + a*c*x^2) - 2*b^2
*Log[1 + E^(-2*ArcTanh[c*x^2])]) + a*(6*a*b*c*x^2 + 2*a^2*c^2*x^4 + 3*a*b*
Log[1 - c*x^2] - 3*a*b*Log[1 + c*x^2] + 6*b^2*Log[1 - c^2*x^4]) + 6*b^3*Po
lyLog[2, -E^(-2*ArcTanh[c*x^2])])/(8*c^2)
```

3.77.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6454, 6452, 6542, 6436, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b \text{arctanh}\left(c x^2\right)\right)^3 \, dx$$

$$\Big\downarrow \text{ 6454}$$

$$\frac{1}{2} \int x^2 \left(a+b \text{arctanh}\left(c x^2\right)\right)^3 dx^2$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2} \left(\frac{1}{2} x^4 \left(a+b \text{arctanh}\left(c x^2\right)\right)^3-\frac{3}{2} b c \int \frac{x^4 \left(a+b \text{arctanh}\left(c x^2\right)\right)^2}{1-c^2 x^4}dx^2\right)$$

$$\Big\downarrow \text{ 6542}$$

$$\frac{1}{2} \left(\frac{1}{2} x^4 \left(a+b \text{arctanh}\left(c x^2\right)\right)^3-\frac{3}{2} b c \left(\frac{\int \frac{\left(a+b \text{arctanh}\left(c x^2\right)\right)^2}{1-c^2 x^4}dx^2}{c^2}-\frac{\int \left(a+b \text{arctanh}\left(c x^2\right)\right)^2dx^2}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 6436}$$

$$\frac{1}{2} \left(\frac{1}{2} x^4 \left(a+b \text{arctanh}\left(c x^2\right)\right)^3-\frac{3}{2} b c \left(\frac{\int \frac{\left(a+b \text{arctanh}\left(c x^2\right)\right)^2}{1-c^2 x^4}dx^2}{c^2}-\frac{x^2 \left(a+b \text{arctanh}\left(c x^2\right)\right)^2-2 b c \int \frac{x^2 \left(a+b \text{arctanh}\left(\right.\right.}{1-c^2 x^4}}{c^2}\right.\right.$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^3}{3bc^3}-\frac{x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-2bc\int\frac{x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{1-c^2x^4}}{c^2}\right.\right.$$

$\big\downarrow$ 6546

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^3}{3bc^3}-\frac{x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-2bc\left(\frac{\int\frac{a+b\text{arctanh}\left(cx^2\right)}{1-cx^2}d}{c}\right.}{c^2}\right.\right.$$

$\big\downarrow$ 6470

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^3}{3bc^3}-\frac{x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-2bc\left(\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\text{arct}\right.}{c}\right.}{c}\right.\right.$$

$\big\downarrow$ 2849

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^3}{3bc^3}-\frac{x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-2bc\left(\frac{b\int\frac{\log\left(\frac{2}{1-cx^2}\right)}{1-\frac{2}{1-cx^2}}d\frac{1}{1-cx^2}}{c}\right.}{}\right.\right.$$

$\big\downarrow$ 2752

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^3}{3bc^3}-\frac{x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2-2bc\left(\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\text{arct}\right.}{c}\right.}{}\right.\right.$$

3.77. $\int x^3\left(a+b\text{arctanh}\left(cx^2\right)\right)^3\,dx$

input
```
Int[x^3*(a + b*ArcTanh[c*x^2])^3,x]
```

output
```
((x^4*(a + b*ArcTanh[c*x^2])^3)/2 - (3*b*c*((a + b*ArcTanh[c*x^2])^3/(3*b*
c^3) - (x^2*(a + b*ArcTanh[c*x^2])^2 - 2*b*c*(-1/2*(a + b*ArcTanh[c*x^2])^
2/(b*c^2) + (((a + b*ArcTanh[c*x^2])*Log[2/(1 - c*x^2)])/c + (b*PolyLog[2,
 1 - 2/(1 - c*x^2)])/(2*c))/c))/c^2))/2)/2
```

3.77.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.77.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.35 (sec) , antiderivative size = 798, normalized size of antiderivative = 5.66

method	result	size
risch	Expression too large to display	798

input
```
int(x^3*(a+b*arctanh(c*x^2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*b^3*(c^2*x^4-1)/c^2*ln(c*x^2+1)^3+3/32*b^2*(-b*c^2*ln(-c*x^2+1)*x^4+2
*a*c^2*x^4+2*b*c*x^2+b*ln(-c*x^2+1)-2*a+2*b)/c^2*ln(c*x^2+1)^2+(3/32*b^3*(
c^2*x^4-1)/c^2*ln(-c*x^2+1)^2-3/32*b^2*(2*a*c*x^2+b)^2/c^2/a*ln(-c*x^2+1)-
3/32*b*(-4*a^3*c^2*x^4-8*a^2*b*c*x^2-4*ln(-c*x^2+1)*a^2*b-4*ln(-c*x^2+1)*a
*b^2-ln(-c*x^2+1)*b^3-4*a*b^2)/a/c^2)*ln(c*x^2+1)-3/8*b/c^2*ln(c*x^2+1)*a^
2+3/4*b^2/c^2*ln(c*x^2+1)*a+3/16/c^2*b^2*a*ln(c*x^2-1)+3/8/c^2*b^3*ln(-c*x
^2+1)-3/4/c*a*b^2*x^2*ln(-c*x^2+1)+1/4*a^3*x^4-3/8/c^2*b^3*ln(c*x^2-1)-3/8
/c^2*b^3*ln(c*x^2+1)-1/32*b^3*x^4*ln(-c*x^2+1)^3-3/16*b^3/c^2*ln(-c*x^2+1)
^2+1/32*b^3/c^2*ln(-c*x^2+1)^3-3/16*b^3/c^2+3/16/c*b^3*x^2*ln(-c*x^2+1)^2-
3/8*a^2*b*x^4*ln(-c*x^2+1)+3/8*a^2*b/c^2*ln(c*x^2-1)+3/16*a*b^2*x^4*ln(-c*
x^2+1)^2+9/16/c^2*a*b^2*ln(-c*x^2+1)-3/16/c^2*a*b^2*ln(-c*x^2+1)^2+3/4/c*a
^2*b*x^2+3/4/c*b^2*Sum(-(ln(x-_alpha)*ln(-c*x^2+1)+2*c*(-1/2*ln(x-_alpha)*
(ln((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_
alpha*c-2,index=1))+ln((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)-x+_alpha)/R
ootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)))/c-1/2*(dilog((RootOf(_Z^2*c+2*_Z*_
alpha*c-2,index=1)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1))+dilog
((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)-x+_alpha)/RootOf(_Z^2*c+2*_Z*_alp
ha*c-2,index=2)))/c))*b/c,_alpha=RootOf(_Z^2*c+1))
```

3.77.5 Fricas [F]

$$\int x^3 (a + b\text{arctanh}(cx^2))^3 \, dx = \int \left(b \operatorname{artanh}\left(cx^2\right) + a\right)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arctanh(c*x^2)^3 + 3*a*b^2*x^3*arctanh(c*x^2)^2 + 3*a^2*b
*x^3*arctanh(c*x^2) + a^3*x^3, x)
```

3.77.6 Sympy [F]

$$\int x^3 (a + b\text{arctanh}(cx^2))^3 \, dx = \int x^3 \left(a + b \operatorname{atanh}\left(cx^2\right)\right)^3 \, dx$$

input
```
integrate(x**3*(a+b*atanh(c*x**2))**3,x)
```

output
```
Integral(x**3*(a + b*atanh(c*x**2))**3, x)
```

3.77.7 Maxima [F]

$$\int x^3\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^3\,dx=\int\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arctanh(c*x^2)^2 + 1/4*a^3*x^4 + 3/8*(2*x^4*arctanh(c*x^2) +
 c*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3))*a^2*b + 3/16*(4*
c*(2*x^2/c^2 - log(c*x^2 + 1)/c^3 + log(c*x^2 - 1)/c^3)*arctanh(c*x^2) - (
2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2 - log(c*x^2 - 1)^
2 - 4*log(c*x^2 - 1))/c^2)*a*b^2 - 1/128*(4*x^4*log(-c*x^2 + 1)^3 + 3*c^3*
(x^4/c^3 + log(c^2*x^4 - 1)/c^5) - 6*c*((c*x^4 + 2*x^2)/c^2 + 2*log(c*x^2
- 1)/c^3)*log(-c*x^2 + 1)^2 + 21*c^2*(2*x^2/c^3 - log(c*x^2 + 1)/c^4 + log
(c*x^2 - 1)/c^4) + c*(6*(c^2*x^4 + 6*c*x^2 + 2*log(c*x^2 - 1)^2 + 6*log(c*
x^2 - 1))*log(-c*x^2 + 1)/c^3 - (3*c^2*x^4 + 42*c*x^2 + 4*log(c*x^2 - 1)^3
 + 18*log(c*x^2 - 1)^2 + 42*log(c*x^2 - 1))/c^3) - 1152*c*integrate(1/4*x^
3*log(c*x^2 + 1)/(c^3*x^4 - c), x) - 2*(12*c*x^2*log(c*x^2 + 1)^2 + 2*(c^2
*x^4 - 1)*log(c*x^2 + 1)^3 - 3*(c^2*x^4 - 2*c*x^2 - 2*(c^2*x^4 - 1)*log(c*
x^2 + 1) + 1)*log(-c*x^2 + 1)^2 + 3*(c^2*x^4 + 6*c*x^2 - 2*(c^2*x^4 - 1)*l
og(c*x^2 + 1)^2 - 8*(c*x^2 + 1)*log(c*x^2 + 1))*log(-c*x^2 + 1))/c^2 + 18*
log(4*c^3*x^4 - 4*c)/c^2 - 384*integrate(1/4*x*log(c*x^2 + 1)/(c^3*x^4 - c
), x))*b^3
```

3.77.8 Giac [F]

$$\int x^3\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^3\,dx=\int\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^3*x^3, x)
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int x^3 (a + b\text{arctanh}(cx^2))^3 \, dx = \int x^3 \left(a + b\,\text{atanh}(c\,x^2)\right)^3 dx$$

input
```
int(x^3*(a + b*atanh(c*x^2))^3,x)
```

output
```
int(x^3*(a + b*atanh(c*x^2))^3, x)
```

3.78 $\int x(a+b\text{arctanh}(cx^2))^3\,dx$

3.78.1 Optimal result

Integrand size = 14, antiderivative size = 134

$$\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^3\,dx=\frac{(a+b\text{arctanh}(cx^2))^3}{2c}+\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-\frac{3b(a+b\text{arctanh}(cx^2))^2\log\left(\frac{2}{1-cx^2}\right)}{2c}-\frac{3b^2(a+b\text{arctanh}(cx^2))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{2c}+\frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{1-cx^2}\right)}{4c}$$

output
```
1/2*(a+b*arctanh(c*x^2))^3/c+1/2*x^2*(a+b*arctanh(c*x^2))^3-3/2*b*(a+b*arc
tanh(c*x^2))^2*ln(2/(-c*x^2+1))/c-3/2*b^2*(a+b*arctanh(c*x^2))*polylog(2,1
-2/(-c*x^2+1))/c+3/4*b^3*polylog(3,1-2/(-c*x^2+1))/c
```

3.78.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.59

$$\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^3\,dx=\frac{a^3x^2}{2}+\frac{3}{2}a^2bx^2\text{arctanh}\left(cx^2\right)+\frac{3a^2b\log\left(1-c^2x^4\right)}{4c}+\frac{3ab^2\left(\text{arctanh}(cx^2)\left(-\text{arctanh}(cx^2)+cx^2\text{arctanh}(cx^2)-2\log\left(1+e^{-2\text{arctanh}(cx^2)}\right)\right)+\text{PolyLog}\left(2,-e\right.\right.}{2c}+\frac{b^3\left(\text{arctanh}(cx^2)^2\left(-\text{arctanh}(cx^2)+cx^2\text{arctanh}(cx^2)-3\log\left(1+e^{-2\text{arctanh}(cx^2)}\right)\right)+3\text{arctanh}(cx^2)\,\text{Po}\right.}{2c}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x^2])^3,x]
```

output
```
(a^3*x^2)/2 + (3*a^2*b*x^2*ArcTanh[c*x^2])/2 + (3*a^2*b*Log[1 - c^2*x^4])/
(4*c) + (3*a*b^2*(ArcTanh[c*x^2]*(-ArcTanh[c*x^2] + c*x^2*ArcTanh[c*x^2] -
 2*Log[1 + E^(-2*ArcTanh[c*x^2])]) + PolyLog[2, -E^(-2*ArcTanh[c*x^2])]))/
(2*c) + (b^3*(ArcTanh[c*x^2]^2*(-ArcTanh[c*x^2] + c*x^2*ArcTanh[c*x^2] - 3
*Log[1 + E^(-2*ArcTanh[c*x^2])]) + 3*ArcTanh[c*x^2]*PolyLog[2, -E^(-2*ArcT
anh[c*x^2])] + (3*PolyLog[3, -E^(-2*ArcTanh[c*x^2])])/2))/(2*c)
```

3.78.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6454, 6436, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^3dx$$

$$\Big\downarrow \text{ 6454}$$

$$\frac{1}{2}\int \left(a+b\text{arctanh}\left(cx^2\right)\right)^3dx^2$$

$$\Big\downarrow \text{ 6436}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-3bc\int \frac{x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{1-c^2x^4}dx^2\right)$$

$$\Big\downarrow \text{ 6546}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-3bc\left(\frac{\int \frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{1-cx^2}dx^2}{c}-\frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^3}{3bc^2}\right)\right)$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-3bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{c}-2b\int \frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)\log\left(\frac{2}{1-cx^2}\right)}{1-c^2x^4}dx^2}{c}-\frac{(a+b}{}\right.\right.$$

$$\Big\downarrow \ 6620$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-3bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{c}-2b\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)}{1-c^2x^4}dx^2-\frac{\text{PolyLog}\left(2,1-\frac{}{1}\right.}{}\right.}{c}\right.\right.$$

$$\Big\downarrow \ 7164$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-3bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{c}-2b\left(\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{1-cx^2}\right)}{4c}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\left(a\right.}{2c}\right.}{c}\right.\right.$$

input
```
Int[x*(a + b*ArcTanh[c*x^2])^3,x]
```

output
```
(x^2*(a + b*ArcTanh[c*x^2])^3 - 3*b*c*(-1/3*(a + b*ArcTanh[c*x^2])^3/(b*c^
2) + (((a + b*ArcTanh[c*x^2])^2*Log[2/(1 - c*x^2)])/c - 2*b*(-1/2*((a + b*
ArcTanh[c*x^2])*PolyLog[2, 1 - 2/(1 - c*x^2)])/c + (b*PolyLog[3, 1 - 2/(1
- c*x^2)])/(4*c)))/c))/2
```

3.78.3.1 Defintions of rubi rules used

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.78.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(124) = 248$.

Time = 2.59 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.98

method	result
derivativedivides	$\frac{a^3 c x^2+b^3 \left(\operatorname{arctanh}(c x^2)^3 (c x^2-1)+2 \operatorname{arctanh}(c x^2)^3-3 \operatorname{arctanh}(c x^2)^2 \ln\left(1+\frac{(c x^2+1)^2}{-c^2 x^4+1}\right)-3 \operatorname{arctanh}(c x^2) \operatorname{polylog}(\right.}{}$
default	$\frac{a^3 c x^2+b^3 \left(\operatorname{arctanh}(c x^2)^3 (c x^2-1)+2 \operatorname{arctanh}(c x^2)^3-3 \operatorname{arctanh}(c x^2)^2 \ln\left(1+\frac{(c x^2+1)^2}{-c^2 x^4+1}\right)-3 \operatorname{arctanh}(c x^2) \operatorname{polylog}(\right.}{}$
parts	$\frac{a^3 x^2}{2}+\frac{b^3 \left(\operatorname{arctanh}(c x^2)^3 (c x^2-1)+2 \operatorname{arctanh}(c x^2)^3-3 \operatorname{arctanh}(c x^2)^2 \ln\left(1+\frac{(c x^2+1)^2}{-c^2 x^4+1}\right)-3 \operatorname{arctanh}(c x^2) \operatorname{polylog}\right.}{2c}$

input
```
int(x*(a+b*arctanh(c*x^2))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/c*(a^3*c*x^2+b^3*(arctanh(c*x^2)^3*(c*x^2-1)+2*arctanh(c*x^2)^3-3*arct
anh(c*x^2)^2*ln(1+(c*x^2+1)^2/(-c^2*x^4+1))-3*arctanh(c*x^2)*polylog(2,-(c
*x^2+1)^2/(-c^2*x^4+1))+3/2*polylog(3,-(c*x^2+1)^2/(-c^2*x^4+1)))+3*a*b^2*
(arctanh(c*x^2)^2*(c*x^2-1)+2*arctanh(c*x^2)^2-2*arctanh(c*x^2)*ln(1+(c*x^
2+1)^2/(-c^2*x^4+1))-polylog(2,-(c*x^2+1)^2/(-c^2*x^4+1)))+3*a^2*b*(c*x^2*
arctanh(c*x^2)+1/2*ln(-c^2*x^4+1)))
```

3.78.5 Fricas [F]

$$\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^3\,dx=\int\left(b\operatorname{artanh}\left(cx^2\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arctanh(c*x^2)^3 + 3*a*b^2*x*arctanh(c*x^2)^2 + 3*a^2*b*x*a
rctanh(c*x^2) + a^3*x, x)
```

3.78.6 Sympy [F]

$$\int x\left(a+b\mathrm{arctanh}\left(cx^2\right)\right)^3\,dx=\int x\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)^3\,dx$$

input
```
integrate(x*(a+b*atanh(c*x**2))**3,x)
```

output
```
Integral(x*(a + b*atanh(c*x**2))**3, x)
```

3.78.7 Maxima [F]

$$\int x\left(a+b\mathrm{arctanh}\left(cx^2\right)\right)^3\,dx=\int \left(b\,\mathrm{artanh}\left(cx^2\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*x^2 + 3/4*(2*c*x^2*arctanh(c*x^2) + log(-c^2*x^4 + 1))*a^2*b/c - 1
/16*((b^3*c*x^2 - b^3)*log(-c*x^2 + 1)^3 - 3*(2*a*b^2*c*x^2 + (b^3*c*x^2 +
 b^3)*log(c*x^2 + 1))*log(-c*x^2 + 1)^2)/c - integrate(-1/8*((b^3*c*x^3 -
b^3*x)*log(c*x^2 + 1)^3 + 6*(a*b^2*c*x^3 - a*b^2*x)*log(c*x^2 + 1)^2 - 3*(
4*a*b^2*c*x^3 + (b^3*c*x^3 - b^3*x)*log(c*x^2 + 1)^2 + 2*((2*a*b^2*c + b^3
*c)*x^3 - (2*a*b^2 - b^3)*x)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^2 - 1),
 x)
```

3.78.8 Giac [F]

$$\int x\left(a+b\mathrm{arctanh}\left(cx^2\right)\right)^3\,dx=\int \left(b\,\mathrm{artanh}\left(cx^2\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^3*x, x)
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^3\,dx=\int x\,\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)^3\,dx$$

input
```
int(x*(a + b*atanh(c*x^2))^3,x)
```

output
```
int(x*(a + b*atanh(c*x^2))^3, x)
```

3.79 $\int \frac{\left(a+b\,\mathrm{arctanh}\left(cx^2\right)\right)^3}{x}\,dx$

3.79.1 Optimal result

Integrand size = 16, antiderivative size = 207

$$\begin{aligned}\int \frac{(a+b\,\mathrm{arctanh}(cx^2))^3}{x}\,dx &= (a+b\,\mathrm{arctanh}(cx^2))^3\,\mathrm{arctanh}\left(1-\frac{2}{1-cx^2}\right)\\
&\quad-\frac{3}{4}b(a+b\,\mathrm{arctanh}(cx^2))^2\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\\
&\quad+\frac{3}{4}b(a+b\,\mathrm{arctanh}(cx^2))^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-cx^2}\right)\\
&\quad+\frac{3}{4}b^2(a+b\,\mathrm{arctanh}(cx^2))\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-cx^2}\right)\\
&\quad-\frac{3}{4}b^2(a+b\,\mathrm{arctanh}(cx^2))\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-cx^2}\right)\\
&\quad-\frac{3}{8}b^3\,\mathrm{PolyLog}\left(4,1-\frac{2}{1-cx^2}\right)\\
&\quad+\frac{3}{8}b^3\,\mathrm{PolyLog}\left(4,-1+\frac{2}{1-cx^2}\right)\end{aligned}$$

output
```
-(a+b*arctanh(c*x^2))^3*arctanh(-1+2/(-c*x^2+1))-3/4*b*(a+b*arctanh(c*x^2)
)^2*polylog(2,1-2/(-c*x^2+1))+3/4*b*(a+b*arctanh(c*x^2))^2*polylog(2,-1+2/
(-c*x^2+1))+3/4*b^2*(a+b*arctanh(c*x^2))*polylog(3,1-2/(-c*x^2+1))-3/4*b^2
*(a+b*arctanh(c*x^2))*polylog(3,-1+2/(-c*x^2+1))-3/8*b^3*polylog(4,1-2/(-c
*x^2+1))+3/8*b^3*polylog(4,-1+2/(-c*x^2+1))
```

3.79.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.79

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x}\,dx = a^3\log(x)+\frac{3}{4}a^2b(-\text{PolyLog}(2,-cx^2)+\text{PolyLog}(2,cx^2)) + \frac{3}{2}ab^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\text{arctanh}(cx^2)^3 - \text{arctanh}(cx^2)^2\log\left(1+e^{-2\text{arctanh}(cx^2)}\right) + \text{arctanh}(cx^2)^2\log\left(1-e^{2\text{arctanh}(cx^2)}\right) + \text{arctanh}(cx^2)\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx^2)}\right) + \text{arctanh}(cx^2)\,\text{PolyLog}\left(2,e^{2\text{arctanh}(cx^2)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx^2)}\right) - \frac{1}{2}\text{PolyLog}\left(3,e^{2\text{arctanh}(cx^2)}\right)\right) + \frac{1}{128}b^3\left(\pi^4 - 32\text{arctanh}(cx^2)^4 - 64\text{arctanh}(cx^2)^3\log\left(1+e^{-2\text{arctanh}(cx^2)}\right) + 64\text{arctanh}(cx^2)^3\log\left(1-e^{2\text{arctanh}(cx^2)}\right) + 96\text{arctanh}(cx^2)^2\,\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx^2)}\right) + 96\text{arctanh}(cx^2)^2\,\text{PolyLog}\left(2,e^{2\text{arctanh}(cx^2)}\right) + 96\text{arctanh}(cx^2)\,\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx^2)}\right) - 96\text{arctanh}(cx^2)\,\text{PolyLog}\left(3,e^{2\text{arctanh}(cx^2)}\right) + 48\,\text{PolyLog}\left(4,-e^{-2\text{arctanh}(cx^2)}\right) + 48\,\text{PolyLog}\left(4,e^{2\text{arctanh}(cx^2)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^3/x,x]
```

output
```
a^3*Log[x] + (3*a^2*b*(-PolyLog[2, -(c*x^2)] + PolyLog[2, c*x^2]))/4 + (3*
a*b^2*((I/24)*Pi^3 - (2*ArcTanh[c*x^2]^3)/3 - ArcTanh[c*x^2]^2*Log[1 + E^(
-2*ArcTanh[c*x^2])] + ArcTanh[c*x^2]^2*Log[1 - E^(2*ArcTanh[c*x^2])] + Arc
Tanh[c*x^2]*PolyLog[2, -E^(-2*ArcTanh[c*x^2])] + ArcTanh[c*x^2]*PolyLog[2,
 E^(2*ArcTanh[c*x^2])] + PolyLog[3, -E^(-2*ArcTanh[c*x^2])]/2 - PolyLog[3,
 E^(2*ArcTanh[c*x^2])]/2))/2 + (b^3*(Pi^4 - 32*ArcTanh[c*x^2]^4 - 64*ArcTa
nh[c*x^2]^3*Log[1 + E^(-2*ArcTanh[c*x^2])] + 64*ArcTanh[c*x^2]^3*Log[1 - E
^(2*ArcTanh[c*x^2])] + 96*ArcTanh[c*x^2]^2*PolyLog[2, -E^(-2*ArcTanh[c*x^2
])] + 96*ArcTanh[c*x^2]^2*PolyLog[2, E^(2*ArcTanh[c*x^2])] + 96*ArcTanh[c*
x^2]*PolyLog[3, -E^(-2*ArcTanh[c*x^2])] - 96*ArcTanh[c*x^2]*PolyLog[3, E^(
2*ArcTanh[c*x^2])] + 48*PolyLog[4, -E^(-2*ArcTanh[c*x^2])] + 48*PolyLog[4,
 E^(2*ArcTanh[c*x^2])]))/128
```

3.79.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6450, 6448, 6614, 6620, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x}dx$$

$$\Big\downarrow \text{6450}$$

$$\frac{1}{2}\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^2}dx^2$$

$$\Big\downarrow \text{6448}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)(a+b\text{arctanh}(cx^2))^3-6bc\int \frac{(a+b\text{arctanh}(cx^2))^2\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)}{1-c^2x^4}dx^2\right)$$

$$\Big\downarrow \text{6614}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)(a+b\text{arctanh}(cx^2))^3-6bc\left(\frac{1}{2}\int \frac{(a+b\text{arctanh}(cx^2))^2\log\left(2-\frac{2}{1-cx^2}\right)}{1-c^2x^4}dx^2-\frac{1}{2}\int \right.\right.$$

$$\Big\downarrow \text{6620}$$

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2c}-b\int\right.\right.\right.$$

$\downarrow$ 6624

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2c}-b\left(\right.\right.\right.\right.$$

$\downarrow$ 7164

$$\frac{1}{2}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^2}\right)\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{2c}-b\left(\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x^2])^3/x,x]
```

output
```
(2*(a + b*ArcTanh[c*x^2])^3*ArcTanh[1 - 2/(1 - c*x^2)] - 6*b*c*((((a + b*A
rcTanh[c*x^2])^2*PolyLog[2, 1 - 2/(1 - c*x^2)])/(2*c) - b*(((a + b*ArcTanh
[c*x^2])*PolyLog[3, 1 - 2/(1 - c*x^2)])/(2*c) - (b*PolyLog[4, 1 - 2/(1 - c
*x^2)])/(4*c)))/2 + (-1/2*((a + b*ArcTanh[c*x^2])^2*PolyLog[2, -1 + 2/(1 -
 c*x^2)])/c + b*(((a + b*ArcTanh[c*x^2])*PolyLog[3, -1 + 2/(1 - c*x^2)])/(
2*c) - (b*PolyLog[4, -1 + 2/(1 - c*x^2)])/(4*c)))/2))/2
```

3.79.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.79.4 Maple [F]

$$\int \frac{(a + b \operatorname{arctanh}(c x^2))^3}{x} dx$$

input
```
int((a+b*arctanh(c*x^2))^3/x,x)
```

output
```
int((a+b*arctanh(c*x^2))^3/x,x)
```

3.79.5 Fricas [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^3}{x} \, dx = \int \frac{(b \operatorname{artanh}(cx^2) + a)^3}{x} \, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctan
h(c*x^2) + a^3)/x, x)
```

3.79.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^3}{x} \, dx = \int \frac{(a + b \operatorname{atanh}(cx^2))^3}{x} \, dx$$

input
```
integrate((a+b*atanh(c*x**2))**3/x,x)
```

output
```
Integral((a + b*atanh(c*x**2))**3/x, x)
```

3.79.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^3}{x} \, dx = \int \frac{(b \operatorname{artanh}(cx^2) + a)^3}{x} \, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + integrate(1/8*b^3*(log(c*x^2 + 1) - log(-c*x^2 + 1))^3/x + 3/
4*a*b^2*(log(c*x^2 + 1) - log(-c*x^2 + 1))^2/x + 3/2*a^2*b*(log(c*x^2 + 1)
 - log(-c*x^2 + 1))/x, x)
```

3.79.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^3}{x} \, dx = \int \frac{(b \operatorname{artanh}(cx^2) + a)^3}{x} \, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^3/x, x)
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^2))^3}{x} \, dx = \int \frac{(a + b \operatorname{atanh}(c\,x^2))^3}{x} \, dx$$

input
```
int((a + b*atanh(c*x^2))^3/x,x)
```

output
```
int((a + b*atanh(c*x^2))^3/x, x)
```

3.80 $\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^3}\,dx$

3.80.1 Optimal result

Integrand size = 16, antiderivative size = 125

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^3}\,dx &= \frac{1}{2}c\big(a+b\text{arctanh}(cx^2)\big)^3 - \frac{(a+b\text{arctanh}(cx^2))^3}{2x^2}\\ &\quad+\frac{3}{2}bc\big(a+b\text{arctanh}(cx^2)\big)^2\log\left(2-\frac{2}{1+cx^2}\right)\\ &\quad-\frac{3}{2}b^2c\big(a+b\text{arctanh}(cx^2)\big)\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx^2}\right)\\ &\quad-\frac{3}{4}b^3c\,\text{PolyLog}\left(3,-1+\frac{2}{1+cx^2}\right)\end{aligned}$$

output
```
1/2*c*(a+b*arctanh(c*x^2))^3-1/2*(a+b*arctanh(c*x^2))^3/x^2+3/2*b*c*(a+b*a
rctanh(c*x^2))^2*ln(2-2/(c*x^2+1))-3/2*b^2*c*(a+b*arctanh(c*x^2))*polylog(
2,-1+2/(c*x^2+1))-3/4*b^3*c*polylog(3,-1+2/(c*x^2+1))
```

3.80.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.78

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^3}\,dx$$
$$= \frac{1}{4}\Bigg(-\frac{2a^3}{x^2} - \frac{6a^2b\text{arctanh}(cx^2)}{x^2} + 12a^2bc\log(x) - 3a^2bc\log\left(1-c^2x^4\right) + 6ab^2c\Bigg(\text{arctanh}(cx^2)\left(\left(1-\frac{1}{cx^2}\right)\text{arctanh}(cx^2) + 2\log\left(1-e^{-2\text{arctanh}(cx^2)}\right)\right) - \text{PolyLog}\left(2,e^{-2\text{arctanh}(cx^2)}\right)\Bigg) + 2b^3c\Bigg(\frac{i\pi^3}{8} - \text{arctanh}(cx^2)^3 - \frac{\text{arctanh}(cx^2)^3}{cx^2} + 3\text{arctanh}(cx^2)^2\log\left(1-e^{2\text{arctanh}(cx^2)}\right) + 3\text{arctanh}(cx^2)\text{PolyLog}\left(2,e^{2\text{arctanh}(cx^2)}\right) - \frac{3}{2}\text{PolyLog}\left(3,e^{2\text{arctanh}(cx^2)}\right)\Bigg)\Bigg)$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^3/x^3,x]
```

output
```
((-2*a^3)/x^2 - (6*a^2*b*ArcTanh[c*x^2])/x^2 + 12*a^2*b*c*Log[x] - 3*a^2*b
*c*Log[1 - c^2*x^4] + 6*a*b^2*c*(ArcTanh[c*x^2]*((1 - 1/(c*x^2))*ArcTanh[c
*x^2] + 2*Log[1 - E^(-2*ArcTanh[c*x^2])]) - PolyLog[2, E^(-2*ArcTanh[c*x^2
])]) + 2*b^3*c*((I/8)*Pi^3 - ArcTanh[c*x^2]^3 - ArcTanh[c*x^2]^3/(c*x^2) +
 3*ArcTanh[c*x^2]^2*Log[1 - E^(2*ArcTanh[c*x^2])] + 3*ArcTanh[c*x^2]*PolyL
og[2, E^(2*ArcTanh[c*x^2])] - (3*PolyLog[3, E^(2*ArcTanh[c*x^2])])/2))/4
```

3.80.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6454, 6452, 6550, 6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^3}\,dx$$

$$\Big\downarrow \text{ 6454}$$

$$\frac{1}{2}\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^3}{x^4}dx^2$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}\left(3bc\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{x^2(1-c^2x^4)}dx^2-\frac{\left(a+b\text{arctanh}(cx^2)\right)^3}{x^2}\right)$$

$$\Big\downarrow \text{ 6550}$$

$$\frac{1}{2}\left(3bc\left(\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{x^2(cx^2+1)}dx^2+\frac{\left(a+b\text{arctanh}(cx^2)\right)^3}{3b}\right)-\frac{\left(a+b\text{arctanh}(cx^2)\right)^3}{x^2}\right)$$

$$\Big\downarrow \text{ 6494}$$

$$\frac{1}{2}\left(3bc\left(-2bc\int \frac{\left(a+b\text{arctanh}(cx^2)\right)\log\left(2-\frac{2}{cx^2+1}\right)}{1-c^2x^4}dx^2+\frac{\left(a+b\text{arctanh}(cx^2)\right)^3}{3b}+\log\left(2-\frac{2}{cx^2+1}\right)(a+\text{bar}\right.\right.$$

$$\Big\downarrow \text{ 6618}$$

$$\frac{1}{2}\left(3bc\left(-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx^2+1}-1\right)\left(a+b\text{arctanh}(cx^2)\right)}{2c}-\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,\frac{2}{cx^2+1}-1\right)}{1-c^2x^4}dx^2\right)+\frac{(a+\text{barcta}}{3}\right.\right.$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{1}{2}\left(3bc\left(-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx^2+1}-1\right)\left(a+b\text{arctanh}(cx^2)\right)}{2c}+\frac{b\,\text{PolyLog}\left(3,\frac{2}{cx^2+1}-1\right)}{4c}\right)+\frac{\left(a+b\text{arctanh}(cx^2\right.}{3b}\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x^2])^3/x^3,x]
```

output
```
(-((a + b*ArcTanh[c*x^2])^3/x^2) + 3*b*c*((a + b*ArcTanh[c*x^2])^3/(3*b) +
 (a + b*ArcTanh[c*x^2])^2*Log[2 - 2/(1 + c*x^2)] - 2*b*c*(((a + b*ArcTanh[
c*x^2])*PolyLog[2, -1 + 2/(1 + c*x^2)])/(2*c) + (b*PolyLog[3, -1 + 2/(1 +
c*x^2)])/(4*c))))/2
```

3.80. $\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^3}{x^3}\,dx$

3.80.3.1 Defintions of rubi rules used

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.80. $\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^3}{x^3}\,dx$

3.80.4 Maple [F]

$$\int \frac{(a+b\ \mathrm{arctanh}\left(c\,x^{2}\right))^{3}}{x^{3}}dx$$

input
```
int((a+b*arctanh(c*x^2))^3/x^3,x)
```

output
```
int((a+b*arctanh(c*x^2))^3/x^3,x)
```

3.80.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arctanh}(cx^2))^3}{x^3}\,dx=\int \frac{(b\,\mathrm{artanh}\left(cx^2\right)+a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctan
h(c*x^2) + a^3)/x^3, x)
```

3.80.6 Sympy [F]

$$\int \frac{(a+b\mathrm{arctanh}(cx^2))^3}{x^3}\,dx=\int \frac{(a+b\,\mathrm{atanh}\left(cx^2\right))^3}{x^3}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))**3/x**3,x)
```

output
```
Integral((a + b*atanh(c*x**2))**3/x**3, x)
```

3.80.7 Maxima [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx^2))^3}{x^3}\,dx = \int \frac{(b\operatorname{artanh}(cx^2) + a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="maxima")
```

output
```
-3/4*(c*(log(c^2*x^4 - 1) - log(x^4)) + 2*arctanh(c*x^2)/x^2)*a^2*b - 1/2*
a^3/x^2 - 1/16*((b^3*c*x^2 - b^3)*log(-c*x^2 + 1)^3 + 3*(2*a*b^2 + (b^3*c*
x^2 + b^3)*log(c*x^2 + 1))*log(-c*x^2 + 1)^2)/x^2 - integrate(-1/8*((b^3*c
*x^2 - b^3)*log(c*x^2 + 1)^3 + 6*(a*b^2*c*x^2 - a*b^2)*log(c*x^2 + 1)^2 +
3*(4*a*b^2*c*x^2 - (b^3*c*x^2 - b^3)*log(c*x^2 + 1)^2 + 2*(b^3*c^2*x^4 + 2
*a*b^2 - (2*a*b^2*c - b^3*c)*x^2)*log(c*x^2 + 1))*log(-c*x^2 + 1))/(c*x^5
- x^3), x)
```

3.80.8 Giac [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx^2))^3}{x^3}\,dx = \int \frac{(b\operatorname{artanh}(cx^2) + a)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^3/x^3, x)
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\operatorname{arctanh}(cx^2))^3}{x^3}\,dx = \int \frac{(a + b\operatorname{atanh}(c\,x^2))^3}{x^3}\,dx$$

input
```
int((a + b*atanh(c*x^2))^3/x^3,x)
```

output
```
int((a + b*atanh(c*x^2))^3/x^3, x)
```

3.81 $\int \frac{\left(a+b\,\text{arctanh}\left(cx^2\right)\right)^3}{x^5}\,dx$

3.81.1 Optimal result

Integrand size = 16, antiderivative size = 139

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^5}\,dx = \frac{3}{4}bc^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^2 - \frac{3bc(a+b\text{arctanh}(cx^2))^2}{4x^2} + \frac{1}{4}c^2\left(a+b\text{arctanh}\left(cx^2\right)\right)^3 - \frac{(a+b\text{arctanh}(cx^2))^3}{4x^4} + \frac{3}{2}b^2c^2\left(a+b\text{arctanh}\left(cx^2\right)\right)\log\left(2-\frac{2}{1+cx^2}\right) - \frac{3}{4}b^3c^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx^2}\right)$$

output
```
3/4*b*c^2*(a+b*arctanh(c*x^2))^2-3/4*b*c*(a+b*arctanh(c*x^2))^2/x^2+1/4*c^
2*(a+b*arctanh(c*x^2))^3-1/4*(a+b*arctanh(c*x^2))^3/x^4+3/2*b^2*c^2*(a+b*a
rctanh(c*x^2))*ln(2-2/(c*x^2+1))-3/4*b^3*c^2*polylog(2,-1+2/(c*x^2+1))
```

3.81.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.57

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^5}\,dx$$
$$= \frac{6b^2(-1+cx^2)(a+acx^2+bcx^2)\,\text{arctanh}(cx^2)^2+2b^3(-1+c^2x^4)\,\text{arctanh}(cx^2)^3-6b\text{arctanh}(cx^2)\left(a^2+2\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^3/x^5,x]
```

output
```
(6*b^2*(-1 + c*x^2)*(a + a*c*x^2 + b*c*x^2)*ArcTanh[c*x^2]^2 + 2*b^3*(-1 +
 c^2*x^4)*ArcTanh[c*x^2]^3 - 6*b*ArcTanh[c*x^2]*(a^2 + 2*a*b*c*x^2 - 2*b^2
*c^2*x^4*Log[1 - E^(-2*ArcTanh[c*x^2])]) + a*(-2*a^2 - 6*a*b*c*x^2 - 3*a*b
*c^2*x^4*Log[1 - c*x^2] + 3*a*b*c^2*x^4*Log[1 + c*x^2] + 12*b^2*c^2*x^4*Lo
g[(c*x^2)/Sqrt[1 - c^2*x^4]]) - 6*b^3*c^2*x^4*PolyLog[2, E^(-2*ArcTanh[c*x
^2])])/(8*x^4)
```

3.81.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6454, 6452, 6544, 6452, 6510, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^5}\,dx$$
$$\Big\downarrow \text{6454}$$
$$\frac{1}{2}\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^6}dx^2$$
$$\Big\downarrow \text{6452}$$
$$\frac{1}{2}\left(\frac{3}{2}bc\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^4(1-c^2x^4)}dx^2-\frac{(a+b\text{arctanh}(cx^2))^3}{2x^4}\right)$$
$$\Big\downarrow \text{6544}$$

$$\frac{1}{2}\left(\frac{3}{2}bc\left(c^2\int \frac{(a+b\text{arctanh}(cx^2))^2}{1-c^2x^4}dx^2+\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^4}dx^2\right)-\frac{(a+b\text{arctanh}(cx^2))^3}{2x^4}\right)$$

$\big\downarrow$ 6452

$$\frac{1}{2}\left(\frac{3}{2}bc\left(c^2\int \frac{(a+b\text{arctanh}(cx^2))^2}{1-c^2x^4}dx^2+2bc\int \frac{a+b\text{arctanh}(cx^2)}{x^2(1-c^2x^4)}dx^2-\frac{(a+b\text{arctanh}(cx^2))^2}{x^2}\right)-\frac{(a+b\text{arcta}}{2x}$$

$\big\downarrow$ 6510

$$\frac{1}{2}\left(\frac{3}{2}bc\left(2bc\int \frac{a+b\text{arctanh}(cx^2)}{x^2(1-c^2x^4)}dx^2+\frac{c(a+b\text{arctanh}(cx^2))^3}{3b}-\frac{(a+b\text{arctanh}(cx^2))^2}{x^2}\right)-\frac{(a+b\text{arctanh}(cx^2)}{2x^4}$$

$\big\downarrow$ 6550

$$\frac{1}{2}\left(\frac{3}{2}bc\left(2bc\left(\int \frac{a+b\text{arctanh}(cx^2)}{x^2(cx^2+1)}dx^2+\frac{(a+b\text{arctanh}(cx^2))^2}{2b}\right)+\frac{c(a+b\text{arctanh}(cx^2))^3}{3b}-\frac{(a+b\text{arctanh}(cx}{x^2}$$

$\big\downarrow$ 6494

$$\frac{1}{2}\left(\frac{3}{2}bc\left(2bc\left(-bc\int \frac{\log\left(2-\frac{2}{cx^2+1}\right)}{1-c^2x^4}dx^2+\frac{(a+b\text{arctanh}(cx^2))^2}{2b}+\log\left(2-\frac{2}{cx^2+1}\right)(a+b\text{arctanh}(cx^2))\right)+\right.\right.$$

$\big\downarrow$ 2897

$$\frac{1}{2}\left(\frac{3}{2}bc\left(2bc\left(\frac{(a+b\text{arctanh}(cx^2))^2}{2b}+\log\left(2-\frac{2}{cx^2+1}\right)(a+b\text{arctanh}(cx^2))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx^2+1}-1\right)\right)\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x^2])^3/x^5,x]
```

output
```
(-1/2*(a + b*ArcTanh[c*x^2])^3/x^4 + (3*b*c*(-((a + b*ArcTanh[c*x^2])^2/x^
2) + (c*(a + b*ArcTanh[c*x^2])^3)/(3*b) + 2*b*c*((a + b*ArcTanh[c*x^2])^2/
(2*b) + (a + b*ArcTanh[c*x^2])*Log[2 - 2/(1 + c*x^2)] - (b*PolyLog[2, -1 +
 2/(1 + c*x^2)])/2)))/2)/2
```

3.81. $\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^5}\,dx$

3.81.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.81. $\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^3}{x^5}\,dx$

3.81.4 Maple [F]

$$\int \frac{(a+b\,\text{arctanh}\,(c\,x^2))^3}{x^5}dx$$

input
```
int((a+b*arctanh(c*x^2))^3/x^5,x)
```

output
```
int((a+b*arctanh(c*x^2))^3/x^5,x)
```

3.81.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^5}\,dx = \int \frac{(b\,\text{artanh}\,(cx^2)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctan
h(c*x^2) + a^3)/x^5, x)
```

3.81.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^5}\,dx = \int \frac{(a+b\,\text{atanh}\,(cx^2))^3}{x^5}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))**3/x**5,x)
```

output
```
Integral((a + b*atanh(c*x**2))**3/x**5, x)
```

3.81.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^5}\,dx=\int \frac{(b\,\text{artanh}\,(cx^2)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="maxima")
```

output
```
3/8*((c*log(c*x^2 + 1) - c*log(c*x^2 - 1) - 2/x^2)*c - 2*arctanh(c*x^2)/x^
4)*a^2*b + 3/16*((2*(log(c*x^2 - 1) - 2)*log(c*x^2 + 1) - log(c*x^2 + 1)^2
 - log(c*x^2 - 1)^2 - 4*log(c*x^2 - 1) + 16*log(x))*c^2 + 4*(c*log(c*x^2 +
 1) - c*log(c*x^2 - 1) - 2/x^2)*c*arctanh(c*x^2))*a*b^2 - 1/32*b^3*(((c^2*
x^4 - 1)*log(-c*x^2 + 1)^3 + 3*(2*c*x^2 - (c^2*x^4 - 1)*log(c*x^2 + 1))*lo
g(-c*x^2 + 1)^2)/x^4 + 4*integrate(-((c*x^2 - 1)*log(c*x^2 + 1)^3 + 3*(2*c
^2*x^4 - (c*x^2 - 1)*log(c*x^2 + 1)^2 - (c^3*x^6 - c*x^2)*log(c*x^2 + 1))*
log(-c*x^2 + 1))/(c*x^7 - x^5), x)) - 3/4*a*b^2*arctanh(c*x^2)^2/x^4 - 1/4
*a^3/x^4
```

3.81.8 Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^5}\,dx=\int \frac{(b\,\text{artanh}\,(cx^2)+a)^3}{x^5}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^3/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^3/x^5, x)
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx^2))^3}{x^5}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x^2))^3}{x^5}\,dx$$

input
```
int((a + b*atanh(c*x^2))^3/x^5,x)
```

output
```
int((a + b*atanh(c*x^2))^3/x^5, x)
```

3.82 $\int (dx)^{5/2} \left(a + b\text{arctanh}(cx^2)\right) dx$

3.82.1 Optimal result

Integrand size = 18, antiderivative size = 317

$$\begin{aligned}\int (dx)^{5/2} \left(a+b\text{arctanh}(cx^2)\right) dx &= \frac{8bd(dx)^{3/2}}{21c}+\frac{2bd^{5/2}\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}} \\ &+\frac{\sqrt{2}bd^{5/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}}-\frac{\sqrt{2}bd^{5/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}} \\ &+\frac{2(dx)^{7/2}\left(a+b\text{arctanh}(cx^2)\right)}{7d}-\frac{2bd^{5/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7c^{7/4}} \\ &-\frac{bd^{5/2}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{7\sqrt{2}c^{7/4}}+\frac{bd^{5/2}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{7\sqrt{2}c^{7/4}}\end{aligned}$$

output
```
8/21*b*d*(d*x)^(3/2)/c+2/7*b*d^(5/2)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c
^(7/4)+2/7*(d*x)^(7/2)*(a+b*arctanh(c*x^2))/d-2/7*b*d^(5/2)*arctanh(c^(1/4
)*(d*x)^(1/2)/d^(1/2))/c^(7/4)-1/14*b*d^(5/2)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)
-c^(1/4)*2^(1/2)*(d*x)^(1/2))/c^(7/4)*2^(1/2)+1/14*b*d^(5/2)*ln(d^(1/2)+x*
c^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/c^(7/4)*2^(1/2)-1/7*b*d^(5/2)
*arctan(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(7/4)-1/7*b*d^(5
/2)*arctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(7/4)
```

3.82.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.76

$$\int (dx)^{5/2} (a + b\text{arctanh}(cx^2)) \, dx = \frac{(dx)^{5/2} \left(16bc^{3/4}x^{3/2} + 12ac^{7/4}x^{7/2} + 6\sqrt{2}b\arctan\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right) - 6\sqrt{2}b\arctan\left(1 +\right.\right.}{}$$

input
```
Integrate[(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]),x]
```

output
```
((d*x)^(5/2)*(16*b*c^(3/4)*x^(3/2) + 12*a*c^(7/4)*x^(7/2) + 6*Sqrt[2]*b*Ar
cTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]] - 6*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)
*Sqrt[x]] + 12*b*ArcTan[c^(1/4)*Sqrt[x]] + 12*b*c^(7/4)*x^(7/2)*ArcTanh[c*
x^2] + 6*b*Log[1 - c^(1/4)*Sqrt[x]] - 6*b*Log[1 + c^(1/4)*Sqrt[x]] - 3*Sqr
t[2]*b*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x] + 3*Sqrt[2]*b*Log[1 +
Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/(42*c^(7/4)*x^(5/2))
```

3.82.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.12, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {6464, 843, 851, 27, 829, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{5/2} (a + b\text{arctanh}(cx^2)) \, dx$$

$$\Big\downarrow \text{6464}$$

$$\frac{2(dx)^{7/2} (a + b\text{arctanh}(cx^2))}{7d} - \frac{4bc \int \frac{(dx)^{9/2}}{1-c^2x^4}dx}{7d^2}$$

$$\Big\downarrow \text{843}$$

$$\frac{2(dx)^{7/2} (a + b\text{arctanh}(cx^2))}{7d} - \frac{4bc \left(\frac{d^4 \int \frac{\sqrt{dx}}{1-c^2x^4}dx}{c^2} - \frac{2d^3(dx)^{3/2}}{3c^2}\right)}{7d^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{2(dx)^{7/2}\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{7d}-\frac{4bc\left(\frac{2d^3\int\frac{d^5x}{d^4-c^2d^4x^4}d\sqrt{dx}}{c^2}-\frac{2d^3(dx)^{3/2}}{3c^2}\right)}{7d^2}$$

$\Big\downarrow$ 27

$$\frac{2(dx)^{7/2}\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{7d}-\frac{4bc\left(\frac{2d^7\int\frac{dx}{d^4-c^2d^4x^4}d\sqrt{dx}}{c^2}-\frac{2d^3(dx)^{3/2}}{3c^2}\right)}{7d^2}$$

$\Big\downarrow$ 829

$$\frac{2(dx)^{7/2}\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\int\frac{dx}{d^2-cd^2x^2}d\sqrt{dx}}{2d^2}+\frac{\int\frac{dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d^2}\right)}{c^2}-\frac{2d^3(dx)^{3/2}}{3c^2}\right)}{7d^2}$$

$\Big\downarrow$ 826

$$\frac{2(dx)^{7/2}\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\int\frac{dx}{d^2-cd^2x^2}d\sqrt{dx}}{2d^2}+\frac{\frac{\int\frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}\right)}{c^2}-\frac{2d^3(dx)^{3/2}}{3c^2}\right)}{7d^2}$$

$\Big\downarrow$ 827

$$\frac{2(dx)^{7/2}\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\frac{\int\frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\int\frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int\frac{1}{\sqrt{c}xd+d}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}\right)}{c^2}-\frac{2d^3(dx)^{3/2}}{3c^2}\right)}{7d^2}$$

$\Big\downarrow$ 218

3.82. $\int(dx)^{5/2}\left(a+b\operatorname{arctanh}(cx^2)\right)\,dx$

$$\frac{2(dx)^{7/2}\left(a+b\text{arctanh}(cx^2)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\frac{\int\frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}+\frac{\frac{\int\frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}\right)}{c^2}-\frac{2d^3(dx)^{3/2}}{3c^2}\right)}{7d^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(dx)^{7/2}\left(a+b\text{arctanh}(cx^2)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\frac{\int\frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}\right)}{c^2}-\frac{2d^3(dx)^{3/2}}{3c^2}\right)}{7d^2}$$

$$\Big\downarrow \text{1476}$$

$$\frac{2(dx)^{7/2}\left(a+b\text{arctanh}(cx^2)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\frac{\frac{\int\frac{1}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}+\frac{\int\frac{1}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}\right)}{c^2}-\frac{2d^3(dx}{3c^2}\right)}{7d^2}$$

$$\Big\downarrow \text{1082}$$

3.82. $\int (dx)^{5/2}\left(a+b\text{arctanh}(cx^2)\right)\,dx$

$$\frac{2(dx)^{7/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}\right)}{c^2}-\frac{2d^3(\cdots}{\cdots}\right)}{7d^2}$$

$\downarrow$ 217

$$\frac{2(dx)^{7/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}\right)}{c^2}-\frac{2d^3(dx)^{3/2}}{3c^2}\cdots\right)}{7d^2}$$

$\downarrow$ 1479

3.82. $\int (dx)^{5/2}\left(a+b\text{arctanh}(cx^2)\right)\,dx$

$$\frac{2(dx)^{7/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}}{2d^2}+\text{arc}\cdots\right)}{c^2}\cdots\right)}{7d^2}$$

$\downarrow$ 25

$$\frac{2(dx)^{7/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}}{2d^2}+\frac{\arctan\cdots}{2\cdots}\cdots\right)}{c^2}\cdots\right)}{7d^2}$$

$\downarrow$ 27

3.82. $\int (dx)^{5/2}\left(a+b\text{arctanh}(cx^2)\right)\,dx$

$$\frac{2(dx)^{7/2}\left(a+b\text{arctanh}(cx^2)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}\sqrt{d}}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\cdots}{2d^2}\cdots\right)}{c^2}\cdots\right)}{7d^2}$$

$\downarrow$ 1103

$$\frac{2(dx)^{7/2}\left(a+b\text{arctanh}(cx^2)\right)}{7d}-\frac{4bc\left(\frac{2d^7\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}+\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{c}dx+\sqrt{2}\sqrt[4]{c}\sqrt{d}\sqrt{dx}+d\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\log\left(\cdots\right)}{\cdots}}{2\sqrt{c}}}{2d^2}\cdots\right)}{c^2}\cdots\right)}{7d^2}$$

input
```
Int[(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]),x]
```

3.82. $\int (dx)^{5/2}\left(a+b\text{arctanh}(cx^2)\right)\,dx$

output
```
(2*(d*x)^(7/2)*(a + b*ArcTanh[c*x^2]))/(7*d) - (4*b*c*((-2*d^3*(d*x)^(3/2)
)/(3*c^2) + (2*d^7*((-1/2*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(c^(3/4)*Sqr
t[d]) + ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*c^(3/4)*Sqrt[d]))/(2*d^2)
+ ((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqr
t[d])) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*
Sqrt[d]))/(2*Sqrt[c]) - (-1/2*Log[d + Sqrt[c]*d*x - Sqrt[2]*c^(1/4)*Sqrt[d
]*Sqrt[d*x]]/(Sqrt[2]*c^(1/4)*Sqrt[d]) + Log[d + Sqrt[c]*d*x + Sqrt[2]*c^(
1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*c^(1/4)*Sqrt[d]))/(2*Sqrt[c]))/(2*d^2))
)/c^2))/(7*d^2)
```

3.82.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 829
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[x^m/(r + s*x^
(n/2)), x], x] + Simp[r/(2*a)   Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ[{
a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.82. $\int (dx)^{5/2} \left(a + b\,\text{arctanh}(cx^2)\right) dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.82.4 Maple [A] (verified)

Time = 4.03 (sec) , antiderivative size = 256, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{\frac{2a(dx)^{\frac{7}{2}}}{7}+2b\left(\frac{(dx)^{\frac{7}{2}}\operatorname{arctanh}\left(c\,x^{2}\right)}{7}-\frac{4c\,d^{2}\left(-\frac{(dx)^{\frac{3}{2}}}{3c^{2}}+\frac{d^{2}\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arcta}\cdots\right.}{16c^{3}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\cdots\right.}{7}\cdots\right.}{d}\cdots$
default	$\frac{\frac{2a(dx)^{\frac{7}{2}}}{7}+2b\left(\frac{(dx)^{\frac{7}{2}}\operatorname{arctanh}\left(c\,x^{2}\right)}{7}-\frac{4c\,d^{2}\left(-\frac{(dx)^{\frac{3}{2}}}{3c^{2}}+\frac{d^{2}\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{arcta}\cdots\right.}{16c^{3}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\cdots\right.}{7}\cdots\right.}{d}\cdots$
parts	$\frac{2a(dx)^{\frac{7}{2}}}{7d}+\frac{2b\left(\frac{(dx)^{\frac{7}{2}}\operatorname{arctanh}\left(c\,x^{2}\right)}{7}-\frac{4c\,d^{2}\left(-\frac{(dx)^{\frac{3}{2}}}{3c^{2}}+\frac{d^{2}\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)+2\,\mathrm{a}\cdots\right.}{16c^{3}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\cdots\right.}{7}\cdots\right.}{d}\cdots$

input
```
int((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

3.82. $\int (dx)^{5/2}\,(a+b\operatorname{arctanh}(cx^2))\,dx$

output
```
2/d*(1/7*a*(d*x)^(7/2)+b*(1/7*(d*x)^(7/2)*arctanh(c*x^2)-4/7*c*d^2*(-1/3*(
d*x)^(3/2)/c^2+1/16*d^2/c^3/(d^2/c)^(1/4)*2^(1/2)*(ln((d*x-(d^2/c)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(
d^2/c)^(1/2)))+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1
/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1))-1/8*d^2/c^3/(d^2/c)^(1/4)*(2*arctan((d*x
)^(1/2)/(d^2/c)^(1/4))-ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)
^(1/4))))))
```

3.82.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.37

$$\int (dx)^{5/2} \left(a + b\operatorname{arctanh}(cx^2)\right) \, dx =$$

$$-\frac{3\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}} c\log\left(\sqrt{dx}b^3d^7 + \left(\frac{b^4d^{10}}{c^7}\right)^{\frac{3}{4}} c^5\right) - 3i\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}} c\log\left(\sqrt{dx}b^3d^7 + i\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{3}{4}} c^5\right) + 3i\left(\frac{b^4d^{10}}{c^7}\right)^{\frac{1}{4}} c\,\mathrm{l}}{}$$

input
```
integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
-1/21*(3*(b^4*d^10/c^7)^(1/4)*c*log(sqrt(d*x)*b^3*d^7 + (b^4*d^10/c^7)^(3/
4)*c^5) - 3*I*(b^4*d^10/c^7)^(1/4)*c*log(sqrt(d*x)*b^3*d^7 + I*(b^4*d^10/c
^7)^(3/4)*c^5) + 3*I*(b^4*d^10/c^7)^(1/4)*c*log(sqrt(d*x)*b^3*d^7 - I*(b^4
*d^10/c^7)^(3/4)*c^5) - 3*(b^4*d^10/c^7)^(1/4)*c*log(sqrt(d*x)*b^3*d^7 - (
b^4*d^10/c^7)^(3/4)*c^5) + 3*(-b^4*d^10/c^7)^(1/4)*c*log(sqrt(d*x)*b^3*d^7
 + (-b^4*d^10/c^7)^(3/4)*c^5) - 3*I*(-b^4*d^10/c^7)^(1/4)*c*log(sqrt(d*x)*
b^3*d^7 + I*(-b^4*d^10/c^7)^(3/4)*c^5) + 3*I*(-b^4*d^10/c^7)^(1/4)*c*log(s
qrt(d*x)*b^3*d^7 - I*(-b^4*d^10/c^7)^(3/4)*c^5) - 3*(-b^4*d^10/c^7)^(1/4)*
c*log(sqrt(d*x)*b^3*d^7 - (-b^4*d^10/c^7)^(3/4)*c^5) - (3*b*c*d^2*x^3*log(
-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a*c*d^2*x^3 + 8*b*d^2*x)*sqrt(d*x))/c
```

3.82.6 Sympy [F]

$$\int (dx)^{5/2} \left(a + b\text{arctanh}\left(cx^2\right)\right) \, dx = \int (dx)^{\frac{5}{2}} \left(a + b \operatorname{atanh}\left(cx^2\right)\right) \, dx$$

input
```
integrate((d*x)**(5/2)*(a+b*atanh(c*x**2)),x)
```

output
```
Integral((d*x)**(5/2)*(a + b*atanh(c*x**2)), x)
```

3.82.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.00

$$\int (dx)^{5/2} (a + b\text{arctanh}(cx^2)) \, dx = \frac{12\,(dx)^{\frac{7}{2}}\,a + \left(12\,(dx)^{\frac{7}{2}}\,\operatorname{artanh}\left(cx^2\right) - \frac{3\,d^6\left(\frac{2\,\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\,\sqrt{dx}\sqrt{c}\right)}{2\,\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}} + \frac{2\,\sqrt{2}\arctan\left(-\cdots\right)}{\cdots} \cdots\right)}{\cdots}\right)\cdots}{\cdots}$$

input
```
integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
1/42*(12*(d*x)^(7/2)*a + (12*(d*x)^(7/2)*arctanh(c*x^2) - (3*d^6*(2*sqrt(2
)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(
sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqr
t(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)
*d)*sqrt(c)) - sqrt(2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d)
 + d)/(c^(3/4)*sqrt(d)) + sqrt(2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1
/4)*sqrt(d) + d)/(c^(3/4)*sqrt(d)))/c^2 - 6*d^6*(2*arctan(sqrt(d*x)*sqrt(c
)/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + log((sqrt(d*x)*sqrt(c) - sq
rt(sqrt(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*sqr
t(c)))/c^2 - 16*(d*x)^(3/2)*d^4/c^2)*c/d^2)*b)/d
```

3.82.8 Giac [F]

$$\int (dx)^{5/2} \left(a+b\text{arctanh}\left(cx^2\right)\right) \, dx=\int (dx)^{\frac{5}{2}} \left(b\,\text{artanh}\left(cx^2\right)+a\right) dx$$

input
```
integrate((d*x)^(5/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
integrate((d*x)^(5/2)*(b*arctanh(c*x^2) + a), x)
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2} \left(a+b\text{arctanh}\left(cx^2\right)\right) \, dx=\int (d\,x)^{5/2} \left(a+b\,\text{atanh}\left(c\,x^2\right)\right) \,dx$$

input
```
int((d*x)^(5/2)*(a + b*atanh(c*x^2)),x)
```

output
```
int((d*x)^(5/2)*(a + b*atanh(c*x^2)), x)
```

3.83 $\int (dx)^{3/2} \left(a+b\text{arctanh}(cx^2)\right) \, dx$

3.83.1 Optimal result

Integrand size = 18, antiderivative size = 317

$$\int (dx)^{3/2} \left(a+b\text{arctanh}(cx^2)\right) \, dx = \frac{8bd\sqrt{dx}}{5c} - \frac{2bd^{3/2}\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} + \frac{\sqrt{2}bd^{3/2}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} - \frac{\sqrt{2}bd^{3/2}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} + \frac{2(dx)^{5/2}\left(a+b\text{arctanh}(cx^2)\right)}{5d} - \frac{2bd^{3/2}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5c^{5/4}} + \frac{bd^{3/2}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{5\sqrt{2}c^{5/4}} - \frac{bd^{3/2}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{5\sqrt{2}c^{5/4}}$$

output
```
-2/5*b*d^(3/2)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c^(5/4)+2/5*(d*x)^(5/2)
*(a+b*arctanh(c*x^2))/d-2/5*b*d^(3/2)*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))
/c^(5/4)+1/10*b*d^(3/2)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)
^(1/2))/c^(5/4)*2^(1/2)-1/10*b*d^(3/2)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4
)*2^(1/2)*(d*x)^(1/2))/c^(5/4)*2^(1/2)-1/5*b*d^(3/2)*arctan(-1+c^(1/4)*2^(
1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(5/4)-1/5*b*d^(3/2)*arctan(1+c^(1/4)*2
^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(5/4)+8/5*b*d*(d*x)^(1/2)/c
```

3.83.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.76

$$\int (dx)^{3/2} (a + b\operatorname{arctanh}(cx^2)) \, dx = \frac{(dx)^{3/2} \left(16b\sqrt[4]{c}\sqrt{x} + 4ac^{5/4}x^{5/2} + 2\sqrt{2}b\arctan\left(1 - \sqrt{2}\sqrt[4]{c}\sqrt{x}\right) - 2\sqrt{2}b\arctan\left(1 + \right.\right.}{}$$

input
```
Integrate[(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]),x]
```

output
```
((d*x)^(3/2)*(16*b*c^(1/4)*Sqrt[x] + 4*a*c^(5/4)*x^(5/2) + 2*Sqrt[2]*b*Arc
Tan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]] - 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*
Sqrt[x]] - 4*b*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*c^(5/4)*x^(5/2)*ArcTanh[c*x^2
] + 2*b*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] + Sqrt[2]*
b*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - Sqrt[2]*b*Log[1 + Sqrt[2]
*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/(10*c^(5/4)*x^(3/2))
```

3.83.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.10, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6464, 843, 851, 758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2} (a + b\operatorname{arctanh}(cx^2)) \, dx$$

$$\Big\downarrow \; 6464$$

$$\frac{2(dx)^{5/2} (a + b\operatorname{arctanh}(cx^2))}{5d} - \frac{4bc \int \frac{(dx)^{7/2}}{1-c^2x^4} dx}{5d^2}$$

$$\Big\downarrow \; 843$$

$$\frac{2(dx)^{5/2} (a + b\operatorname{arctanh}(cx^2))}{5d} - \frac{4bc \left(\frac{d^4 \int \frac{1}{\sqrt{dx}(1-c^2x^4)} dx}{c^2} - \frac{2d^3\sqrt{dx}}{c^2}\right)}{5d^2}$$

$$\Big\downarrow \; 851$$

$$\frac{2(dx)^{5/2}\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\int\frac{1}{1-c^2x^4}d\sqrt{dx}}{c^2}-\frac{2d^3\sqrt{dx}}{c^2}\right)}{5d^2}$$

$$\Big\downarrow \text{758}$$

$$\frac{2(dx)^{5/2}\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\int\frac{1}{d^2-cd^2x^2}d\sqrt{dx}+\frac{1}{2}d^2\int\frac{1}{cx^2d^2+d^2}d\sqrt{dx}\right)}{c^2}-\frac{2d^3\sqrt{dx}}{c^2}\right)}{5d^2}$$

$$\Big\downarrow \text{755}$$

$$\frac{2(dx)^{5/2}\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\int\frac{1}{d^2-cd^2x^2}d\sqrt{dx}+\frac{1}{2}d^2\left(\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int\frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}\right)\right)}{c^2}-\frac{2d^3\sqrt{dx}}{c^2}\right)}{5d^2}$$

$$\Big\downarrow \text{756}$$

$$\frac{2(dx)^{5/2}\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\left(\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int\frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\int\frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2d}+\frac{\int\frac{1}{\sqrt{c}xd+d}d\sqrt{dx}}{2d}\right)\right)}{c^2}-\frac{2d^3\sqrt{dx}}{c^2}\right)}{5d^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{2(dx)^{5/2}\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\left(\frac{\int\frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2d}+\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\right)+\frac{1}{2}d^2\left(\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int\frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}\right)\right)}{c^2}-\frac{2d^3\sqrt{dx}}{c^2}\right)}{5d^2}$$

$$\Big\downarrow \text{221}$$

3.83. $\int(dx)^{3/2}\left(a+b\operatorname{arctanh}(cx^2)\right)\,dx$

$$\frac{2(dx)^{5/2}\left(a+b\,\text{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\left(\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int\frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\right)\right)}{c^2}-\frac{2d^3\sqrt{dx}}{c^2}\right)}{5d^2}$$

$\Big\downarrow$ 1476

$$\frac{2(dx)^{5/2}\left(a+b\,\text{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\left(\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\int\frac{1}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}+\frac{\int\frac{1}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\cdots\right)\right)}{c^2}\cdots\right)}{5d^2}$$

$\Big\downarrow$ 1082

$$\frac{2(dx)^{5/2}\left(a+b\,\text{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\left(\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\cdots\right)\right)}{c^2}\cdots\right)}{5d^2}$$

$\Big\downarrow$ 217

3.83. $\int(dx)^{3/2}\left(a+b\,\text{arctanh}(cx^2)\right)\,dx$

$$\frac{2(dx)^{5/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\left(\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\right)\right)}{c^2}\right.}{5d^2}$$

$$\Big\downarrow \; 1479$$

$$\frac{2(dx)^{5/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\left(\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}\right.\right.}{c^2}\right.}{5d^2}$$

$$\Big\downarrow \; 25$$

3.83. $\int (dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)\,dx$

$$\frac{2(dx)^{5/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}\right)+\frac{1}{2}\cdots\right)}{c^2}\cdots\right)}{5d^2}\cdots$$

$\downarrow$ 27

$$\frac{2(dx)^{5/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d}-\frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\left(\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\arctan\left(\cdots\right)}{2\sqrt[4]{c}\cdots}\cdots\right)\cdots\right)}{c^2}\cdots\right)}{5d^2}\cdots$$

$\downarrow$ 1103

3.83. $\int (dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)\,dx$

$$\frac{2(dx)^{5/2}\left(a+b\text{arctanh}(cx^2)\right)}{5d} - \frac{4bc\left(\frac{2d^3\left(\frac{1}{2}d^2\left(\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\right)+\frac{1}{2}d^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\log\left(\sqrt{c}dx+\sqrt{2}\sqrt[4]{c}\sqrt{d}\sqrt{\quad}\right.}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}\right.\right)}{c^2}\right.}{5d^2}$$

input
```
Int[(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(2*(d*x)^(5/2)*(a + b*ArcTanh[c*x^2]))/(5*d) - (4*b*c*((-2*d^3*Sqrt[d*x])/
c^2 + (2*d^3*((d^2*(ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*c^(1/4)*d^(3/2)
) + ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*c^(1/4)*d^(3/2))))/2 + (d^2*((
-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d]
)) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt
[d]))/(2*d) + (-1/2*Log[d + Sqrt[c]*d*x - Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x
]]/(Sqrt[2]*c^(1/4)*Sqrt[d]) + Log[d + Sqrt[c]*d*x + Sqrt[2]*c^(1/4)*Sqrt[
d]*Sqrt[d*x]]/(2*Sqrt[2]*c^(1/4)*Sqrt[d]))/(2*d)))/2))/c^2))/(5*d^2)
```

3.83.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.83. $\int (dx)^{3/2} \left(a + b\text{arctanh}(cx^2)\right) \, dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.83.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{2b\left(\frac{(dx)^{\frac{5}{2}}\operatorname{arctanh}(cx^2)}{5}-\frac{4cd^2\left(-\frac{\sqrt{dx}}{c^2}+\frac{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)\right)}{8c^2}+\frac{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{5\cdots}\cdots\right)}{d}$ with leading term $\frac{2(dx)^{\frac{5}{2}}a}{5}+2b\left(\cdots\right)$
default	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dx)^{\frac{5}{2}}\operatorname{arctanh}(cx^2)}{5}-\frac{4cd^2\left(-\frac{\sqrt{dx}}{c^2}+\frac{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)\right)}{8c^2}+\frac{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right)\cdots\right)}{\cdots}\cdots\right)}{5\cdots}\cdots\right)}{d}$
parts	$\frac{2a(dx)^{\frac{5}{2}}}{5d}+\frac{2b\left(\frac{(dx)^{\frac{5}{2}}\operatorname{arctanh}(cx^2)}{5}-\frac{4cd^2\left(-\frac{\sqrt{dx}}{c^2}+\frac{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)\right)}{8c^2}+\frac{\left(\frac{d^2}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\cdots\right)}{\cdots}\cdots\right)}{5\cdots}\cdots\right)}{d}$

input
```
int((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

3.83. $\int (dx)^{3/2}\left(a+b\operatorname{arctanh}(cx^2)\right)\,dx$

output
```
2/d*(1/5*(d*x)^(5/2)*a+b*(1/5*(d*x)^(5/2)*arctanh(c*x^2)-4/5*c*d^2*(-1/c^2
*(d*x)^(1/2)+1/8/c^2*(d^2/c)^(1/4)*(ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^
(1/2)-(d^2/c)^(1/4)))+2*arctan((d*x)^(1/2)/(d^2/c)^(1/4)))+1/16/c^2*(d^2/c
)^(1/4)*2^(1/2)*(ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/
(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+2*arctan(2^(1/2)/(d
^2/c)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)))
))
```

3.83.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.19

$$\int (dx)^{3/2} \left(a + b\operatorname{arctanh}(cx^2)\right) dx =$$

$$-\frac{\left(\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}} c\log\left(\sqrt{dx}bd + \left(\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}} c\right) + i\left(\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}} c\log\left(\sqrt{dx}bd + i\left(\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}} c\right) - i\left(\frac{b^4d^6}{c^5}\right)^{\frac{1}{4}} c\log\left(\sqrt{dx}bd - \right.}{}$$

input
```
integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
-1/5*((b^4*d^6/c^5)^(1/4)*c*log(sqrt(d*x)*b*d + (b^4*d^6/c^5)^(1/4)*c) + I
*(b^4*d^6/c^5)^(1/4)*c*log(sqrt(d*x)*b*d + I*(b^4*d^6/c^5)^(1/4)*c) - I*(b
^4*d^6/c^5)^(1/4)*c*log(sqrt(d*x)*b*d - I*(b^4*d^6/c^5)^(1/4)*c) - (b^4*d^
6/c^5)^(1/4)*c*log(sqrt(d*x)*b*d - (b^4*d^6/c^5)^(1/4)*c) + (-b^4*d^6/c^5)
^(1/4)*c*log(sqrt(d*x)*b*d + (-b^4*d^6/c^5)^(1/4)*c) + I*(-b^4*d^6/c^5)^(1
/4)*c*log(sqrt(d*x)*b*d + I*(-b^4*d^6/c^5)^(1/4)*c) - I*(-b^4*d^6/c^5)^(1/
4)*c*log(sqrt(d*x)*b*d - I*(-b^4*d^6/c^5)^(1/4)*c) - (-b^4*d^6/c^5)^(1/4)*
c*log(sqrt(d*x)*b*d - (-b^4*d^6/c^5)^(1/4)*c) - (b*c*d*x^2*log(-(c*x^2 + 1
)/(c*x^2 - 1)) + 2*a*c*d*x^2 + 8*b*d)*sqrt(d*x))/c
```

3.83.6 Sympy [F]

$$\int (dx)^{3/2} \left(a + b\text{arctanh}\left(cx^2\right)\right) \, dx = \int (dx)^{\frac{3}{2}} \left(a + b\,\text{atanh}\left(cx^2\right)\right) \, dx$$

input
```
integrate((d*x)**(3/2)*(a+b*atanh(c*x**2)),x)
```

output
```
Integral((d*x)**(3/2)*(a + b*atanh(c*x**2)), x)
```

3.83.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.98

$$\int (dx)^{3/2} \left(a + b\text{arctanh}\left(cx^2\right)\right) \, dx = \frac{4\,(dx)^{\frac{5}{2}}\,a + \Bigg(4\,(dx)^{\frac{5}{2}}\,\text{artanh}\left(cx^2\right) + \frac{\Bigg(\frac{16\,\sqrt{dx}\,d^4}{c^2} - \frac{\frac{2\,\sqrt{2}d^5 \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\,\sqrt{dx}\sqrt{c}\right)}{2\,\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}} + \frac{2\,\sqrt{2}d^5\,\text{arc}}{}}{}}{}}{}$$

input
```
integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
1/10*(4*(d*x)^(5/2)*a + (4*(d*x)^(5/2)*arctanh(c*x^2) + (16*sqrt(d*x)*d^4/
c^2 - (2*sqrt(2)*d^5*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(
d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) + 2*sqrt(2)*d^5*arctan(-1/2
*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/
sqrt(sqrt(c)*d) + sqrt(2)*d^(9/2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1
/4)*sqrt(d) + d)/c^(1/4) - sqrt(2)*d^(9/2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(
d*x)*c^(1/4)*sqrt(d) + d)/c^(1/4))/c^2 - 2*(2*d^5*arctan(sqrt(d*x)*sqrt(c)
/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) - d^5*log((sqrt(d*x)*sqrt(c) - sqrt(sqrt
(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/sqrt(sqrt(c)*d))/c^2)*c/d^2
)*b)/d
```

3.83.8 Giac [F]

$$\int (dx)^{3/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\int (dx)^{\frac{3}{2}}\left(b\,\text{artanh}\left(cx^2\right)+a\right)dx$$

input
```
integrate((d*x)^(3/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
integrate((d*x)^(3/2)*(b*arctanh(c*x^2) + a), x)
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx=\int (d\,x)^{3/2}\left(a+b\,\text{atanh}\left(c\,x^2\right)\right)\,dx$$

input
```
int((d*x)^(3/2)*(a + b*atanh(c*x^2)),x)
```

output
```
int((d*x)^(3/2)*(a + b*atanh(c*x^2)), x)
```

3.84 $\int \sqrt{dx}(a + b\text{arctanh}(cx^2))\,dx$

3.84.1 Optimal result

Integrand size = 18, antiderivative size = 301

$$\int \sqrt{dx}(a+b\text{arctanh}(cx^2))\,dx = \frac{2b\sqrt{d}\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} - \frac{\sqrt{2}b\sqrt{d}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{\sqrt{2}b\sqrt{d}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{2(dx)^{3/2}(a+b\text{arctanh}(cx^2))}{3d} - \frac{2b\sqrt{d}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3c^{3/4}} + \frac{b\sqrt{d}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{3\sqrt{2}c^{3/4}} - \frac{b\sqrt{d}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{3\sqrt{2}c^{3/4}}$$

output
```
2/3*(d*x)^(3/2)*(a+b*arctanh(c*x^2))/d+2/3*b*arctan(c^(1/4)*(d*x)^(1/2)/d^
(1/2))*d^(1/2)/c^(3/4)-2/3*b*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))*d^(1/2)/
c^(3/4)+1/6*b*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1/2))*d^
(1/2)/c^(3/4)*2^(1/2)-1/6*b*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(
d*x)^(1/2))*d^(1/2)/c^(3/4)*2^(1/2)+1/3*b*arctan(-1+c^(1/4)*2^(1/2)*(d*x)^
(1/2)/d^(1/2))*2^(1/2)*d^(1/2)/c^(3/4)+1/3*b*arctan(1+c^(1/4)*2^(1/2)*(d*x
)^(1/2)/d^(1/2))*2^(1/2)*d^(1/2)/c^(3/4)
```

3.84.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.75

$$\int \sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right) \, dx$$
$$= \frac{\sqrt{dx}\left(4ac^{3/4}x^{3/2}-2\sqrt{2}b\arctan\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)+2\sqrt{2}b\arctan\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)+4b\arctan\left(\sqrt[4]{c}\sqrt{x}\right)+4b\right.}{}$$

input
```
Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2]),x]
```

output
```
(Sqrt[d*x]*(4*a*c^(3/4)*x^(3/2) - 2*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*c^(1/4)*S
qrt[x]] + 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] + 4*b*ArcTan[c^(
1/4)*Sqrt[x]] + 4*b*c^(3/4)*x^(3/2)*ArcTanh[c*x^2] + 2*b*Log[1 - c^(1/4)*S
qrt[x]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] + Sqrt[2]*b*Log[1 - Sqrt[2]*c^(1/4)
*Sqrt[x] + Sqrt[c]*x] - Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c
]*x]))/(6*c^(3/4)*Sqrt[x])
```

3.84.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6464, 851, 27, 830, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right) \, dx$$
$$\Big\downarrow \text{6464}$$
$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d}-\frac{4bc\int \frac{(dx)^{5/2}}{1-c^2x^4}dx}{3d^2}$$
$$\Big\downarrow \text{851}$$
$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d}-\frac{8bc\int \frac{d^7x^3}{d^4-c^2d^4x^4}d\sqrt{dx}}{3d^3}$$
$$\Big\downarrow \text{27}$$
$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d}-\frac{8}{3}bcd\int \frac{d^3x^3}{d^4-c^2d^4x^4}d\sqrt{dx}$$

$$\Big\downarrow \text{830}$$

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\int \frac{dx}{d^2-cd^2x^2}d\sqrt{dx}}{2c}-\frac{\int \frac{dx}{cx^2d^2+d^2}d\sqrt{dx}}{2c}\right)$$

$$\Big\downarrow \text{826}$$

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\int \frac{dx}{d^2-cd^2x^2}d\sqrt{dx}}{2c}-\frac{\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{c}xd+d}d\sqrt{dx}}{2\sqrt{c}}}{2c}-\frac{\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)$$

$$\Big\downarrow \text{218}$$

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)$$

$$\Big\downarrow \text{1476}$$

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\int \frac{1}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}+\frac{\int \frac{1}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)$$

3.84. $\int \sqrt{dx}(a+b\text{arctanh}(cx^2))\,dx$

$\Big\downarrow$ 1082

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)$$

$\Big\downarrow$ 217

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)$$

$\Big\downarrow$ 1479

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2c}\right.$$

$\Big\downarrow$ 25

3.84. $\int\sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)\,dx$

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\text{arctan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\text{arctan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\text{arctan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{\cdots}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}\cdots}{2c}\cdots\right.$$

$\downarrow$ 27

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\text{arctan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\text{arctan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\text{arctan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int\cdots}{\cdots}}{2\sqrt{c}}\cdots}{2c}\cdots\right.$$

$\downarrow$ 1103

$$\frac{2(dx)^{3/2}\left(a+b\text{arctanh}(cx^2)\right)}{3d}-\frac{8}{3}bcd\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\text{arctan}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\text{arctan}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\text{arctan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{c}dx+\sqrt{2}\sqrt[4]{c}\sqrt{d}\sqrt{dx}+d\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\cdots}{2\sqrt{c}}\cdots}{2c}\cdots\right.$$

input
```
Int[Sqrt[d*x]*(a + b*ArcTanh[c*x^2]),x]
```

3.84. $\int\sqrt{dx}(a+b\text{arctanh}(cx^2))\,dx$

output
```
(2*(d*x)^(3/2)*(a + b*ArcTanh[c*x^2]))/(3*d) - (8*b*c*d*((-1/2*ArcTan[(c^(
1/4)*Sqrt[d*x])/Sqrt[d]]/(c^(3/4)*Sqrt[d]) + ArcTanh[(c^(1/4)*Sqrt[d*x])/S
qrt[d]]/(2*c^(3/4)*Sqrt[d]))/(2*c) - ((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[
d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sq
rt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d]))/(2*Sqrt[c]) - (-1/2*Log[d + S
qrt[c]*d*x - Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*c^(1/4)*Sqrt[d])
+ Log[d + Sqrt[c]*d*x + Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*c^(1
/4)*Sqrt[d]))/(2*Sqrt[c]))/(2*c)))/3
```

3.84.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.84. $\int \sqrt{dx}(a + b\text{arctanh}(cx^2))\,dx$

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.84.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dx)^{\frac{3}{2}}\operatorname{arctanh}\left(cx^{2}\right)}{3}-\frac{4c\,d^{2}\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{8c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)\cdots\right.}{\cdots}\right.}{3}\cdots\right.}{d}$
default	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dx)^{\frac{3}{2}}\operatorname{arctanh}\left(cx^{2}\right)}{3}-\frac{4c\,d^{2}\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{8c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)\cdots\right.}{\cdots}\right.}{3}\cdots\right.}{d}$
parts	$\frac{2a(dx)^{\frac{3}{2}}}{3d}+\frac{2b\left(\frac{(dx)^{\frac{3}{2}}\operatorname{arctanh}\left(cx^{2}\right)}{3}-\frac{4c\,d^{2}\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{8c^{2}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\cdots}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\cdots}}\right)\cdots\right.}{\cdots}\right.}{3}\cdots\right.}{d}$

input
```
int((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x,method=_RETURNVERBOSE)
```

3.84. $\int\sqrt{dx}(a+b\operatorname{arctanh}(cx^2))\,dx$

output
```
2/d*(1/3*(d*x)^(3/2)*a+b*(1/3*(d*x)^(3/2)*arctanh(c*x^2)-4/3*c*d^2*(-1/8/c
^2/(d^2/c)^(1/4)*(2*arctan((d*x)^(1/2)/(d^2/c)^(1/4))-ln(((d*x)^(1/2)+(d^2
/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4))))-1/16/c^2/(d^2/c)^(1/4)*2^(1/2)*(l
n((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/4)
*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^
(1/2)+1)+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)))))
```

3.84.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \sqrt{dx}(a + b\text{arctanh}(cx^2)) \, dx = & \frac{1}{3}\left(bx\log\left(-\frac{cx^2+1}{cx^2-1}\right) + 2\,ax\right)\sqrt{dx} \\ & - \frac{1}{3}\left(\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}b^3d + \left(\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right) \\ & + \frac{1}{3}i\left(\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}b^3d + i\left(\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right) \\ & - \frac{1}{3}i\left(\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}b^3d - i\left(\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right) \\ & + \frac{1}{3}\left(\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}b^3d - \left(\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right) \\ & + \frac{1}{3}\left(-\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}b^3d + \left(-\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right) \\ & - \frac{1}{3}i\left(-\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}b^3d + i\left(-\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right) \\ & + \frac{1}{3}i\left(-\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}b^3d - i\left(-\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right) \\ & - \frac{1}{3}\left(-\frac{b^4d^2}{c^3}\right)^{\frac{1}{4}}\log\left(\sqrt{dx}b^3d - \left(-\frac{b^4d^2}{c^3}\right)^{\frac{3}{4}}c^2\right)\end{aligned}$$

input
```
integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
1/3*(b*x*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a*x)*sqrt(d*x) - 1/3*(b^4*d^2/c
^3)^(1/4)*log(sqrt(d*x)*b^3*d + (b^4*d^2/c^3)^(3/4)*c^2) + 1/3*I*(b^4*d^2/
c^3)^(1/4)*log(sqrt(d*x)*b^3*d + I*(b^4*d^2/c^3)^(3/4)*c^2) - 1/3*I*(b^4*d
^2/c^3)^(1/4)*log(sqrt(d*x)*b^3*d - I*(b^4*d^2/c^3)^(3/4)*c^2) + 1/3*(b^4*
d^2/c^3)^(1/4)*log(sqrt(d*x)*b^3*d - (b^4*d^2/c^3)^(3/4)*c^2) + 1/3*(-b^4*
d^2/c^3)^(1/4)*log(sqrt(d*x)*b^3*d + (-b^4*d^2/c^3)^(3/4)*c^2) - 1/3*I*(-b
^4*d^2/c^3)^(1/4)*log(sqrt(d*x)*b^3*d + I*(-b^4*d^2/c^3)^(3/4)*c^2) + 1/3*
I*(-b^4*d^2/c^3)^(1/4)*log(sqrt(d*x)*b^3*d - I*(-b^4*d^2/c^3)^(3/4)*c^2) -
 1/3*(-b^4*d^2/c^3)^(1/4)*log(sqrt(d*x)*b^3*d - (-b^4*d^2/c^3)^(3/4)*c^2)
```

3.84.6 Sympy [F]

$$\int \sqrt{dx}(a + b\text{arctanh}(cx^2)) \, dx = \int \sqrt{dx}\left(a + b\,\text{atanh}\left(cx^2\right)\right) \, dx$$

input
```
integrate((d*x)**(1/2)*(a+b*atanh(c*x**2)),x)
```

output
```
Integral(sqrt(d*x)*(a + b*atanh(c*x**2)), x)
```

3.84.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.00

$$\int \sqrt{dx}(a+b\operatorname{arctanh}(cx^2))\,dx$$

$$= \frac{4\,(dx)^{\frac{3}{2}}\,a + \left(4\,(dx)^{\frac{3}{2}}\operatorname{artanh}\left(cx^2\right) + \frac{\left(\frac{d^4\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}} - \frac{\sqrt{2}\log \cdots}{\cdots}\right)}{c}\cdots\right)}{\cdots}\cdots\right)}{6\,d}$$

input
```
integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
1/6*(4*(d*x)^(3/2)*a + (4*(d*x)^(3/2)*arctanh(c*x^2) + (d^4*(2*sqrt(2)*arc
tan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(
c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*
c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*s
qrt(c)) - sqrt(2)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)
/(c^(3/4)*sqrt(d)) + sqrt(2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*s
qrt(d) + d)/(c^(3/4)*sqrt(d)))/c + 2*d^4*(2*arctan(sqrt(d*x)*sqrt(c)/sqrt(
sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + log((sqrt(d*x)*sqrt(c) - sqrt(sqrt
(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*sqrt(c)))/
c)*c/d^2)*b)/d
```

3.84.8 Giac [F]

$$\int \sqrt{dx}(a + b\text{arctanh}(cx^2)) \, dx = \int \sqrt{dx}(b \,\text{artanh}\,(cx^2) + a) \, dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*(b*arctanh(c*x^2) + a), x)
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a + b\text{arctanh}(cx^2)) \, dx = \int \sqrt{d\,x}\,\left(a + b\,\text{atanh}(c\,x^2)\right) \, dx$$

input
```
int((d*x)^(1/2)*(a + b*atanh(c*x^2)),x)
```

output
```
int((d*x)^(1/2)*(a + b*atanh(c*x^2)), x)
```

3.85 $\int \frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx$

3.85.1 Optimal result

Integrand size = 18, antiderivative size = 285

$$\int \frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx = -\frac{2b\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} - \frac{\sqrt{2}b\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{\sqrt{2}b\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} + \frac{2\sqrt{dx}(a+b\text{arctanh}(cx^2))}{d} - \frac{2b\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt[4]{c}\sqrt{d}} - \frac{b\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}} + \frac{b\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}$$

output
```
-2*b*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/c^(1/4)/d^(1/2)-2*b*arctanh(c^(1/
4)*(d*x)^(1/2)/d^(1/2))/c^(1/4)/d^(1/2)-1/2*b*ln(d^(1/2)+x*c^(1/2)*d^(1/2)
-c^(1/4)*2^(1/2)*(d*x)^(1/2))/c^(1/4)*2^(1/2)/d^(1/2)+1/2*b*ln(d^(1/2)+x*c
^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/c^(1/4)*2^(1/2)/d^(1/2)+b*arct
an(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(1/4)/d^(1/2)+b*arcta
n(1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/c^(1/4)/d^(1/2)+2*(a+b*ar
ctanh(c*x^2))*(d*x)^(1/2)/d
```

3.85.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.80

$$\int \frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx$$
$$= \sqrt{x}\Big(4a\sqrt[4]{c}\sqrt{x} - 2\sqrt{2}b\arctan\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right) + 2\sqrt{2}b\arctan\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}\right) - 4b\arctan\left(\sqrt[4]{c}\sqrt{x}\right) + 4b\sqrt[4]{}$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/Sqrt[d*x],x]
```

output
```
(Sqrt[x]*(4*a*c^(1/4)*Sqrt[x] - 2*Sqrt[2]*b*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqr
t[x]] + 2*Sqrt[2]*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] - 4*b*ArcTan[c^(1/
4)*Sqrt[x]] + 4*b*c^(1/4)*Sqrt[x]*ArcTanh[c*x^2] + 2*b*Log[1 - c^(1/4)*Sqr
t[x]] - 2*b*Log[1 + c^(1/4)*Sqrt[x]] - Sqrt[2]*b*Log[1 - Sqrt[2]*c^(1/4)*S
qrt[x] + Sqrt[c]*x] + Sqrt[2]*b*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqrt[c]*
x]))/(2*c^(1/4)*Sqrt[d*x])
```

3.85.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6464, 851, 27, 830, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx$$

$$\Big\downarrow \text{6464}$$

$$\frac{2\sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{d} - \frac{4bc\int \frac{(dx)^{3/2}}{1-c^2x^4}dx}{d^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{2\sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{d} - \frac{8bc\int \frac{d^6x^2}{d^4-c^2d^4x^4}d\sqrt{dx}}{d^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{d} - 8bcd\int \frac{d^2x^2}{d^4-c^2d^4x^4}d\sqrt{dx}$$

$\Big\downarrow$ 830

$$\frac{2\sqrt{dx}\left(a+b\,\text{arctanh}\left(cx^2\right)\right)}{d}-8bcd\left(\frac{\int \frac{1}{d^2-cd^2x^2}d\sqrt{dx}}{2c}-\frac{\int \frac{1}{cx^2d^2+d^2}d\sqrt{dx}}{2c}\right)$$

$\Big\downarrow$ 755

$$\frac{2\sqrt{dx}\left(a+b\,\text{arctanh}\left(cx^2\right)\right)}{d}-8bcd\left(\frac{\int \frac{1}{d^2-cd^2x^2}d\sqrt{dx}}{2c}-\frac{\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}}{2c}\right)$$

$\Big\downarrow$ 756

$$\frac{2\sqrt{dx}\left(a+b\,\text{arctanh}\left(cx^2\right)\right)}{d}-8bcd\left(\frac{\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2d}+\frac{\int \frac{1}{\sqrt{c}xd+d}d\sqrt{dx}}{2d}}{2c}-\frac{\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}}{2c}\right)$$

$\Big\downarrow$ 218

$$\frac{2\sqrt{dx}\left(a+b\,\text{arctanh}\left(cx^2\right)\right)}{d}-8bcd\left(\frac{\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2d}+\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}}{2c}\right)$$

$\Big\downarrow$ 221

$$\frac{2\sqrt{dx}\left(a+b\,\text{arctanh}\left(cx^2\right)\right)}{d}-8bcd\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}}{2c}\right)$$

$\Big\downarrow$ 1476

3.85. $\int \frac{a+b\,\text{arctanh}\left(cx^2\right)}{\sqrt{dx}}\,dx$

$$\frac{2\sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{d}-8bcd\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\int\frac{1}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}+\frac{\int\frac{1}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}}{2d}}{2c}\right)$$

$\downarrow$ 1082

$$\frac{2\sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{d}-8bcd\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}}{2c}\right)$$

$\downarrow$ 217

$$\frac{2\sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{d}-8bcd\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}}{2c}\right)$$

$\downarrow$ 1479

$$\frac{2\sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)}{d}-8bcd\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\arctan(\cdots}{\cdots}}{2c}\cdots$$

$\downarrow$ 25

3.85. $\int\frac{a+b\text{arctanh}\left(cx^2\right)}{\sqrt{dx}}dx$

$$\frac{2\sqrt{dx}(a+b\text{arctanh}(cx^2))}{d}-$$
$$8bcd\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{\ldots}}{\ldots}\right)}{\sqrt{2}\sqrt[4]{\ldots}}}{\ldots}}{2c}\right.$$

$\downarrow$ 27

$$\frac{2\sqrt{dx}(a+b\text{arctanh}(cx^2))}{d}-$$
$$8bcd\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\ldots}{2d}}{2c}\right.$$

$\downarrow$ 1103

$$\frac{2\sqrt{dx}(a+b\text{arctanh}(cx^2))}{d}-$$
$$8bcd\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{c}dx+\sqrt{2}\sqrt[4]{c}\sqrt{d}\sqrt{dx}+d\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\ldots}{2d}}{2c}\right.$$

input
```
Int[(a + b*ArcTanh[c*x^2])/Sqrt[d*x],x]
```

output
```
(2*Sqrt[d*x]*(a + b*ArcTanh[c*x^2]))/d - 8*b*c*d*((ArcTan[(c^(1/4)*Sqrt[d*
x])/Sqrt[d]]/(2*c^(1/4)*d^(3/2)) + ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2
*c^(1/4)*d^(3/2)))/(2*c) - ((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt
[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/S
qrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d]))/(2*d) + (-1/2*Log[d + Sqrt[c]*d*x - Sqr
t[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*c^(1/4)*Sqrt[d]) + Log[d + Sqrt[c
]*d*x + Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*c^(1/4)*Sqrt[d]))/(2
*d))/(2*c))
```

3.85. $\int\frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx$

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.85. $\int \frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx$

3.85.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{2\sqrt{dx}\,a+2b\left(\sqrt{dx}\,\operatorname{arctanh}\left(c\,x^{2}\right)-4c\,d^{2}\left(-\frac{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right)\right)}{16c\,d^{2}}\cdots\right)\right)}{d}$
default	$\frac{2\sqrt{dx}\,a+2b\left(\sqrt{dx}\,\operatorname{arctanh}\left(c\,x^{2}\right)-4c\,d^{2}\left(-\frac{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right)\right)}{16c\,d^{2}}\cdots\right)\right)}{d}$
parts	$\frac{2a\sqrt{dx}}{d}+\frac{2b\left(\sqrt{dx}\,\operatorname{arctanh}\left(c\,x^{2}\right)-4c\,d^{2}\left(-\frac{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right)\right)}{16c\,d^{2}}\cdots\right)\right)}{d}$

input
```
int((a+b*arctanh(c*x^2))/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*((d*x)^(1/2)*a+b*((d*x)^(1/2)*arctanh(c*x^2)-4*c*d^2*(-1/16/c*(d^2/c)^
(1/4)/d^2*2^(1/2)*(ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)
)/(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+2*arctan(2^(1/2)/
(d^2/c)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)
)+1/8/c*(d^2/c)^(1/4)/d^2*(ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^
2/c)^(1/4)))+2*arctan((d*x)^(1/2)/(d^2/c)^(1/4))))))
```

3.85.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.24

$$\int \frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx = -\frac{\left(\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\log\left(\sqrt{dx}b+\left(\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\right)+i\left(\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\log\left(\sqrt{dx}b+i\left(\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\right)-i\left(\frac{b^4}{cd^2}\right)^{\frac{1}{4}} d\log\left(\sqrt{dx}b-i\left(\right.\right.}{}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
-((b^4/(c*d^2))^(1/4)*d*log(sqrt(d*x)*b + (b^4/(c*d^2))^(1/4)*d) + I*(b^4/
(c*d^2))^(1/4)*d*log(sqrt(d*x)*b + I*(b^4/(c*d^2))^(1/4)*d) - I*(b^4/(c*d^
2))^(1/4)*d*log(sqrt(d*x)*b - I*(b^4/(c*d^2))^(1/4)*d) - (b^4/(c*d^2))^(1/
4)*d*log(sqrt(d*x)*b - (b^4/(c*d^2))^(1/4)*d) - (-b^4/(c*d^2))^(1/4)*d*log
(sqrt(d*x)*b + (-b^4/(c*d^2))^(1/4)*d) - I*(-b^4/(c*d^2))^(1/4)*d*log(sqrt
(d*x)*b + I*(-b^4/(c*d^2))^(1/4)*d) + I*(-b^4/(c*d^2))^(1/4)*d*log(sqrt(d*
x)*b - I*(-b^4/(c*d^2))^(1/4)*d) + (-b^4/(c*d^2))^(1/4)*d*log(sqrt(d*x)*b
- (-b^4/(c*d^2))^(1/4)*d) - sqrt(d*x)*(b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2
*a))/d
```

3.85.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx = \int \frac{a+b\,\text{atanh}\,(cx^2)}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))/(d*x)**(1/2),x)
```

output
```
Integral((a + b*atanh(c*x**2))/sqrt(d*x), x)
```

3.85.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.04

$$\int \frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx$$
$$= \frac{\left(4\sqrt{dx}\,\text{artanh}\left(cx^2\right) + \frac{c\left(\frac{\frac{2\sqrt{2}d^3\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}}+\frac{2\sqrt{2}d^3\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}}+\frac{\sqrt{2}d^{\frac{5}{2}}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{dx}\cdots\right.}{c^{\frac{1}{4}}}\cdots}{c}\cdots\right)}{d^2}\cdots\right)}{2\,d}\cdots$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*((4*sqrt(d*x)*arctanh(c*x^2) + c*((2*sqrt(2)*d^3*arctan(1/2*sqrt(2)*(s
qrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c
)*d) + 2*sqrt(2)*d^3*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt
(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) + sqrt(2)*d^(5/2)*log(sqrt
(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/c^(1/4) - sqrt(2)*d^(5/2)
*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/c^(1/4))/c - 2*(
2*d^3*arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) - d^3*log(
(sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)
))/sqrt(sqrt(c)*d))/c)/d^2)*b + 4*sqrt(d*x)*a)/d
```

3.85.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(198) = 396.

Time = 0.29 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.73

$$\int \frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx$$
$$= \frac{\left(cd^2\left(\frac{2\sqrt{2}(c^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}+\frac{2\sqrt{2}(c^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2}-\frac{2\sqrt{2}(-c^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}}{\cdots}\right.}{c^2d^{\cdots}}\right.\right.}{\cdots}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*((c*d^2*(2*sqrt(2)*(c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)
^(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^2*d^2) + 2*sqrt(2)*(c^3*d^2)^(1/4)
*arctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/
(c^2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c
)^(1/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c^2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(1
/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1
/4))/(c^2*d^2) + sqrt(2)*(c^3*d^2)^(1/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/
c)^(1/4) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(c^3*d^2)^(1/4)*log(d*x - sqrt
(2)*sqrt(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(-c^3*d^2)^
(1/4)*log(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c^2*d^2)
 + sqrt(2)*(-c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + s
qrt(-d^2/c))/(c^2*d^2)) + 2*sqrt(d*x)*log(-(c*x^2 + 1)/(c*x^2 - 1)))*b + 4
*sqrt(d*x)*a)/d
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx^2)}{\sqrt{dx}}\,dx = \int \frac{a+b\,\mathrm{atanh}(c\,x^2)}{\sqrt{d\,x}}\,dx$$

input
```
int((a + b*atanh(c*x^2))/(d*x)^(1/2),x)
```

output
```
int((a + b*atanh(c*x^2))/(d*x)^(1/2), x)
```

3.86 $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx$

3.86.1 Optimal result

Integrand size = 18, antiderivative size = 285

$$\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx=-\frac{2b\sqrt[4]{c}\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{\sqrt{2}b\sqrt[4]{c}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{2}b\sqrt[4]{c}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}+\frac{2b\sqrt[4]{c}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{b\sqrt[4]{c}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt{2}d^{3/2}}-\frac{b\sqrt[4]{c}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt{2}d^{3/2}}$$

output
```
-2*b*c^(1/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+2*b*c^(1/4)*arcta
nh(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(3/2)+1/2*b*c^(1/4)*ln(d^(1/2)+x*c^(1/2)
*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(3/2)*2^(1/2)-1/2*b*c^(1/4)*ln(d^(
1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(3/2)*2^(1/2)+b*c^(1
/4)*arctan(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)+b*c^(1/
4)*arctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(3/2)-2*(a+b*ar
ctanh(c*x^2))/d/(d*x)^(1/2)
```

3.86.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.94

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{3/2}}\,dx =$$

$$-\frac{x\left(4a+2\sqrt{2}b\sqrt[4]{c}\sqrt{x}\arctan\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-2\sqrt{2}b\sqrt[4]{c}\sqrt{x}\arctan\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)+4b\sqrt[4]{c}\sqrt{x}\arctan\left(\sqrt[4]{c}\sqrt{x}\right.\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(3/2),x]
```

output
```
-1/2*(x*(4*a + 2*Sqrt[2]*b*c^(1/4)*Sqrt[x]*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt
[x]] - 2*Sqrt[2]*b*c^(1/4)*Sqrt[x]*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] + 4
*b*c^(1/4)*Sqrt[x]*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*ArcTanh[c*x^2] + 2*b*c^(1
/4)*Sqrt[x]*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*c^(1/4)*Sqrt[x]*Log[1 + c^(1/4)
*Sqrt[x]] - Sqrt[2]*b*c^(1/4)*Sqrt[x]*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sq
rt[c]*x] + Sqrt[2]*b*c^(1/4)*Sqrt[x]*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqr
t[c]*x]))/(d*x)^(3/2)
```

3.86.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.14, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6464, 851, 27, 829, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6464}$$

$$\frac{4bc\int \frac{\sqrt{dx}}{1-c^2x^4}dx}{d^2}-\frac{2\left(a+b\operatorname{arctanh}(cx^2)\right)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 851}$$

$$\frac{8bc\int \frac{d^5x}{d^4-c^2d^4x^4}d\sqrt{dx}}{d^3}-\frac{2\left(a+b\operatorname{arctanh}(cx^2)\right)}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 27}$$

$$8bcd\int \frac{dx}{d^4-c^2d^4x^4}d\sqrt{dx}-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$\downarrow$ 829

$$8bcd\left(\frac{\int \frac{dx}{d^2-cd^2x^2}d\sqrt{dx}}{2d^2}+\frac{\int \frac{dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d^2}\right)-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$\downarrow$ 826

$$8bcd\left(\frac{\int \frac{dx}{d^2-cd^2x^2}d\sqrt{dx}}{2d^2}+\frac{\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}\right)-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$\downarrow$ 827

$$8bcd\left(\frac{\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{c}xd+d}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}\right)-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$\downarrow$ 218

$$8bcd\left(\frac{\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}+\frac{\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}\right)-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$\downarrow$ 221

$$8bcd\left(\frac{\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}\right)-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$\downarrow$ 1476

$$8bcd\left(\frac{\frac{\frac{\int \frac{1}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}+\frac{\int \frac{1}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}\right)-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

3.86. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx$

$\downarrow$ 1082

$$8bcd\left(\frac{\frac{\frac{\int \frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int \frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}\right)-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$\downarrow$ 217

$$8bcd\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}\right)-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$\downarrow$ 1479

$$8bcd\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}}{2d^2}+ \text{arc}\ldots\right)-\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$\downarrow$ 25

3.86. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx$

$$8bcd\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}}{2d^2}+\frac{\text{arctanh}\ldots}{2c\ldots}\right.$$

$$\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$$\Big\downarrow \; 27$$

$$8bcd\left(\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int\frac{\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}\sqrt{d}}}{2\sqrt{c}}}{2d^2}+\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}\ldots}{2d\ldots}\right.$$

$$\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

$$\Big\downarrow \; 1103$$

$$8bcd\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2d^2}+\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{c}dx+\sqrt{2}\sqrt[4]{c}\sqrt{d}\sqrt{dx}+d\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\log\ldots}{\ldots}}{2\sqrt{c}\ldots}}{2d^2}\right.$$

$$\frac{2(a+b\text{arctanh}(cx^2))}{d\sqrt{dx}}$$

input
```
Int[(a + b*ArcTanh[c*x^2])/(d*x)^(3/2),x]
```

3.86. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx$

output
```
(-2*(a + b*ArcTanh[c*x^2]))/(d*Sqrt[d*x]) + 8*b*c*d*((-1/2*ArcTan[(c^(1/4)
*Sqrt[d*x])/Sqrt[d]]/(c^(3/4)*Sqrt[d]) + ArcTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[
d]]/(2*c^(3/4)*Sqrt[d]))/(2*d^2) + ((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*
x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*c^(1/4)*Sqrt
[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d]))/(2*Sqrt[c]) - (-1/2*Log[d + Sqr
t[c]*d*x - Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*c^(1/4)*Sqrt[d]) +
Log[d + Sqrt[c]*d*x + Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*c^(1/4
)*Sqrt[d]))/(2*Sqrt[c]))/(2*d^2))
```

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 829
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[x^m/(r + s*x^
(n/2)), x], x] + Simp[r/(2*a)   Int[x^m/(r - s*x^(n/2)), x], x]] /; FreeQ[{
a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n/2] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.86. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx$

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.86.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.86

method	result
derivativedivides	$\frac{-\frac{2a}{\sqrt{dx}}+2b\left(-\frac{\operatorname{arctanh}\left(c x^{2}\right)}{\sqrt{dx}}+4c d^{2}\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{8d^{2}c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\,a\right.}{16d^{2}}\right.\right.}{d}$
default	$\frac{-\frac{2a}{\sqrt{dx}}+2b\left(-\frac{\operatorname{arctanh}\left(c x^{2}\right)}{\sqrt{dx}}+4c d^{2}\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{8d^{2}c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\,a\right.}{16d^{2}}\right.\right.}{d}$
parts	$-\frac{2a}{\sqrt{dx}\,d}+\frac{2b\left(-\frac{\operatorname{arctanh}\left(c x^{2}\right)}{\sqrt{dx}}+4c d^{2}\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{8d^{2}c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)\right.}{}\right.\right.}{d}$

input
```
int((a+b*arctanh(c*x^2))/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/d*(-a/(d*x)^(1/2)+b*(-1/(d*x)^(1/2)*arctanh(c*x^2)+4*c*d^2*(-1/8/d^2/c/(
d^2/c)^(1/4)*(2*arctan((d*x)^(1/2)/(d^2/c)^(1/4))-ln(((d*x)^(1/2)+(d^2/c)^
(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4))))+1/16/d^2/c/(d^2/c)^(1/4)*2^(1/2)*(ln(
(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/4)*(
d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1
/2)+1)+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1)))))
```

3.86.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.36

$$\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx = \frac{d^2x\left(\frac{b^4c}{d^6}\right)^{\frac{1}{4}}\log\left(d^5\left(\frac{b^4c}{d^6}\right)^{\frac{3}{4}}+\sqrt{dx}b^3c\right)-i\,d^2x\left(\frac{b^4c}{d^6}\right)^{\frac{1}{4}}\log\left(i\,d^5\left(\frac{b^4c}{d^6}\right)^{\frac{3}{4}}+\sqrt{dx}b^3c\right)}{}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
(d^2*x*(b^4*c/d^6)^(1/4)*log(d^5*(b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*c) - I*
d^2*x*(b^4*c/d^6)^(1/4)*log(I*d^5*(b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*c) + I
*d^2*x*(b^4*c/d^6)^(1/4)*log(-I*d^5*(b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*c) -
 d^2*x*(b^4*c/d^6)^(1/4)*log(-d^5*(b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*c) + d
^2*x*(-b^4*c/d^6)^(1/4)*log(d^5*(-b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*c) - I*
d^2*x*(-b^4*c/d^6)^(1/4)*log(I*d^5*(-b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*c) +
 I*d^2*x*(-b^4*c/d^6)^(1/4)*log(-I*d^5*(-b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*
c) - d^2*x*(-b^4*c/d^6)^(1/4)*log(-d^5*(-b^4*c/d^6)^(3/4) + sqrt(d*x)*b^3*
c) - sqrt(d*x)*(b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a))/(d^2*x)
```

3.86.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx = \int \frac{a+b\,\text{atanh}\,(cx^2)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))/(d*x)**(3/2),x)
```

output
```
Integral((a + b*atanh(c*x**2))/(d*x)**(3/2), x)
```

3.86.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.04

$$\int \frac{a + b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx =$$

$$-\frac{b\left(\frac{4\,\text{artanh}(cx^2)}{\sqrt{dx}} - \frac{\left(d^2\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}} + \frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}} - \frac{\sqrt{2}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}} + \cdots}{d^2}\right.}{2\,d}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-1/2*(b*(4*arctanh(c*x^2)/sqrt(d*x) - (d^2*(2*sqrt(2)*arctan(1/2*sqrt(2)*(
sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt
(c)*d)*sqrt(c)) + 2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) -
 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) - sqrt(2)
*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sqrt(d)
) + sqrt(2)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(3
/4)*sqrt(d))) - 2*d^2*(2*arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)*d))/(sqrt(s
qrt(c)*d)*sqrt(c)) + log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*
sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*sqrt(c))))*c/d^2) + 4*a/sqrt(
d*x))/d
```

3.86.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(198) = 396$.

Time = 0.50 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.77

$$\int \frac{a + b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx =$$

$$-\frac{\frac{2\,b\log\left(-\frac{cd^2x^2+d^2}{cd^2x^2-d^2}\right)}{\sqrt{dx}} + \frac{4\,a}{\sqrt{dx}} - \frac{2\sqrt{2}(c^3d^2)^{\frac{3}{4}}\,b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2} - \frac{2\sqrt{2}(c^3d^2)^{\frac{3}{4}}\,b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{c^2d^2} - \cdots}{\cdots}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(2*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/sqrt(d*x) + 4*a/sqrt(d
*x) - 2*sqrt(2)*(c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4
) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^2*d^2) - 2*sqrt(2)*(c^3*d^2)^(3/4)*b*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c^
2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)
^(1/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c^2*d^2) - 2*sqrt(2)*(-c^3*d^2)^(3/
4)*b*arctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(
1/4))/(c^2*d^2) + sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x + sqrt(2)*sqrt(d*x)*(d
^2/c)^(1/4) + sqrt(d^2/c))/(c^2*d^2) - sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x -
 sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c^2*d^2) + sqrt(2)*(-c^3*
d^2)^(3/4)*b*log(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c
^2*d^2) - sqrt(2)*(-c^3*d^2)^(3/4)*b*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^
(1/4) + sqrt(-d^2/c))/(c^2*d^2))/d
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^2)}{(dx)^{3/2}}\,dx = \int \frac{a + b\,\text{atanh}(c\,x^2)}{(d\,x)^{3/2}}\,dx$$

input
```
int((a + b*atanh(c*x^2))/(d*x)^(3/2),x)
```

output
```
int((a + b*atanh(c*x^2))/(d*x)^(3/2), x)
```

3.87 $\int \frac{a+b\,\text{arctanh}(cx^2)}{(dx)^{5/2}}\,dx$

3.87.1 Optimal result

Integrand size = 18, antiderivative size = 301

$$\int \frac{a+b\,\text{arctanh}(cx^2)}{(dx)^{5/2}}\,dx = \frac{2bc^{3/4}\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{\sqrt{2}bc^{3/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{\sqrt{2}bc^{3/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{2(a+b\,\text{arctanh}(cx^2))}{3d(dx)^{3/2}} + \frac{2bc^{3/4}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{bc^{3/4}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{3\sqrt{2}d^{5/2}} + \frac{bc^{3/4}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{3\sqrt{2}d^{5/2}}$$

output
```
2/3*b*c^(3/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(5/2)-2/3*(a+b*arctanh
(c*x^2))/d/(d*x)^(3/2)+2/3*b*c^(3/4)*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))/
d^(5/2)-1/6*b*c^(3/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)^(
1/2))/d^(5/2)*2^(1/2)+1/6*b*c^(3/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4)*2
^(1/2)*(d*x)^(1/2))/d^(5/2)*2^(1/2)+1/3*b*c^(3/4)*arctan(-1+c^(1/4)*2^(1/2
)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(5/2)+1/3*b*c^(3/4)*arctan(1+c^(1/4)*2^(1
/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(5/2)
```

3.87.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.89

$$\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{5/2}}\,dx =$$
$$-\frac{x\left(4a+2\sqrt{2}bc^{3/4}x^{3/2}\arctan\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-2\sqrt{2}bc^{3/4}x^{3/2}\arctan\left(1+\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-4bc^{3/4}x^{3/2}\arctan\left(\right.\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(5/2),x]
```

output
```
-1/6*(x*(4*a + 2*Sqrt[2]*b*c^(3/4)*x^(3/2)*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt
[x]] - 2*Sqrt[2]*b*c^(3/4)*x^(3/2)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] - 4
*b*c^(3/4)*x^(3/2)*ArcTan[c^(1/4)*Sqrt[x]] + 4*b*ArcTanh[c*x^2] + 2*b*c^(3
/4)*x^(3/2)*Log[1 - c^(1/4)*Sqrt[x]] - 2*b*c^(3/4)*x^(3/2)*Log[1 + c^(1/4)
*Sqrt[x]] + Sqrt[2]*b*c^(3/4)*x^(3/2)*Log[1 - Sqrt[2]*c^(1/4)*Sqrt[x] + Sq
rt[c]*x] - Sqrt[2]*b*c^(3/4)*x^(3/2)*Log[1 + Sqrt[2]*c^(1/4)*Sqrt[x] + Sqr
t[c]*x]))/(d*x)^(5/2)
```

3.87.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.778, Rules used = {6464, 851, 758, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{5/2}}\,dx$$
$$\Big\downarrow \text{6464}$$
$$\frac{4bc\int \frac{1}{\sqrt{dx}(1-c^2x^4)}dx}{3d^2}-\frac{2(a+b\text{arctanh}(cx^2))}{3d(dx)^{3/2}}$$
$$\Big\downarrow \text{851}$$
$$\frac{8bc\int \frac{1}{1-c^2x^4}d\sqrt{dx}}{3d^3}-\frac{2(a+b\text{arctanh}(cx^2))}{3d(dx)^{3/2}}$$
$$\Big\downarrow \text{758}$$

$$\frac{8bc\left(\frac{1}{2}d^2\int \frac{1}{d^2-cd^2x^2}d\sqrt{dx}+\frac{1}{2}d^2\int \frac{1}{cx^2d^2+d^2}d\sqrt{dx}\right)}{3d^3}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 755$$

$$\frac{8bc\left(\frac{1}{2}d^2\int \frac{1}{d^2-cd^2x^2}d\sqrt{dx}+\frac{1}{2}d^2\left(\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}\right)\right)}{3d^3}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 756$$

$$\frac{8bc\left(\frac{1}{2}d^2\left(\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2d}+\frac{\int \frac{1}{\sqrt{c}xd+d}d\sqrt{dx}}{2d}\right)\right)}{3d^3}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{8bc\left(\frac{1}{2}d^2\left(\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2d}+\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\right)+\frac{1}{2}d^2\left(\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}\right)\right)}{3d^3}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{8bc\left(\frac{1}{2}d^2\left(\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\right)\right)}{3d^3}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 1476$$

$$\frac{8bc\left(\frac{1}{2}d^2\left(\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\int \frac{1}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}+\frac{\int \frac{1}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\right)\right)}{3d^3}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \; 1082$$

3.87. $\int \frac{a+b\text{arctanh}\left(cx^2\right)}{(dx)^{5/2}}\,dx$

$$\frac{8bc\left(\frac{1}{2}d^2\left(\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\right.\right.}{3d^3}$$

$$\frac{2(a+b\operatorname{arctanh}(cx^2))}{3d(dx)^{3/2}}$$

$\downarrow$ 217

$$\frac{8bc\left(\frac{1}{2}d^2\left(\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\right.\right.}{3d^3}$$

$$\frac{2(a+b\operatorname{arctanh}(cx^2))}{3d(dx)^{3/2}}$$

$\downarrow$ 1479

$$\frac{8bc\left(\frac{1}{2}d^2\left(\frac{-\frac{\int-\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int-\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}\right)\right.}{3d^3}$$

$$\frac{2(a+b\operatorname{arctanh}(cx^2))}{3d(dx)^{3/2}}$$

$\downarrow$ 25

3.87. $\int\frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{5/2}}\,dx$

$$\frac{8bc\left(\frac{1}{2}d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}\right)+\frac{1}{2}\cdots}{3d^3}$$

$$\frac{2(a+b\text{arctanh}(cx^2))}{3d(dx)^{3/2}}$$

$\downarrow$ 27

$$\frac{8bc\left(\frac{1}{2}d^2\left(\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}\right)+\frac{1}{2}d^2\left(\frac{\arctan\left(\cdots\right.}{2\sqrt[4]{\cdots}}\right.\cdots}{3d^3}$$

$$\frac{2(a+b\text{arctanh}(cx^2))}{3d(dx)^{3/2}}$$

$\downarrow$ 1103

$$\frac{8bc\left(\frac{1}{2}d^2\left(\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}\right)+\frac{1}{2}d^2\left(\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\log\left(\sqrt{c}dx+\sqrt{2}\sqrt[4]{c}\cdots\right.}{2\sqrt{2}\sqrt[4]{c}\cdots}\cdots\right.}{3d^3}$$

$$\frac{2(a+b\text{arctanh}(cx^2))}{3d(dx)^{3/2}}$$

input
```
Int[(a + b*ArcTanh[c*x^2])/(d*x)^(5/2),x]
```

3.87. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{5/2}}\,dx$

output
```
(-2*(a + b*ArcTanh[c*x^2]))/(3*d*(d*x)^(3/2)) + (8*b*c*((d^2*(ArcTan[(c^(1
/4)*Sqrt[d*x])/Sqrt[d]]/(2*c^(1/4)*d^(3/2)) + ArcTanh[(c^(1/4)*Sqrt[d*x])/
Sqrt[d]]/(2*c^(1/4)*d^(3/2))))/2 + (d^2*((-(ArcTan[1 - (Sqrt[2]*c^(1/4)*Sq
rt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d])) + ArcTan[1 + (Sqrt[2]*c^(1/4)
*Sqrt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d]))/(2*d) + (-1/2*Log[d + Sqrt
[c]*d*x - Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt[2]*c^(1/4)*Sqrt[d]) + L
og[d + Sqrt[c]*d*x + Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x]]/(2*Sqrt[2]*c^(1/4)
*Sqrt[d]))/(2*d)))/2))/(3*d^3)
```

3.87.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 758
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b
, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^(n/2)),
 x], x] + Simp[r/(2*a)   Int[1/(r + s*x^(n/2)), x], x]] /; FreeQ[{a, b}, x]
 && IGtQ[n/4, 1] &&  !GtQ[a/b, 0]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

3.87. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{5/2}} \, dx$

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.87.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{3(dx)^{\frac{3}{2}}}+\frac{4c\,d^{2}\left(\frac{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{d^{2}}{\cdots}\right)}\cdots\right.\right.}{16d^{4}}\cdots\right)}{3}\cdots\right)}{d}$
default	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}+2b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{3(dx)^{\frac{3}{2}}}+\frac{4c\,d^{2}\left(\frac{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\cdots}{\left(\frac{d^{2}}{\cdots}\right)}\cdots\right.\right.}{16d^{4}}\cdots\right)}{3}\cdots\right)}{d}$
parts	$-\frac{2a}{3(dx)^{\frac{3}{2}}d}+\frac{2b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{3(dx)^{\frac{3}{2}}}+\frac{4c\,d^{2}\left(\frac{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\cdots\right.\right.}{16d^{4}}\cdots\right)}{3}\cdots\right)}{d}$

input
```
int((a+b*arctanh(c*x^2))/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

3.87. $\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{5/2}}\,dx$

output
```
2/d*(-1/3*a/(d*x)^(3/2)+b*(-1/3/(d*x)^(3/2)*arctanh(c*x^2)+4/3*c*d^2*(1/16
/d^4*(d^2/c)^(1/4)*2^(1/2)*(ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2
/c)^(1/2))/(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+2*arctan
(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)
^(1/2)-1))+1/8/d^4*(d^2/c)^(1/4)*(ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1
/2)-(d^2/c)^(1/4)))+2*arctan((d*x)^(1/2)/(d^2/c)^(1/4))))))
```

3.87.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.40

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{5/2}}\,dx = \frac{d^3x^2\left(\frac{b^4c^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(d^3\left(\frac{b^4c^3}{d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}bc\right)+i\,d^3x^2\left(\frac{b^4c^3}{d^{10}}\right)^{\frac{1}{4}}\log\left(i\,d^3\left(\frac{b^4c^3}{d^{10}}\right)^{\frac{1}{4}}+\sqrt{dx}\right.}{}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(d^3*x^2*(b^4*c^3/d^10)^(1/4)*log(d^3*(b^4*c^3/d^10)^(1/4) + sqrt(d*x)
*b*c) + I*d^3*x^2*(b^4*c^3/d^10)^(1/4)*log(I*d^3*(b^4*c^3/d^10)^(1/4) + sq
rt(d*x)*b*c) - I*d^3*x^2*(b^4*c^3/d^10)^(1/4)*log(-I*d^3*(b^4*c^3/d^10)^(1
/4) + sqrt(d*x)*b*c) - d^3*x^2*(b^4*c^3/d^10)^(1/4)*log(-d^3*(b^4*c^3/d^10
)^(1/4) + sqrt(d*x)*b*c) + d^3*x^2*(-b^4*c^3/d^10)^(1/4)*log(d^3*(-b^4*c^3
/d^10)^(1/4) + sqrt(d*x)*b*c) + I*d^3*x^2*(-b^4*c^3/d^10)^(1/4)*log(I*d^3*
(-b^4*c^3/d^10)^(1/4) + sqrt(d*x)*b*c) - I*d^3*x^2*(-b^4*c^3/d^10)^(1/4)*l
og(-I*d^3*(-b^4*c^3/d^10)^(1/4) + sqrt(d*x)*b*c) - d^3*x^2*(-b^4*c^3/d^10)
^(1/4)*log(-d^3*(-b^4*c^3/d^10)^(1/4) + sqrt(d*x)*b*c) - sqrt(d*x)*(b*log(
-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a))/(d^3*x^2)
```

3.87.6 Sympy [F]

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{5/2}}\,dx = \int \frac{a+b\operatorname{atanh}(cx^2)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))/(d*x)**(5/2),x)
```

output
```
Integral((a + b*atanh(c*x**2))/(d*x)**(5/2), x)
```

3.87.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.92

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{5/2}}\,dx = \frac{b\left(\frac{\frac{2\sqrt{2}d\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}}+\frac{2\sqrt{2}d\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}}+\frac{\sqrt{2}\sqrt{d}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{\cdots}\right.}{c^{\frac{1}{4}}}}{d^2}\right.}{\cdots}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(b*((2*sqrt(2)*d*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sqrt(
d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sqrt(sqrt(c)*d) + 2*sqrt(2)*d*arctan(-1/2*s
qrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/sq
rt(sqrt(c)*d) + sqrt(2)*sqrt(d)*log(sqrt(c)*d*x + sqrt(2)*sqrt(d*x)*c^(1/4
)*sqrt(d) + d)/c^(1/4) - sqrt(2)*sqrt(d)*log(sqrt(c)*d*x - sqrt(2)*sqrt(d*
x)*c^(1/4)*sqrt(d) + d)/c^(1/4) + 4*d*arctan(sqrt(d*x)*sqrt(c)/sqrt(sqrt(c
)*d))/sqrt(sqrt(c)*d) - 2*d*log((sqrt(d*x)*sqrt(c) - sqrt(sqrt(c)*d))/(sqr
t(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/sqrt(sqrt(c)*d))*c/d^2 - 4*arctanh(c*x^
2)/(d*x)^(3/2)) - 4*a/(d*x)^(3/2))/d
```

3.87.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. 2(200) = 400.

Time = 0.68 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.71

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{5/2}}\,dx = \frac{bcd^2\left(\frac{2\sqrt{2}(c^3d^2)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^4}+\frac{2\sqrt{2}(c^3d^2)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right.}{cd^4}\cdots\right.}{\cdots}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(5/2),x, algorithm="giac")
```

output
```
1/6*(b*c*d^2*(2*sqrt(2)*(c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c
)^(1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d^4) + 2*sqrt(2)*(c^3*d^2)^(1/4)*
arctan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(
c*d^4) + 2*sqrt(2)*(-c^3*d^2)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(
1/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^4) + 2*sqrt(2)*(-c^3*d^2)^(1/4)*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/
(c*d^4) + sqrt(2)*(c^3*d^2)^(1/4)*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4
) + sqrt(d^2/c))/(c*d^4) - sqrt(2)*(c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt(
d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c*d^4) + sqrt(2)*(-c^3*d^2)^(1/4)*log(d
*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c*d^4) - sqrt(2)*(-
c^3*d^2)^(1/4)*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/
(c*d^4)) - 2*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*d*x) -
 4*a/(sqrt(d*x)*d*x))/d
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^2)}{(dx)^{5/2}} \, dx = \int \frac{a + b\,\text{atanh}(c\,x^2)}{(d\,x)^{5/2}} \, dx$$

input
```
int((a + b*atanh(c*x^2))/(d*x)^(5/2),x)
```

output
```
int((a + b*atanh(c*x^2))/(d*x)^(5/2), x)
```

3.88 $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx$

3.88.1 Optimal result

Integrand size = 18, antiderivative size = 317

$$\begin{aligned}\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx =& -\frac{8bc}{5d^3\sqrt{dx}} - \frac{2bc^{5/4}\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &+ \frac{\sqrt{2}bc^{5/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{\sqrt{2}bc^{5/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &- \frac{2(a+b\text{arctanh}(cx^2))}{5d(dx)^{5/2}} + \frac{2bc^{5/4}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{5d^{7/2}} \\ &- \frac{bc^{5/4}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{5\sqrt{2}d^{7/2}} + \frac{bc^{5/4}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{5\sqrt{2}d^{7/2}}\end{aligned}$$

output
```
-2/5*b*c^(5/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2))/d^(7/2)-2/5*(a+b*arctan
h(c*x^2))/d/(d*x)^(5/2)+2/5*b*c^(5/4)*arctanh(c^(1/4)*(d*x)^(1/2)/d^(1/2))
/d^(7/2)-1/10*b*c^(5/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)-c^(1/4)*2^(1/2)*(d*x)
^(1/2))/d^(7/2)*2^(1/2)+1/10*b*c^(5/4)*ln(d^(1/2)+x*c^(1/2)*d^(1/2)+c^(1/4
)*2^(1/2)*(d*x)^(1/2))/d^(7/2)*2^(1/2)-1/5*b*c^(5/4)*arctan(-1+c^(1/4)*2^(
1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(7/2)-1/5*b*c^(5/4)*arctan(1+c^(1/4)*2
^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(7/2)-8/5*b*c/d^3/(d*x)^(1/2)
```

3.88.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.87

$$\int \frac{a+b\,\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx = \frac{x\left(-4a-16bcx^2+2\sqrt{2}bc^{5/4}x^{5/2}\arctan\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-2\sqrt{2}bc^{5/4}x^{5/2}\arctan\left(\right.\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(7/2),x]
```

output
```
(x*(-4*a - 16*b*c*x^2 + 2*Sqrt[2]*b*c^(5/4)*x^(5/2)*ArcTan[1 - Sqrt[2]*c^(
1/4)*Sqrt[x]] - 2*Sqrt[2]*b*c^(5/4)*x^(5/2)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqr
t[x]] - 4*b*c^(5/4)*x^(5/2)*ArcTan[c^(1/4)*Sqrt[x]] - 4*b*ArcTanh[c*x^2] -
 2*b*c^(5/4)*x^(5/2)*Log[1 - c^(1/4)*Sqrt[x]] + 2*b*c^(5/4)*x^(5/2)*Log[1
+ c^(1/4)*Sqrt[x]] - Sqrt[2]*b*c^(5/4)*x^(5/2)*Log[1 - Sqrt[2]*c^(1/4)*Sqr
t[x] + Sqrt[c]*x] + Sqrt[2]*b*c^(5/4)*x^(5/2)*Log[1 + Sqrt[2]*c^(1/4)*Sqrt
[x] + Sqrt[c]*x]))/(10*(d*x)^(7/2))
```

3.88.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.11, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {6464, 847, 851, 27, 830, 826, 827, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\,\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx$$

$$\Big\downarrow \text{6464}$$

$$\frac{4bc\int \frac{1}{(dx)^{3/2}(1-c^2x^4)}dx}{5d^2}-\frac{2(a+b\,\text{arctanh}(cx^2))}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{847}$$

$$\frac{4bc\left(\frac{c^2\int \frac{(dx)^{5/2}}{1-c^2x^4}dx}{d^4}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2(a+b\,\text{arctanh}(cx^2))}{5d(dx)^{5/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{4bc\left(\frac{2c^2\int \frac{d^7x^3}{d^4-c^2d^4x^4}d\sqrt{dx}}{d^5}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{4bc\left(\frac{2c^2\int \frac{d^3x^3}{d^4-c^2d^4x^4}d\sqrt{dx}}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 830$$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\int \frac{dx}{d^2-cd^2x^2}d\sqrt{dx}}{2c}-\frac{\int \frac{dx}{cx^2d^2+d^2}d\sqrt{dx}}{2c}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 826$$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\int \frac{dx}{d^2-cd^2x^2}d\sqrt{dx}}{2c}-\frac{\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 827$$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{1}{\sqrt{c}xd+d}d\sqrt{dx}}{2\sqrt{c}}}{2c}-\frac{\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 218$$

3.88. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\int\frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2\sqrt{c}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\int\frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2(a+b\text{arctanh}(cx^2))}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\int\frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2(a+b\text{arctanh}(cx^2))}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 1476$$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\int\frac{1}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}+\frac{\int\frac{1}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-\frac{2(a+b\text{arctanh}(cx^2))}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 1082$$

3.88. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)}{d}-\frac{2}{d\sqrt{d}}\right.}{5d^2}$$

$$\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 217$$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2\sqrt{c}}}{2c}\right)}{d}-\frac{2}{d\sqrt{dx}}\right)}{5d^2}-$$

$$\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

$$\Big\downarrow \; 1479$$

3.88. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}\cdots}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}\cdots}{2\cdots}}{2c}\cdots\right)}{d}\cdots\right)}{5d^2}\cdots$$

$$\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

$\downarrow$ 25

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\cdots}{2\sqrt{\cdots}}}{2c}\cdots\right)}{d}\cdots\right)}{5d^2}\cdots$$

$$\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

$\downarrow$ 27

3.88. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\int\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int\frac{\sqrt{d}+\cdots}{xd+\frac{d}{\sqrt{\cdots}}}\cdots}{2\sqrt{c}}\cdots}{2c}\cdots\right)}{d}\cdots\right)}{5d^2}\cdots$$

$$\frac{2\left(a+b\,\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

$\Big\downarrow$ 1103

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}-\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2c^{3/4}\sqrt{d}}}{2c}-\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2\sqrt{c}}-\frac{\frac{\log\left(\sqrt{c}dx+\sqrt{2}\sqrt[4]{c}\sqrt{d}\sqrt{dx}+d\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\log\left(\sqrt{\cdots}\right)\cdots}{\cdots}}{2\sqrt{c}}\cdots}{2c}\cdots\right)}{d}\cdots\right)}{5d^2}\cdots$$

$$\frac{2\left(a+b\,\text{arctanh}\left(cx^2\right)\right)}{5d(dx)^{5/2}}$$

input
```
Int[(a + b*ArcTanh[c*x^2])/(d*x)^(7/2),x]
```

3.88. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx$

output
```
(-2*(a + b*ArcTanh[c*x^2]))/(5*d*(d*x)^(5/2)) + (4*b*c*(-2/(d*Sqrt[d*x]) +
 (2*c^2*((-1/2*ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(c^(3/4)*Sqrt[d]) + Arc
Tanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*c^(3/4)*Sqrt[d]))/(2*c) - ((-(ArcTan[
1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d])) + ArcT
an[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d]))/(2*
Sqrt[c]) - (-1/2*Log[d + Sqrt[c]*d*x - Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x]]/
(Sqrt[2]*c^(1/4)*Sqrt[d]) + Log[d + Sqrt[c]*d*x + Sqrt[2]*c^(1/4)*Sqrt[d]*
Sqrt[d*x]]/(2*Sqrt[2]*c^(1/4)*Sqrt[d]))/(2*Sqrt[c]))/(2*c)))/d))/(5*d^2)
```

3.88.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.88. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.88.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{-\frac{2a}{5(dx)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{5(dx)^{\frac{5}{2}}}+\frac{4c\,d^{2}\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{8d^{4}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\cdots\right.}{16\cdots}\cdots\right.}{5\cdots}\cdots\right.}{d\cdots}$
default	$\frac{-\frac{2a}{5(dx)^{\frac{5}{2}}}+2b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{5(dx)^{\frac{5}{2}}}+\frac{4c\,d^{2}\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{8d^{4}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}}}\right)+2\cdots\right.}{16\cdots}\cdots\right.}{5\cdots}\cdots\right.}{d\cdots}$
parts	$-\frac{2a}{5(dx)^{\frac{5}{2}}d}+\frac{2b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{5(dx)^{\frac{5}{2}}}+\frac{4c\,d^{2}\left(-\frac{2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)-\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)}{8d^{4}\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}-\frac{\sqrt{2}\left(\ln\left(\frac{dx-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}\cdots}}{dx+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{dx}\,\sqrt{2}+\sqrt{\frac{d^{2}}{c}\cdots}}\right)\cdots\right.}{\cdots}\cdots\right.}{5\cdots}\cdots\right.}{d\cdots}$

input
```
int((a+b*arctanh(c*x^2))/(d*x)^(7/2),x,method=_RETURNVERBOSE)
```

3.88. $\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{7/2}}\,dx$

output
```
2/d*(-1/5*a/(d*x)^(5/2)+b*(-1/5/(d*x)^(5/2)*arctanh(c*x^2)+4/5*c*d^2*(-1/8
/d^4/(d^2/c)^(1/4)*(2*arctan((d*x)^(1/2)/(d^2/c)^(1/4))-ln(((d*x)^(1/2)+(d
^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(1/4))))-1/16/d^4/(d^2/c)^(1/4)*2^(1/2)*
(ln((d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2))/(d*x+(d^2/c)^(1/
4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x
)^(1/2)+1)+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)-1))-1/d^4/(d*x)^(1/2
))))
```

3.88.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.45

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{7/2}}\,dx = \frac{d^4x^3\left(\frac{b^4c^5}{d^{14}}\right)^{\frac{1}{4}}\log\left(d^{11}\left(\frac{b^4c^5}{d^{14}}\right)^{\frac{3}{4}}+\sqrt{dx}b^3c^4\right)-i\,d^4x^3\left(\frac{b^4c^5}{d^{14}}\right)^{\frac{1}{4}}\log\left(i\,d^{11}\left(\frac{b^4c^5}{d^{14}}\right)^{\frac{3}{4}}+\sqrt{\quad}\right.}{}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="fricas")
```

output
```
1/5*(d^4*x^3*(b^4*c^5/d^14)^(1/4)*log(d^11*(b^4*c^5/d^14)^(3/4) + sqrt(d*x
)*b^3*c^4) - I*d^4*x^3*(b^4*c^5/d^14)^(1/4)*log(I*d^11*(b^4*c^5/d^14)^(3/4
) + sqrt(d*x)*b^3*c^4) + I*d^4*x^3*(b^4*c^5/d^14)^(1/4)*log(-I*d^11*(b^4*c
^5/d^14)^(3/4) + sqrt(d*x)*b^3*c^4) - d^4*x^3*(b^4*c^5/d^14)^(1/4)*log(-d^
11*(b^4*c^5/d^14)^(3/4) + sqrt(d*x)*b^3*c^4) - d^4*x^3*(-b^4*c^5/d^14)^(1/
4)*log(d^11*(-b^4*c^5/d^14)^(3/4) + sqrt(d*x)*b^3*c^4) + I*d^4*x^3*(-b^4*c
^5/d^14)^(1/4)*log(I*d^11*(-b^4*c^5/d^14)^(3/4) + sqrt(d*x)*b^3*c^4) - I*d
^4*x^3*(-b^4*c^5/d^14)^(1/4)*log(-I*d^11*(-b^4*c^5/d^14)^(3/4) + sqrt(d*x)
*b^3*c^4) + d^4*x^3*(-b^4*c^5/d^14)^(1/4)*log(-d^11*(-b^4*c^5/d^14)^(3/4)
+ sqrt(d*x)*b^3*c^4) - (8*b*c*x^2 + b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 2*a)
*sqrt(d*x))/(d^4*x^3)
```

3.88.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx = \int \frac{a+b\,\text{atanh}\,(cx^2)}{(dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))/(d*x)**(7/2),x)
```

output
```
Integral((a + b*atanh(c*x**2))/(d*x)**(7/2), x)
```

3.88.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx = -\frac{b\left(\frac{\left(c\left(\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}}+\frac{2\sqrt{2}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}\sqrt{c}}-\frac{\sqrt{2}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\sqrt{d}+d\right)}{c^{\frac{3}{4}}\sqrt{d}}+\frac{\sqrt{2}\log\left(\sqrt{c}dx-\sqrt{2}\sqrt{dx}c^{\frac{1}{4}} \cdots\right.}{c^{\frac{3}{4}}\sqrt{d}} \cdots}{d^2} \cdots\right)}{10\,d}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="maxima")
```

output
```
-1/10*(b*((c*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*sq
rt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + 2*sqrt(2)*ar
ctan(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqr
t(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) - sqrt(2)*log(sqrt(c)*d*x + sqrt(2)*sqr
t(d*x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sqrt(d)) + sqrt(2)*log(sqrt(c)*d*x -
sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/(c^(3/4)*sqrt(d))) + 2*c*(2*arctan(
sqrt(d*x)*sqrt(c)/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*sqrt(c)) + log((sqrt(d
*x)*sqrt(c) - sqrt(sqrt(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqr
t(sqrt(c)*d)*sqrt(c))) + 16/sqrt(d*x))*c/d^2 + 4*arctanh(c*x^2)/(d*x)^(5/2
)) + 4*a/(d*x)^(5/2))/d
```

3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. $2(212) = 424$.

Time = 1.79 (sec) , antiderivative size = 532, normalized size of antiderivative = 1.68

$$\int \frac{a + b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx =$$

$$-\frac{\frac{2\sqrt{2}(c^3d^2)^{\frac{3}{4}} b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^4} + \frac{2\sqrt{2}(c^3d^2)^{\frac{3}{4}} b\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{cd^4} - \frac{2\sqrt{2}(-c^3d^2)^{\frac{3}{4}} b\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\cdots\right)\right)}{2\left(\cdots\right)}\right)}{cd^4} \cdots}{\cdots}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(7/2),x, algorithm="giac")
```

output
```
-1/10*(2*sqrt(2)*(c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/
4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d^4) + 2*sqrt(2)*(c^3*d^2)^(3/4)*b*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/(c*d
^4) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1
/4) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/(c*d^4) - 2*sqrt(2)*(-c^3*d^2)^(3/4)*b*
arctan(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))
/(c*d^4) - sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(
1/4) + sqrt(d^2/c))/(c*d^4) + sqrt(2)*(c^3*d^2)^(3/4)*b*log(d*x - sqrt(2)*
sqrt(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/(c*d^4) + sqrt(2)*(-c^3*d^2)^(3/4)*
b*log(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/(c*d^4) - sqr
t(2)*(-c^3*d^2)^(3/4)*b*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(
-d^2/c))/(c*d^4) + 2*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x
)*d^2*x^2) + 4*(4*b*c*d^2*x^2 + a*d^2)/(sqrt(d*x)*d^4*x^2))/d
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^2)}{(dx)^{7/2}}\,dx = \int \frac{a + b\,\text{atanh}(c\,x^2)}{(d\,x)^{7/2}}\,dx$$

input
```
int((a + b*atanh(c*x^2))/(d*x)^(7/2),x)
```

output
```
int((a + b*atanh(c*x^2))/(d*x)^(7/2), x)
```

3.89 $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{9/2}}\,dx$

3.89.1 Optimal result

Integrand size = 18, antiderivative size = 317

$$\begin{aligned}\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{9/2}}\,dx =& -\frac{8bc}{21d^3(dx)^{3/2}}+\frac{2bc^{7/4}\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}\\&+\frac{\sqrt{2}bc^{7/4}\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}-\frac{\sqrt{2}bc^{7/4}\arctan\left(1+\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}\\&-\frac{2(a+b\text{arctanh}(cx^2))}{7d(dx)^{7/2}}+\frac{2bc^{7/4}\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{7d^{9/2}}\\&+\frac{bc^{7/4}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x-\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{7\sqrt{2}d^{9/2}}-\frac{bc^{7/4}\log\left(\sqrt{d}+\sqrt{c}\sqrt{d}x+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{7\sqrt{2}d^{9/2}}\end{aligned}$$

output
```
-8/21*b*c/d^3/(d*x)^(3/2)+2/7*b*c^(7/4)*arctan(c^(1/4)*(d*x)^(1/2)/d^(1/2)
)/d^(9/2)-2/7*(a+b*arctanh(c*x^2))/d/(d*x)^(7/2)+2/7*b*c^(7/4)*arctanh(c^(
1/4)*(d*x)^(1/2)/d^(1/2))/d^(9/2)+1/14*b*c^(7/4)*ln(d^(1/2)+x*c^(1/2)*d^(1
/2)-c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(9/2)*2^(1/2)-1/14*b*c^(7/4)*ln(d^(1/2)
+x*c^(1/2)*d^(1/2)+c^(1/4)*2^(1/2)*(d*x)^(1/2))/d^(9/2)*2^(1/2)-1/7*b*c^(7
/4)*arctan(-1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(9/2)-1/7*b*c
^(7/4)*arctan(1+c^(1/4)*2^(1/2)*(d*x)^(1/2)/d^(1/2))*2^(1/2)/d^(9/2)
```

3.89.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.89

$$\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{9/2}}\,dx = \frac{\sqrt{dx}\left(-12a-16bcx^2+6\sqrt{2}bc^{7/4}x^{7/2}\arctan\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)-6\sqrt{2}bc^{7/4}x^{7/2}\text{arct}\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])/(d*x)^(9/2),x]
```

output
```
(Sqrt[d*x]*(-12*a - 16*b*c*x^2 + 6*Sqrt[2]*b*c^(7/4)*x^(7/2)*ArcTan[1 - Sq
rt[2]*c^(1/4)*Sqrt[x]] - 6*Sqrt[2]*b*c^(7/4)*x^(7/2)*ArcTan[1 + Sqrt[2]*c^
(1/4)*Sqrt[x]] + 12*b*c^(7/4)*x^(7/2)*ArcTan[c^(1/4)*Sqrt[x]] - 12*b*ArcTa
nh[c*x^2] - 6*b*c^(7/4)*x^(7/2)*Log[1 - c^(1/4)*Sqrt[x]] + 6*b*c^(7/4)*x^(
7/2)*Log[1 + c^(1/4)*Sqrt[x]] + 3*Sqrt[2]*b*c^(7/4)*x^(7/2)*Log[1 - Sqrt[2
]*c^(1/4)*Sqrt[x] + Sqrt[c]*x] - 3*Sqrt[2]*b*c^(7/4)*x^(7/2)*Log[1 + Sqrt[
2]*c^(1/4)*Sqrt[x] + Sqrt[c]*x]))/(42*d^5*x^4)
```

3.89.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.10, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.889, Rules used = {6464, 847, 851, 27, 830, 755, 756, 218, 221, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{9/2}}dx$$

$$\Big\downarrow \text{6464}$$

$$\frac{4bc\int \frac{1}{(dx)^{5/2}(1-c^2x^4)}dx}{7d^2}-\frac{2(a+b\text{arctanh}(cx^2))}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{847}$$

$$\frac{4bc\left(\frac{c^2\int \frac{(dx)^{3/2}}{1-c^2x^4}dx}{d^4}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2(a+b\text{arctanh}(cx^2))}{7d(dx)^{7/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{4bc\left(\frac{2c^2\int \frac{d^6x^2}{d^4-c^2d^4x^4}d\sqrt{dx}}{d^5}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$\big\downarrow$ 27

$$\frac{4bc\left(\frac{2c^2\int \frac{d^2x^2}{d^4-c^2d^4x^4}d\sqrt{dx}}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$\big\downarrow$ 830

$$\frac{4bc\left(\frac{2c^2\left(\frac{\int \frac{1}{d^2-cd^2x^2}d\sqrt{dx}}{2c}-\frac{\int \frac{1}{cx^2d^2+d^2}d\sqrt{dx}}{2c}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$\big\downarrow$ 755

$$\frac{4bc\left(\frac{2c^2\left(\frac{\int \frac{1}{d^2-cd^2x^2}d\sqrt{dx}}{2c}-\frac{\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}}{2c}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$\big\downarrow$ 756

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2d}+\frac{\int \frac{1}{\sqrt{c}xd+d}d\sqrt{dx}}{2d}}{2c}-\frac{\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}}{2c}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$\big\downarrow$ 218

3.89. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{9/2}}\,dx$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\int \frac{1}{d-\sqrt{c}dx}d\sqrt{dx}}{2d}+\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}}{2c}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \; 221$$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\int \frac{\sqrt{c}xd+d}{cx^2d^2+d^2}d\sqrt{dx}}{2d}}{2c}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$$\Big\downarrow \; 1476$$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\int \frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\int \frac{1}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}+\frac{\int \frac{1}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}}}{2d}}{2c}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

3.89. $\int \frac{a+b\text{arctanh}\left(cx^2\right)}{(dx)^{9/2}}\,dx$

$\downarrow$ 1082

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\int\frac{1}{-dx-1}d\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int\frac{1}{-dx-1}d\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}}{2c}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$\downarrow$ 217

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\int\frac{d-\sqrt{c}dx}{cx^2d^2+d^2}d\sqrt{dx}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}}{2c}\right)}{d}-\frac{2}{3d(dx)^{3/2}}\right)}{7d^2}-\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$\downarrow$ 1479

3.89. $\int \frac{a+b\text{arctanh}\left(cx^2\right)}{(dx)^{9/2}}\,dx$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{-\frac{\int -\frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\int -\frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\arctan\left(\frac{\sqrt{2}\cdots}{\cdots}\right)}{\sqrt{2}\cdots}}{2c}\cdots\right)}{d}\cdots\right)}{7d^2}\cdots$$

$$\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$\big\downarrow$ 25

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}+\frac{\int \frac{\sqrt{2}\left(\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}\right)}{\sqrt[4]{c}\left(xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}\right)}d\sqrt{dx}}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\cdots}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\cdots}}{2c}\cdots\right)}{d}\cdots\right)}{7d^2}\cdots$$

$$\frac{2\left(a+b\text{arctanh}\left(cx^2\right)\right)}{7d(dx)^{7/2}}$$

$\big\downarrow$ 27

3.89. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{9/2}}\,dx$

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\frac{\int \frac{\sqrt{2}\sqrt{d}-2\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}-\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{2}\sqrt{c}\sqrt{d}}+\frac{\int \frac{\sqrt{d}+\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{xd+\frac{d}{\sqrt{c}}+\frac{\sqrt{2}\sqrt{dx}\sqrt{d}}{\sqrt[4]{c}}}d\sqrt{dx}}{2\sqrt{c}\sqrt{d}}}{2d}+\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\cdots}{2d}}{2c}\cdots\right)}{d}\cdots\right)}{7d^2}$$

$$\frac{2(a+b\text{arctanh}(cx^2))}{7d(dx)^{7/2}}$$

$\Big\downarrow$ 1103

$$\frac{4bc\left(\frac{2c^2\left(\frac{\frac{\arctan\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}+\frac{\text{arctanh}\left(\frac{\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt[4]{c}d^{3/2}}}{2c}-\frac{\frac{\frac{\arctan\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}+1\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\arctan\left(1-\frac{\sqrt{2}\sqrt[4]{c}\sqrt{dx}}{\sqrt{d}}\right)}{\sqrt{2}\sqrt[4]{c}\sqrt{d}}}{2d}+\frac{\frac{\log\left(\sqrt{c}dx+\sqrt{2}\sqrt[4]{c}\sqrt{d}\sqrt{dx}+d\right)}{2\sqrt{2}\sqrt[4]{c}\sqrt{d}}-\frac{\log\left(\sqrt{\cdots}\right)}{\cdots}}{2d}}{2c}\cdots\right)}{d}\cdots\right)}{7d^2}$$

$$\frac{2(a+b\text{arctanh}(cx^2))}{7d(dx)^{7/2}}$$

input
```
Int[(a + b*ArcTanh[c*x^2])/(d*x)^(9/2),x]
```

3.89. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{9/2}}\,dx$

output
```
(-2*(a + b*ArcTanh[c*x^2]))/(7*d*(d*x)^(7/2)) + (4*b*c*(-2/(3*d*(d*x)^(3/2
)) + (2*c^2*((ArcTan[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*c^(1/4)*d^(3/2)) + Ar
cTanh[(c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(2*c^(1/4)*d^(3/2)))/(2*c) - ((-(ArcTan
[1 - (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d])) + Arc
Tan[1 + (Sqrt[2]*c^(1/4)*Sqrt[d*x])/Sqrt[d]]/(Sqrt[2]*c^(1/4)*Sqrt[d]))/(2
*d) + (-1/2*Log[d + Sqrt[c]*d*x - Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[d*x]]/(Sqrt
[2]*c^(1/4)*Sqrt[d]) + Log[d + Sqrt[c]*d*x + Sqrt[2]*c^(1/4)*Sqrt[d]*Sqrt[
d*x]]/(2*Sqrt[2]*c^(1/4)*Sqrt[d]))/(2*d))/(2*c)))/d))/(7*d^2)
```

3.89.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 830
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[Rt
[-a/b, 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[x^(m - n/2)/(
r + s*x^(n/2)), x], x] - Simp[s/(2*b)   Int[x^(m - n/2)/(r - s*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LeQ[n/2, m] && L
tQ[m, n] &&  !GtQ[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

3.89. $\int \frac{a+b\text{arctanh}(cx^2)}{(dx)^{9/2}}\,dx$

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.89.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{-\frac{2a}{7(dx)^{\frac{7}{2}}}+2b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{7(dx)^{\frac{7}{2}}}+\frac{4c\,d^{2}\left(\frac{c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)\right)}{8d^{6}}-\frac{1}{3d^{4}(dx)^{\frac{3}{2}}}-\frac{c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right.\right.}{\cdots}\right)}{7\cdots}\cdots\right)}{d}\cdots$
default	$\frac{-\frac{2a}{7(dx)^{\frac{7}{2}}}+2b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{7(dx)^{\frac{7}{2}}}+\frac{4c\,d^{2}\left(\frac{c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)\right)}{8d^{6}}-\frac{1}{3d^{4}(dx)^{\frac{3}{2}}}-\frac{c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\cdots\right.\right.}{\cdots}\right)}{7\cdots}\cdots\right)}{d}\cdots$
parts	$-\frac{2a}{7(dx)^{\frac{7}{2}}d}+\frac{2b\left(-\frac{\operatorname{arctanh}\left(c\,x^{2}\right)}{7(dx)^{\frac{7}{2}}}+\frac{4c\,d^{2}\left(\frac{c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\left(\ln\left(\frac{\sqrt{dx}+\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}{\sqrt{dx}-\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)+2\arctan\left(\frac{\sqrt{dx}}{\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}}\right)\right)}{8d^{6}}-\frac{1}{3d^{4}(dx)^{\frac{3}{2}}}-\frac{c\left(\frac{d^{2}}{c}\right)^{\frac{1}{4}}\sqrt{2}\cdots}{\cdots}\right)}{7\cdots}\cdots\right)}{d}\cdots$

input
```
int((a+b*arctanh(c*x^2))/(d*x)^(9/2),x,method=_RETURNVERBOSE)
```

3.89. $\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{9/2}}\,dx$

output
```
2/d*(-1/7*a/(d*x)^(7/2)+b*(-1/7/(d*x)^(7/2)*arctanh(c*x^2)+4/7*c*d^2*(1/8/
d^6*c*(d^2/c)^(1/4)*(ln(((d*x)^(1/2)+(d^2/c)^(1/4))/((d*x)^(1/2)-(d^2/c)^(
1/4)))+2*arctan((d*x)^(1/2)/(d^2/c)^(1/4)))-1/3/d^4/(d*x)^(3/2)-1/16/d^6*c
*(d^2/c)^(1/4)*2^(1/2)*(ln((d*x+(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^
(1/2))/(d*x-(d^2/c)^(1/4)*(d*x)^(1/2)*2^(1/2)+(d^2/c)^(1/2)))+2*arctan(2^(
1/2)/(d^2/c)^(1/4)*(d*x)^(1/2)+1)+2*arctan(2^(1/2)/(d^2/c)^(1/4)*(d*x)^(1/
2)-1)))))
```

3.89.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 447, normalized size of antiderivative = 1.41

$$\int \frac{a + b\text{arctanh}(cx^2)}{(dx)^{9/2}}\,dx = \frac{3\,d^5x^4\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}\log\left(d^5\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}} + \sqrt{dx}bc^2\right) + 3i\,d^5x^4\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}}\log\left(i\,d^5\left(\frac{b^4c^7}{d^{18}}\right)^{\frac{1}{4}} + \sqrt{}\right.}{}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="fricas")
```

output
```
1/21*(3*d^5*x^4*(b^4*c^7/d^18)^(1/4)*log(d^5*(b^4*c^7/d^18)^(1/4) + sqrt(d
*x)*b*c^2) + 3*I*d^5*x^4*(b^4*c^7/d^18)^(1/4)*log(I*d^5*(b^4*c^7/d^18)^(1/
4) + sqrt(d*x)*b*c^2) - 3*I*d^5*x^4*(b^4*c^7/d^18)^(1/4)*log(-I*d^5*(b^4*c
^7/d^18)^(1/4) + sqrt(d*x)*b*c^2) - 3*d^5*x^4*(b^4*c^7/d^18)^(1/4)*log(-d^
5*(b^4*c^7/d^18)^(1/4) + sqrt(d*x)*b*c^2) - 3*d^5*x^4*(-b^4*c^7/d^18)^(1/4
)*log(d^5*(-b^4*c^7/d^18)^(1/4) + sqrt(d*x)*b*c^2) - 3*I*d^5*x^4*(-b^4*c^7
/d^18)^(1/4)*log(I*d^5*(-b^4*c^7/d^18)^(1/4) + sqrt(d*x)*b*c^2) + 3*I*d^5*
x^4*(-b^4*c^7/d^18)^(1/4)*log(-I*d^5*(-b^4*c^7/d^18)^(1/4) + sqrt(d*x)*b*c
^2) + 3*d^5*x^4*(-b^4*c^7/d^18)^(1/4)*log(-d^5*(-b^4*c^7/d^18)^(1/4) + sqr
t(d*x)*b*c^2) - (8*b*c*x^2 + 3*b*log(-(c*x^2 + 1)/(c*x^2 - 1)) + 6*a)*sqrt
(d*x))/(d^5*x^4)
```

3.89.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arctanh}(cx^2)}{(dx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**2))/(d*x)**(9/2),x)
```

output
```
Timed out
```

3.89.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 297, normalized size of antiderivative = 0.94

$$\int \frac{a + b\operatorname{arctanh}(cx^2)}{(dx)^{9/2}}\,dx =$$

$$-\frac{b\left(\frac{c\left(\frac{6\sqrt{2}c\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}+2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}d}+\frac{6\sqrt{2}c\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}c^{\frac{1}{4}}\sqrt{d}-2\sqrt{dx}\sqrt{c}\right)}{2\sqrt{\sqrt{c}d}}\right)}{\sqrt{\sqrt{c}d}d}+\frac{3\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{c}dx+\sqrt{2}\sqrt{dx}c^{\frac{1}{4}}\sqrt{d}+d\right)}{d^{\frac{3}{2}}}-\frac{3\sqrt{2}c^{\frac{3}{4}}\log\left(\sqrt{c}dx-\right.}{d^{\frac{3}{2}}}}{d^2}\right.}{42\,d}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="maxima")
```

output
```
-1/42*(b*(c*(6*sqrt(2)*c*arctan(1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) + 2*s
qrt(d*x)*sqrt(c))/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*d) + 6*sqrt(2)*c*arcta
n(-1/2*sqrt(2)*(sqrt(2)*c^(1/4)*sqrt(d) - 2*sqrt(d*x)*sqrt(c))/sqrt(sqrt(c
)*d))/(sqrt(sqrt(c)*d)*d) + 3*sqrt(2)*c^(3/4)*log(sqrt(c)*d*x + sqrt(2)*sq
rt(d*x)*c^(1/4)*sqrt(d) + d)/d^(3/2) - 3*sqrt(2)*c^(3/4)*log(sqrt(c)*d*x -
 sqrt(2)*sqrt(d*x)*c^(1/4)*sqrt(d) + d)/d^(3/2) - 12*c*arctan(sqrt(d*x)*sq
rt(c)/sqrt(sqrt(c)*d))/(sqrt(sqrt(c)*d)*d) + 6*c*log((sqrt(d*x)*sqrt(c) -
sqrt(sqrt(c)*d))/(sqrt(d*x)*sqrt(c) + sqrt(sqrt(c)*d)))/(sqrt(sqrt(c)*d)*d
) + 16/(d*x)^(3/2))/d^2 + 12*arctanh(c*x^2)/(d*x)^(7/2)) + 12*a/(d*x)^(7/2
))/d
```

3.89.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(212) = 424$.

Time = 5.38 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.64

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{9/2}}\,dx = -\frac{\frac{6\sqrt{2}(c^3d^2)^{\frac{1}{4}}bc\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}+2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d^4}+\frac{6\sqrt{2}(c^3d^2)^{\frac{1}{4}}bc\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(\frac{d^2}{c}\right)^{\frac{1}{4}}-2\sqrt{dx}\right)}{2\left(\frac{d^2}{c}\right)^{\frac{1}{4}}}\right)}{d^4}-\frac{6\sqrt{2}(-c^3d^2)^{\frac{1}{4}}bc\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\right.}{}\right.}{d^4}}{}$$

input
```
integrate((a+b*arctanh(c*x^2))/(d*x)^(9/2),x, algorithm="giac")
```

output
```
-1/42*(6*sqrt(2)*(c^3*d^2)^(1/4)*b*c*arctan(1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(
1/4) + 2*sqrt(d*x))/(d^2/c)^(1/4))/d^4 + 6*sqrt(2)*(c^3*d^2)^(1/4)*b*c*arc
tan(-1/2*sqrt(2)*(sqrt(2)*(d^2/c)^(1/4) - 2*sqrt(d*x))/(d^2/c)^(1/4))/d^4
- 6*sqrt(2)*(-c^3*d^2)^(1/4)*b*c*arctan(1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4
) + 2*sqrt(d*x))/(-d^2/c)^(1/4))/d^4 - 6*sqrt(2)*(-c^3*d^2)^(1/4)*b*c*arct
an(-1/2*sqrt(2)*(sqrt(2)*(-d^2/c)^(1/4) - 2*sqrt(d*x))/(-d^2/c)^(1/4))/d^4
 + 3*sqrt(2)*(c^3*d^2)^(1/4)*b*c*log(d*x + sqrt(2)*sqrt(d*x)*(d^2/c)^(1/4)
 + sqrt(d^2/c))/d^4 - 3*sqrt(2)*(c^3*d^2)^(1/4)*b*c*log(d*x - sqrt(2)*sqrt
(d*x)*(d^2/c)^(1/4) + sqrt(d^2/c))/d^4 - 3*sqrt(2)*(-c^3*d^2)^(1/4)*b*c*lo
g(d*x + sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2/c))/d^4 + 3*sqrt(2)*(
-c^3*d^2)^(1/4)*b*c*log(d*x - sqrt(2)*sqrt(d*x)*(-d^2/c)^(1/4) + sqrt(-d^2
/c))/d^4 + 6*b*log(-(c*d^2*x^2 + d^2)/(c*d^2*x^2 - d^2))/(sqrt(d*x)*d^3*x^
3) + 4*(4*b*c*d^2*x^2 + 3*a*d^2)/(sqrt(d*x)*d^5*x^3))/d
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arctanh}(cx^2)}{(dx)^{9/2}}\,dx = \int \frac{a+b\,\mathrm{atanh}(c\,x^2)}{(d\,x)^{9/2}}\,dx$$

input
```
int((a + b*atanh(c*x^2))/(d*x)^(9/2),x)
```

output
```
int((a + b*atanh(c*x^2))/(d*x)^(9/2), x)
```

3.90 $\int \sqrt{dx}(a+b\text{arctanh}(cx^2))^2\,dx$

3.90.1 Optimal result

Integrand size = 20, antiderivative size = 6327

$$\int \sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\text{Too large to display}$$

output
```
-8/9*a*b*x*(d*x)^(1/2)+4/9*b^2*x*ln(-c*x^2+1)*(d*x)^(1/2)+4/9*b*x*(2*a-b*l
n(-c*x^2+1))*(d*x)^(1/2)+1/6*b^2*x*ln(c*x^2+1)^2*(d*x)^(1/2)+1/3*I*b^2*pol
ylog(2,1-2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(1/4))/(1-I*(-c)
^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1+2*c^
(1/4)*(1-(-c)^(1/4)*x^(1/2))/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))
*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1/3*I*b^2*polylog(2,1-2*c^(1/4)*(1+(-c)^(1/4)
*x^(1/2))/(I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4
)/x^(1/2)+1/3*I*b^2*polylog(2,1-2*(-c)^(1/4)*(1+c^(1/4)*x^(1/2))/((-c)^(1/
4)+I*c^(1/4))/(1-I*(-c)^(1/4)*x^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2)+1/3
*I*b^2*polylog(2,1+2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*
x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1/3
*I*b^2*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*
x^(1/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)+1/3*
I*b^2*polylog(2,1+2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4
)*x^(1/2))/(-(-c)^(1/4)+I*(-c^(1/2))^(1/2)))*(d*x)^(1/2)/(-c)^(3/4)/x^(1/2
)+1/6*x*(2*a-b*ln(-c*x^2+1))^2*(d*x)^(1/2)-1/3*I*b^2*polylog(2,1-(1+I)*(1-
c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-2/3*I*
b^2*polylog(2,1-2/(1+I*c^(1/4)*x^(1/2)))*(d*x)^(1/2)/c^(3/4)/x^(1/2)-1/3*I
*b^2*polylog(2,1+(-1+I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*(d*x)^(
1/2)/c^(3/4)/x^(1/2)-2/3*I*b^2*arctan((-c)^(1/4)*x^(1/2))^2*(d*x)^(1/2)...
```

3.90.2 Mathematica [F]

$$\int \sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)^2 dx=\int \sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)^2 dx$$

input
```
Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
Integrate[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2, x]
```

3.90.3 Rubi [A] (warning: unable to verify)

Time = 11.56 (sec) , antiderivative size = 5360, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6466, 6458, 6456, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)^2 dx$$

$$\Big\downarrow \text{6466}$$

$$\frac{\sqrt{dx}\int \sqrt{x}\left(a+b\text{arctanh}\left(cx^2\right)\right)^2 dx}{\sqrt{x}}$$

$$\Big\downarrow \text{6458}$$

$$\frac{2\sqrt{dx}\int x\left(a+b\text{arctanh}\left(cx^2\right)\right)^2 d\sqrt{x}}{\sqrt{x}}$$

$$\Big\downarrow \text{6456}$$

$$\frac{2\sqrt{dx}\int \left(\frac{1}{4}x\left(2a-b\log\left(1-cx^2\right)\right)^2+\frac{1}{4}b^2x\log^2\left(cx^2+1\right)-\frac{1}{2}bx\left(b\log\left(1-cx^2\right)-2a\right)\log\left(cx^2+1\right)\right)d\sqrt{x}}{\sqrt{x}}$$

$$\Big\downarrow \text{2009}$$

$$2\sqrt{dx}\left(-\frac{i\arctan\left(\sqrt[4]{-c}\sqrt{x}\right)^2b^2}{3(-c)^{3/4}}-\frac{i\arctan\left(\sqrt[4]{c}\sqrt{x}\right)^2b^2}{3c^{3/4}}-\frac{\operatorname{arctanh}\left(\sqrt[4]{-c}\sqrt{x}\right)^2b^2}{3(-c)^{3/4}}-\frac{\operatorname{arctanh}\left(\sqrt[4]{c}\sqrt{x}\right)^2b^2}{3c^{3/4}}+\frac{1}{12}x^{3/2}\log^2\left(cx^2\right.\right.$$

input
```
Int[Sqrt[d*x]*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
(2*Sqrt[d*x]*((-4*a*b*x^(3/2))/9 - (Sqrt[2]*a*b*ArcTan[1 - Sqrt[2]*c^(1/4)
*Sqrt[x]])/(3*c^(3/4)) + (Sqrt[2]*a*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]])
/(3*c^(3/4)) - ((I/3)*b^2*ArcTan[(-c)^(1/4)*Sqrt[x]]^2)/(-c)^(3/4) - ((I/3
)*b^2*ArcTan[c^(1/4)*Sqrt[x]]^2)/c^(3/4) - (b^2*ArcTanh[(-c)^(1/4)*Sqrt[x]
]^2)/(3*(-c)^(3/4)) - (b^2*ArcTanh[c^(1/4)*Sqrt[x]]^2)/(3*c^(3/4)) + (2*b^
2*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/(3*(-c)^(3/
4)) + (2*b^2*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])
/(3*(-c)^(3/4)) - (b^2*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 -
Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-c)^(1/4
)*Sqrt[x]))])/(3*(-c)^(3/4)) - (b^2*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)
^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 -
 I*(-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)) + (b^2*ArcTan[(-c)^(1/4)*Sqrt[x]]
*Log[((1 + I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/(3*(-
c)^(3/4)) - (2*b^2*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt
[x])])/(3*(-c)^(3/4)) - (2*b^2*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)
^(1/4)*Sqrt[x])])/(3*(-c)^(3/4)) - (b^2*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-
2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4)
)*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)) - (b^2*ArcTanh[(-c)^(1/4)*Sqr
t[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] +
 (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(3*(-c)^(3/4)) + (b^2*ArcTanh[...
```

3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6456
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6458
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
With[{k = Denominator[m]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*ArcT
anh[c*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1
] && IGtQ[n, 0] && FractionQ[m]
```

rule 6466
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_)*(x_))^(m_), x_Sym
bol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a +
b*ArcTanh[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] &
& (EqQ[p, 1] || RationalQ[m, n])
```

3.90.4 Maple [F]

$$\int \sqrt{dx}\,\left(a+b\,\operatorname{arctanh}\left(c\,x^{2}\right)\right)^{2}dx$$

input
```
int((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x)
```

output
```
int((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x)
```

3.90.5 Fricas [F]

$$\int \sqrt{dx}(a+b\text{arctanh}(cx^2))^2\,dx=\int \sqrt{dx}\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x), x)
```

3.90.6 Sympy [F]

$$\int \sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int \sqrt{dx}\left(a+b\,\text{atanh}\left(cx^2\right)\right)^2\,dx$$

input
```
integrate((d*x)**(1/2)*(a+b*atanh(c*x**2))**2,x)
```

output
```
Integral(sqrt(d*x)*(a + b*atanh(c*x**2))**2, x)
```

3.90.7 Maxima [F]

$$\int \sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)^2\,dx=\int \sqrt{dx}\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2\,dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*sqrt(d)*x^(3/2)*log(-c*x^2 + 1)^2 + 1/6*a^2*c*sqrt(d)*(4*x^(3/2)/c
 - 3*(I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(3/4)
 - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(3/4))/c
) + 3*b^2*c*sqrt(d)*integrate(1/12*x^(5/2)*log(c*x^2 + 1)^2/(c*x^2 - 1), x
) - 6*b^2*c*sqrt(d)*integrate(1/12*x^(5/2)*log(c*x^2 + 1)*log(-c*x^2 + 1)/
(c*x^2 - 1), x) + 12*a*b*c*sqrt(d)*integrate(1/12*x^(5/2)*log(c*x^2 + 1)/(
c*x^2 - 1), x) - 12*a*b*c*sqrt(d)*integrate(1/12*x^(5/2)*log(-c*x^2 + 1)/(
c*x^2 - 1), x) - 8*b^2*c*sqrt(d)*integrate(1/12*x^(5/2)*log(-c*x^2 + 1)/(c
*x^2 - 1), x) + 1/2*a^2*sqrt(d)*(I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^
(1/4)*sqrt(x) + 1))/c^(3/4) - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqr
t(x) + c^(1/4)))/c^(3/4)) - 3*b^2*sqrt(d)*integrate(1/12*sqrt(x)*log(c*x^2
 + 1)^2/(c*x^2 - 1), x) + 6*b^2*sqrt(d)*integrate(1/12*sqrt(x)*log(c*x^2 +
 1)*log(-c*x^2 + 1)/(c*x^2 - 1), x) - 12*a*b*sqrt(d)*integrate(1/12*sqrt(x
)*log(c*x^2 + 1)/(c*x^2 - 1), x) + 12*a*b*sqrt(d)*integrate(1/12*sqrt(x)*l
og(-c*x^2 + 1)/(c*x^2 - 1), x)
```

3.90.8 Giac [F]

$$\int \sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int \sqrt{dx}\left(b\,\text{artanh}\left(cx^2\right)+a\right)^2 \, dx$$

input
```
integrate((d*x)^(1/2)*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x)*(b*arctanh(c*x^2) + a)^2, x)
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\left(a+b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int \sqrt{d\,x}\,\left(a+b\,\text{atanh}\left(c\,x^2\right)\right)^2 \,dx$$

input
```
int((d*x)^(1/2)*(a + b*atanh(c*x^2))^2,x)
```

output
```
int((d*x)^(1/2)*(a + b*atanh(c*x^2))^2, x)
```

3.91 $\int \frac{\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx$

3.91.1 Optimal result

Integrand size = 20, antiderivative size = 6177

$$\int \frac{\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2}{\sqrt{dx}}\,dx = \text{Too large to display}$$

output
```
2*a^2*x/(d*x)^(1/2)-b^2*polylog(2,1-2*(-c)^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/
2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/(-c)^(1/
4)/(d*x)^(1/2)+b^2*polylog(2,1-2*c^(1/4)*(1+x^(1/2)*(-c^(1/2))^(1/2))/(1+c
^(1/4)*x^(1/2))/(c^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)-b^
2*x*ln(-c*x^2+1)*ln(c*x^2+1)/(d*x)^(1/2)+I*b^2*polylog(2,1-(1+I)*(1-(-c)^(
1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+I*b
^2*polylog(2,1+(-1+I)*(1+(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2)))*x^(
1/2)/(-c)^(1/4)/(d*x)^(1/2)+I*b^2*polylog(2,1-(1+I)*(1-c^(1/4)*x^(1/2))/(1
-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+I*b^2*polylog(2,1+(-1+I)*
(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*a
*b*arctan(1+c^(1/4)*2^(1/2)*x^(1/2))*2^(1/2)*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2
*I*b^2*arctan((-c)^(1/4)*x^(1/2))^2*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*I*b^2
*arctan(c^(1/4)*x^(1/2))^2*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*I*b^2*polylog(2,1
-2/(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*I*b^2*polylo
g(2,1-2/(1+I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)+2*I*b^2*p
olylog(2,1-2/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+2*I*b^2*po
lylog(2,1-2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/c^(1/4)/(d*x)^(1/2)+1/2*b^2*x*l
n(-c*x^2+1)^2/(d*x)^(1/2)+1/2*b^2*x*ln(c*x^2+1)^2/(d*x)^(1/2)-b^2*polylog(
2,1-2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(1/4))/(1+(-c)^(1/4)*x^
(1/2)))*x^(1/2)/(-c)^(1/4)/(d*x)^(1/2)-b^2*polylog(2,1+2*c^(1/4)*(1-(-c...
```

3.91.2 Mathematica [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx^2))^2}{\sqrt{dx}}\,dx=\int \frac{(a+b\operatorname{arctanh}(cx^2))^2}{\sqrt{dx}}\,dx$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x],x]
```

output
```
Integrate[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x], x]
```

3.91.3 Rubi [A] (verified)

Time = 9.88 (sec) , antiderivative size = 5216, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6466, 6458, 6438, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{6466}$$

$$\frac{\sqrt{x}\int \frac{\left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2}{\sqrt{x}}dx}{\sqrt{dx}}$$

$$\Big\downarrow \text{6458}$$

$$\frac{2\sqrt{x}\int \left(a+b\operatorname{arctanh}\left(cx^2\right)\right)^2d\sqrt{x}}{\sqrt{dx}}$$

$$\Big\downarrow \text{6438}$$

$$\frac{2\sqrt{x}\int \left(a^2-b\log\left(1-cx^2\right)a+b\log\left(cx^2+1\right)a+\frac{1}{4}b^2\log^2\left(1-cx^2\right)+\frac{1}{4}b^2\log^2\left(cx^2+1\right)-\frac{1}{2}b^2\log\left(1-cx^2\right)\log}{\sqrt{dx}}$$

$$\Big\downarrow \text{2009}$$

$$2\sqrt{x}\left(\sqrt{x}a^2-\frac{\sqrt{2}b\arctan\left(1-\sqrt{2}\sqrt[4]{c}\sqrt{x}\right)a}{\sqrt[4]{c}}+\frac{\sqrt{2}b\arctan\left(\sqrt{2}\sqrt[4]{c}\sqrt{x}+1\right)a}{\sqrt[4]{c}}-\frac{2b\arctan\left(\sqrt[4]{c}\sqrt{x}\right)a}{\sqrt[4]{c}}-\frac{2b\text{arctanh}\left(\sqrt[4]{c}\sqrt{x}\right)a}{\sqrt[4]{c}}-\frac{b\log\left(\sqrt{\right.}}{}\right.$$

input
```
Int[(a + b*ArcTanh[c*x^2])^2/Sqrt[d*x],x]
```

output
```
(2*Sqrt[x]*(a^2*Sqrt[x] - (Sqrt[2]*a*b*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]]
)/c^(1/4) + (Sqrt[2]*a*b*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]])/c^(1/4) + (I
*b^2*ArcTan[(-c)^(1/4)*Sqrt[x]]^2)/(-c)^(1/4) - (2*a*b*ArcTan[c^(1/4)*Sqrt
[x]])/c^(1/4) + (I*b^2*ArcTan[c^(1/4)*Sqrt[x]]^2)/c^(1/4) - (b^2*ArcTanh[(
-c)^(1/4)*Sqrt[x]]^2)/(-c)^(1/4) - (2*a*b*ArcTanh[c^(1/4)*Sqrt[x]])/c^(1/4
) - (b^2*ArcTanh[c^(1/4)*Sqrt[x]]^2)/c^(1/4) + (2*b^2*ArcTanh[(-c)^(1/4)*S
qrt[x]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/(-c)^(1/4) - (2*b^2*ArcTan[(-c)^(
1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])/(-c)^(1/4) + (b^2*ArcTan[
(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*S
qrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(-c)^(1/4) + (b^
2*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]
))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/(-c)^(1/
4) - (b^2*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - (-c)^(1/4)*Sqrt[x])
)/(1 - I*(-c)^(1/4)*Sqrt[x])])/(-c)^(1/4) + (2*b^2*ArcTan[(-c)^(1/4)*Sqrt[
x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/(-c)^(1/4) - (2*b^2*ArcTanh[(-c)^(1
/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])/(-c)^(1/4) - (b^2*ArcTanh[(-
c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt
[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(-c)^(1/4) - (b^2*Ar
cTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))
/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/(-c)^(1/4)...
```

3.91.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6438
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandI
ntegrand[(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p, x], x] /; Fre
eQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

rule 6458
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
With[{k = Denominator[m]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*ArcT
anh[c*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1
] && IGtQ[n, 0] && FractionQ[m]
```

rule 6466
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_)*(x_))^(m_), x_Sym
bol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a +
b*ArcTanh[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] &
& (EqQ[p, 1] || RationalQ[m, n])
```

3.91.4 Maple [F]

$$\int \frac{\left(a+b \,\operatorname{arctanh}\left(c \,x^{2}\right)\right)^{2}}{\sqrt{d x}}d x$$

input
```
int((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x)
```

output
```
int((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x)
```

3.91.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{\sqrt{dx}}\,dx=\int \frac{\left(b \operatorname{artanh}\left(cx^{2}\right)+a\right)^{2}}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d*
x), x)
```

3.91.6 Sympy [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx^2))^2}{\sqrt{dx}}\,dx = \int \frac{(a + b\operatorname{atanh}(cx^2))^2}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))**2/(d*x)**(1/2),x)
```

output
```
Integral((a + b*atanh(c*x**2))**2/sqrt(d*x), x)
```

3.91.7 Maxima [F]

$$\int \frac{(a + b\operatorname{arctanh}(cx^2))^2}{\sqrt{dx}}\,dx = \int \frac{(b\operatorname{artanh}(cx^2) + a)^2}{\sqrt{dx}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*a^2*c*((-I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))
/c^(1/4) - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^
(1/4))/(c*sqrt(d)) - 4*sqrt(x)/(c*sqrt(d))) + b^2*c*integrate(1/4*x^(3/2)*
log(c*x^2 + 1)^2/(c*sqrt(d)*x^2 - sqrt(d)), x) - 2*b^2*c*integrate(1/4*x^(
3/2)*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) + 4*a*b*
c*integrate(1/4*x^(3/2)*log(c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) - 4*a
*b*c*integrate(1/4*x^(3/2)*log(-c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)), x) -
 8*b^2*c*integrate(1/4*x^(3/2)*log(-c*x^2 + 1)/(c*sqrt(d)*x^2 - sqrt(d)),
x) + 1/2*b^2*sqrt(x)*log(-c*x^2 + 1)^2/sqrt(d) - b^2*integrate(1/4*log(c*x
^2 + 1)^2/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) + 2*b^2*integrate(1/4*lo
g(c*x^2 + 1)*log(-c*x^2 + 1)/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) - 4*a
*b*integrate(1/4*log(c*x^2 + 1)/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x) +
4*a*b*integrate(1/4*log(-c*x^2 + 1)/((c*sqrt(d)*x^2 - sqrt(d))*sqrt(x)), x
) + 1/2*a^2*(-I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))
/c^(1/4) - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^
(1/4))/sqrt(d)
```

3.91.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{\sqrt{dx}}\, dx = \int \frac{(b\,\text{artanh}\,(cx^2) + a)^2}{\sqrt{dx}}\, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/(d*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2/sqrt(d*x), x)
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{\sqrt{dx}}\, dx = \int \frac{(a + b\,\text{atanh}(c\,x^2))^2}{\sqrt{d\,x}}\, dx$$

input
```
int((a + b*atanh(c*x^2))^2/(d*x)^(1/2),x)
```

output
```
int((a + b*atanh(c*x^2))^2/(d*x)^(1/2), x)
```

3.92 $\int \frac{(a+b\,\mathrm{arctanh}(cx^2))^2}{(dx)^{3/2}}\,dx$

3.92.1 Optimal result

Integrand size = 20, antiderivative size = 6334

$$\int \frac{(a+b\mathrm{arctanh}(cx^2))^2}{(dx)^{3/2}}\,dx = \text{Too large to display}$$

output
```
-2*a*b*ln(c*x^2+1)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(
(1-I)*(1+c^(1/4)*x^(1/2))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b
^2*c^(1/4)*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^
(1/2))/(1-I*c^(1/4)*x^(1/2))/(-c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(
d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctanh((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(
1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+(-(-c)^
(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2)
)*ln(-2*c^(1/4)*(1-x^(1/2)*(-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(-c^(1
/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(1/4)*arctan(c^(1/
4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1/2))/(1-I*c^(1/4)*x^(1
/2))/(c^(1/4)+I*(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/
4)*arctanh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1+x^(1/2)*(-(-c)^(1/2))^(1
/2))/(1+(-c)^(1/4)*x^(1/2))/((-c)^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(1/2)/d/(d
*x)^(1/2)-2*b^2*c^(1/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+x^(1/2)*(
-(-c)^(1/2))^(1/2))/(1+c^(1/4)*x^(1/2))/(c^(1/4)+(-(-c)^(1/2))^(1/2)))*x^(
1/2)/d/(d*x)^(1/2)+2*b^2*(-c)^(1/4)*arctan((-c)^(1/4)*x^(1/2))*ln(-2*(-c)^
(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))/(-(-c)^(1/4)+I
*(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)-2*b^2*(-c)^(1/4)*arctanh((-c)^(1
/4)*x^(1/2))*ln(-2*(-c)^(1/4)*(1-x^(1/2)*(-c^(1/2))^(1/2))/(1+(-c)^(1/4)*x
^(1/2))/(-(-c)^(1/4)+(-c^(1/2))^(1/2)))*x^(1/2)/d/(d*x)^(1/2)+2*b^2*c^(...
```

3.92.2 Mathematica [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{(dx)^{3/2}}\,dx = \int \frac{(a+b\text{arctanh}(cx^2))^2}{(dx)^{3/2}}\,dx$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2),x]
```

output
```
Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2), x]
```

3.92.3 Rubi [A] (verified)

Time = 9.86 (sec) , antiderivative size = 5171, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6466, 6458, 6456, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{(dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 6466}$$

$$\frac{\sqrt{x}\int \frac{(a+b\text{arctanh}(cx^2))^2}{x^{3/2}}dx}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 6458}$$

$$\frac{2\sqrt{x}\int \frac{(a+b\text{arctanh}(cx^2))^2}{x}d\sqrt{x}}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 6456}$$

$$\frac{2\sqrt{x}\int \left(\frac{(2a-b\log(1-cx^2))^2}{4x}+\frac{b^2\log^2(cx^2+1)}{4x}-\frac{b(b\log(1-cx^2)-2a)\log(cx^2+1)}{2x}\right)d\sqrt{x}}{d\sqrt{dx}}$$

$$\Big\downarrow \text{ 2009}$$

$$2\sqrt{x}\left(i\sqrt[4]{-c}\arctan\left(\sqrt[4]{-c}\sqrt{x}\right)^2b^2+i\sqrt[4]{c}\arctan\left(\sqrt[4]{c}\sqrt{x}\right)^2b^2+\sqrt[4]{-c}\text{carctanh}\left(\sqrt[4]{-c}\sqrt{x}\right)^2b^2+\sqrt[4]{c}\text{carctanh}\left(\sqrt[4]{c}\sqrt{x}\right)^2b\right.$$

input
```
Int[(a + b*ArcTanh[c*x^2])^2/(d*x)^(3/2),x]
```

output
```
(2*Sqrt[x]*(-(Sqrt[2]*a*b*c^(1/4)*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]]) + S
qrt[2]*a*b*c^(1/4)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]] + I*b^2*(-c)^(1/4)*
ArcTan[(-c)^(1/4)*Sqrt[x]]^2 + I*b^2*c^(1/4)*ArcTan[c^(1/4)*Sqrt[x]]^2 + b
^2*(-c)^(1/4)*ArcTanh[(-c)^(1/4)*Sqrt[x]]^2 + b^2*c^(1/4)*ArcTanh[c^(1/4)*
Sqrt[x]]^2 - 2*b^2*(-c)^(1/4)*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - (-c)^
(1/4)*Sqrt[x])] - 2*b^2*(-c)^(1/4)*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 - I
*(-c)^(1/4)*Sqrt[x])] + b^2*(-c)^(1/4)*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(-2*
(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*
(1 - I*(-c)^(1/4)*Sqrt[x]))] + b^2*(-c)^(1/4)*ArcTan[(-c)^(1/4)*Sqrt[x]]*L
og[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*Sqrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(
1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))] - b^2*(-c)^(1/4)*ArcTan[(-c)^(1/4)*Sqrt
[x]]*Log[((1 + I)*(1 - (-c)^(1/4)*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])] +
2*b^2*(-c)^(1/4)*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x
])] + 2*b^2*(-c)^(1/4)*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*S
qrt[x])] + b^2*(-c)^(1/4)*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(
1 - Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1
/4)*Sqrt[x]))] + b^2*(-c)^(1/4)*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1
/4)*(1 + Sqrt[-Sqrt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-
c)^(1/4)*Sqrt[x]))] - b^2*(-c)^(1/4)*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(
-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/((Sqrt[-Sqrt[c]] - (-c)^(1/4))*...
```

3.92.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6456
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6458
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
With[{k = Denominator[m]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*ArcT
anh[c*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1
] && IGtQ[n, 0] && FractionQ[m]
```

rule 6466
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_)*(x_))^(m_), x_Sym
bol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a +
b*ArcTanh[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] &
& (EqQ[p, 1] || RationalQ[m, n])
```

3.92.4 Maple [F]

$$\int \frac{(a+b\,\operatorname{arctanh}(c\,x^2))^2}{(dx)^{\frac{3}{2}}}dx$$

input
```
int((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x)
```

output
```
int((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x)
```

3.92.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx^2))^2}{(dx)^{3/2}}\,dx = \int \frac{(b\operatorname{artanh}(cx^2)+a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d^
2*x^2), x)
```

3.92.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{(dx)^{3/2}}\,dx = \int \frac{(a + b\,\text{atanh}\,(cx^2))^2}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))**2/(d*x)**(3/2),x)
```

output
```
Integral((a + b*atanh(c*x**2))**2/(d*x)**(3/2), x)
```

3.92.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{(dx)^{3/2}}\,dx = \int \frac{(b\,\text{artanh}\,(cx^2) + a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
b^2*c*integrate(1/4*x^(3/2)*log(c*x^2 + 1)^2/(c*d^(3/2)*x^3 - d^(3/2)*x),
x) - 2*b^2*c*integrate(1/4*x^(3/2)*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c*d^(3/
2)*x^3 - d^(3/2)*x), x) + 4*a*b*c*integrate(1/4*x^(3/2)*log(c*x^2 + 1)/(c*
d^(3/2)*x^3 - d^(3/2)*x), x) - 4*a*b*c*integrate(1/4*x^(3/2)*log(-c*x^2 +
1)/(c*d^(3/2)*x^3 - d^(3/2)*x), x) + 8*b^2*c*integrate(1/4*x^(3/2)*log(-c*
x^2 + 1)/(c*d^(3/2)*x^3 - d^(3/2)*x), x) + 1/2*a^2*(c*(I*(log(I*c^(1/4)*sq
rt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(3/4) - log((sqrt(c)*sqrt(x) -
 c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(3/4))/d^(3/2) - 4/(d^(3/2)*sqrt(
x))) - b^2*integrate(1/4*log(c*x^2 + 1)^2/((c*d^(3/2)*x^3 - d^(3/2)*x)*sqr
t(x)), x) + 2*b^2*integrate(1/4*log(c*x^2 + 1)*log(-c*x^2 + 1)/((c*d^(3/2)
*x^3 - d^(3/2)*x)*sqrt(x)), x) - 4*a*b*integrate(1/4*log(c*x^2 + 1)/((c*d^
(3/2)*x^3 - d^(3/2)*x)*sqrt(x)), x) + 4*a*b*integrate(1/4*log(-c*x^2 + 1)/
((c*d^(3/2)*x^3 - d^(3/2)*x)*sqrt(x)), x) - 1/2*a^2*c*(I*(log(I*c^(1/4)*sq
rt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1))/c^(3/4) - log((sqrt(c)*sqrt(x) -
 c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4)))/c^(3/4))/d^(3/2) - 1/2*b^2*log(-c*x
^2 + 1)^2/(d^(3/2)*sqrt(x))
```

3.92.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{(dx)^{3/2}}\, dx = \int \frac{(b\,\text{artanh}\,(cx^2) + a)^2}{(dx)^{\frac{3}{2}}}\, dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2/(d*x)^(3/2), x)
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{(dx)^{3/2}}\, dx = \int \frac{(a + b\,\text{atanh}(c\,x^2))^2}{(d\,x)^{3/2}}\, dx$$

input
```
int((a + b*atanh(c*x^2))^2/(d*x)^(3/2),x)
```

output
```
int((a + b*atanh(c*x^2))^2/(d*x)^(3/2), x)
```

3.93 $\int \frac{(a+b\,\mathrm{arctanh}(cx^2))^2}{(dx)^{5/2}}\,dx$

3.93.1 Optimal result

Integrand size = 20, antiderivative size = 6520

$$\int \frac{(a+b\,\mathrm{arctanh}(cx^2))^2}{(dx)^{5/2}}\,dx = \text{Too large to display}$$

output
```
-1/6*(2*a-b*ln(-c*x^2+1))^2/d^2/x/(d*x)^(1/2)+2/3*b^2*(-c)^(3/4)*arctan((-
c)^(1/4)*x^(1/2))*ln((1-I)*(1+(-c)^(1/4)*x^(1/2))/(1-I*(-c)^(1/4)*x^(1/2))
)*x^(1/2)/d^2/(d*x)^(1/2)-4/3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(2/(1
-c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arctan((-c)^
(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-I*c^(1/4))/
(1-I*(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^2*(-c)^(3/4)*arcta
nh((-c)^(1/4)*x^(1/2))*ln(2*(-c)^(1/4)*(1-c^(1/4)*x^(1/2))/((-c)^(1/4)-c^(
1/4))/(1+(-c)^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+4/3*b^2*c^(3/4)*arct
an(c^(1/4)*x^(1/2))*ln(2/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/
3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2))
/(I*(-c)^(1/4)-c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3
*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/(
I*(-c)^(1/4)+c^(1/4))/(1-I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+2/3*b
^2*c^(3/4)*arctan(c^(1/4)*x^(1/2))*ln((1+I)*(1-c^(1/4)*x^(1/2))/(1-I*c^(1/
4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-4/3*b^2*c^(3/4)*arctan(c^(1/4)*x^(1/2
))*ln(2/(1+I*c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)+4/3*b^2*c^(3/4)*arc
tanh(c^(1/4)*x^(1/2))*ln(2/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/
3*b^2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(-2*c^(1/4)*(1-(-c)^(1/4)*x^(1/2)
)/((-c)^(1/4)-c^(1/4))/(1+c^(1/4)*x^(1/2)))*x^(1/2)/d^2/(d*x)^(1/2)-2/3*b^
2*c^(3/4)*arctanh(c^(1/4)*x^(1/2))*ln(2*c^(1/4)*(1+(-c)^(1/4)*x^(1/2))/...
```

3.93.2 Mathematica [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{(dx)^{5/2}}\,dx=\int \frac{(a+b\text{arctanh}(cx^2))^2}{(dx)^{5/2}}\,dx$$

input
```
Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2),x]
```

output
```
Integrate[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2), x]
```

3.93.3 Rubi [A] (warning: unable to verify)

Time = 9.76 (sec) , antiderivative size = 5305, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6466, 6458, 6456, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{(dx)^{5/2}}\,dx$$

$$\Big\downarrow \textbf{6466}$$

$$\frac{\sqrt{x}\int \frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{x^{5/2}}dx}{d^2\sqrt{dx}}$$

$$\Big\downarrow \textbf{6458}$$

$$\frac{2\sqrt{x}\int \frac{\left(a+b\text{arctanh}\left(cx^2\right)\right)^2}{x^2}d\sqrt{x}}{d^2\sqrt{dx}}$$

$$\Big\downarrow \textbf{6456}$$

$$\frac{2\sqrt{x}\int \left(\frac{\left(2a-b\log\left(1-cx^2\right)\right)^2}{4x^2}+\frac{b^2\log^2\left(cx^2+1\right)}{4x^2}-\frac{b\left(b\log\left(1-cx^2\right)-2a\right)\log\left(cx^2+1\right)}{2x^2}\right)d\sqrt{x}}{d^2\sqrt{dx}}$$

$$\Big\downarrow \textbf{2009}$$

$$2\sqrt{x}\left(-\frac{1}{3}i(-c)^{3/4}\arctan\left(\sqrt[4]{-c}\sqrt{x}\right)^2b^2-\frac{1}{3}ic^{3/4}\arctan\left(\sqrt[4]{c}\sqrt{x}\right)^2b^2+\frac{1}{3}(-c)^{3/4}\text{arctanh}\left(\sqrt[4]{-c}\sqrt{x}\right)^2b^2+\frac{1}{3}c^{3/4}\text{arct}\right.$$

input
```
Int[(a + b*ArcTanh[c*x^2])^2/(d*x)^(5/2),x]
```

output
```
(2*Sqrt[x]*(-1/3*(Sqrt[2]*a*b*c^(3/4)*ArcTan[1 - Sqrt[2]*c^(1/4)*Sqrt[x]])
 + (Sqrt[2]*a*b*c^(3/4)*ArcTan[1 + Sqrt[2]*c^(1/4)*Sqrt[x]])/3 - (I/3)*b^2
*(-c)^(3/4)*ArcTan[(-c)^(1/4)*Sqrt[x]]^2 - (I/3)*b^2*c^(3/4)*ArcTan[c^(1/4
)*Sqrt[x]]^2 + (b^2*(-c)^(3/4)*ArcTanh[(-c)^(1/4)*Sqrt[x]]^2)/3 + (b^2*c^(
3/4)*ArcTanh[c^(1/4)*Sqrt[x]]^2)/3 - (2*b^2*(-c)^(3/4)*ArcTanh[(-c)^(1/4)*
Sqrt[x]]*Log[2/(1 - (-c)^(1/4)*Sqrt[x])])/3 + (2*b^2*(-c)^(3/4)*ArcTan[(-c
)^(1/4)*Sqrt[x]]*Log[2/(1 - I*(-c)^(1/4)*Sqrt[x])])/3 - (b^2*(-c)^(3/4)*Ar
cTan[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[c]]*Sqrt[x]))/
((I*Sqrt[-Sqrt[c]] - (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/3 - (b^2*(-
c)^(3/4)*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sqrt[c]]*
Sqrt[x]))/((I*Sqrt[-Sqrt[c]] + (-c)^(1/4))*(1 - I*(-c)^(1/4)*Sqrt[x]))])/3
 + (b^2*(-c)^(3/4)*ArcTan[(-c)^(1/4)*Sqrt[x]]*Log[((1 + I)*(1 - (-c)^(1/4)
*Sqrt[x]))/(1 - I*(-c)^(1/4)*Sqrt[x])])/3 - (2*b^2*(-c)^(3/4)*ArcTan[(-c)^
(1/4)*Sqrt[x]]*Log[2/(1 + I*(-c)^(1/4)*Sqrt[x])])/3 + (2*b^2*(-c)^(3/4)*Ar
cTanh[(-c)^(1/4)*Sqrt[x]]*Log[2/(1 + (-c)^(1/4)*Sqrt[x])])/3 + (b^2*(-c)^(
3/4)*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*(1 - Sqrt[-Sqrt[-c]]*S
qrt[x]))/((Sqrt[-Sqrt[-c]] - (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x]))])/3 + (
b^2*(-c)^(3/4)*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(2*(-c)^(1/4)*(1 + Sqrt[-Sq
rt[-c]]*Sqrt[x]))/((Sqrt[-Sqrt[-c]] + (-c)^(1/4))*(1 + (-c)^(1/4)*Sqrt[x])
)])/3 - (b^2*(-c)^(3/4)*ArcTanh[(-c)^(1/4)*Sqrt[x]]*Log[(-2*(-c)^(1/4)*...
```

3.93.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6456
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + c*x^n]/2) - b*(Log[1 - c*x^n]/2))^p
, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && IntegerQ[m]
```

3.93. $\int \frac{\left(a+b\text{arctanh}(cx^2)\right)^2}{(dx)^{5/2}}\,dx$

rule 6458
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
With[{k = Denominator[m]}, Simp[k   Subst[Int[x^(k*(m + 1) - 1)*(a + b*ArcT
anh[c*x^(k*n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1
] && IGtQ[n, 0] && FractionQ[m]
```

rule 6466
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_)*(x_))^(m_), x_Sym
bol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a +
b*ArcTanh[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] &
& (EqQ[p, 1] || RationalQ[m, n])
```

3.93.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arctanh}\,(c\,x^{2}))^{2}}{(dx)^{\frac{5}{2}}}dx$$

input
```
int((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x)
```

output
```
int((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x)
```

3.93.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arctanh}(cx^2))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\,\mathrm{artanh}\,(cx^2)+a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*sqrt(d*x)/(d^
3*x^3), x)
```

3.93.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{(dx)^{5/2}}\,dx = \int \frac{(a+b\operatorname{atanh}(cx^2))^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*atanh(c*x**2))**2/(d*x)**(5/2),x)
```

output
```
Integral((a + b*atanh(c*x**2))**2/(d*x)**(5/2), x)
```

3.93.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx^2))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\operatorname{artanh}(cx^2)+a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
3*b^2*c*integrate(1/12*x^(3/2)*log(c*x^2 + 1)^2/(c*d^(5/2)*x^4 - d^(5/2)*x
^2), x) - 6*b^2*c*integrate(1/12*x^(3/2)*log(c*x^2 + 1)*log(-c*x^2 + 1)/(c
*d^(5/2)*x^4 - d^(5/2)*x^2), x) + 12*a*b*c*integrate(1/12*x^(3/2)*log(c*x^
2 + 1)/(c*d^(5/2)*x^4 - d^(5/2)*x^2), x) - 12*a*b*c*integrate(1/12*x^(3/2)
*log(-c*x^2 + 1)/(c*d^(5/2)*x^4 - d^(5/2)*x^2), x) + 8*b^2*c*integrate(1/1
2*x^(3/2)*log(-c*x^2 + 1)/(c*d^(5/2)*x^4 - d^(5/2)*x^2), x) + 1/6*a^2*(3*(
-I*c^(3/4)*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) + 1)) - c^
(3/4)*log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4))))/d^(5/2
) - 4/(d^(5/2)*x^(3/2))) - 3*b^2*integrate(1/12*log(c*x^2 + 1)^2/((c*d^(5/
2)*x^4 - d^(5/2)*x^2)*sqrt(x)), x) + 6*b^2*integrate(1/12*log(c*x^2 + 1)*l
og(-c*x^2 + 1)/((c*d^(5/2)*x^4 - d^(5/2)*x^2)*sqrt(x)), x) - 12*a*b*integr
ate(1/12*log(c*x^2 + 1)/((c*d^(5/2)*x^4 - d^(5/2)*x^2)*sqrt(x)), x) + 12*a
*b*integrate(1/12*log(-c*x^2 + 1)/((c*d^(5/2)*x^4 - d^(5/2)*x^2)*sqrt(x)),
 x) - 1/2*a^2*c*(-I*(log(I*c^(1/4)*sqrt(x) + 1) - log(-I*c^(1/4)*sqrt(x) +
 1))/c^(1/4) - log((sqrt(c)*sqrt(x) - c^(1/4))/(sqrt(c)*sqrt(x) + c^(1/4))
)/c^(1/4))/d^(5/2) - 1/6*b^2*log(-c*x^2 + 1)^2/(d^(5/2)*x^(3/2))
```

3.93.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\,\text{artanh}\,(cx^2) + a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^2))^2/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2/(d*x)^(5/2), x)
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^2))^2}{(dx)^{5/2}}\,dx = \int \frac{(a + b\,\text{atanh}(c\,x^2))^2}{(d\,x)^{5/2}}\,dx$$

input
```
int((a + b*atanh(c*x^2))^2/(d*x)^(5/2),x)
```

output
```
int((a + b*atanh(c*x^2))^2/(d*x)^(5/2), x)
```

3.94 $\int (dx)^m \left(a+b\operatorname{arctanh}(cx^2)\right)^3 dx$

3.94.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\operatorname{arctanh}(cx^2)\right)^3 dx = \operatorname{Int}\left((dx)^m \left(a+b\operatorname{arctanh}(cx^2)\right)^3, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

3.94.2 Mathematica [N/A]

Not integrable

Time = 1.61 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\operatorname{arctanh}(cx^2)\right)^3 dx = \int (dx)^m \left(a+b\operatorname{arctanh}(cx^2)\right)^3 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^3, x]
```

3.94.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right)^3 \, dx$$

$$\Big\downarrow \text{ 6468}$$

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right)^3dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x^2])^3,x]
```

output
```
$Aborted
```

3.94.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.94.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(c\,x^2\right)\right)^3dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x^2))^3,x)
```

3.94.5 Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right)^3 \, dx = \int \left(b\,\text{artanh}\left(cx^2\right)+a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x^2)^3 + 3*a*b^2*arctanh(c*x^2)^2 + 3*a^2*b*arctan
h(c*x^2) + a^3)*(d*x)^m, x)
```

3.94.6 Sympy [N/A]

Not integrable

Time = 52.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right)^3 \, dx = \int (dx)^m \left(a+b\,\text{atanh}\left(cx^2\right)\right)^3 \, dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x**2))**3,x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c*x**2))**3, x)
```

3.94.7 Maxima [N/A]

Not integrable

Time = 2.65 (sec) , antiderivative size = 413, normalized size of antiderivative = 22.94

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right)^3 \, dx = \int \left(b\,\text{artanh}\left(cx^2\right)+a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="maxima")
```

output
```
-1/8*b^3*d^m*x*x^m*log(-c*x^2 + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1
)) + integrate(1/8*((b^3*c*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(c*x^
2 + 1)^3 + 6*(a*b^2*c*d^m*(m + 1)*x^2 - a*b^2*d^m*(m + 1))*x^m*log(c*x^2 +
 1)^2 + 12*(a^2*b*c*d^m*(m + 1)*x^2 - a^2*b*d^m*(m + 1))*x^m*log(c*x^2 + 1
) + 3*((b^3*c*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(c*x^2 + 1) - 2*(a
*b^2*d^m*(m + 1) - (a*b^2*c*d^m*(m + 1) + b^3*c*d^m)*x^2)*x^m)*log(-c*x^2
+ 1)^2 - 3*((b^3*c*d^m*(m + 1)*x^2 - b^3*d^m*(m + 1))*x^m*log(c*x^2 + 1)^2
 + 4*(a*b^2*c*d^m*(m + 1)*x^2 - a*b^2*d^m*(m + 1))*x^m*log(c*x^2 + 1) + 4*
(a^2*b*c*d^m*(m + 1)*x^2 - a^2*b*d^m*(m + 1))*x^m)*log(-c*x^2 + 1))/(c*(m
+ 1)*x^2 - m - 1), x)
```

3.94.8 Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right)^3 \, dx=\int \left(b\,\text{artanh}\left(cx^2\right)+a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^2))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^3*(d*x)^m, x)
```

3.94.9 Mupad [N/A]

Not integrable

Time = 3.79 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right)^3 \, dx=\int {\left(d\,x\right)}^m\,{\left(a+b\,\mathrm{atanh}\left(c\,x^2\right)\right)}^3 \,d x$$

input
```
int((d*x)^m*(a + b*atanh(c*x^2))^3,x)
```

output
```
int((d*x)^m*(a + b*atanh(c*x^2))^3, x)
```

3.95 $\int (dx)^m \left(a+b\,\mathrm{arctanh}(cx^2)\right)^2 dx$

3.95.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\,\mathrm{arctanh}\left(cx^2\right)\right)^2 dx = \mathrm{Int}\left((dx)^m \left(a+b\,\mathrm{arctanh}\left(cx^2\right)\right)^2, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

3.95.2 Mathematica [N/A]

Not integrable

Time = 1.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\,\mathrm{arctanh}\left(cx^2\right)\right)^2 dx = \int (dx)^m \left(a+b\,\mathrm{arctanh}\left(cx^2\right)\right)^2 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2])^2, x]
```

3.95.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{arctanh}(cx^2)\right)^2 \, dx$$
$$\Big\downarrow \ 6468$$
$$\int (dx)^m \left(a+b\text{arctanh}(cx^2)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
$Aborted
```

3.95.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.95.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b \,\text{arctanh}\left(c\,x^2\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x^2))^2,x)
```

3.95.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int \left(b \operatorname{artanh}\left(cx^2\right)+a\right)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2)*(d*x)^m, x)
```

3.95.6 Sympy [N/A]

Not integrable

Time = 33.33 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int (dx)^m \left(a+b \operatorname{atanh}\left(cx^2\right)\right)^2 \, dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x**2))**2,x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c*x**2))**2, x)
```

3.95.7 Maxima [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 236, normalized size of antiderivative = 13.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int \left(b \operatorname{artanh}\left(cx^2\right)+a\right)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*d^m*x*x^m*log(-c*x^2 + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1)
) - integrate(-1/4*((b^2*c*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(c*x^
2 + 1)^2 + 4*(a*b*c*d^m*(m + 1)*x^2 - a*b*d^m*(m + 1))*x^m*log(c*x^2 + 1)
- 2*((b^2*c*d^m*(m + 1)*x^2 - b^2*d^m*(m + 1))*x^m*log(c*x^2 + 1) - 2*(a*b
*d^m*(m + 1) - (a*b*c*d^m*(m + 1) + b^2*c*d^m)*x^2)*x^m)*log(-c*x^2 + 1))/
(c*(m + 1)*x^2 - m - 1), x)
```

3.95.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a + b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int \left(b\,\text{artanh}\left(cx^2\right) + a\right)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)^2*(d*x)^m, x)
```

3.95.9 Mupad [N/A]

Not integrable

Time = 3.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a + b\text{arctanh}\left(cx^2\right)\right)^2 \, dx = \int (d\,x)^m \left(a + b\,\text{atanh}\left(c\,x^2\right)\right)^2 \, dx$$

input
```
int((d*x)^m*(a + b*atanh(c*x^2))^2,x)
```

output
```
int((d*x)^m*(a + b*atanh(c*x^2))^2, x)
```

3.96 $\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right) \, dx$

3.96.1 Optimal result

Integrand size = 16, antiderivative size = 74

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right) \, dx=\frac{(dx)^{1+m} \left(a+b\text{arctanh}\left(cx^2\right)\right)}{d(1+m)}-\frac{2bc(dx)^{3+m} \text{Hypergeometric2F1}\left(1,\frac{3+m}{4},\frac{7+m}{4},c^2x^4\right)}{d^3(1+m)(3+m)}$$

output
```
(d*x)^(1+m)*(a+b*arctanh(c*x^2))/d/(1+m)-2*b*c*(d*x)^(3+m)*hypergeom([1, 3
/4+1/4*m],[7/4+1/4*m],c^2*x^4)/d^3/(1+m)/(3+m)
```

3.96.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right) \, dx = -\frac{x(dx)^m \left(-\left((3+m) \left(a+b\text{arctanh}\left(cx^2\right)\right)\right)+2bcx^2 \text{Hypergeometric2F1}\left(1,\frac{3+m}{4},\frac{7+m}{4},c^2x^4\right)\right)}{(1+m)(3+m)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^2]),x]
```

output
```
-((x*(d*x)^m*(-((3 + m)*(a + b*ArcTanh[c*x^2])) + 2*b*c*x^2*Hypergeometric
2F1[1, (3 + m)/4, (7 + m)/4, c^2*x^4]))/((1 + m)*(3 + m)))
```

3.96.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6464, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^2\right)\right) \, dx$$

$$\Big\downarrow \text{6464}$$

$$\frac{(dx)^{m+1} \left(a+b\text{arctanh}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc\int \frac{(dx)^{m+2}}{1-c^2x^4}dx}{d^2(m+1)}$$

$$\Big\downarrow \text{888}$$

$$\frac{(dx)^{m+1} \left(a+b\text{arctanh}\left(cx^2\right)\right)}{d(m+1)}-\frac{2bc(dx)^{m+3} \text{Hypergeometric2F1}\left(1,\frac{m+3}{4},\frac{m+7}{4},c^2x^4\right)}{d^3(m+1)(m+3)}$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x^2]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcTanh[c*x^2]))/(d*(1 + m)) - (2*b*c*(d*x)^(3 + m)*
Hypergeometric2F1[1, (3 + m)/4, (7 + m)/4, c^2*x^4])/(d^3*(1 + m)*(3 + m))
```

3.96.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.96.4 Maple [F]

$$\int (dx)^m \left(a+b\ \mathrm{arctanh}\left(c\,x^2\right)\right)dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x^2)),x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x^2)),x)
```

3.96.5 Fricas [F]

$$\int (dx)^m \left(a+b\mathrm{arctanh}\left(cx^2\right)\right)\,dx=\int \left(b\,\mathrm{artanh}\left(cx^2\right)+a\right)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x^2) + a)*(d*x)^m, x)
```

3.96.6 Sympy [F]

$$\int (dx)^m \left(a+b\mathrm{arctanh}\left(cx^2\right)\right)\,dx=\int (dx)^m\left(a+b\,\mathrm{atanh}\left(cx^2\right)\right)\,dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x**2)),x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c*x**2)), x)
```

3.96.7 Maxima [F]

$$\int (dx)^m \left(a+b\operatorname{arctanh}\left(cx^2\right)\right) dx=\int \left(b\operatorname{artanh}\left(cx^2\right)+a\right)(dx)^m dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
1/2*(4*c*d^m*integrate(x^2*x^m/(c^2*(m + 1)*x^4 - m - 1), x) + (d^m*x*x^m*
log(c*x^2 + 1) - d^m*x*x^m*log(-c*x^2 + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(
d*(m + 1))
```

3.96.8 Giac [F]

$$\int (dx)^m \left(a+b\operatorname{arctanh}\left(cx^2\right)\right) dx=\int \left(b\operatorname{artanh}\left(cx^2\right)+a\right)(dx)^m dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^2) + a)*(d*x)^m, x)
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\operatorname{arctanh}\left(cx^2\right)\right) dx=\int (d\,x)^m \left(a+b\operatorname{atanh}\left(c\,x^2\right)\right) dx$$

input
```
int((d*x)^m*(a + b*atanh(c*x^2)),x)
```

output
```
int((d*x)^m*(a + b*atanh(c*x^2)), x)
```

3.97 $\int \frac{(dx)^m}{a+b\text{arctanh}(cx^2)}\,dx$

3.97.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^2)}\,dx=\text{Int}\left(\frac{(dx)^m}{a+b\text{arctanh}(cx^2)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctanh(c*x^2)),x)
```

3.97.2 Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^2)}\,dx=\int \frac{(dx)^m}{a+b\text{arctanh}(cx^2)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2]), x]
```

3.97.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(cx^2\right)} \, dx$$

$$\Big\downarrow \; 6468$$

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(cx^2\right)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTanh[c*x^2]),x]
```

output
```
$Aborted
```

3.97.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.97.4 Maple [N/A] (verified)

Not integrable

Time = 0.12 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\,\text{arctanh}\left(c\,x^2\right)}dx$$

input
```
int((d*x)^m/(a+b*arctanh(c*x^2)),x)
```

output
```
int((d*x)^m/(a+b*arctanh(c*x^2)),x)
```

3.97.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\text{arctanh}\,(cx^2)}\,dx = \int \frac{(dx)^m}{b\,\text{artanh}\,(cx^2) + a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

3.97.6 Sympy [N/A]

Not integrable

Time = 42.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^m}{a + b\text{arctanh}\,(cx^2)}\,dx = \int \frac{(dx)^m}{a + b\,\text{atanh}\,(cx^2)}\,dx$$

input
```
integrate((d*x)**m/(a+b*atanh(c*x**2)),x)
```

output
```
Integral((d*x)**m/(a + b*atanh(c*x**2)), x)
```

3.97.7 Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\text{arctanh}\,(cx^2)}\,dx = \int \frac{(dx)^m}{b\,\text{artanh}\,(cx^2) + a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

3.97.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^2)}\,dx=\int \frac{(dx)^m}{b\,\text{artanh}(cx^2)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^2)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x^2) + a), x)
```

3.97.9 Mupad [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^2)}\,dx=\int \frac{(d\,x)^m}{a+b\,\text{atanh}(c\,x^2)}\,dx$$

input
```
int((d*x)^m/(a + b*atanh(c*x^2)),x)
```

output
```
int((d*x)^m/(a + b*atanh(c*x^2)), x)
```

3.98 $\int \frac{(dx)^m}{\left(a+b\,\textbf{arctanh}(cx^2)\right)^2}\,dx$

3.98.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{(a+b\,\text{arctanh}\,(cx^2))^2}\,dx=\text{Int}\left(\frac{(dx)^m}{(a+b\,\text{arctanh}\,(cx^2))^2},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
```

3.98.2 Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\,\text{arctanh}\,(cx^2))^2}\,dx=\int \frac{(dx)^m}{(a+b\,\text{arctanh}\,(cx^2))^2}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^2])^2, x]
```

3.98.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^2))^2} \, dx$$

$$\Big\downarrow \text{ 6468}$$

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^2))^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTanh[c*x^2])^2,x]
```

output
```
$Aborted
```

3.98.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.98.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\,\text{arctanh}\,(c\,x^2))^2}dx$$

input
```
int((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
```

output
```
int((d*x)^m/(a+b*arctanh(c*x^2))^2,x)
```

3.98.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\text{arctanh}\,(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\,\text{artanh}\,(cx^2)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arctanh(c*x^2)^2 + 2*a*b*arctanh(c*x^2) + a^2), x)
```

3.98.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+b\text{arctanh}\,(cx^2))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atanh(c*x**2))**2,x)
```

output
```
Timed out
```

3.98.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 7.44

$$\int \frac{(dx)^m}{(a+b\text{arctanh}\,(cx^2))^2}\,dx = \int \frac{(dx)^m}{(b\,\text{artanh}\,(cx^2)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="maxima")
```

output
```
(c^2*d^m*x^4 - d^m)*x^m/(b^2*c*x*log(c*x^2 + 1) - b^2*c*x*log(-c*x^2 + 1)
+ 2*a*b*c*x) + integrate(-(c^2*d^m*(m + 3)*x^4 - d^m*(m - 1))*x^m/(b^2*c*x
^2*log(c*x^2 + 1) - b^2*c*x^2*log(-c*x^2 + 1) + 2*a*b*c*x^2), x)
```

3.98.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a + b\text{arctanh}(cx^2))^2} \, dx = \int \frac{(dx)^m}{(b \, \text{artanh}(cx^2) + a)^2} \, dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^2))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x^2) + a)^2, x)
```

3.98.9 Mupad [N/A]

Not integrable

Time = 3.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a + b\text{arctanh}(cx^2))^2} \, dx = \int \frac{(d\,x)^m}{(a + b \, \text{atanh}(c\,x^2))^2} \, dx$$

input
```
int((d*x)^m/(a + b*atanh(c*x^2))^2,x)
```

output
```
int((d*x)^m/(a + b*atanh(c*x^2))^2, x)
```

3.99 $\int x^{11}(a+b\text{arctanh}(cx^3))\,dx$

3.99.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^{11}(a+b\text{arctanh}(cx^3))\,dx=\frac{bx^3}{12c^3}+\frac{bx^9}{36c}-\frac{b\text{arctanh}(cx^3)}{12c^4}+\frac{1}{12}x^{12}(a+b\text{arctanh}(cx^3))$$

output
```
1/12*b*x^3/c^3+1/36*b*x^9/c-1/12*b*arctanh(c*x^3)/c^4+1/12*x^12*(a+b*arcta
nh(c*x^3))
```

3.99.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int x^{11}(a+b\text{arctanh}(cx^3))\,dx=\frac{bx^3}{12c^3}+\frac{bx^9}{36c}+\frac{ax^{12}}{12}+\frac{1}{12}bx^{12}\text{arctanh}(cx^3)+\frac{b\log(1-cx^3)}{24c^4}-\frac{b\log(1+cx^3)}{24c^4}$$

input
```
Integrate[x^11*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(b*x^3)/(12*c^3) + (b*x^9)/(36*c) + (a*x^12)/12 + (b*x^12*ArcTanh[c*x^3])/
12 + (b*Log[1 - c*x^3])/(24*c^4) - (b*Log[1 + c*x^3])/(24*c^4)
```

3.99.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 807, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}(a+b\text{arctanh}(cx^3))\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{12}x^{12}(a+b\text{arctanh}(cx^3))-\frac{1}{4}bc\int\frac{x^{14}}{1-c^2x^6}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{12}x^{12}(a+b\text{arctanh}(cx^3))-\frac{1}{12}bc\int\frac{x^{12}}{1-c^2x^6}dx^3$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{12}x^{12}(a+b\text{arctanh}(cx^3))-\frac{1}{12}bc\int\left(-\frac{x^6}{c^2}+\frac{1}{c^4(1-c^2x^6)}-\frac{1}{c^4}\right)dx^3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{12}x^{12}(a+b\text{arctanh}(cx^3))-\frac{1}{12}bc\left(\frac{\text{arctanh}(cx^3)}{c^5}-\frac{x^3}{c^4}-\frac{x^9}{3c^2}\right)$$

input
```
Int[x^11*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(x^12*(a + b*ArcTanh[c*x^3]))/12 - (b*c*(-(x^3/c^4) - x^9/(3*c^2) + ArcTan
h[c*x^3]/c^5))/12
```

3.99.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.99.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.04

method	result	size
parallelrisch	$-\frac{-3b\,\mathrm{arctanh}(c\,x^3)x^{12}c^4-3a\,c^4x^{12}-b\,c^3x^9-3bc\,x^3+3b\,\mathrm{arctanh}(c\,x^3)}{36c^4}$	56
default	$\frac{a\,x^{12}}{12}+\frac{b\,x^{12}\,\mathrm{arctanh}(c\,x^3)}{12}+\frac{b\,x^9}{36c}+\frac{b\,x^3}{12c^3}-\frac{b\ln(c\,x^3+1)}{24c^4}+\frac{b\ln(c\,x^3-1)}{24c^4}$	66
parts	$\frac{a\,x^{12}}{12}+\frac{b\,x^{12}\,\mathrm{arctanh}(c\,x^3)}{12}+\frac{b\,x^9}{36c}+\frac{b\,x^3}{12c^3}-\frac{b\ln(c\,x^3+1)}{24c^4}+\frac{b\ln(c\,x^3-1)}{24c^4}$	66
risch	$\frac{x^{12}b\ln(c\,x^3+1)}{24}-\frac{x^{12}b\ln(-c\,x^3+1)}{24}+\frac{a\,x^{12}}{12}+\frac{b\,x^9}{36c}+\frac{b\,x^3}{12c^3}+\frac{b\ln(c\,x^3-1)}{24c^4}-\frac{b\ln(c\,x^3+1)}{24c^4}$	83

input
```
int(x^11*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
-1/36*(-3*b*arctanh(c*x^3)*x^12*c^4-3*a*c^4*x^12-b*c^3*x^9-3*b*c*x^3+3*b*a
rctanh(c*x^3))/c^4
```

3.99.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.19

$$\int x^{11}\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx=\frac{6\,ac^4x^{12}+2\,bc^3x^9+6\,bcx^3+3\,\left(bc^4x^{12}-b\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{72\,c^4}$$

input
```
integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

output
```
1/72*(6*a*c^4*x^12 + 2*b*c^3*x^9 + 6*b*c*x^3 + 3*(b*c^4*x^12 - b)*log(-(c*
x^3 + 1)/(c*x^3 - 1)))/c^4
```

3.99.6 Sympy [F(-1)]

Timed out.

$$\int x^{11}\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**11*(a+b*atanh(c*x**3)),x)
```

output
```
Timed out
```

3.99.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\begin{aligned}\int x^{11}\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx &= \frac{1}{12}\,ax^{12}\\ &\quad+\frac{1}{72}\left(6\,x^{12}\operatorname{artanh}\left(cx^3\right)+c\left(\frac{2\,\left(c^2x^9+3\,x^3\right)}{c^4}-\frac{3\,\log\left(cx^3+1\right)}{c^5}+\frac{3\,\log\left(cx^3-1\right)}{c^5}\right)\right)b\end{aligned}$$

input
```
integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

output
```
1/12*a*x^12 + 1/72*(6*x^12*arctanh(c*x^3) + c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3
*log(c*x^3 + 1)/c^5 + 3*log(c*x^3 - 1)/c^5))*b
```

3.99.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.44

$$\int x^{11}(a+b\operatorname{arctanh}(cx^3))\,dx=\frac{1}{24}\,bx^{12}\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{12}\,ax^{12}+\frac{bx^9}{36\,c}+\frac{bx^3}{12\,c^3}-\frac{b\log\left(cx^3+1\right)}{24\,c^4}+\frac{b\log\left(cx^3-1\right)}{24\,c^4}$$

input
```
integrate(x^11*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

output
```
1/24*b*x^12*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/12*a*x^12 + 1/36*b*x^9/c + 1
/12*b*x^3/c^3 - 1/24*b*log(c*x^3 + 1)/c^4 + 1/24*b*log(c*x^3 - 1)/c^4
```

3.99.9 Mupad [B] (verification not implemented)

Time = 3.85 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28

$$\int x^{11}(a+b\operatorname{arctanh}(cx^3))\,dx=\frac{a\,x^{12}}{12}+\frac{b\,x^3}{12\,c^3}+\frac{b\,x^9}{36\,c}+\frac{b\,x^{12}\,\ln\left(c\,x^3+1\right)}{24}-\frac{b\,x^{12}\,\ln\left(1-c\,x^3\right)}{24}+\frac{b\,\mathrm{atan}(c\,x^3\,1\mathrm{i})\,1\mathrm{i}}{12\,c^4}$$

input
```
int(x^11*(a + b*atanh(c*x^3)),x)
```

output
```
(a*x^12)/12 + (b*x^3)/(12*c^3) + (b*x^9)/(36*c) + (b*atan(c*x^3*1i)*1i)/(1
2*c^4) + (b*x^12*log(c*x^3 + 1))/24 - (b*x^12*log(1 - c*x^3))/24
```

3.100 $\int x^8(a + b\text{arctanh}(cx^3))\,dx$

3.100.1 Optimal result

Integrand size = 14, antiderivative size = 48

$$\int x^8(a + b\text{arctanh}(cx^3))\,dx = \frac{bx^6}{18c} + \frac{1}{9}x^9(a + b\text{arctanh}(cx^3)) + \frac{b\log(1 - c^2x^6)}{18c^3}$$

output
```
1/18*b*x^6/c+1/9*x^9*(a+b*arctanh(c*x^3))+1/18*b*ln(-c^2*x^6+1)/c^3
```

3.100.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int x^8(a + b\text{arctanh}(cx^3))\,dx = \frac{bx^6}{18c} + \frac{ax^9}{9} + \frac{1}{9}bx^9\text{arctanh}(cx^3) + \frac{b\log(1 - c^2x^6)}{18c^3}$$

input
```
Integrate[x^8*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(b*x^6)/(18*c) + (a*x^9)/9 + (b*x^9*ArcTanh[c*x^3])/9 + (b*Log[1 - c^2*x^6
])/(18*c^3)
```

3.100.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 798, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 (a+b\text{arctanh}(cx^3)) \, dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{9}x^9(a+b\text{arctanh}(cx^3))-\frac{1}{3}bc\int \frac{x^{11}}{1-c^2x^6}dx$$

$$\Big\downarrow \text{ 798}$$

$$\frac{1}{9}x^9(a+b\text{arctanh}(cx^3))-\frac{1}{18}bc\int \frac{x^6}{1-c^2x^6}dx^6$$

$$\Big\downarrow \text{ 49}$$

$$\frac{1}{9}x^9(a+b\text{arctanh}(cx^3))-\frac{1}{18}bc\int \left(-\frac{1}{c^2}-\frac{1}{c^2(c^2x^6-1)}\right)dx^6$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{9}x^9(a+b\text{arctanh}(cx^3))-\frac{1}{18}bc\left(-\frac{x^6}{c^2}-\frac{\log(1-c^2x^6)}{c^4}\right)$$

input
```
Int[x^8*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(x^9*(a + b*ArcTanh[c*x^3]))/9 - (b*c*(-(x^6/c^2) - Log[1 - c^2*x^6]/c^4))
/18
```

3.100.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.100.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{a\,x^{9}}{9}+\frac{b\,x^{9}\operatorname{arctanh}\left(c\,x^{3}\right)}{9}+\frac{b\,x^{6}}{18c}+\frac{b\ln\left(c^{2}x^{6}-1\right)}{18c^{3}}$	45
parts	$\frac{a\,x^{9}}{9}+\frac{b\,x^{9}\operatorname{arctanh}\left(c\,x^{3}\right)}{9}+\frac{b\,x^{6}}{18c}+\frac{b\ln\left(c^{2}x^{6}-1\right)}{18c^{3}}$	45
parallelrisch	$\frac{2b\operatorname{arctanh}\left(c\,x^{3}\right)x^{9}c^{3}+2a\,c^{3}x^{9}+b\,c^{2}x^{6}+2b\ln\left(c\,x^{3}-1\right)+2b\operatorname{arctanh}\left(c\,x^{3}\right)}{18c^{3}}$	59
risch	$\frac{x^{9}b\ln\left(c\,x^{3}+1\right)}{18}-\frac{x^{9}b\ln\left(-c\,x^{3}+1\right)}{18}+\frac{a\,x^{9}}{9}+\frac{b\,x^{6}}{18c}+\frac{b\ln\left(c^{2}x^{6}-1\right)}{18c^{3}}$	62

input
```
int(x^8*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/9*a*x^9+1/9*b*x^9*arctanh(c*x^3)+1/18*b*x^6/c+1/18*b/c^3*ln(c^2*x^6-1)
```

3.100.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int x^8(a+b\text{arctanh}(cx^3))\,dx=\frac{bc^3x^9\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,ac^3x^9+bc^2x^6+b\log\left(c^2x^6-1\right)}{18\,c^3}$$

input
```
integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

output
```
1/18*(b*c^3*x^9*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a*c^3*x^9 + b*c^2*x^6 +
b*log(c^2*x^6 - 1))/c^3
```

3.100.6 Sympy [F(-1)]

Timed out.

$$\int x^8(a+b\text{arctanh}(cx^3))\,dx=\text{Timed out}$$

input
```
integrate(x**8*(a+b*atanh(c*x**3)),x)
```

output
```
Timed out
```

3.100.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int x^8(a+b\text{arctanh}(cx^3))\,dx=\frac{1}{9}\,ax^9+\frac{1}{18}\left(2\,x^9\,\text{artanh}\left(cx^3\right)+\left(\frac{x^6}{c^2}+\frac{\log\left(c^2x^6-1\right)}{c^4}\right)c\right)b$$

input
```
integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

output
```
1/9*a*x^9 + 1/18*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)*
c)*b
```

3.100.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int x^8(a+b\operatorname{arctanh}(cx^3))\,dx=\frac{1}{18}\,bx^9\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{9}\,ax^9+\frac{bx^6}{18\,c}+\frac{b\log\left(c^2x^6-1\right)}{18\,c^3}$$

input
```
integrate(x^8*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

output
```
1/18*b*x^9*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/9*a*x^9 + 1/18*b*x^6/c + 1/18
*b*log(c^2*x^6 - 1)/c^3
```

3.100.9 Mupad [B] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.27

$$\int x^8(a+b\operatorname{arctanh}(cx^3))\,dx=\frac{a\,x^9}{9}+\frac{b\,\ln\left(c^2\,x^6-1\right)}{18\,c^3}+\frac{b\,x^6}{18\,c}+\frac{b\,x^9\,\ln\left(c\,x^3+1\right)}{18}-\frac{b\,x^9\,\ln\left(1-c\,x^3\right)}{18}$$

input
```
int(x^8*(a + b*atanh(c*x^3)),x)
```

output
```
(a*x^9)/9 + (b*log(c^2*x^6 - 1))/(18*c^3) + (b*x^6)/(18*c) + (b*x^9*log(c*
x^3 + 1))/18 - (b*x^9*log(1 - c*x^3))/18
```

3.101 $\int x^5(a+b\text{arctanh}(cx^3))\,dx$

3.101.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^5(a+b\text{arctanh}(cx^3))\,dx=\frac{bx^3}{6c}-\frac{b\text{arctanh}(cx^3)}{6c^2}+\frac{1}{6}x^6(a+b\text{arctanh}(cx^3))$$

output
```
1/6*b*x^3/c-1/6*b*arctanh(c*x^3)/c^2+1/6*x^6*(a+b*arctanh(c*x^3))
```

3.101.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.56

$$\int x^5(a+b\text{arctanh}(cx^3))\,dx=\frac{bx^3}{6c}+\frac{ax^6}{6}+\frac{1}{6}bx^6\text{arctanh}(cx^3)+\frac{b\log(1-cx^3)}{12c^2}-\frac{b\log(1+cx^3)}{12c^2}$$

input
```
Integrate[x^5*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(b*x^3)/(6*c) + (a*x^6)/6 + (b*x^6*ArcTanh[c*x^3])/6 + (b*Log[1 - c*x^3])/
(12*c^2) - (b*Log[1 + c*x^3])/(12*c^2)
```

3.101.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 807, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{6}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{2}bc\int \frac{x^8}{1-c^2x^6}dx$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{6}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{6}bc\int \frac{x^6}{1-c^2x^6}dx^3$$

$$\Big\downarrow \text{ 262}$$

$$\frac{1}{6}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{6}bc\left(\frac{\int \frac{1}{1-c^2x^6}dx^3}{c^2}-\frac{x^3}{c^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{6}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{6}bc\left(\frac{\text{arctanh}\left(cx^3\right)}{c^3}-\frac{x^3}{c^2}\right)$$

input
```
Int[x^5*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(x^6*(a + b*ArcTanh[c*x^3]))/6 - (b*c*(-(x^3/c^2) + ArcTanh[c*x^3]/c^3))/6
```

3.101.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.101.4 Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

method	result	size
parallelrisch	$-\frac{-\operatorname{arctanh}\left(c\,x^{3}\right) b\,c^{2} x^{6}-a\,c^{2} x^{6}-b c\,x^{3}+b\,\operatorname{arctanh}\left(c\,x^{3}\right)}{6 c^{2}}$	46
default	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6} \operatorname{arctanh}\left(c\,x^{3}\right)}{6}+\frac{b\,x^{3}}{6 c}-\frac{b \ln\left(c\,x^{3}+1\right)}{12 c^{2}}+\frac{b \ln\left(c\,x^{3}-1\right)}{12 c^{2}}$	57
parts	$\frac{a\,x^{6}}{6}+\frac{b\,x^{6} \operatorname{arctanh}\left(c\,x^{3}\right)}{6}+\frac{b\,x^{3}}{6 c}-\frac{b \ln\left(c\,x^{3}+1\right)}{12 c^{2}}+\frac{b \ln\left(c\,x^{3}-1\right)}{12 c^{2}}$	57
risch	$\frac{b\,x^{6} \ln\left(c\,x^{3}+1\right)}{12}-\frac{b\,x^{6} \ln\left(-c\,x^{3}+1\right)}{12}+\frac{a\,x^{6}}{6}+\frac{b\,x^{3}}{6 c}-\frac{b \ln\left(c\,x^{3}+1\right)}{12 c^{2}}+\frac{b \ln\left(c\,x^{3}-1\right)}{12 c^{2}}+\frac{b^{2}}{24 a\,c^{2}}$	85

input
```
int(x^5*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-arctanh(c*x^3)*b*c^2*x^6-a*c^2*x^6-b*c*x^3+b*arctanh(c*x^3))/c^2
```

3.101.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.26

$$\int x^5(a+b\text{arctanh}(cx^3))\,dx=\frac{2\,ac^2x^6+2\,bcx^3+(bc^2x^6-b)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{12\,c^2}$$

input
```
integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

output
```
1/12*(2*a*c^2*x^6 + 2*b*c*x^3 + (b*c^2*x^6 - b)*log(-(c*x^3 + 1)/(c*x^3 -
1)))/c^2
```

3.101.6 Sympy [F(-1)]

Timed out.

$$\int x^5(a+b\text{arctanh}(cx^3))\,dx=\text{Timed out}$$

input
```
integrate(x**5*(a+b*atanh(c*x**3)),x)
```

output
```
Timed out
```

3.101.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.35

$$\int x^5(a+b\text{arctanh}(cx^3))\,dx=\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\,\text{artanh}\left(cx^3\right)+c\left(\frac{2\,x^3}{c^2}-\frac{\log\left(cx^3+1\right)}{c^3}+\frac{\log\left(cx^3-1\right)}{c^3}\right)\right)b$$

input
```
integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/12*(2*x^6*arctanh(c*x^3) + c*(2*x^3/c^2 - log(c*x^3 + 1)/c^3
 + log(c*x^3 - 1)/c^3))*b
```

3.101.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(37) = 74$.

Time = 0.30 (sec) , antiderivative size = 181, normalized size of antiderivative = 4.21

$$\int x^5(a + b\text{arctanh}(cx^3))\,dx = \frac{1}{3}c\left(\frac{(cx^3+1)b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{(cx^3-1)\left(\frac{(cx^3+1)^2c^3}{(cx^3-1)^2} - \frac{2(cx^3+1)c^3}{cx^3-1} + c^3\right)} + \frac{\frac{2(cx^3+1)a}{cx^3-1} + \frac{(cx^3+1)b}{cx^3-1} - b}{\frac{(cx^3+1)^2c^3}{(cx^3-1)^2} - \frac{2(cx^3+1)c^3}{cx^3-1} + c^3}\right)$$

input
```
integrate(x^5*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

output
```
1/3*c*((c*x^3 + 1)*b*log(-(c*x^3 + 1)/(c*x^3 - 1))/((c*x^3 - 1)*((c*x^3 +
1)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) + c^3)) + (2*(c*x^3
 + 1)*a/(c*x^3 - 1) + (c*x^3 + 1)*b/(c*x^3 - 1) - b)/((c*x^3 + 1)^2*c^3/(c
*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) + c^3))
```

3.101.9 Mupad [B] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int x^5(a + b\text{arctanh}(cx^3))\,dx = \frac{a\,x^6}{6} + \frac{b\,x^3}{6\,c} + \frac{b\,x^6\,\ln\left(c\,x^3+1\right)}{12} - \frac{b\,x^6\,\ln\left(1-c\,x^3\right)}{12} + \frac{b\,\text{atan}(c\,x^3\,1\text{i})\,1\text{i}}{6\,c^2}$$

input
```
int(x^5*(a + b*atanh(c*x^3)),x)
```

output
```
(a*x^6)/6 + (b*x^3)/(6*c) + (b*atan(c*x^3*1i)*1i)/(6*c^2) + (b*x^6*log(c*x
^3 + 1))/12 - (b*x^6*log(1 - c*x^3))/12
```

3.102 $\int x^2(a + b\text{arctanh}(cx^3))\,dx$

3.102.1 Optimal result

Integrand size = 14, antiderivative size = 37

$$\int x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx=\frac{1}{3}x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}$$

output
```
1/3*x^3*(a+b*arctanh(c*x^3))+1/6*b*ln(-c^2*x^6+1)/c
```

3.102.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx=\frac{ax^3}{3}+\frac{1}{3}bx^3\text{arctanh}\left(cx^3\right)+\frac{b\log\left(1-c^2x^6\right)}{6c}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(a*x^3)/3 + (b*x^3*ArcTanh[c*x^3])/3 + (b*Log[1 - c^2*x^6])/(6*c)
```

3.102.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6452, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a + b\text{arctanh}(cx^3)) \, dx$$

$$\Big\downarrow \; 6452$$

$$\frac{1}{3}x^3 (a + b\text{arctanh}(cx^3)) - bc \int \frac{x^5}{1 - c^2x^6} dx$$

$$\Big\downarrow \; 792$$

$$\frac{1}{3}x^3 (a + b\text{arctanh}(cx^3)) + \frac{b \log (1 - c^2x^6)}{6c}$$

input
```
Int[x^2*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(x^3*(a + b*ArcTanh[c*x^3]))/3 + (b*Log[1 - c^2*x^6])/(6*c)
```

3.102.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.102.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{a\,x^{3}}{3}+\frac{b\,x^{3}\operatorname{arctanh}\left(c\,x^{3}\right)}{3}+\frac{b\ln\left(-c^{2}x^{6}+1\right)}{6c}$	37
derivativedivides	$\frac{ac\,x^{3}+b\left(c\,x^{3}\operatorname{arctanh}\left(c\,x^{3}\right)+\frac{\ln\left(-c^{2}x^{6}+1\right)}{2}\right)}{3c}$	40
default	$\frac{ac\,x^{3}+b\left(c\,x^{3}\operatorname{arctanh}\left(c\,x^{3}\right)+\frac{\ln\left(-c^{2}x^{6}+1\right)}{2}\right)}{3c}$	40
parallelrisch	$\frac{b\operatorname{arctanh}\left(c\,x^{3}\right)x^{3}c+ac\,x^{3}+b\ln\left(c\,x^{3}-1\right)+b\operatorname{arctanh}\left(c\,x^{3}\right)}{3c}$	43
risch	$\frac{b\,x^{3}\ln\left(c\,x^{3}+1\right)}{6}-\frac{b\,x^{3}\ln\left(-c\,x^{3}+1\right)}{6}+\frac{a\,x^{3}}{3}+\frac{b\ln\left(c^{2}x^{6}-1\right)}{6c}$	53

input
```
int(x^2*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/3*b*x^3*arctanh(c*x^3)+1/6*b*ln(-c^2*x^6+1)/c
```

3.102.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.35

$$\int x^2(a+b\text{arctanh}(cx^3))\,dx=\frac{bcx^3\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,acx^3+b\log\left(c^2x^6-1\right)}{6\,c}$$

input
```
integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

output
```
1/6*(b*c*x^3*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a*c*x^3 + b*log(c^2*x^6 - 1
))/c
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*atanh(c*x**3)),x)
```

output
```
Timed out
```

3.102.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx=\frac{1}{3}\,ax^3+\frac{\left(2\,cx^3\operatorname{artanh}\left(cx^3\right)+\log\left(-c^2x^6+1\right)\right)b}{6\,c}$$

input
```
integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*c*x^3*arctanh(c*x^3) + log(-c^2*x^6 + 1))*b/c
```

3.102.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(33) = 66.

Time = 0.28 (sec) , antiderivative size = 188, normalized size of antiderivative = 5.08

$$\int x^2\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx$$

$$=\frac{1}{3}\,ax^3+\frac{1}{3}\,bc\left(\frac{\log\left(\frac{\left|-cx^3-1\right|}{\left|cx^3-1\right|}\right)}{c^2}-\frac{\log\left(\left|-\frac{cx^3+1}{cx^3-1}+1\right|\right)}{c^2}+\frac{\log\left(-\frac{\frac{c\left(\frac{cx^3+1}{cx^3-1}+1\right)}{\frac{\left(cx^3+1\right)c}{cx^3-1}-c}+1}{\frac{c\left(\frac{cx^3+1}{cx^3-1}+1\right)}{\frac{\left(cx^3+1\right)c}{cx^3-1}-c}-1}\right)}{c^2\left(\frac{cx^3+1}{cx^3-1}-1\right)}\right)$$

input
```
integrate(x^2*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

output
```
1/3*a*x^3 + 1/3*b*c*(log(abs(-c*x^3 - 1)/abs(c*x^3 - 1))/c^2 - log(abs(-(c
*x^3 + 1)/(c*x^3 - 1) + 1))/c^2 + log(-(c*((c*x^3 + 1)/(c*x^3 - 1) + 1)/((
c*x^3 + 1)*c/(c*x^3 - 1) - c) + 1)/(c*((c*x^3 + 1)/(c*x^3 - 1) + 1)/((c*x^
3 + 1)*c/(c*x^3 - 1) - c) - 1))/(c^2*((c*x^3 + 1)/(c*x^3 - 1) - 1)))
```

3.102.9 Mupad [B] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int x^2(a+b\text{arctanh}(cx^3))\ dx = \frac{a\,x^3}{3}+\frac{b\,\ln\left(c^2\,x^6-1\right)}{6\,c}+\frac{b\,x^3\,\ln\left(c\,x^3+1\right)}{6}-\frac{b\,x^3\,\ln\left(1-c\,x^3\right)}{6}$$

input
```
int(x^2*(a + b*atanh(c*x^3)),x)
```

output
```
(a*x^3)/3 + (b*log(c^2*x^6 - 1))/(6*c) + (b*x^3*log(c*x^3 + 1))/6 - (b*x^3
*log(1 - c*x^3))/6
```

3.103 $\int \frac{a+b\text{arctanh}(cx^3)}{x} \, dx$

3.103.1 Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \frac{a+b\text{arctanh}(cx^3)}{x} \, dx = a\log(x) - \frac{1}{6}b\,\text{PolyLog}\left(2,-cx^3\right) + \frac{1}{6}b\,\text{PolyLog}\left(2,cx^3\right)$$

output
```
a*ln(x)-1/6*b*polylog(2,-c*x^3)+1/6*b*polylog(2,c*x^3)
```

3.103.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{a+b\text{arctanh}(cx^3)}{x} \, dx = a\log(x) + \frac{1}{6}b\left(-\text{PolyLog}\left(2,-cx^3\right) + \text{PolyLog}\left(2,cx^3\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])/x,x]
```

output
```
a*Log[x] + (b*(-PolyLog[2, -(c*x^3)] + PolyLog[2, c*x^3]))/6
```

3.103.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6450, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^3)}{x}dx$$

$$\Big\downarrow \text{ 6450}$$

$$\frac{1}{3}\int \frac{a+b\text{arctanh}(cx^3)}{x^3}dx^3$$

$$\Big\downarrow \text{ 6446}$$

$$\frac{1}{3}\left(a\log\left(x^3\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,-cx^3\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,cx^3\right)\right)$$

input
```
Int[(a + b*ArcTanh[c*x^3])/x,x]
```

output
```
(a*Log[x^3] - (b*PolyLog[2, -(c*x^3)])/2 + (b*PolyLog[2, c*x^3])/2)/3
```

3.103.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.103.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.25 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.07

method	result
default	$a\ln(x)+b\ln(x)\operatorname{arctanh}(c\,x^3)+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(c\_Z^3-1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{b\left(\_R\right.}{}$
parts	$a\ln(x)+b\ln(x)\operatorname{arctanh}(c\,x^3)+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(c\_Z^3-1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}-\frac{b\left(\_R\right.}{}$
risch	$a\ln(x)-\frac{\ln(-c\,x^3+1)\ln(x)b}{2}+\frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(c\_Z^3-1\right)}\left(\ln(x)\ln\left(\frac{\_R1-x}{\_R1}\right)+\mathrm{dilog}\left(\frac{\_R1-x}{\_R1}\right)\right)\right)}{2}+\frac{\ln(x)\ln(c\,x^3+1)}{2}$

input
```
int((a+b*arctanh(c*x^3))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*ln(x)*arctanh(c*x^3)+1/2*b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-
x)/_R1),_R1=RootOf(_Z^3*c-1))-1/2*b*sum(ln(x)*ln((_R1-x)/_R1)+dilog((_R1-x
)/_R1),_R1=RootOf(_Z^3*c+1))
```

3.103.5 Fricas [F]

$$\int \frac{a+b\text{arctanh}(cx^3)}{x}\,dx=\int \frac{b\,\text{artanh}\,(cx^3)+a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))/x,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x^3) + a)/x, x)
```

3.103.6 Sympy [F]

$$\int \frac{a + b\text{arctanh}(cx^3)}{x}\,dx = \int \frac{a + b\,\text{atanh}\,(cx^3)}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x**3))/x,x)
```

output
```
Integral((a + b*atanh(c*x**3))/x, x)
```

3.103.7 Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx^3)}{x}\,dx = \int \frac{b\,\text{artanh}\,(cx^3) + a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))/x,x, algorithm="maxima")
```

output
```
1/2*b*integrate((log(c*x^3 + 1) - log(-c*x^3 + 1))/x, x) + a*log(x)
```

3.103.8 Giac [F]

$$\int \frac{a + b\text{arctanh}(cx^3)}{x}\,dx = \int \frac{b\,\text{artanh}\,(cx^3) + a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)/x, x)
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^3)}{x}\,dx = \int \frac{a + b\,\text{atanh}(c\,x^3)}{x}\,dx$$

input

```
int((a + b*atanh(c*x^3))/x,x)
```

output

```
int((a + b*atanh(c*x^3))/x, x)
```

3.104 $\int \frac{a+b\text{arctanh}(cx^3)}{x^4} dx$

3.104.1 Optimal result

Integrand size = 14, antiderivative size = 40

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^4} dx = -\frac{a+b\text{arctanh}(cx^3)}{3x^3} + bc\log(x) - \frac{1}{6}bc\log(1-c^2x^6)$$

output
```
1/3*(-a-b*arctanh(c*x^3))/x^3+b*c*ln(x)-1/6*b*c*ln(-c^2*x^6+1)
```

3.104.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^4} dx = -\frac{a}{3x^3} - \frac{b\text{arctanh}(cx^3)}{3x^3} + bc\log(x) - \frac{1}{6}bc\log(1-c^2x^6)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*ArcTanh[c*x^3])/(3*x^3) + b*c*Log[x] - (b*c*Log[1 - c^2*x^
6])/6
```

3.104.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6452, 798, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(cx^3\right)}{x^4}dx$$

$$\Big\downarrow \text{6452}$$

$$bc\int \frac{1}{x\left(1-c^2x^6\right)}dx-\frac{a+b\text{arctanh}\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{6}bc\int \frac{1}{x^6\left(1-c^2x^6\right)}dx^6-\frac{a+b\text{arctanh}\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{6}bc\left(c^2\int \frac{1}{1-c^2x^6}dx^6+\int \frac{1}{x^6}dx^6\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{14}$$

$$\frac{1}{6}bc\left(c^2\int \frac{1}{1-c^2x^6}dx^6+\log\left(x^6\right)\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{3x^3}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{6}bc\left(\log\left(x^6\right)-\log\left(1-c^2x^6\right)\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{3x^3}$$

input
```
Int[(a + b*ArcTanh[c*x^3])/x^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c*x^3])/x^3 + (b*c*(Log[x^6] - Log[1 - c^2*x^6]))/6
```

3.104.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.104.4 Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.18

method	result	size
default	$-\frac{a}{3x^{3}}+b\left(-\frac{\operatorname{arctanh}\left(c\,x^{3}\right)}{3x^{3}}+c\left(\ln\left(x\right)-\frac{\ln\left(c\,x^{3}-1\right)}{6}-\frac{\ln\left(c\,x^{3}+1\right)}{6}\right)\right)$	47
parts	$-\frac{a}{3x^{3}}+b\left(-\frac{\operatorname{arctanh}\left(c\,x^{3}\right)}{3x^{3}}+c\left(\ln\left(x\right)-\frac{\ln\left(c\,x^{3}-1\right)}{6}-\frac{\ln\left(c\,x^{3}+1\right)}{6}\right)\right)$	47
parallelrisch	$\frac{3bc\ln\left(x\right)x^{3}-\ln\left(c\,x^{3}-1\right)x^{3}bc-b\ \operatorname{arctanh}\left(c\,x^{3}\right)x^{3}c-b\ \operatorname{arctanh}\left(c\,x^{3}\right)-a}{3x^{3}}$	56
risch	$-\frac{b\ln\left(c\,x^{3}+1\right)}{6x^{3}}+\frac{6bc\ln\left(x\right)x^{3}-bc\ln\left(c^{2}x^{6}-1\right)x^{3}+b\ln\left(-c\,x^{3}+1\right)-2a}{6x^{3}}$	62

input
```
int((a+b*arctanh(c*x^3))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+b*(-1/3/x^3*arctanh(c*x^3)+c*(ln(x)-1/6*ln(c*x^3-1)-1/6*ln(c*x^
3+1)))
```

3.104.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^4}\,dx = -\frac{bcx^3\log\left(c^2x^6-1\right)-6\,bcx^3\log\left(x\right)+b\log\left(-\frac{cx^3+1}{cx^3-1}\right)+2\,a}{6\,x^3}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(b*c*x^3*log(c^2*x^6 - 1) - 6*b*c*x^3*log(x) + b*log(-(c*x^3 + 1)/(c*
x^3 - 1)) + 2*a)/x^3
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))/x**4,x)
```

output
```
Timed out
```

3.104.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^4}\,dx = -\frac{1}{6}\left(c\left(\log\left(c^2x^6-1\right)-\log\left(x^6\right)\right)+\frac{2\,\text{artanh}\left(cx^3\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="maxima")
```

output
```
-1/6*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*b - 1/3*a/x^
3
```

3.104.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^4} \, dx = -\frac{1}{6} bc \log \left(c^2 x^6 - 1\right) + bc \log \left(x\right) - \frac{b \log \left(-\frac{cx^3+1}{cx^3-1}\right)}{6 \, x^3} - \frac{a}{3 \, x^3}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^4,x, algorithm="giac")
```

output
```
-1/6*b*c*log(c^2*x^6 - 1) + b*c*log(x) - 1/6*b*log(-(c*x^3 + 1)/(c*x^3 - 1
))/x^3 - 1/3*a/x^3
```

3.104.9 Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.38

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^4} \, dx = b \, c \, \ln \left(x\right) - \frac{a}{3 \, x^3} - \frac{b \, c \, \ln \left(c^2 \, x^6 - 1\right)}{6} - \frac{b \, \ln \left(c \, x^3 + 1\right)}{6 \, x^3} + \frac{b \, \ln \left(1 - c \, x^3\right)}{6 \, x^3}$$

input
```
int((a + b*atanh(c*x^3))/x^4,x)
```

output
```
b*c*log(x) - a/(3*x^3) - (b*c*log(c^2*x^6 - 1))/6 - (b*log(c*x^3 + 1))/(6*
x^3) + (b*log(1 - c*x^3))/(6*x^3)
```

3.105 $\int \frac{a+b\text{arctanh}(cx^3)}{x^7}\,dx$

3.105.1 Optimal result

Integrand size = 14, antiderivative size = 41

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^7}\,dx = -\frac{bc}{6x^3}+\frac{1}{6}bc^2\text{arctanh}(cx^3)-\frac{a+b\text{arctanh}(cx^3)}{6x^6}$$

output
```
-1/6*b*c/x^3+1/6*b*c^2*arctanh(c*x^3)+1/6*(-a-b*arctanh(c*x^3))/x^6
```

3.105.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^7}\,dx = -\frac{a}{6x^6}-\frac{bc}{6x^3}-\frac{b\text{arctanh}(cx^3)}{6x^6} -\frac{1}{12}bc^2\log\left(1-cx^3\right)+\frac{1}{12}bc^2\log\left(1+cx^3\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])/x^7,x]
```

output
```
-1/6*a/x^6 - (b*c)/(6*x^3) - (b*ArcTanh[c*x^3])/(6*x^6) - (b*c^2*Log[1 - c
*x^3])/12 + (b*c^2*Log[1 + c*x^3])/12
```

3.105.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 807, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^7}dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}bc\int \frac{1}{x^4(1-c^2x^6)}dx-\frac{a+b\text{arctanh}(cx^3)}{6x^6}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{1}{6}bc\int \frac{1}{x^6(1-c^2x^6)}dx^3-\frac{a+b\text{arctanh}(cx^3)}{6x^6}$$

$$\Big\downarrow \text{ 264}$$

$$\frac{1}{6}bc\left(c^2\int \frac{1}{1-c^2x^6}dx^3-\frac{1}{x^3}\right)-\frac{a+b\text{arctanh}(cx^3)}{6x^6}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{6}bc\left(c\text{arctanh}(cx^3)-\frac{1}{x^3}\right)-\frac{a+b\text{arctanh}(cx^3)}{6x^6}$$

input
```
Int[(a + b*ArcTanh[c*x^3])/x^7,x]
```

output
```
-1/6*(a + b*ArcTanh[c*x^3])/x^6 + (b*c*(-x^(-3) + c*ArcTanh[c*x^3]))/6
```

3.105.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.105.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

method	result	size
parallelrisch	$-\frac{-\operatorname{arctanh}\left(c\,x^{3}\right) b\,c^{2} x^{6}+a\,c^{2} x^{6}+b c\,x^{3}+b \,\operatorname{arctanh}\left(c\,x^{3}\right)+a}{6 x^{6}}$	45
default	$-\frac{a}{6 x^{6}}-\frac{b \,\operatorname{arctanh}\left(c\,x^{3}\right)}{6 x^{6}}-\frac{b c}{6 x^{3}}-\frac{b\,c^{2} \ln \left(c\,x^{3}-1\right)}{12}+\frac{b\,c^{2} \ln \left(c\,x^{3}+1\right)}{12}$	55
parts	$-\frac{a}{6 x^{6}}-\frac{b \,\operatorname{arctanh}\left(c\,x^{3}\right)}{6 x^{6}}-\frac{b c}{6 x^{3}}-\frac{b\,c^{2} \ln \left(c\,x^{3}-1\right)}{12}+\frac{b\,c^{2} \ln \left(c\,x^{3}+1\right)}{12}$	55
risch	$-\frac{b \ln \left(c\,x^{3}+1\right)}{12 x^{6}}+\frac{b\,c^{2} \ln \left(c\,x^{3}+1\right) x^{6}-b\,c^{2} \ln \left(c\,x^{3}-1\right) x^{6}-2 b c\,x^{3}+b \ln \left(-c\,x^{3}+1\right)-2 a}{12 x^{6}}$	76

input
```
int((a+b*arctanh(c*x^3))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-arctanh(c*x^3)*b*c^2*x^6+a*c^2*x^6+b*c*x^3+b*arctanh(c*x^3)+a)/x^6
```

3.105.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^7}\,dx = -\frac{2\,bcx^3 - (bc^2x^6 - b)\log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2\,a}{12\,x^6}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="fricas")
```

output
```
-1/12*(2*b*c*x^3 - (b*c^2*x^6 - b)*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^
6
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))/x**7,x)
```

output
```
Timed out
```

3.105.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.24

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^7}\,dx = \frac{1}{12}\left(\left(c\log\left(cx^3+1\right) - c\log\left(cx^3-1\right) - \frac{2}{x^3}\right)c - \frac{2\,\text{artanh}\left(cx^3\right)}{x^6}\right)b - \frac{a}{6\,x^6}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="maxima")
```

output
```
1/12*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x
^6)*b - 1/6*a/x^6
```

3.105.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.63

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^7}\,dx = \frac{1}{12}\,bc^2\log\left(cx^3+1\right) - \frac{1}{12}\,bc^2\log\left(cx^3-1\right) - \frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{12\,x^6} - \frac{bcx^3+a}{6\,x^6}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^7,x, algorithm="giac")
```

output
```
1/12*b*c^2*log(c*x^3 + 1) - 1/12*b*c^2*log(c*x^3 - 1) - 1/12*b*log(-(c*x^3
 + 1)/(c*x^3 - 1))/x^6 - 1/6*(b*c*x^3 + a)/x^6
```

3.105.9 Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.27

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^7}\,dx = \frac{b\,c^2\,\text{atanh}(c\,x^3)}{6} - \frac{\frac{a}{6}+\frac{b\,\ln(c\,x^3+1)}{12}-\frac{b\,\ln(1-c\,x^3)}{12}+\frac{b\,c\,x^3}{6}}{x^6}$$

input
```
int((a + b*atanh(c*x^3))/x^7,x)
```

output
```
(b*c^2*atanh(c*x^3))/6 - (a/6 + (b*log(c*x^3 + 1))/12 - (b*log(1 - c*x^3))
/12 + (b*c*x^3)/6)/x^6
```

3.106 $\int \frac{a+b\text{arctanh}(cx^3)}{x^{10}}\,dx$

3.106.1 Optimal result

Integrand size = 14, antiderivative size = 56

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^{10}}\,dx=-\frac{bc}{18x^6}-\frac{a+b\text{arctanh}(cx^3)}{9x^9}+\frac{1}{3}bc^3\log(x)-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)$$

output
```
-1/18*b*c/x^6+1/9*(-a-b*arctanh(c*x^3))/x^9+1/3*b*c^3*ln(x)-1/18*b*c^3*ln(
-c^2*x^6+1)
```

3.106.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^{10}}\,dx=-\frac{a}{9x^9}-\frac{bc}{18x^6}-\frac{b\text{arctanh}(cx^3)}{9x^9}+\frac{1}{3}bc^3\log(x)-\frac{1}{18}bc^3\log\left(1-c^2x^6\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])/x^10,x]
```

output
```
-1/9*a/x^9 - (b*c)/(18*x^6) - (b*ArcTanh[c*x^3])/(9*x^9) + (b*c^3*Log[x])/
3 - (b*c^3*Log[1 - c^2*x^6])/18
```

3.106.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 798, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(cx^3\right)}{x^{10}}dx$$

$$\Big\downarrow \ 6452$$

$$\frac{1}{3}bc\int \frac{1}{x^7\left(1-c^2x^6\right)}dx-\frac{a+b\text{arctanh}\left(cx^3\right)}{9x^9}$$

$$\Big\downarrow \ 798$$

$$\frac{1}{18}bc\int \frac{1}{x^{12}\left(1-c^2x^6\right)}dx^6-\frac{a+b\text{arctanh}\left(cx^3\right)}{9x^9}$$

$$\Big\downarrow \ 54$$

$$\frac{1}{18}bc\int \left(-\frac{c^4}{c^2x^6-1}+\frac{c^2}{x^6}+\frac{1}{x^{12}}\right)dx^6-\frac{a+b\text{arctanh}\left(cx^3\right)}{9x^9}$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{18}bc\left(c^2\log\left(x^6\right)-c^2\log\left(1-c^2x^6\right)-\frac{1}{x^6}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{9x^9}$$

input
```
Int[(a + b*ArcTanh[c*x^3])/x^10,x]
```

output
```
-1/9*(a + b*ArcTanh[c*x^3])/x^9 + (b*c*(-x^(-6) + c^2*Log[x^6] - c^2*Log[1
 - c^2*x^6]))/18
```

3.106.3.1 Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.106.4 Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{a}{9x^{9}}+b\left(-\frac{\operatorname{arctanh}\left(c\,x^{3}\right)}{9x^{9}}+\frac{c\left(-\frac{1}{6x^{6}}+c^{2}\ln\left(x\right)-\frac{c^{2}\ln\left(c\,x^{3}-1\right)}{6}-\frac{c^{2}\ln\left(c\,x^{3}+1\right)}{6}\right)}{3}\right)$	63
parts	$-\frac{a}{9x^{9}}+b\left(-\frac{\operatorname{arctanh}\left(c\,x^{3}\right)}{9x^{9}}+\frac{c\left(-\frac{1}{6x^{6}}+c^{2}\ln\left(x\right)-\frac{c^{2}\ln\left(c\,x^{3}-1\right)}{6}-\frac{c^{2}\ln\left(c\,x^{3}+1\right)}{6}\right)}{3}\right)$	63
risch	$-\frac{b\ln\left(c\,x^{3}+1\right)}{18x^{9}}+\frac{6b\,c^{3}\ln\left(x\right)x^{9}-b\,c^{3}\ln\left(c^{2}x^{6}-1\right)x^{9}-bc\,x^{3}+b\ln\left(-c\,x^{3}+1\right)-2a}{18x^{9}}$	73
parallelrisch	$\frac{6b\,c^{3}\ln\left(x\right)x^{9}-2\ln\left(c\,x^{3}-1\right)x^{9}b\,c^{3}-2b\operatorname{arctanh}\left(c\,x^{3}\right)x^{9}c^{3}-b\,c^{3}x^{9}-bc\,x^{3}-2b\operatorname{arctanh}\left(c\,x^{3}\right)-2a}{18x^{9}}$	78

input
```
int((a+b*arctanh(c*x^3))/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a/x^9+b*(-1/9/x^9*arctanh(c*x^3)+1/3*c*(-1/6/x^6+c^2*ln(x)-1/6*c^2*ln
(c*x^3-1)-1/6*c^2*ln(c*x^3+1)))
```

3.106.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^{10}}\,dx = -\frac{bc^3x^9\log\left(c^2x^6-1\right) - 6\,bc^3x^9\log\left(x\right) + bcx^3 + b\log\left(-\frac{cx^3+1}{cx^3-1}\right) + 2\,a}{18\,x^9}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="fricas")
```

output
```
-1/18*(b*c^3*x^9*log(c^2*x^6 - 1) - 6*b*c^3*x^9*log(x) + b*c*x^3 + b*log(-
(c*x^3 + 1)/(c*x^3 - 1)) + 2*a)/x^9
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^{10}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))/x**10,x)
```

output
```
Timed out
```

3.106.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^{10}}\,dx = -\frac{1}{18}\left(\left(c^2\log\left(c^2x^6-1\right) - c^2\log\left(x^6\right) + \frac{1}{x^6}\right)c + \frac{2\,\text{artanh}\left(cx^3\right)}{x^9}\right)b - \frac{a}{9\,x^9}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="maxima")
```

output
```
-1/18*((c^2*log(c^2*x^6 - 1) - c^2*log(x^6) + 1/x^6)*c + 2*arctanh(c*x^3)/
x^9)*b - 1/9*a/x^9
```

3.106.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^{10}}\,dx = -\frac{1}{18}\,bc^3\log\left(c^2x^6-1\right)+\frac{1}{3}\,bc^3\log\left(x\right)-\frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{18\,x^9}-\frac{bcx^3+2\,a}{18\,x^9}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^10,x, algorithm="giac")
```

output
```
-1/18*b*c^3*log(c^2*x^6 - 1) + 1/3*b*c^3*log(x) - 1/18*b*log(-(c*x^3 + 1)/
(c*x^3 - 1))/x^9 - 1/18*(b*c*x^3 + 2*a)/x^9
```

3.106.9 Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^{10}}\,dx = \frac{b\,c^3\,\ln\left(x\right)}{3}-\frac{b\,c^3\,\ln\left(c^2\,x^6-1\right)}{18}-\frac{a}{9\,x^9} -\frac{b\,c}{18\,x^6}-\frac{b\,\ln\left(c\,x^3+1\right)}{18\,x^9}+\frac{b\,\ln\left(1-c\,x^3\right)}{18\,x^9}$$

input
```
int((a + b*atanh(c*x^3))/x^10,x)
```

output
```
(b*c^3*log(x))/3 - (b*c^3*log(c^2*x^6 - 1))/18 - a/(9*x^9) - (b*c)/(18*x^6
) - (b*log(c*x^3 + 1))/(18*x^9) + (b*log(1 - c*x^3))/(18*x^9)
```

3.107 $\int x^3(a + b\,\mathrm{arctanh}(cx^3))\,dx$

3.107.1 Optimal result

Integrand size = 14, antiderivative size = 174

$$\int x^3(a+b\,\mathrm{arctanh}(cx^3))\,dx = \frac{3bx}{4c} + \frac{\sqrt{3}b\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{\sqrt{3}b\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8c^{4/3}} - \frac{b\,\mathrm{arctanh}(\sqrt[3]{c}x)}{4c^{4/3}} + \frac{1}{4}x^4(a+b\,\mathrm{arctanh}(cx^3)) + \frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}} - \frac{b\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}$$

output
```
3/4*b*x/c-1/4*b*arctanh(c^(1/3)*x)/c^(4/3)+1/4*x^4*(a+b*arctanh(c*x^3))+1/
16*b*ln(1-c^(1/3)*x+c^(2/3)*x^2)/c^(4/3)-1/16*b*ln(1+c^(1/3)*x+c^(2/3)*x^2
)/c^(4/3)-1/8*b*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)
-1/8*b*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(4/3)
```

3.107.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.13

$$\int x^3(a+b\text{arctanh}(cx^3))\,dx=\frac{3bx}{4c}+\frac{ax^4}{4}-\frac{\sqrt{3}b\arctan\left(\frac{-1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8c^{4/3}}-\frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{8c^{4/3}}+\frac{1}{4}bx^4\text{arctanh}(cx^3)+\frac{b\log\left(1-\sqrt[3]{c}x\right)}{8c^{4/3}}-\frac{b\log\left(1+\sqrt[3]{c}x\right)}{8c^{4/3}}+\frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}-\frac{b\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)}{16c^{4/3}}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(3*b*x)/(4*c) + (a*x^4)/4 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])
/(8*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(8*c^(4/3)) +
 (b*x^4*ArcTanh[c*x^3])/4 + (b*Log[1 - c^(1/3)*x])/(8*c^(4/3)) - (b*Log[1
+ c^(1/3)*x])/(8*c^(4/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/
3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(16*c^(4/3))
```

3.107.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6452, 843, 754, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+b\text{arctanh}(cx^3))\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^3))-\frac{3}{4}bc\int\frac{x^6}{1-c^2x^6}dx$$

$$\Big\downarrow \text{ 843}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^3))-\frac{3}{4}bc\left(\frac{\int \frac{1}{1-c^2x^6}dx}{c^2}-\frac{x}{c^2}\right)$$

$\downarrow$ 754

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^3))-\frac{3}{4}bc\left(\frac{\frac{1}{3}\int \frac{1}{1-c^{2/3}x^2}dx+\frac{1}{3}\int \frac{2-\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}dx+\frac{1}{3}\int \frac{\sqrt[3]{c}x+2}{2\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}dx}{c^2}-\frac{x}{c^2}\right)$$

$\downarrow$ 27

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^3))-\frac{3}{4}bc\left(\frac{\frac{1}{3}\int \frac{1}{1-c^{2/3}x^2}dx+\frac{1}{6}\int \frac{2-\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{1}{6}\int \frac{\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{c^2}-\frac{x}{c^2}\right)$$

$\downarrow$ 219

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^3))-\frac{3}{4}bc\left(\frac{\frac{1}{6}\int \frac{2-\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{1}{6}\int \frac{\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3\sqrt[3]{c}}}{c^2}-\frac{x}{c^2}\right)$$

$\downarrow$ 1142

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^3))-\frac{3}{4}bc\left(\frac{\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{\int -\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx+\frac{\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+1\right)}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{\text{arctanh}(}{3\sqrt[3]{c}}}{c^2}\right.$$

$\downarrow$ 25

3.107. $\int x^3(a+b\text{arctanh}(cx^3))\,dx$

$$\frac{1}{4}x^4(a+b\operatorname{arctanh}(cx^3))-$$

$$\frac{3}{4}bc\left(\frac{\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{\int\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx+\frac{\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+1\right)}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{\operatorname{arctanh}(\sqrt[3]{\ldots}}{3\sqrt[3]{c}}}{c^2}\right.$$

$\downarrow$ 27

$$\frac{1}{4}x^4(a+b\operatorname{arctanh}(cx^3))-$$

$$\frac{3}{4}bc\left(\frac{\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx\right)+\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx+\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx\right)+\frac{\operatorname{arctan}\ldots}{3\ldots}}{c^2}\right.$$

$\downarrow$ 1082

$$\frac{1}{4}x^4(a+b\operatorname{arctanh}(cx^3))-$$

$$\frac{3}{4}bc\left(\frac{\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{3\int\frac{1}{-\left(1-2\sqrt[3]{c}x\right)^2-3}d\left(1-2\sqrt[3]{c}x\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{3\int\frac{1}{-\left(2\sqrt[3]{c}x+1\right)^2-3}d\left(2\sqrt[3]{\ldots}\right.}{\sqrt[3]{c}}\right.}{c^2}\right.$$

$\downarrow$ 217

$$\frac{1}{4}x^4(a+b\operatorname{arctanh}(cx^3))-$$

$$\frac{3}{4}bc\left(\frac{\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}x+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}\right)+\frac{\operatorname{arctanh}\ldots}{3\sqrt[3]{\ldots}}}{c^2}\right.$$

$\downarrow$ 1103

3.107. $\int x^3(a+b\operatorname{arctanh}(cx^3))\,dx$

$$\frac{1}{4}x^4(a+b\text{arctanh}(cx^3)) - \frac{3}{4}bc\left(\frac{\frac{1}{6}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}x+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}+\frac{\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}\right)+\frac{\text{arctanh}(}{3\sqrt[3]{c}}}{c^2}\right.$$

input
```
Int[x^3*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(x^4*(a + b*ArcTanh[c*x^3]))/4 - (3*b*c*(-(x/c^2) + (ArcTanh[c^(1/3)*x]/(3
*c^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*x)/Sqrt[3]])/c^(1/3)) - Log[
1 - c^(1/3)*x + c^(2/3)*x^2]/(2*c^(1/3)))/6 + ((Sqrt[3]*ArcTan[(1 + 2*c^(1
/3)*x)/Sqrt[3]])/c^(1/3) + Log[1 + c^(1/3)*x + c^(2/3)*x^2]/(2*c^(1/3)))/6
)/c^2))/4
```

3.107.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.107. $\int x^3(a + b\text{arctanh}(cx^3))\,dx$

3.107.4 Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.06

method	result
default	$\frac{a\,x^4}{4}+\frac{b\,x^4\,\mathrm{arctanh}(c\,x^3)}{4}+\frac{3bx}{4c}+\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln(}{8c}$
parts	$\frac{a\,x^4}{4}+\frac{b\,x^4\,\mathrm{arctanh}(c\,x^3)}{4}+\frac{3bx}{4c}+\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln(}{8c}$
risch	$\frac{b\,x^4\ln(c\,x^3+1)}{8}+\frac{a\,x^4}{4}-\frac{b\,x^4\ln(-c\,x^3+1)}{8}+\frac{3bx}{4c}+\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)}\right.}{3}\right.}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}$

input
```
int(x^3*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/4*b*x^4*arctanh(c*x^3)+3/4*b*x/c+1/8*b/c^2/(1/c)^(2/3)*ln(x-(1
/c)^(1/3))-1/16*b/c^2/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b/
c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/8*b/c^2/
(1/c)^(2/3)*ln(x+(1/c)^(1/3))+1/16*b/c^2/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+
(1/c)^(2/3))-1/8*b/c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/
3)*x-1))
```

3.107.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 981, normalized size of antiderivative = 5.64

$$\int x^3\left(a+b\mathrm{arctanh}\left(cx^3\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

output
```
[1/16*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 + sqrt(3)*b
*c*sqrt((-c)^(1/3)/c)*log((2*c*x^3 - sqrt(3)*(2*c*x^2 + (-c)^(2/3)*x + (-c
)^(1/3))*sqrt((-c)^(1/3)/c) + 3*(-c)^(1/3)*x - 1)/(c*x^3 + 1)) + sqrt(3)*b
*c*sqrt(-1/c^(2/3))*log((2*c*x^3 - sqrt(3)*(2*c*x^2 - c^(2/3)*x - c^(1/3))
*sqrt(-1/c^(2/3)) - 3*c^(1/3)*x + 1)/(c*x^3 - 1)) + 12*b*c*x + b*(-c)^(2/3
)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x
 + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x -
 c^(2/3)))/c^2, 1/16*(2*b*c^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*
x^4 - 2*sqrt(3)*b*c*sqrt(-(-c)^(1/3)/c)*arctan(1/3*sqrt(3)*(2*(-c)^(2/3)*x
 + (-c)^(1/3))*sqrt(-(-c)^(1/3)/c)) + sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*
c*x^3 - sqrt(3)*(2*c*x^2 - c^(2/3)*x - c^(1/3))*sqrt(-1/c^(2/3)) - 3*c^(1/
3)*x + 1)/(c*x^3 - 1)) + 12*b*c*x + b*(-c)^(2/3)*log(c*x^2 - (-c)^(2/3)*x
- (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(2/3
)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c*x - c^(2/3)))/c^2, 1/16*(2*b*c
^2*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^4 + sqrt(3)*b*c*sqrt((-c)
^(1/3)/c)*log((2*c*x^3 - sqrt(3)*(2*c*x^2 + (-c)^(2/3)*x + (-c)^(1/3))*sqr
t((-c)^(1/3)/c) + 3*(-c)^(1/3)*x - 1)/(c*x^3 + 1)) - 2*sqrt(3)*b*c^(2/3)*a
rctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3)) + 12*b*c*x + b*(-c)^(2/
3)*log(c*x^2 - (-c)^(2/3)*x - (-c)^(1/3)) - b*c^(2/3)*log(c*x^2 + c^(2/3)*
x + c^(1/3)) - 2*b*(-c)^(2/3)*log(c*x + (-c)^(2/3)) + 2*b*c^(2/3)*log(c...
```

3.107.6 Sympy [F(-1)]

Timed out.

$$\int x^3(a+b\text{arctanh}(cx^3))\,dx=\text{Timed out}$$

input
```
integrate(x**3*(a+b*atanh(c*x**3)),x)
```

output
```
Timed out
```

3.107.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.93

$$\int x^3 \left(a + b\text{arctanh}\left(cx^3\right)\right) \, dx = \frac{1}{4}\,ax^4 + \frac{1}{16}\left(4\,x^4\,\text{artanh}\left(cx^3\right) - c\left(\frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x + c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}} + \frac{2\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x - c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}} - \frac{12\,x}{c^2} + \frac{\log}{}\right.\right.$$

input
```
integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/16*(4*x^4*arctanh(c*x^3) - c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(
2*c^(2/3)*x + c^(1/3))/c^(1/3))/c^(7/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*
c^(2/3)*x - c^(1/3))/c^(1/3))/c^(7/3) - 12*x/c^2 + log(c^(2/3)*x^2 + c^(1/
3)*x + 1)/c^(7/3) - log(c^(2/3)*x^2 - c^(1/3)*x + 1)/c^(7/3) + 2*log((c^(1
/3)*x + 1)/c^(1/3))/c^(7/3) - 2*log((c^(1/3)*x - 1)/c^(1/3))/c^(7/3)))*b
```

3.107.8 Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.19

$$\int x^3 \left(a + b\text{arctanh}\left(cx^3\right)\right) \, dx = \frac{1}{16}\,bc^7\left(\frac{2\left(-\frac{1}{c}\right)^{\frac{1}{3}}\log\left(\left|x - \left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{c^8} - \frac{2\,\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\,\sqrt{3}c^{\frac{1}{3}}\left(2\,x + \frac{1}{c^{\frac{1}{3}}}\right)\right)}{c^9} - \frac{2\,\sqrt{3}\left(-c^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\right.}{}\right.}{c^9}\right.$$
$$+ \frac{1}{8}\,bx^4\log\left(-\frac{cx^3 + 1}{cx^3 - 1}\right) + \frac{1}{4}\,ax^4 + \frac{3\,bx}{4\,c}$$

input
```
integrate(x^3*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

output
```
1/16*b*c^7*(2*(-1/c)^(1/3)*log(abs(x - (-1/c)^(1/3)))/c^8 - 2*sqrt(3)*abs(
c)^(2/3)*arctan(1/3*sqrt(3)*c^(1/3)*(2*x + 1/c^(1/3)))/c^9 - 2*sqrt(3)*(-c
^2)^(1/3)*arctan(1/3*sqrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(1/3))/c^9 - abs(
c)^(2/3)*log(x^2 + x/c^(1/3) + 1/c^(2/3))/c^9 + 2*log(abs(x - 1/c^(1/3)))/
c^(25/3) - (-c^2)^(1/3)*log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/c^9) + 1/
8*b*x^4*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/4*a*x^4 + 3/4*b*x/c
```

3.107.9 Mupad [B] (verification not implemented)

Time = 4.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.72

$$\int x^3(a+b\text{arctanh}(cx^3))\,dx = \frac{a\,x^4}{4}+\frac{b\left(-\frac{\text{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\text{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\text{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{4\,c^{4/3}} + \frac{3\,b\,x}{4\,c}+\frac{b\,x^4\,\ln\left(c\,x^3+1\right)}{8}-\frac{b\,x^4\,\ln\left(1-c\,x^3\right)}{8} - \frac{\sqrt{3}\,b\left(\text{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)+\text{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)\right)}{8\,c^{4/3}}$$

input
```
int(x^3*(a + b*atanh(c*x^3)),x)
```

output
```
(a*x^4)/4 + (b*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)/2 - atan((c^(1/3)*x*(3^
(1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/(4*c^(4/3)) + (3*b*x)/(4*c) +
(b*x^4*log(c*x^3 + 1))/8 - (b*x^4*log(1 - c*x^3))/8 - (3^(1/2)*b*(atan((c^
(1/3)*x*(3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/(8*c^(4/
3))
```

3.108 $\int (a + b\text{arctanh}(cx^3))\, dx$

3.108.1 Optimal result

Integrand size = 10, antiderivative size = 101

$$\int (a + b\text{arctanh}(cx^3))\, dx = ax + \frac{\sqrt{3}b\arctan\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{2\sqrt[3]{c}} + bx\text{arctanh}(cx^3) + \frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}} - \frac{b\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)}{4\sqrt[3]{c}}$$

output
```
a*x+b*x*arctanh(c*x^3)+1/2*b*ln(1-c^(2/3)*x^2)/c^(1/3)-1/4*b*ln(1+c^(2/3)*
x^2+c^(4/3)*x^4)/c^(1/3)+1/2*b*arctan(1/3*(1+2*c^(2/3)*x^2)*3^(1/2))*3^(1/
2)/c^(1/3)
```

3.108.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.35

$$\int (a + b\text{arctanh}(cx^3))\, dx = ax + bx\text{arctanh}(cx^3) - \frac{b\left(-2\sqrt{3}\arctan\left(\frac{-1+2\sqrt[3]{c}x}{\sqrt{3}}\right) + 2\sqrt{3}\arctan\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right) - 2\log\left(1-\sqrt[3]{c}x\right) - 2\log\left(1+\sqrt[3]{c}x\right) + \log (1 - \right.}{4\sqrt[3]{c}}$$

input
```
Integrate[a + b*ArcTanh[c*x^3],x]
```

output
```
a*x + b*x*ArcTanh[c*x^3] - (b*(-2*Sqrt[3]*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3
]] + 2*Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]] - 2*Log[1 - c^(1/3)*x] -
2*Log[1 + c^(1/3)*x] + Log[1 - c^(1/3)*x + c^(2/3)*x^2] + Log[1 + c^(1/3)*
x + c^(2/3)*x^2]))/(4*c^(1/3))
```

3.108.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax+\frac{\sqrt{3}b\arctan\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{2\sqrt[3]{c}}+bx\text{arctanh}\left(cx^3\right)+\frac{b\log\left(1-c^{2/3}x^2\right)}{2\sqrt[3]{c}}-\frac{b\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{4\sqrt[3]{c}}$$

input
```
Int[a + b*ArcTanh[c*x^3],x]
```

output
```
a*x + (Sqrt[3]*b*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/(2*c^(1/3)) + b*x*Ar
cTanh[c*x^3] + (b*Log[1 - c^(2/3)*x^2])/(2*c^(1/3)) - (b*Log[1 + c^(2/3)*x
^2 + c^(4/3)*x^4])/(4*c^(1/3))
```

3.108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.108.4 Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.98

method	result
default	$ax+bx\ \operatorname{arctanh}\left(c\,x^{3}\right)+\frac{b\ln\left(x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{4}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$
parts	$ax+bx\ \operatorname{arctanh}\left(c\,x^{3}\right)+\frac{b\ln\left(x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{4}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}$
risch	$ax+\frac{bx\ln\left(c\,x^{3}+1\right)}{2}+\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x^{2}-\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\ \arctan\left(\frac{\sqrt{3}\left(-2x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{bx\ln\left(-c\,x^{3}+1\right)}{2}$

input
```
int(a+b*arctanh(c*x^3),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arctanh(c*x^3)+1/2*b/c/(1/c^2)^(1/3)*ln(x^2-(1/c^2)^(1/3))-1/4*b/c
/(1/c^2)^(1/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b*3^(1/2)/c/(1/
c^2)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2+1))
```

3.108.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.57

$$\int \left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx$$
$$=\left[\frac{\sqrt{3}bc\sqrt{-\frac{1}{c^{\frac{2}{3}}}}\log\left(\frac{2\,c^2x^6-3\,c^{\frac{2}{3}}x^2+\sqrt{3}\left(2\,c^{\frac{5}{3}}x^4-cx^2-c^{\frac{1}{3}}\right)\sqrt{-\frac{1}{c^{\frac{2}{3}}}}+1}{c^2x^6-1}\right)+2\,bcx\log\left(-\frac{cx^3+1}{cx^3-1}\right)+4\,acx-bc^{\frac{2}{3}}\log\left(c^2x^4\right.}{4\,c}\right.$$

input
```
integrate(a+b*arctanh(c*x^3),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(3)*b*c*sqrt(-1/c^(2/3))*log((2*c^2*x^6 - 3*c^(2/3)*x^2 + sqrt(3
)*(2*c^(5/3)*x^4 - c*x^2 - c^(1/3))*sqrt(-1/c^(2/3)) + 1)/(c^2*x^6 - 1)) +
 2*b*c*x*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 +
 c^(4/3)*x^2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2 - c^(1/3)))/c, 1/4*(2*b*c*
x*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 2*sqrt(3)*b*c^(2/3)*arctan(1/3*sqrt(3)*(
2*c*x^2 + c^(1/3))/c^(1/3)) + 4*a*c*x - b*c^(2/3)*log(c^2*x^4 + c^(4/3)*x^
2 + c^(2/3)) + 2*b*c^(2/3)*log(c*x^2 - c^(1/3)))/c]
```

3.108.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(a+b*atanh(c*x**3),x)
```

output
```
Timed out
```

3.108.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.89

$$\int \left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx$$
$$=\frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{4}{3}}}+\frac{2\,\log\left(\frac{c^{\frac{2}{3}}x^2-1}{c^{\frac{2}{3}}}\right)}{c^{\frac{4}{3}}}\right)+4\,x\,\text{artanh}\left(cx^3\right)\right)b$$
$$+ax$$

input
```
integrate(a+b*arctanh(c*x^3),x, algorithm="maxima")
```

output
```
1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2/3))/c^
(4/3) - log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1)/c^(4/3) + 2*log((c^(2/3)*x^2 -
1)/c^(2/3))/c^(4/3)) + 4*x*arctanh(c*x^3))*b + a*x
```

3.108.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.08

$$\int \left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx$$
$$=\frac{1}{4}\left(c\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2}-\frac{|c|^{\frac{2}{3}}\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2}+\frac{2\,\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{4}{3}}}\right)-\right.$$
$$+ax$$

input
```
integrate(a+b*arctanh(c*x^3),x, algorithm="giac")
```

output
```
1/4*(c*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))
*abs(c)^(2/3))/c^2 - abs(c)^(2/3)*log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4
/3))/c^2 + 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(4/3)) + 2*x*log(-(c*x^
3 + 1)/(c*x^3 - 1)))*b + a*x
```

3.108.9 Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.06

$$\int (a+b\operatorname{arctanh}(cx^3))\,dx = a\,x+\frac{b\,\ln\left(c^{2/3}\,x^2-1\right)}{2\,c^{1/3}}-\frac{\ln\left(4\,c^{2/3}\,x^2+2-\sqrt{3}\,2\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}} -\frac{\ln\left(4\,c^{2/3}\,x^2+2+\sqrt{3}\,2\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{4\,c^{1/3}} +\frac{b\,x\,\ln\left(c\,x^3+1\right)}{2}-\frac{b\,x\,\ln\left(1-c\,x^3\right)}{2}$$

input
```
int(a + b*atanh(c*x^3),x)
```

output
```
a*x + (b*log(c^(2/3)*x^2 - 1))/(2*c^(1/3)) - (log(4*c^(2/3)*x^2 - 3^(1/2)*
2i + 2)*(b + 3^(1/2)*b*1i))/(4*c^(1/3)) - (log(3^(1/2)*2i + 4*c^(2/3)*x^2
+ 2)*(b - 3^(1/2)*b*1i))/(4*c^(1/3)) + (b*x*log(c*x^3 + 1))/2 - (b*x*log(1
 - c*x^3))/2
```

3.109 $\int \frac{a+b\,\text{arctanh}(cx^3)}{x^3}\,dx$

3.109.1 Optimal result

Integrand size = 14, antiderivative size = 165

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^3}\,dx = -\frac{1}{4}\sqrt{3}bc^{2/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}bc^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)+\frac{1}{2}bc^{2/3}\text{arctanh}(\sqrt[3]{c}x)-\frac{a+b\text{arctanh}(cx^3)}{2x^2}-\frac{1}{8}bc^{2/3}\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}bc^{2/3}\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)$$

output
```
1/2*b*c^(2/3)*arctanh(c^(1/3)*x)+1/2*(-a-b*arctanh(c*x^3))/x^2-1/8*b*c^(2/
3)*ln(1-c^(1/3)*x+c^(2/3)*x^2)+1/8*b*c^(2/3)*ln(1+c^(1/3)*x+c^(2/3)*x^2)+1
/4*b*c^(2/3)*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)+1/4*b*c^(2
/3)*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)
```

3.109.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^3}\,dx = -\frac{a}{2x^2}+\frac{1}{4}\sqrt{3}bc^{2/3}\arctan\left(\frac{-1+2\sqrt[3]{c}x}{\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}bc^{2/3}\arctan\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)-\frac{b\text{arctanh}(cx^3)}{2x^2}-\frac{1}{4}bc^{2/3}\log\left(1-\sqrt[3]{c}x\right)+\frac{1}{4}bc^{2/3}\log\left(1+\sqrt[3]{c}x\right)-\frac{1}{8}bc^{2/3}\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{8}bc^{2/3}\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])/x^3,x]
```

output
```
-1/2*a/x^2 + (Sqrt[3]*b*c^(2/3)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/4 + (S
qrt[3]*b*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/4 - (b*ArcTanh[c*x^3])
/(2*x^2) - (b*c^(2/3)*Log[1 - c^(1/3)*x])/4 + (b*c^(2/3)*Log[1 + c^(1/3)*x
])/4 - (b*c^(2/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/8 + (b*c^(2/3)*Log[1 +
 c^(1/3)*x + c^(2/3)*x^2])/8
```

3.109.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6452, 754, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^3}dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{3}{2}bc\int \frac{1}{1-c^2x^6}dx-\frac{a+b\text{arctanh}(cx^3)}{2x^2}$$

$$\Big\downarrow \text{ 754}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{1}{1-c^{2/3}x^2}dx+\frac{1}{3}\int \frac{2-\sqrt[3]{c}x}{2\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}dx+\frac{1}{3}\int \frac{\sqrt[3]{c}x+2}{2\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}dx\right)-\frac{a+b\text{arctanh}(cx^3)}{2x^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{1}{1-c^{2/3}x^2}dx+\frac{1}{6}\int \frac{2-\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{1}{6}\int \frac{\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx\right)-\frac{a+b\text{arctanh}(cx^3)}{2x^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3}{2}bc\left(\frac{1}{6}\int \frac{2-\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{1}{6}\int \frac{\sqrt[3]{c}x+2}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx+\frac{\text{arctanh}(\sqrt[3]{c}x)}{3\sqrt[3]{c}}\right)-\frac{a+b\text{arctanh}(cx^3)}{2x^2}$$

$$\Big\downarrow \text{1142}$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{\int-\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx+\frac{\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+1\right)}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{\int\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx+\frac{\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+1\right)}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}\right)\right.$$

$$\frac{a+b\text{arctanh}\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx\right)+\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx+\frac{1}{2}\int\frac{2\sqrt[3]{c}x}{c^{2/3}x^2+}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{1082}$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{3\int\frac{1}{-\left(1-2\sqrt[3]{c}x\right)^2-3}d\left(1-2\sqrt[3]{c}x\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{3\int}{}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{217}$$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx+\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}}{}\right.}{\sqrt[3]{c}}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^3\right)}{2x^2}$$

$$\Big\downarrow \text{1103}$$

3.109. $\int\frac{a+b\text{arctanh}(cx^3)}{x^3}\,dx$

$$\frac{3}{2}bc\left(\frac{1}{6}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}x+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}+\frac{\log\left(c^{2/3}x^2+\sqrt[3]{c}x\cdots\right.}{2\sqrt[3]{c}}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^3\right)}{2x^2}$$

input
```
Int[(a + b*ArcTanh[c*x^3])/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c*x^3])/x^2 + (3*b*c*(ArcTanh[c^(1/3)*x]/(3*c^(1/3)) +
 (-((Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*x)/Sqrt[3]])/c^(1/3)) - Log[1 - c^(1/3)
*x + c^(2/3)*x^2]/(2*c^(1/3)))/6 + ((Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt
[3]])/c^(1/3) + Log[1 + c^(1/3)*x + c^(2/3)*x^2]/(2*c^(1/3)))/6))/2
```

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.109.4 Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.96

method	result
default	$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{2x^2}-\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right.}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}$
parts	$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{2x^2}-\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right.}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}$
risch	$-\frac{b\ln(c\,x^3+1)}{4x^2}-\frac{a}{2x^2}+\frac{b\ln(-c\,x^3+1)}{4x^2}-\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+$

input
```
int((a+b*arctanh(c*x^3))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2-1/2*b/x^2*arctanh(c*x^3)-1/4*b/(1/c)^(2/3)*ln(x-(1/c)^(1/3))+1/
8*b/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b/(1/c)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))+1/4*b/(1/c)^(2/3)*ln(x+(1/c)^(1/
3))-1/8*b/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b/(1/c)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

3.109.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.38

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^3}\,dx =$$

$$-\frac{2\sqrt{3}(-c^2)^{\frac{1}{3}}\,bx^2\arctan\left(\frac{2\sqrt{3}(-c^2)^{\frac{2}{3}}x+\sqrt{3}c}{3c}\right)-2\sqrt{3}b(c^2)^{\frac{1}{3}}x^2\arctan\left(\frac{2\sqrt{3}(c^2)^{\frac{2}{3}}x-\sqrt{3}c}{3c}\right)+(-c^2)^{\frac{1}{3}}\,bx^2\log\left(c^2\right.}{}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(3)*(-c^2)^(1/3)*b*x^2*arctan(1/3*(2*sqrt(3)*(-c^2)^(2/3)*x +
sqrt(3)*c)/c) - 2*sqrt(3)*b*(c^2)^(1/3)*x^2*arctan(1/3*(2*sqrt(3)*(c^2)^(2
/3)*x - sqrt(3)*c)/c) + (-c^2)^(1/3)*b*x^2*log(c^2*x^2 - (-c^2)^(1/3)*c*x
+ (-c^2)^(2/3)) + b*(c^2)^(1/3)*x^2*log(c^2*x^2 - (c^2)^(1/3)*c*x + (c^2)^
(2/3)) - 2*(-c^2)^(1/3)*b*x^2*log(c*x + (-c^2)^(1/3)) - 2*b*(c^2)^(1/3)*x^
2*log(c*x + (c^2)^(1/3)) + 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^2
```

3.109.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\,\text{arctanh}(cx^3)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))/x**3,x)
```

output
```
Timed out
```

3.109.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.94

$$\int \frac{a+b\,\text{arctanh}(cx^3)}{x^3}\,dx = \frac{1}{8}\left(\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x^2+c^{\frac{1}{3}}x+1\right)}{c^{\frac{1}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x^2 -\right.}{c}\right.\right. - \frac{a}{2x^2}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="maxima")
```

output
```
1/8*((2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3))/c^(1/3
) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x - c^(1/3))/c^(1/3))/c^(1/3)
+ log(c^(2/3)*x^2 + c^(1/3)*x + 1)/c^(1/3) - log(c^(2/3)*x^2 - c^(1/3)*x +
 1)/c^(1/3) + 2*log((c^(1/3)*x + 1)/c^(1/3))/c^(1/3) - 2*log((c^(1/3)*x -
1)/c^(1/3))/c^(1/3))*c - 4*arctanh(c*x^3)/x^2)*b - 1/2*a/x^2
```

3.109.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^3}\,dx$$
$$= \frac{1}{8}\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x + \frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}} + \frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x - \frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}} + \frac{\log\left(x^2 + \frac{x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right.}{|c|^{\frac{1}{3}}}\right.$$
$$- \frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{4x^2} - \frac{a}{2x^2}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^3,x, algorithm="giac")
```

output
```
1/8*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs(c)^(1/3))/abs
(c)^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x - 1/abs(c)^(1/3))*abs(c)^(1/
3))/abs(c)^(1/3) + log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3)
 - log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(1/3) + 2*log(abs(x +
 1/abs(c)^(1/3)))/abs(c)^(1/3) - 2*log(abs(x - 1/abs(c)^(1/3)))/abs(c)^(1/
3))*b*c - 1/4*b*log(-(c*x^3 + 1)/(c*x^3 - 1))/x^2 - 1/2*a/x^2
```

3.109.9 Mupad [B] (verification not implemented)

Time = 4.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.72

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^3}\,dx$$
$$= \frac{b\ln(1 - c\,x^3)}{4\,x^2} - \frac{b\,c^{2/3}\left(-\frac{\text{atan}\left(\frac{c^{1/3}\,x\,(\sqrt{3}-\text{i})}{2}\right)}{2} + \frac{\text{atan}\left(\frac{c^{1/3}\,x\,(\sqrt{3}+1\text{i})}{2}\right)}{2} + \text{atan}(c^{1/3}\,x\,1\text{i})\right)1\text{i}}{2}$$
$$- \frac{b\ln(c\,x^3+1)}{4\,x^2} - \frac{a}{2\,x^2} + \frac{\sqrt{3}\,b\,c^{2/3}\left(\text{atan}\left(\frac{c^{1/3}\,x\,(\sqrt{3}-\text{i})}{2}\right) + \text{atan}\left(\frac{c^{1/3}\,x\,(\sqrt{3}+1\text{i})}{2}\right)\right)}{4}$$

input
```
int((a + b*atanh(c*x^3))/x^3,x)
```

output
```
(b*log(1 - c*x^3))/(4*x^2) - (b*c^(2/3)*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2
)/2 - atan((c^(1/3)*x*(3^(1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/2 - (
b*log(c*x^3 + 1))/(4*x^2) - a/(2*x^2) + (3^(1/2)*b*c^(2/3)*(atan((c^(1/3)*
x*(3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/4
```

3.109. $\int \frac{a+b\text{arctanh}(cx^3)}{x^3}\,dx$

3.110 $\int \frac{a+b\text{arctanh}(cx^3)}{x^6}\,dx$

3.110.1 Optimal result

Integrand size = 14, antiderivative size = 115

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^6}\,dx = -\frac{3bc}{10x^2}-\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)-\frac{a+b\text{arctanh}(cx^3)}{5x^5} -\frac{1}{10}bc^{5/3}\log\left(1-c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)$$

output
```
-3/10*b*c/x^2+1/5*(-a-b*arctanh(c*x^3))/x^5-1/10*b*c^(5/3)*ln(1-c^(2/3)*x^
2)+1/20*b*c^(5/3)*ln(1+c^(2/3)*x^2+c^(4/3)*x^4)-1/10*b*c^(5/3)*arctan(1/3*
(1+2*c^(2/3)*x^2)*3^(1/2))*3^(1/2)
```

3.110.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.70

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^6}\,dx = -\frac{a}{5x^5}-\frac{3bc}{10x^2} -\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\frac{-1+2\sqrt[3]{c}x}{\sqrt{3}}\right)+\frac{1}{10}\sqrt{3}bc^{5/3}\arctan\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)-\frac{b\text{arctanh}(cx^3)}{5x^5} -\frac{1}{10}bc^{5/3}\log\left(1-\sqrt[3]{c}x\right)-\frac{1}{10}bc^{5/3}\log\left(1+\sqrt[3]{c}x\right)+\frac{1}{20}bc^{5/3}\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{20}bc^{5/3}\log\left(1+\sqrt[3]{c}x+c^{2/\right.$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])/x^6,x]
```

output
```
-1/5*a/x^5 - (3*b*c)/(10*x^2) - (Sqrt[3]*b*c^(5/3)*ArcTan[(-1 + 2*c^(1/3)*
x)/Sqrt[3]])/10 + (Sqrt[3]*b*c^(5/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/10
 - (b*ArcTanh[c*x^3])/(5*x^5) - (b*c^(5/3)*Log[1 - c^(1/3)*x])/10 - (b*c^(
5/3)*Log[1 + c^(1/3)*x])/10 + (b*c^(5/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])
/20 + (b*c^(5/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/20
```

3.110.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {6452, 807, 847, 821, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^6}dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{3}{5}bc\int \frac{1}{x^3(1-c^2x^6)}dx-\frac{a+b\text{arctanh}(cx^3)}{5x^5}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{3}{10}bc\int \frac{1}{x^4(1-c^2x^6)}dx^2-\frac{a+b\text{arctanh}(cx^3)}{5x^5}$$

$$\Big\downarrow \text{ 847}$$

$$\frac{3}{10}bc\left(c^2\int \frac{x^2}{1-c^2x^6}dx^2-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx^3)}{5x^5}$$

$$\Big\downarrow \text{ 821}$$

$$\frac{3}{10}bc\left(c^2\left(\frac{\int \frac{1}{1-c^{2/3}x^2}dx^2}{3c^{2/3}}-\frac{\int \frac{1-c^{2/3}x^2}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2}{3c^{2/3}}\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx^3)}{5x^5}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{3}{10}bc\left(c^2\left(-\frac{\int \frac{1-c^{2/3}x^2}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{4/3}}\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}(cx^3)}{5x^5}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{3}{10}bc\left(c^2\left(-\frac{\frac{3}{2}\int \frac{1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2-\frac{\int \frac{c^{2/3}\left(2c^{2/3}x^2+1\right)}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2}{2c^{2/3}}}{3c^{2/3}}-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{4/3}}\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{5x^5}$$

$\Big\downarrow$ 27

$$\frac{3}{10}bc\left(c^2\left(-\frac{\frac{3}{2}\int \frac{1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2-\frac{1}{2}\int \frac{2c^{2/3}x^2+1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{4/3}}\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{5x^5}$$

$\Big\downarrow$ 1082

$$\frac{3}{10}bc\left(c^2\left(-\frac{-\frac{3\int \frac{1}{-x^4-3}d\left(2c^{2/3}x^2+1\right)}{c^{2/3}}-\frac{1}{2}\int \frac{2c^{2/3}x^2+1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{4/3}}\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{5x^5}$$

$\Big\downarrow$ 217

$$\frac{3}{10}bc\left(c^2\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{c^{2/3}}-\frac{1}{2}\int \frac{2c^{2/3}x^2+1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2}{3c^{2/3}}-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{4/3}}\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{5x^5}$$

$\Big\downarrow$ 1103

$$\frac{3}{10}bc\left(c^2\left(-\frac{\frac{\sqrt{3}\arctan\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{c^{2/3}}-\frac{\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{2c^{2/3}}}{3c^{2/3}}-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{4/3}}\right)-\frac{1}{x^2}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{5x^5}$$

input
```
Int[(a + b*ArcTanh[c*x^3])/x^6,x]
```

output
```
-1/5*(a + b*ArcTanh[c*x^3])/x^5 + (3*b*c*(-x^(-2) + c^2*(-1/3*Log[1 - c^(2
/3)*x^2]/c^(4/3) - ((Sqrt[3]*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/c^(2/3)
- Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4]/(2*c^(2/3)))/(3*c^(2/3)))))/10
```

3.110. $\int \frac{a+b\text{arctanh}(cx^3)}{x^6}\,dx$

3.110.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

3.110. $\int \frac{a+b\text{arctanh}(cx^3)}{x^6} dx$

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.110.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.50

method	result
default	$-\frac{a}{5x^5}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{5x^5}-\frac{3bc}{10x^2}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}-$
parts	$-\frac{a}{5x^5}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{5x^5}-\frac{3bc}{10x^2}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}-$
risch	$-\frac{b\ln(c\,x^3+1)}{10x^5}-\frac{a}{5x^5}+\frac{b\ln(-c\,x^3+1)}{10x^5}-\frac{3bc}{10x^2}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{20\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2}{\left(\frac{1}{c}\right.}\right.}{3}\right.}{10\left(\frac{1}{c}\right)^{\frac{2}{3}}}$

input
```
int((a+b*arctanh(c*x^3))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5-1/5*b/x^5*arctanh(c*x^3)-3/10*b*c/x^2-1/10*b*c/(1/c)^(2/3)*ln(x
-(1/c)^(1/3))+1/20*b*c/(1/c)^(2/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/10*
b*c/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/10*b*c/(
1/c)^(2/3)*ln(x+(1/c)^(1/3))+1/20*b*c/(1/c)^(2/3)*ln(x^2-(1/c)^(1/3)*x+(1/
c)^(2/3))-1/10*b*c/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x
-1))
```

3.110.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.31

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^6} \, dx =$$
$$-\frac{2\sqrt{3}(-c^2)^{\frac{1}{3}}\, bcx^5 \arctan\left(\frac{2}{3}\sqrt{3}(-c^2)^{\frac{1}{3}}\, x^2 - \frac{1}{3}\sqrt{3}\right) + (-c^2)^{\frac{1}{3}}\, bcx^5 \log\left(c^2x^4 + (-c^2)^{\frac{2}{3}}\, x^2 - (-c^2)^{\frac{1}{3}}\right) - 2\,(}{20\, x^5}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="fricas")
```

output
```
-1/20*(2*sqrt(3)*(-c^2)^(1/3)*b*c*x^5*arctan(2/3*sqrt(3)*(-c^2)^(1/3)*x^2
- 1/3*sqrt(3)) + (-c^2)^(1/3)*b*c*x^5*log(c^2*x^4 + (-c^2)^(2/3)*x^2 - (-c
^2)^(1/3)) - 2*(-c^2)^(1/3)*b*c*x^5*log(c^2*x^2 - (-c^2)^(2/3)) + 6*b*c*x^
3 + 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^5
```

3.110.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^6} \, dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))/x**6,x)
```

output
```
Timed out
```

3.110.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.87

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^6} \, dx =$$
$$-\frac{1}{20}\left(\left(\left(2\sqrt{3}c^{\frac{2}{3}} \arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{4}{3}}x^2 + c^{\frac{2}{3}}\right)}{3\,c^{\frac{2}{3}}}\right) - c^{\frac{2}{3}} \log\left(c^{\frac{4}{3}}x^4 + c^{\frac{2}{3}}x^2 + 1\right) + 2\,c^{\frac{2}{3}} \log\left(\frac{c^{\frac{2}{3}}x^2 - 1}{c^{\frac{2}{3}}}\right) + \frac{6}{x^2}\right.\right.\right.$$
$$-\frac{a}{5\,x^5}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="maxima")
```

output
```
-1/20*((2*sqrt(3)*c^(2/3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(
2/3)) - c^(2/3)*log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1) + 2*c^(2/3)*log((c^(2/3
)*x^2 - 1)/c^(2/3)) + 6/x^2)*c + 4*arctanh(c*x^3)/x^5)*b - 1/5*a/x^5
```

3.110.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^6} \, dx =$$
$$-\frac{1}{20}\,bc^3\left(\frac{2\sqrt{3}|c|^{\frac{2}{3}} \arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x^2 + \frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^2} - \frac{|c|^{\frac{2}{3}} \log\left(x^4 + \frac{x^2}{|c|^{\frac{2}{3}}} + \frac{1}{|c|^{\frac{4}{3}}}\right)}{c^2} + \frac{2 \log\left(\left|x^2 - \frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{4}{3}}}\right.$$
$$-\frac{b \log\left(-\frac{cx^3+1}{cx^3-1}\right)}{10\,x^5} - \frac{3\,bcx^3 + 2\,a}{10\,x^5}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^6,x, algorithm="giac")
```

output
```
-1/20*b*c^3*(2*sqrt(3)*abs(c)^(2/3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(
2/3))*abs(c)^(2/3))/c^2 - abs(c)^(2/3)*log(x^4 + x^2/abs(c)^(2/3) + 1/abs(
c)^(4/3))/c^2 + 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(4/3)) - 1/10*b*lo
g(-(c*x^3 + 1)/(c*x^3 - 1))/x^5 - 1/10*(3*b*c*x^3 + 2*a)/x^5
```

3.110.9 Mupad [B] (verification not implemented)

Time = 6.23 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^6}\,dx = \frac{b\ln\left(1-c\,x^3\right)}{10\,x^5}-\frac{b\,c^{5/3}\ln\left(c^{2/3}\,x^2-1\right)}{10}-\frac{b\ln\left(c\,x^3+1\right)}{10\,x^5}-\frac{\frac{3\,b\,c\,x^3}{2}+a}{5\,x^5}+\frac{b\,c^{5/3}\ln\left(\sqrt{3}\,c^{2/3}\,x^2-c^{2/3}\,x^2\,1\mathrm{i}-2\mathrm{i}\right)\left(1+\sqrt{3}\,1\mathrm{i}\right)}{20}-\frac{b\,c^{5/3}\ln\left(-c^{2/3}\,x^2\,1\mathrm{i}-\sqrt{3}\,c^{2/3}\,x^2-2\mathrm{i}\right)\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{20}$$

input
```
int((a + b*atanh(c*x^3))/x^6,x)
```

output
```
(b*log(1 - c*x^3))/(10*x^5) - (b*c^(5/3)*log(c^(2/3)*x^2 - 1))/10 - (b*log
(c*x^3 + 1))/(10*x^5) - (a + (3*b*c*x^3)/2)/(5*x^5) + (b*c^(5/3)*log(3^(1/
2)*c^(2/3)*x^2 - c^(2/3)*x^2*1i - 2i)*(3^(1/2)*1i + 1))/20 - (b*c^(5/3)*lo
g(- c^(2/3)*x^2*1i - 3^(1/2)*c^(2/3)*x^2 - 2i)*(3^(1/2)*1i - 1))/20
```

3.111 $\int x^7(a+b\operatorname{arctanh}(cx^3))\,dx$

3.111.1 Optimal result

Integrand size = 14, antiderivative size = 176

$$\int x^7(a+b\operatorname{arctanh}(cx^3))\,dx = \frac{3bx^5}{40c}-\frac{\sqrt{3}b\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16c^{8/3}}+\frac{\sqrt{3}b\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16c^{8/3}}-\frac{b\operatorname{arctanh}(\sqrt[3]{c}x)}{8c^{8/3}}+\frac{1}{8}x^8(a+b\operatorname{arctanh}(cx^3))+\frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}-\frac{b\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}$$

output
```
3/40*b*x^5/c-1/8*b*arctanh(c^(1/3)*x)/c^(8/3)+1/8*x^8*(a+b*arctanh(c*x^3))
+1/32*b*ln(1-c^(1/3)*x+c^(2/3)*x^2)/c^(8/3)-1/32*b*ln(1+c^(1/3)*x+c^(2/3)*
x^2)/c^(8/3)+1/16*b*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(
8/3)+1/16*b*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(8/3)
```

3.111.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.12

$$\int x^7 (a + b\text{arctanh}(cx^3)) \, dx = \frac{3bx^5}{40c} + \frac{ax^8}{8} + \frac{\sqrt{3}b \arctan\left(\frac{-1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16c^{8/3}} + \frac{\sqrt{3}b \arctan\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{16c^{8/3}} + \frac{1}{8}bx^8\text{arctanh}(cx^3) + \frac{b\log\left(1-\sqrt[3]{c}x\right)}{16c^{8/3}} - \frac{b\log\left(1+\sqrt[3]{c}x\right)}{16c^{8/3}} + \frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}} - \frac{b\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)}{32c^{8/3}}$$

input
```
Integrate[x^7*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(3*b*x^5)/(40*c) + (a*x^8)/8 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3
]])/(16*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(16*c^(8/
3)) + (b*x^8*ArcTanh[c*x^3])/8 + (b*Log[1 - c^(1/3)*x])/(16*c^(8/3)) - (b*
Log[1 + c^(1/3)*x])/(16*c^(8/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(3
2*c^(8/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(32*c^(8/3))
```

3.111.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.09, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {6452, 843, 825, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 (a + b\text{arctanh}(cx^3)) \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{8}x^8(a + b\text{arctanh}(cx^3)) - \frac{3}{8}bc\int \frac{x^{10}}{1-c^2x^6}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{8}x^8\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{\int \frac{x^4}{1-c^2x^6}dx}{c^2}-\frac{x^5}{5c^2}\right)$$

$$\Big\downarrow \text{825}$$

$$\frac{1}{8}x^8\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{\frac{\int \frac{1}{1-c^{2/3}x^2}dx}{3c^{4/3}}+\frac{\int -\frac{\sqrt[3]{c}x+1}{2\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}+\frac{\int -\frac{1-\sqrt[3]{c}x}{2\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}}{c^2}-\frac{x^5}{5c^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}x^8\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{\frac{\int \frac{1}{1-c^{2/3}x^2}dx}{3c^{4/3}}-\frac{\int \frac{\sqrt[3]{c}x+1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{1-\sqrt[3]{c}x}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}}{c^2}-\frac{x^5}{5c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}x^8\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{-\frac{\int \frac{\sqrt[3]{c}x+1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{1-\sqrt[3]{c}x}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}}{c^2}-\frac{x^5}{5c^2}\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{8}x^8\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{8}bc\left(\frac{-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{\int -\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+1\right)}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}}{c^2}-\frac{x^5}{5c^2}\right)$$

$$\Big\downarrow \text{25}$$

3.111. $\int x^7\left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx$

$$\frac{1}{8}x^8(a+b\text{arctanh}(cx^3))-\frac{3}{8}bc\left(\frac{-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{\int\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+1\right)}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}}{c^2}-\frac{x^5}{5c^2}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{8}x^8(a+b\text{arctanh}(cx^3))-\frac{3}{8}bc\left(\frac{-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}}{c^2}-\frac{x^5}{5c^2}\right)$$

$$\Big\downarrow \; 1082$$

$$\frac{1}{8}x^8(a+b\text{arctanh}(cx^3))-\frac{3}{8}bc\left(\frac{-\frac{\frac{3\int\frac{1}{-\left(1-2\sqrt[3]{c}x\right)^2-3}d\left(1-2\sqrt[3]{c}x\right)}{\sqrt[3]{c}}-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{-\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{3\int\frac{1}{-\left(2\sqrt[3]{c}x+1\right)^2-3}d\left(2\sqrt[3]{c}x+1\right)}{\sqrt[3]{c}}}{6c^{4/3}}+\text{arcta}}{c^2}\right.$$

$$\Big\downarrow \; 217$$

$$\frac{1}{8}x^8(a+b\text{arctanh}(cx^3))-\frac{3}{8}bc\left(\frac{-\frac{-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}x+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}}{c^2}-\frac{x^5}{5c^2}\right)$$

$$\Big\downarrow \; 1103$$

3.111. $\int x^7(a+b\text{arctanh}(cx^3))\,dx$

$$\frac{1}{8}x^8(a+b\text{arctanh}(cx^3))-\frac{3}{8}bc\left(\frac{-\frac{\frac{\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}x+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}}{c^2}-\frac{x^5}{5c^2}\right)$$

input
```
Int[x^7*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(x^8*(a + b*ArcTanh[c*x^3]))/8 - (3*b*c*(-1/5*x^5/c^2 + (ArcTanh[c^(1/3)*x
]/(3*c^(5/3)) - (-((Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*x)/Sqrt[3]])/c^(1/3)) +
Log[1 - c^(1/3)*x + c^(2/3)*x^2]/(2*c^(1/3)))/(6*c^(4/3)) - ((Sqrt[3]*ArcT
an[(1 + 2*c^(1/3)*x)/Sqrt[3]])/c^(1/3) - Log[1 + c^(1/3)*x + c^(2/3)*x^2]/
(2*c^(1/3)))/(6*c^(4/3)))/c^2))/8
```

3.111.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.111. $\int x^7(a+b\text{arctanh}(cx^3))\,dx$

3.111.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.06

method	result
default	$\frac{x^8 a}{8}+\frac{x^8 b \operatorname{arctanh}(c x^3)}{8}+\frac{3 b x^5}{40 c}+\frac{b \ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{16 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b \ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}} x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{32 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{16 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b \ln}{1}$
parts	$\frac{x^8 a}{8}+\frac{x^8 b \operatorname{arctanh}(c x^3)}{8}+\frac{3 b x^5}{40 c}+\frac{b \ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{16 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b \ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}} x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{32 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{16 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b \ln}{1}$
risch	$\frac{x^8 b \ln(c x^3+1)}{16}+\frac{x^8 a}{8}-\frac{b x^8 \ln(-c x^3+1)}{16}+\frac{3 b x^5}{40 c}+\frac{b \ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{16 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b \ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}} x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{32 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2}{\left(\frac{1}{c}\right.}\right.}{3}\right.}{16 c^3 \left(\frac{1}{c}\right)^{\frac{1}{3}}}$

input
```
int(x^7*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^8*a+1/8*x^8*b*arctanh(c*x^3)+3/40*b*x^5/c+1/16*b/c^3/(1/c)^(1/3)*ln(
x-(1/c)^(1/3))-1/32*b/c^3/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/
16*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/16*
b/c^3/(1/c)^(1/3)*ln(x+(1/c)^(1/3))+1/32*b/c^3/(1/c)^(1/3)*ln(x^2-(1/c)^(1
/3)*x+(1/c)^(2/3))+1/16*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1
/c)^(1/3)*x-1))
```

3.111.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.41

$$\int x^7 (a + b\text{arctanh}(cx^3))\ dx$$

$$= \frac{10\, bc^4 x^8 \log\left(-\frac{cx^3+1}{cx^3-1}\right) + 20\, ac^4 x^8 + 12\, bc^3 x^5 + 10\, \sqrt{3} bc \sqrt{-\left(-c^2\right)^{\frac{1}{3}}} \arctan\left(\frac{\sqrt{3}\left(2\, cx + \left(-c^2\right)^{\frac{1}{3}}\right)\sqrt{-\left(-c^2\right)^{\frac{1}{3}}}}{3\, c}\right) +}{}$$

input
```
integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

output
```
1/160*(10*b*c^4*x^8*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 20*a*c^4*x^8 + 12*b*c^
3*x^5 + 10*sqrt(3)*b*c*sqrt(-(-c^2)^(1/3))*arctan(1/3*sqrt(3)*(2*c*x + (-c
^2)^(1/3))*sqrt(-(-c^2)^(1/3))/c) + 10*sqrt(3)*b*(c^2)^(1/6)*c*arctan(1/3*
sqrt(3)*(c^2)^(1/6)*(2*c*x + (c^2)^(1/3))/c) + 5*(-c^2)^(2/3)*b*log(c^2*x^
2 + (-c^2)^(1/3)*c*x + (-c^2)^(2/3)) - 5*b*(c^2)^(2/3)*log(c^2*x^2 + (c^2)
^(1/3)*c*x + (c^2)^(2/3)) - 10*(-c^2)^(2/3)*b*log(c*x - (-c^2)^(1/3)) + 10
*b*(c^2)^(2/3)*log(c*x - (c^2)^(1/3)))/c^4
```

3.111.6 Sympy [F(-1)]

Timed out.

$$\int x^7\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**7*(a+b*atanh(c*x**3)),x)
```

output
```
Timed out
```

3.111.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.93

$$\int x^7\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx=\frac{1}{8}\,ax^8+\frac{1}{160}\left(20\,x^8\operatorname{artanh}\left(cx^3\right)+\left(\frac{12\,x^5}{c^2}+\frac{10\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}+\frac{10\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}-\right.\right.$$

input
```
integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

output
```
1/8*a*x^8 + 1/160*(20*x^8*arctanh(c*x^3) + (12*x^5/c^2 + 10*sqrt(3)*arctan
(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/3))/c^(11/3) + 10*sqrt(3)*arctan
(1/3*sqrt(3)*(2*c^(2/3)*x - c^(1/3))/c^(1/3))/c^(11/3) - 5*log(c^(2/3)*x^2
 + c^(1/3)*x + 1)/c^(11/3) + 5*log(c^(2/3)*x^2 - c^(1/3)*x + 1)/c^(11/3) -
 10*log((c^(1/3)*x + 1)/c^(1/3))/c^(11/3) + 10*log((c^(1/3)*x - 1)/c^(1/3)
)/c^(11/3))*c)*b
```

3.111.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.18

$$\begin{aligned}\int x^7(a+b\operatorname{arctanh}(cx^3))\,dx &= \frac{1}{16}\,bx^8\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{8}\,ax^8 \\ &\quad+\frac{3\,bx^5}{40\,c}-\frac{b\left(-\frac{1}{c}\right)^{\frac{2}{3}}\log\left(\left|x-\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right|\right)}{16\,c^2} \\ &\quad+\frac{\sqrt{3}b\arctan\left(\frac{\sqrt{3}\left(2\,x+\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{16\left(-c^2\right)^{\frac{1}{3}}c^2} \\ &\quad+\frac{\sqrt{3}b\arctan\left(\frac{1}{3}\sqrt{3}c^{\frac{1}{3}}\left(2\,x+\frac{1}{c^{\frac{1}{3}}}\right)\right)}{16\,c^2|c|^{\frac{2}{3}}} \\ &\quad-\frac{b\log\left(x^2+x\left(-\frac{1}{c}\right)^{\frac{1}{3}}+\left(-\frac{1}{c}\right)^{\frac{2}{3}}\right)}{32\left(-c^2\right)^{\frac{1}{3}}c^2} \\ &\quad-\frac{b\log\left(x^2+\frac{x}{c^{\frac{1}{3}}}+\frac{1}{c^{\frac{2}{3}}}\right)}{32\,c^2|c|^{\frac{2}{3}}}+\frac{b\log\left(\left|x-\frac{1}{c^{\frac{1}{3}}}\right|\right)}{16\,c^{\frac{8}{3}}}\end{aligned}$$

input
```
integrate(x^7*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

output
```
1/16*b*x^8*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/8*a*x^8 + 3/40*b*x^5/c - 1/16
*b*(-1/c)^(2/3)*log(abs(x - (-1/c)^(1/3)))/c^2 + 1/16*sqrt(3)*b*arctan(1/3
*sqrt(3)*(2*x + (-1/c)^(1/3))/(-1/c)^(1/3))/((-c^2)^(1/3)*c^2) + 1/16*sqrt
(3)*b*arctan(1/3*sqrt(3)*c^(1/3)*(2*x + 1/c^(1/3)))/(c^2*abs(c)^(2/3)) - 1
/32*b*log(x^2 + x*(-1/c)^(1/3) + (-1/c)^(2/3))/((-c^2)^(1/3)*c^2) - 1/32*b
*log(x^2 + x/c^(1/3) + 1/c^(2/3))/(c^2*abs(c)^(2/3)) + 1/16*b*log(abs(x -
1/c^(1/3)))/c^(8/3)
```

3.111.9 Mupad [B] (verification not implemented)

Time = 3.68 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.72

$$\int x^7\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx$$
$$=\frac{a\,x^8}{8}+\frac{b\left(-\frac{\operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\operatorname{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{8\,c^{8/3}}$$
$$+\frac{3\,b\,x^5}{40\,c}+\frac{b\,x^8\,\ln\left(c\,x^3+1\right)}{16}-\frac{b\,x^8\,\ln\left(1-c\,x^3\right)}{16}$$
$$+\frac{\sqrt{3}\,b\left(\operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)+\operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)\right)}{16\,c^{8/3}}$$

input
```
int(x^7*(a + b*atanh(c*x^3)),x)
```

output
```
(a*x^8)/8 + (b*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)/2 - atan((c^(1/3)*x*(3^
(1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/(8*c^(8/3)) + (3*b*x^5)/(40*c)
 + (b*x^8*log(c*x^3 + 1))/16 - (b*x^8*log(1 - c*x^3))/16 + (3^(1/2)*b*(ata
n((c^(1/3)*x*(3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/(16
*c^(8/3))
```

3.112 $\int x^4(a+b\text{arctanh}(cx^3))\,dx$

3.112.1 Optimal result

Integrand size = 14, antiderivative size = 117

$$\int x^4(a+b\text{arctanh}(cx^3))\,dx=\frac{3bx^2}{10c}-\frac{\sqrt{3}b\arctan\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{1}{5}x^5(a+b\text{arctanh}(cx^3))+\frac{b\log\left(1-c^{2/3}x^2\right)}{10c^{5/3}}-\frac{b\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)}{20c^{5/3}}$$

output
```
3/10*b*x^2/c+1/5*x^5*(a+b*arctanh(c*x^3))+1/10*b*ln(1-c^(2/3)*x^2)/c^(5/3)
-1/20*b*ln(1+c^(2/3)*x^2+c^(4/3)*x^4)/c^(5/3)-1/10*b*arctan(1/3*(1+2*c^(2/
3)*x^2)*3^(1/2))*3^(1/2)/c^(5/3)
```

3.112.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.69

$$\int x^4(a+b\text{arctanh}(cx^3))\,dx=\frac{3bx^2}{10c}+\frac{ax^5}{5}-\frac{\sqrt{3}b\arctan\left(\frac{-1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{10c^{5/3}}+\frac{1}{5}bx^5\text{arctanh}(cx^3)+\frac{b\log\left(1-\sqrt[3]{c}x\right)}{10c^{5/3}}+\frac{b\log\left(1+\sqrt[3]{c}x\right)}{10c^{5/3}}-\frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{20c^{5/3}}-\frac{b\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)}{20c^{5/3}}$$

input
```
Integrate[x^4*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(3*b*x^2)/(10*c) + (a*x^5)/5 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3
]])/(10*c^(5/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(10*c^(5/
3)) + (b*x^5*ArcTanh[c*x^3])/5 + (b*Log[1 - c^(1/3)*x])/(10*c^(5/3)) + (b*
Log[1 + c^(1/3)*x])/(10*c^(5/3)) - (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(2
0*c^(5/3)) - (b*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/(20*c^(5/3))
```

3.112.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6452, 807, 843, 750, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 (a + b\text{arctanh}(cx^3)) \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{5}x^5(a + b\text{arctanh}(cx^3)) - \frac{3}{5}bc\int \frac{x^7}{1-c^2x^6}dx$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{5}x^5(a + b\text{arctanh}(cx^3)) - \frac{3}{10}bc\int \frac{x^6}{1-c^2x^6}dx^2$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{5}x^5(a + b\text{arctanh}(cx^3)) - \frac{3}{10}bc\left(\frac{\int \frac{1}{1-c^2x^6}dx^2}{c^2} - \frac{x^2}{c^2}\right)$$

$$\Big\downarrow \text{750}$$

$$\frac{1}{5}x^5(a + b\text{arctanh}(cx^3)) - \frac{3}{10}bc\left(\frac{\frac{1}{3}\int \frac{1}{1-c^{2/3}x^2}dx^2 + \frac{1}{3}\int \frac{c^{2/3}x^2+2}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2}{c^2} - \frac{x^2}{c^2}\right)$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{5}x^5(a + b\text{arctanh}(cx^3)) - \frac{3}{10}bc\left(\frac{\frac{1}{3}\int \frac{c^{2/3}x^2+2}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2 - \frac{\log(1-c^{2/3}x^2)}{3c^{2/3}}}{c^2} - \frac{x^2}{c^2}\right)$$

$$\Big\downarrow \ 1142$$

$$\frac{1}{5}x^5\left(a+b\,\text{arctanh}\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2+\frac{\int\frac{c^{2/3}\left(2c^{2/3}x^2+1\right)}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2}{2c^{2/3}}\right)-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}}{c^2}-\frac{x^2}{c^2}\right)$$

$$\Big\downarrow \ 27$$

$$\frac{1}{5}x^5\left(a+b\,\text{arctanh}\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{\frac{1}{3}\left(\frac{3}{2}\int\frac{1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2+\frac{1}{2}\int\frac{2c^{2/3}x^2+1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2\right)-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}}{c^2}-\frac{x^2}{c^2}\right)$$

$$\Big\downarrow \ 1082$$

$$\frac{1}{5}x^5\left(a+b\,\text{arctanh}\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{\frac{1}{3}\left(\frac{1}{2}\int\frac{2c^{2/3}x^2+1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2-\frac{3\int\frac{1}{-x^4-3}d\left(2c^{2/3}x^2+1\right)}{c^{2/3}}\right)-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}}{c^2}-\frac{x^2}{c^2}\right)$$

$$\Big\downarrow \ 217$$

$$\frac{1}{5}x^5\left(a+b\,\text{arctanh}\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{\frac{1}{3}\left(\frac{1}{2}\int\frac{2c^{2/3}x^2+1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2+\frac{\sqrt{3}\arctan\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{c^{2/3}}\right)-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}}{c^2}-\frac{x^2}{c^2}\right)$$

$$\Big\downarrow \ 1103$$

$$\frac{1}{5}x^5\left(a+b\,\text{arctanh}\left(cx^3\right)\right)-\frac{3}{10}bc\left(\frac{\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{c^{2/3}}+\frac{\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{2c^{2/3}}\right)-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}}{c^2}-\frac{x^2}{c^2}\right)$$

3.112. $\int x^4(a+b\,\text{arctanh}(cx^3))\,dx$

input
```
Int[x^4*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(x^5*(a + b*ArcTanh[c*x^3]))/5 - (3*b*c*(-(x^2/c^2) + (-1/3*Log[1 - c^(2/3
)*x^2]/c^(2/3) + ((Sqrt[3]*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/c^(2/3) +
Log[1 + c^(2/3)*x^2 + c^(4/3)*x^4]/(2*c^(2/3)))/3)/c^2))/10
```

3.112.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.112.4 Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

method	result
default	$\frac{a x^5}{5}+\frac{b x^5 \operatorname{arctanh}(c x^3)}{5}+\frac{3b x^2}{10c}+\frac{b \ln\left(x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b \ln\left(x^4+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$
parts	$\frac{a x^5}{5}+\frac{b x^5 \operatorname{arctanh}(c x^3)}{5}+\frac{3b x^2}{10c}+\frac{b \ln\left(x^2-\left(\frac{1}{c^2}\right)^{\frac{1}{3}}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b \ln\left(x^4+\left(\frac{1}{c^2}\right)^{\frac{1}{3}}x^2+\left(\frac{1}{c^2}\right)^{\frac{2}{3}}\right)}{20c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x^2}{\left(\frac{1}{c^2}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{10c^3\left(\frac{1}{c^2}\right)^{\frac{2}{3}}}$
risch	$\frac{b x^5 \ln(c x^3+1)}{10}+\frac{a x^5}{5}-\frac{b x^5 \ln(-c x^3+1)}{10}+\frac{3b x^2}{10c}+\frac{b \ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{10c^2\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b \ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{20c^2\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(-\ldots\right)}{\ldots}\right)}{10c^2\left(\frac{1}{c}\right)^{\frac{1}{3}}}$

input
```
int(x^4*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+1/5*b*x^5*arctanh(c*x^3)+3/10*b*x^2/c+1/10*b/c^3/(1/c^2)^(2/3)*l
n(x^2-(1/c^2)^(1/3))-1/20*b/c^3/(1/c^2)^(2/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/
c^2)^(2/3))-1/10*b/c^3/(1/c^2)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)
^(1/3)*x^2+1))
```

3.112.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.42

$$\int x^4(a+b\operatorname{arctanh}(cx^3))\,dx$$

$$=\frac{2\,bc^3x^5\log\left(-\frac{cx^3+1}{cx^3-1}\right)+4\,ac^3x^5+6\,bc^2x^2-2\,\sqrt{3}b(c^2)^{\frac{1}{6}}c\arctan\left(-\frac{\sqrt{3}\left(4\,c^2x^4-2\,(c^2)^{\frac{2}{3}}x^2+(c^2)^{\frac{1}{3}}\right)(c^2)^{\frac{1}{6}}}{8\,c^3x^6+c}\right)-b(c^2)\ldots}{20\,c^3}$$

input
```
integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

output
```
1/20*(2*b*c^3*x^5*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^3*x^5 + 6*b*c^2*x^
2 - 2*sqrt(3)*b*(c^2)^(1/6)*c*arctan(-sqrt(3)*(4*c^2*x^4 - 2*(c^2)^(2/3)*x
^2 + (c^2)^(1/3))*(c^2)^(1/6)/(8*c^3*x^6 + c)) - b*(c^2)^(2/3)*log(c^2*x^4
 + (c^2)^(2/3)*x^2 + (c^2)^(1/3)) + 2*b*(c^2)^(2/3)*log(c^2*x^2 - (c^2)^(2
/3)))/c^3
```

3.112.6 Sympy [F(-1)]

Timed out.

$$\int x^4(a+b\text{arctanh}(cx^3))\,dx=\text{Timed out}$$

input
```
integrate(x**4*(a+b*atanh(c*x**3)),x)
```

output
```
Timed out
```

3.112.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.88

$$\int x^4(a+b\text{arctanh}(cx^3))\,dx=\frac{1}{5}ax^5$$
$$+\frac{1}{20}\left(4x^5\,\text{artanh}\left(cx^3\right)+c\left(\frac{6x^2}{c^2}-\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{8}{3}}}-\frac{\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{8}{3}}}+\frac{2\log\left(\frac{c^{\frac{2}{3}}}{}\right.}{c^{\frac{8}{3}}}\right.\right.$$

input
```
integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/20*(4*x^5*arctanh(c*x^3) + c*(6*x^2/c^2 - 2*sqrt(3)*arctan(1
/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2/3))/c^(8/3) - log(c^(4/3)*x^4 +
c^(2/3)*x^2 + 1)/c^(8/3) + 2*log((c^(2/3)*x^2 - 1)/c^(2/3))/c^(8/3)))*b
```

3.112.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.08

$$\int x^4\left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx = -\frac{1}{20}\,bc^9\left(\frac{2\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{c^{10}|c|^{\frac{2}{3}}}+\frac{\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{c^{10}|c|^{\frac{2}{3}}}-\frac{2\,\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{c^{10}|c|^{\frac{2}{3}}}\right) + \frac{1}{10}\,bx^5\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{5}\,ax^5+\frac{3\,bx^2}{10\,c}$$

input
```
integrate(x^4*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

output
```
-1/20*b*c^9*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*abs(c)^
(2/3))/(c^10*abs(c)^(2/3)) + log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/
(c^10*abs(c)^(2/3)) - 2*log(abs(x^2 - 1/abs(c)^(2/3)))/(c^10*abs(c)^(2/3))
) + 1/10*b*x^5*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/5*a*x^5 + 3/10*b*x^2/c
```

3.112.9 Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.06

$$\int x^4\left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx = \frac{a\,x^5}{5}+\frac{b\,\ln\left(1-c^{2/3}\,x^2\right)}{10\,c^{5/3}}+\frac{3\,b\,x^2}{10\,c} - \frac{\ln\left(2\,c^{2/3}\,x^2+1-\sqrt{3}\,1\mathrm{i}\right)\,\left(b-\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}} - \frac{\ln\left(2\,c^{2/3}\,x^2+1+\sqrt{3}\,1\mathrm{i}\right)\,\left(b+\sqrt{3}\,b\,1\mathrm{i}\right)}{20\,c^{5/3}} + \frac{b\,x^5\,\ln\left(c\,x^3+1\right)}{10}-\frac{b\,x^5\,\ln\left(1-c\,x^3\right)}{10}$$

input
```
int(x^4*(a + b*atanh(c*x^3)),x)
```

output
```
(a*x^5)/5 + (b*log(1 - c^(2/3)*x^2))/(10*c^(5/3)) + (3*b*x^2)/(10*c) - (lo
g(2*c^(2/3)*x^2 - 3^(1/2)*1i + 1)*(b - 3^(1/2)*b*1i))/(20*c^(5/3)) - (log(
3^(1/2)*1i + 2*c^(2/3)*x^2 + 1)*(b + 3^(1/2)*b*1i))/(20*c^(5/3)) + (b*x^5*
log(c*x^3 + 1))/10 - (b*x^5*log(1 - c*x^3))/10
```

3.113 $\int x(a+b\text{arctanh}(cx^3))\,dx$

3.113.1 Optimal result

Integrand size = 12, antiderivative size = 165

$$\int x(a+b\text{arctanh}(cx^3))\,dx = -\frac{\sqrt{3}b\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}b\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}}-\frac{b\text{arctanh}(\sqrt[3]{c}x)}{2c^{2/3}}+\frac{1}{2}x^2(a+b\text{arctanh}(cx^3))+\frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}-\frac{b\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}$$

output
```
-1/2*b*arctanh(c^(1/3)*x)/c^(2/3)+1/2*x^2*(a+b*arctanh(c*x^3))+1/8*b*ln(1-
c^(1/3)*x+c^(2/3)*x^2)/c^(2/3)-1/8*b*ln(1+c^(1/3)*x+c^(2/3)*x^2)/c^(2/3)+1
/4*b*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)+1/4*b*arct
an(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)/c^(2/3)
```

3.113.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.13

$$\int x(a+b\text{arctanh}(cx^3))\,dx = \frac{ax^2}{2}+\frac{\sqrt{3}b\arctan\left(\frac{-1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)}{4c^{2/3}}+\frac{1}{2}bx^2\text{arctanh}(cx^3)+\frac{b\log\left(1-\sqrt[3]{c}x\right)}{4c^{2/3}}-\frac{b\log\left(1+\sqrt[3]{c}x\right)}{4c^{2/3}}+\frac{b\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}-\frac{b\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)}{8c^{2/3}}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(a*x^2)/2 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (
Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/(4*c^(2/3)) + (b*x^2*ArcTanh[
c*x^3])/2 + (b*Log[1 - c^(1/3)*x])/(4*c^(2/3)) - (b*Log[1 + c^(1/3)*x])/(4
*c^(2/3)) + (b*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3)) - (b*Log[1 +
c^(1/3)*x + c^(2/3)*x^2])/(8*c^(2/3))
```

3.113.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6452, 825, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{2}bc\int \frac{x^4}{1-c^2x^6}dx$$

$$\Big\downarrow \text{825}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{2}bc\left(\frac{\int \frac{1}{1-c^{2/3}x^2}dx}{3c^{4/3}}+\frac{\int -\frac{\sqrt[3]{c}x+1}{2\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}+\frac{\int -\frac{1-\sqrt[3]{c}x}{2\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{2}bc\left(\frac{\int \frac{1}{1-c^{2/3}x^2}dx}{3c^{4/3}}-\frac{\int \frac{\sqrt[3]{c}x+1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{1-\sqrt[3]{c}x}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{\int \frac{\sqrt[3]{c}x+1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{1-\sqrt[3]{c}x}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right)$$

$\downarrow$ 1142

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{\int-\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+1\right)}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right)$$

$\downarrow$ 25

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{\int\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+1\right)}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right)$$

$\downarrow$ 27

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right.$$

$\downarrow$ 1082

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{3}{2}bc\left(-\frac{\frac{3\int\frac{1}{-\left(1-2\sqrt[3]{c}x\right)^2-3}d\left(1-2\sqrt[3]{c}x\right)}{\sqrt[3]{c}}-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{-\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{3\int\frac{1}{-\left(2\sqrt[3]{c}x+1\right)^2-3}d\left(2\sqrt[3]{c}x+1\right)}{\sqrt[3]{c}}}{6c^{4/3}}\right.$$

$\downarrow$ 217

3.113. $\int x(a+b\text{arctanh}(cx^3))\,dx$

$$\frac{1}{2}x^2(a+b\operatorname{arctanh}(cx^3))-$$
$$\frac{3}{2}bc\left(-\frac{-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}x+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\operatorname{arctanh}(\sqrt[3]{c}x}{3c^{5/3}}\right.$$

$\downarrow$ 1103

$$\frac{1}{2}x^2(a+b\operatorname{arctanh}(cx^3))-$$
$$\frac{3}{2}bc\left(-\frac{\frac{\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}x+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\operatorname{arctanh}(\sqrt[3]{c}x)}{3c^{5/3}}\right)$$

input
```
Int[x*(a + b*ArcTanh[c*x^3]),x]
```

output
```
(x^2*(a + b*ArcTanh[c*x^3]))/2 - (3*b*c*(ArcTanh[c^(1/3)*x]/(3*c^(5/3)) -
(-((Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*x)/Sqrt[3]])/c^(1/3)) + Log[1 - c^(1/3)*
x + c^(2/3)*x^2]/(2*c^(1/3)))/(6*c^(4/3)) - ((Sqrt[3]*ArcTan[(1 + 2*c^(1/3
)*x)/Sqrt[3]])/c^(1/3) - Log[1 + c^(1/3)*x + c^(2/3)*x^2]/(2*c^(1/3)))/(6*
c^(4/3))))/2
```

3.113.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.113. $\int x(a+b\text{arctanh}(cx^3))\,dx$

3.113.4 Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07

method	result
default	$\frac{a\,x^{2}}{2}+\frac{b\,x^{2}\operatorname{arctanh}\left(c\,x^{3}\right)}{2}+\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{2}+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right.}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$
parts	$\frac{a\,x^{2}}{2}+\frac{b\,x^{2}\operatorname{arctanh}\left(c\,x^{3}\right)}{2}+\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{2}+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\right.}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$
risch	$\frac{b\,x^{2}\ln\left(c\,x^{3}+1\right)}{4}+\frac{a\,x^{2}}{2}-\frac{b\,x^{2}\ln\left(-c\,x^{3}+1\right)}{4}+\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x^{2}+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right.}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$

input
```
int(x*(a+b*arctanh(c*x^3)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/2*b*x^2*arctanh(c*x^3)+1/4*b/c/(1/c)^(1/3)*ln(x-(1/c)^(1/3))-1
/8*b/c/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*3^(1/2)/c/(1/c)
^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))-1/4*b/c/(1/c)^(1/3)*ln(x+(1
/c)^(1/3))+1/8*b/c/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/3))+1/4*b*3^(
1/2)/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

3.113.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.44

$$\int x\left(a+b\operatorname{arctanh}\left(cx^{3}\right)\right)\,dx$$

$$=\frac{2\,bc^{2}x^{2}\log\left(-\frac{cx^{3}+1}{cx^{3}-1}\right)+4\,ac^{2}x^{2}+2\,\sqrt{3}bc\sqrt{-\left(-c^{2}\right)^{\frac{1}{3}}}\arctan\left(\frac{\sqrt{3}\left(2\,cx+\left(-c^{2}\right)^{\frac{1}{3}}\right)\sqrt{-\left(-c^{2}\right)^{\frac{1}{3}}}}{3\,c}\right)+2\,\sqrt{3}b\left(c^{2}\right)^{\frac{1}{6}}c\,\mathrm{ar}}{}$$

input
```
integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

output
```
1/8*(2*b*c^2*x^2*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 4*a*c^2*x^2 + 2*sqrt(3)*b
*c*sqrt(-(-c^2)^(1/3))*arctan(1/3*sqrt(3)*(2*c*x + (-c^2)^(1/3))*sqrt(-(-c
^2)^(1/3))/c) + 2*sqrt(3)*b*(c^2)^(1/6)*c*arctan(1/3*sqrt(3)*(c^2)^(1/6)*(
2*c*x + (c^2)^(1/3))/c) + (-c^2)^(2/3)*b*log(c^2*x^2 + (-c^2)^(1/3)*c*x +
(-c^2)^(2/3)) - b*(c^2)^(2/3)*log(c^2*x^2 + (c^2)^(1/3)*c*x + (c^2)^(2/3))
 - 2*(-c^2)^(2/3)*b*log(c*x - (-c^2)^(1/3)) + 2*b*(c^2)^(2/3)*log(c*x - (c
^2)^(1/3)))/c^2
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*atanh(c*x**3)),x)
```

output
```
Timed out
```

3.113.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.94

$$\int x\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx=\frac{1}{2}ax^2+\frac{1}{8}\left(4x^2\operatorname{artanh}\left(cx^3\right)+c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}x-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}}-\frac{\log\left(c^{\frac{2}{3}}x^2+\right.}{c^{\frac{5}{3}}}\right.\right.$$

input
```
integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/8*(4*x^2*arctanh(c*x^3) + c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2
*c^(2/3)*x + c^(1/3))/c^(1/3))/c^(5/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c
^(2/3)*x - c^(1/3))/c^(1/3))/c^(5/3) - log(c^(2/3)*x^2 + c^(1/3)*x + 1)/c^
(5/3) + log(c^(2/3)*x^2 - c^(1/3)*x + 1)/c^(5/3) - 2*log((c^(1/3)*x + 1)/c
^(1/3))/c^(5/3) + 2*log((c^(1/3)*x - 1)/c^(1/3))/c^(5/3)))*b
```

3.113.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.05

$$\int x(a+b\operatorname{arctanh}(cx^3))\,dx = \frac{1}{4}bx^2\log\left(-\frac{cx^3+1}{cx^3-1}\right)+\frac{1}{2}ax^2 + \frac{\sqrt{3}bc\arctan\left(\frac{1}{3}\sqrt{3}\left(2x+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{4|c|^{\frac{5}{3}}} + \frac{\sqrt{3}bc\arctan\left(\frac{1}{3}\sqrt{3}\left(2x-\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{4|c|^{\frac{5}{3}}} - \frac{bc\log\left(x^2+\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{8|c|^{\frac{5}{3}}} + \frac{bc\log\left(x^2-\frac{x}{|c|^{\frac{1}{3}}}+\frac{1}{|c|^{\frac{2}{3}}}\right)}{8|c|^{\frac{5}{3}}} - \frac{bc\log\left(\left|x+\frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{4|c|^{\frac{5}{3}}} + \frac{bc\log\left(\left|x-\frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{4|c|^{\frac{5}{3}}}$$

input
```
integrate(x*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

output
```
1/4*b*x^2*log(-(c*x^3 + 1)/(c*x^3 - 1)) + 1/2*a*x^2 + 1/4*sqrt(3)*b*c*arct
an(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(5/3) + 1/4*sqr
t(3)*b*c*arctan(1/3*sqrt(3)*(2*x - 1/abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(5
/3) - 1/8*b*c*log(x^2 + x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(5/3) + 1/
8*b*c*log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(5/3) - 1/4*b*c*lo
g(abs(x + 1/abs(c)^(1/3)))/abs(c)^(5/3) + 1/4*b*c*log(abs(x - 1/abs(c)^(1/
3)))/abs(c)^(5/3)
```

3.113.9 Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.72

$$\int x\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\,dx$$
$$=\frac{a\,x^2}{2}+\frac{b\left(-\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2}+\frac{\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2}+\mathrm{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{2\,c^{2/3}}$$
$$+\frac{b\,x^2\,\ln\left(c\,x^3+1\right)}{4}-\frac{b\,x^2\,\ln\left(1-c\,x^3\right)}{4}$$
$$+\frac{\sqrt{3}\,b\,\left(\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)+\mathrm{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)\right)}{4\,c^{2/3}}$$

input
```
int(x*(a + b*atanh(c*x^3)),x)
```

output
```
(a*x^2)/2 + (b*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2)/2 - atan((c^(1/3)*x*(3^
(1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/(2*c^(2/3)) + (b*x^2*log(c*x^3
 + 1))/4 - (b*x^2*log(1 - c*x^3))/4 + (3^(1/2)*b*(atan((c^(1/3)*x*(3^(1/2)
 - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2)))/(4*c^(2/3))
```

3.114 $\int \frac{a+b\operatorname{arctanh}(cx^3)}{x^2}\,dx$

3.114.1 Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{a+b\operatorname{arctanh}(cx^3)}{x^2}\,dx = \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\frac{1+2c^{2/3}x^2}{\sqrt{3}}\right) - \frac{a+b\operatorname{arctanh}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c}\log\left(1-c^{2/3}x^2\right) + \frac{1}{4}b\sqrt[3]{c}\log\left(1+c^{2/3}x^2+c^{4/3}x^4\right)$$

output
```
(-a-b*arctanh(c*x^3))/x-1/2*b*c^(1/3)*ln(1-c^(2/3)*x^2)+1/4*b*c^(1/3)*ln(1
+c^(2/3)*x^2+c^(4/3)*x^4)+1/2*b*c^(1/3)*arctan(1/3*(1+2*c^(2/3)*x^2)*3^(1/
2))*3^(1/2)
```

3.114.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.76

$$\int \frac{a+b\operatorname{arctanh}(cx^3)}{x^2}\,dx = -\frac{a}{x} + \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\frac{-1+2\sqrt[3]{c}x}{\sqrt{3}}\right) - \frac{1}{2}\sqrt{3}b\sqrt[3]{c}\arctan\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right) - \frac{b\operatorname{arctanh}(cx^3)}{x} - \frac{1}{2}b\sqrt[3]{c}\log\left(1-\sqrt[3]{c}x\right) - \frac{1}{2}b\sqrt[3]{c}\log\left(1+\sqrt[3]{c}x\right) + \frac{1}{4}b\sqrt[3]{c}\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right) + \frac{1}{4}b\sqrt[3]{c}\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])/x^2,x]
```

output
```
-(a/x) + (Sqrt[3]*b*c^(1/3)*ArcTan[(-1 + 2*c^(1/3)*x)/Sqrt[3]])/2 - (Sqrt[
3]*b*c^(1/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/2 - (b*ArcTanh[c*x^3])/x -
 (b*c^(1/3)*Log[1 - c^(1/3)*x])/2 - (b*c^(1/3)*Log[1 + c^(1/3)*x])/2 + (b*
c^(1/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/4 + (b*c^(1/3)*Log[1 + c^(1/3)*x
 + c^(2/3)*x^2])/4
```

3.114.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6452, 807, 750, 16, 1142, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^2}dx$$

$$\Big\downarrow \text{ 6452}$$

$$3bc\int \frac{x}{1-c^2x^6}dx-\frac{a+b\text{arctanh}(cx^3)}{x}$$

$$\Big\downarrow \text{ 807}$$

$$\frac{3}{2}bc\int \frac{1}{1-c^2x^6}dx^2-\frac{a+b\text{arctanh}(cx^3)}{x}$$

$$\Big\downarrow \text{ 750}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{1}{1-c^{2/3}x^2}dx^2+\frac{1}{3}\int \frac{c^{2/3}x^2+2}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2\right)-\frac{a+b\text{arctanh}(cx^3)}{x}$$

$$\Big\downarrow \text{ 16}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\int \frac{c^{2/3}x^2+2}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}\right)-\frac{a+b\text{arctanh}(cx^3)}{x}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2+\frac{\int \frac{c^{2/3}(2c^{2/3}x^2+1)}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2}{2c^{2/3}}\right)-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{3}{2}\int \frac{1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2+\frac{1}{2}\int \frac{2c^{2/3}x^2+1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2\right)-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 1082$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{2c^{2/3}x^2+1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2-\frac{3\int \frac{1}{-x^4-3}d\left(2c^{2/3}x^2+1\right)}{c^{2/3}}\right)-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 217$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{1}{2}\int \frac{2c^{2/3}x^2+1}{c^{4/3}x^4+c^{2/3}x^2+1}dx^2+\frac{\sqrt{3}\arctan\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{c^{2/3}}\right)-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{x}$$

$$\Big\downarrow \; 1103$$

$$\frac{3}{2}bc\left(\frac{1}{3}\left(\frac{\sqrt{3}\arctan\left(\frac{2c^{2/3}x^2+1}{\sqrt{3}}\right)}{c^{2/3}}+\frac{\log\left(c^{4/3}x^4+c^{2/3}x^2+1\right)}{2c^{2/3}}\right)-\frac{\log\left(1-c^{2/3}x^2\right)}{3c^{2/3}}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{x}$$

input
```
Int[(a + b*ArcTanh[c*x^3])/x^2,x]
```

output
```
-((a + b*ArcTanh[c*x^3])/x) + (3*b*c*(-1/3*Log[1 - c^(2/3)*x^2]/c^(2/3) +
((Sqrt[3]*ArcTan[(1 + 2*c^(2/3)*x^2)/Sqrt[3]])/c^(2/3) + Log[1 + c^(2/3)*x
^2 + c^(4/3)*x^4]/(2*c^(2/3)))/3))/2
```

3.114. $\int \frac{a+b\text{arctanh}(cx^3)}{x^2}\,dx$

3.114.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 750
```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Simp[1/(3*Rt[a, 3]^2)   Int[1/
(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]^2)   Int[(2*Rt[a, 3] -
 Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /;
 FreeQ[{a, b}, x]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.114.4 Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

method	result
default	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^{3}\right)}{x}-\frac{b\ln\left(x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x^{4}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}$
parts	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^{3}\right)}{x}-\frac{b\ln\left(x^{2}-\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x^{4}+\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}x^{2}+\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x^{2}}{\left(\frac{1}{c^{2}}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c^{2}}\right)^{\frac{2}{3}}}$
risch	$-\frac{b\ln\left(c\,x^{3}+1\right)}{2x}-\frac{a}{x}+\frac{b\ln\left(-c\,x^{3}+1\right)}{2x}-\frac{b\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\ln\left(x^{2}+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2\left(\frac{1}{c}\right)^{\frac{1}{3}}}-$

input
```
int((a+b*arctanh(c*x^3))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-b/x*arctanh(c*x^3)-1/2*b/c/(1/c^2)^(2/3)*ln(x^2-(1/c^2)^(1/3))+1/4*b/
c/(1/c^2)^(2/3)*ln(x^4+(1/c^2)^(1/3)*x^2+(1/c^2)^(2/3))+1/2*b/c/(1/c^2)^(2
/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c^2)^(1/3)*x^2+1))
```

3.114.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^2}\,dx = -\frac{2\sqrt{3}b(-c)^{\frac{1}{3}}\,x\arctan\left(\frac{2}{3}\sqrt{3}(-c)^{\frac{2}{3}}\,x^2+\frac{1}{3}\sqrt{3}\right)+b(-c)^{\frac{1}{3}}\,x\log\left(c^2x^4-(-c)^{\frac{1}{3}}\,cx^2+(-c)^{\frac{2}{3}}\right)-2\,b(-c)^{\frac{1}{3}}\,x}{4\,x}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*b*(-c)^(1/3)*x*arctan(2/3*sqrt(3)*(-c)^(2/3)*x^2 + 1/3*sqr
t(3)) + b*(-c)^(1/3)*x*log(c^2*x^4 - (-c)^(1/3)*c*x^2 + (-c)^(2/3)) - 2*b*
(-c)^(1/3)*x*log(c*x^2 + (-c)^(1/3)) + 2*b*log(-(c*x^3 + 1)/(c*x^3 - 1)) +
 4*a)/x
```

3.114.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))/x**2,x)
```

output
```
Timed out
```

3.114.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^2}\,dx = \frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{4}{3}}x^2+c^{\frac{2}{3}}\right)}{3c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}+\frac{\log\left(c^{\frac{4}{3}}x^4+c^{\frac{2}{3}}x^2+1\right)}{c^{\frac{2}{3}}}-\frac{2\log\left(\frac{c^{\frac{2}{3}}x^2-1}{c^{\frac{2}{3}}}\right)}{c^{\frac{2}{3}}}\right)-\frac{4\,\text{artanh}\left(cx^3\right)}{x}\right)b - \frac{a}{x}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="maxima")
```

output
```
1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(4/3)*x^2 + c^(2/3))/c^(2/3))/c^
(2/3) + log(c^(4/3)*x^4 + c^(2/3)*x^2 + 1)/c^(2/3) - 2*log((c^(2/3)*x^2 -
1)/c^(2/3))/c^(2/3)) - 4*arctanh(c*x^3)/x)*b - a/x
```

3.114.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.02

$$\int \frac{a+b\operatorname{arctanh}(cx^3)}{x^2}\,dx = \frac{1}{4}\,bc\left(\frac{2\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(2\,x^2+\frac{1}{|c|^{\frac{2}{3}}}\right)|c|^{\frac{2}{3}}\right)}{|c|^{\frac{2}{3}}}+\frac{\log\left(x^4+\frac{x^2}{|c|^{\frac{2}{3}}}+\frac{1}{|c|^{\frac{4}{3}}}\right)}{|c|^{\frac{2}{3}}}-\frac{2\,\log\left(\left|x^2-\frac{1}{|c|^{\frac{2}{3}}}\right|\right)}{|c|^{\frac{2}{3}}}\right) - \frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{2\,x}-\frac{a}{x}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^2,x, algorithm="giac")
```

output
```
1/4*b*c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1/abs(c)^(2/3))*abs(c)^(2/3
))/abs(c)^(2/3) + log(x^4 + x^2/abs(c)^(2/3) + 1/abs(c)^(4/3))/abs(c)^(2/3
) - 2*log(abs(x^2 - 1/abs(c)^(2/3)))/abs(c)^(2/3)) - 1/2*b*log(-(c*x^3 + 1
)/(c*x^3 - 1))/x - a/x
```

3.114.9 Mupad [B] (verification not implemented)

Time = 4.95 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.12

$$\int \frac{a+b\operatorname{arctanh}(cx^3)}{x^2}\,dx = \frac{b\,\ln\left(1-c\,x^3\right)}{2\,x}-\frac{b\,c^{1/3}\,\ln\left(1-c^{2/3}\,x^2\right)}{2}-\frac{b\,\ln\left(c\,x^3+1\right)}{2\,x} -\frac{a}{x}-\frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}-c^{2/3}\,x^2\,2\mathrm{i}-\mathrm{i}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{4} +\frac{b\,c^{1/3}\,\ln\left(-\sqrt{3}+c^{2/3}\,x^2\,2\mathrm{i}+1\mathrm{i}\right)\,\left(1+\sqrt{3}\,1\mathrm{i}\right)}{4}$$

input
```
int((a + b*atanh(c*x^3))/x^2,x)
```

output
```
(b*log(1 - c*x^3))/(2*x) - (b*c^(1/3)*log(1 - c^(2/3)*x^2))/2 - (b*log(c*x
^3 + 1))/(2*x) - a/x - (b*c^(1/3)*log(- 3^(1/2) - c^(2/3)*x^2*2i - 1i)*(3^
(1/2)*1i - 1))/4 + (b*c^(1/3)*log(c^(2/3)*x^2*2i - 3^(1/2) + 1i)*(3^(1/2)*
1i + 1))/4
```

3.114. $\int \frac{a+b\text{arctanh}(cx^3)}{x^2}\,dx$

3.115 $\int \frac{a+b\,\text{arctanh}(cx^3)}{x^5}\,dx$

3.115.1 Optimal result

Integrand size = 14, antiderivative size = 174

$$\int \frac{a+b\,\text{arctanh}(cx^3)}{x^5}\,dx = -\frac{3bc}{4x}+\frac{1}{8}\sqrt{3}bc^{4/3}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)-\frac{1}{8}\sqrt{3}bc^{4/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{c}x}{\sqrt{3}}\right)+\frac{1}{4}bc^{4/3}\text{arctanh}\left(\sqrt[3]{c}x\right)-\frac{a+b\,\text{arctanh}(cx^3)}{4x^4}-\frac{1}{16}bc^{4/3}\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{16}bc^{4/3}\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)$$

output
```
-3/4*b*c/x+1/4*b*c^(4/3)*arctanh(c^(1/3)*x)+1/4*(-a-b*arctanh(c*x^3))/x^4-
1/16*b*c^(4/3)*ln(1-c^(1/3)*x+c^(2/3)*x^2)+1/16*b*c^(4/3)*ln(1+c^(1/3)*x+c
^(2/3)*x^2)-1/8*b*c^(4/3)*arctan(-1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/
2)-1/8*b*c^(4/3)*arctan(1/3*3^(1/2)+2/3*c^(1/3)*x*3^(1/2))*3^(1/2)
```

3.115.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.13

$$\int \frac{a+b\,\text{arctanh}(cx^3)}{x^5}\,dx = -\frac{a}{4x^4}-\frac{3bc}{4x}-\frac{1}{8}\sqrt{3}bc^{4/3}\arctan\left(\frac{-1+2\sqrt[3]{c}x}{\sqrt{3}}\right)-\frac{1}{8}\sqrt{3}bc^{4/3}\arctan\left(\frac{1+2\sqrt[3]{c}x}{\sqrt{3}}\right)-\frac{b\,\text{arctanh}(cx^3)}{4x^4}-\frac{1}{8}bc^{4/3}\log\left(1-\sqrt[3]{c}x\right)+\frac{1}{8}bc^{4/3}\log\left(1+\sqrt[3]{c}x\right)-\frac{1}{16}bc^{4/3}\log\left(1-\sqrt[3]{c}x+c^{2/3}x^2\right)+\frac{1}{16}bc^{4/3}\log\left(1+\sqrt[3]{c}x+c^{2/3}x^2\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])/x^5,x]
```

output
```
-1/4*a/x^4 - (3*b*c)/(4*x) - (Sqrt[3]*b*c^(4/3)*ArcTan[(-1 + 2*c^(1/3)*x)/
Sqrt[3]])/8 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*x)/Sqrt[3]])/8 - (b
*ArcTanh[c*x^3])/(4*x^4) - (b*c^(4/3)*Log[1 - c^(1/3)*x])/8 + (b*c^(4/3)*L
og[1 + c^(1/3)*x])/8 - (b*c^(4/3)*Log[1 - c^(1/3)*x + c^(2/3)*x^2])/16 + (
b*c^(4/3)*Log[1 + c^(1/3)*x + c^(2/3)*x^2])/16
```

3.115.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {6452, 847, 825, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(cx^3\right)}{x^5}dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{3}{4}bc\int \frac{1}{x^2\left(1-c^2x^6\right)}dx-\frac{a+b\text{arctanh}\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{4}bc\left(c^2\int \frac{x^4}{1-c^2x^6}dx-\frac{1}{x}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{825}$$

$$\frac{3}{4}bc\left(c^2\left(\frac{\int \frac{1}{1-c^{2/3}x^2}dx}{3c^{4/3}}+\frac{\int -\frac{\sqrt[3]{c}x+1}{2\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}+\frac{\int -\frac{1-\sqrt[3]{c}x}{2\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}dx}{3c^{4/3}}\right)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{3}{4}bc\left(c^2\left(\frac{\int \frac{1}{1-c^{2/3}x^2}dx}{3c^{4/3}}-\frac{\int \frac{\sqrt[3]{c}x+1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{1-\sqrt[3]{c}x}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}\right)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{4x^4}$$

$$\downarrow 219$$

$$\frac{3}{4}bc\left(c^2\left(-\frac{\int \frac{\sqrt[3]{c}x+1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\int \frac{1-\sqrt[3]{c}x}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right)-\frac{1}{x}\right)-\frac{a+b\text{arctanh}\left(cx^3\right)}{4x^4}$$

$$\downarrow 1142$$

$$\frac{3}{4}bc\left(c^2\left(-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx+\frac{\int -\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+1\right)}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right.}{3c^{5/3}}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^3\right)}{4x^4}$$

$$\downarrow 25$$

$$\frac{3}{4}bc\left(c^2\left(-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{\int \frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}x\right)}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}x+1\right)}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^3\right)}{4x^4}$$

$$\downarrow 27$$

$$\frac{3}{4}bc\left(c^2\left(-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{1}{2}\int \frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{1}{2}\int \frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\text{arctanh}(}{3c^{5/}}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^3\right)}{4x^4}$$

$$\downarrow 1082$$

3.115. $\int \frac{a+b\text{arctanh}(cx^3)}{x^5}\,dx$

$$\frac{3}{4}bc\left(c^2\left(-\frac{\frac{3\int\frac{1}{-\left(1-2\sqrt[3]{c}x\right)^2-3}d\left(1-2\sqrt[3]{c}x\right)}{\sqrt[3]{c}}-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx}{6c^{4/3}}-\frac{-\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx-\frac{3\int\frac{1}{-\left(2\sqrt[3]{c}x+1\right)^2-3}d\left(2\sqrt[3]{c}\cdots\right.}{\sqrt[3]{c}}}{6c^{4/3}}\cdots\right.\right.$$

$$\frac{a+b\operatorname{arctanh}\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \; 217$$

$$\frac{3}{4}bc\left(c^2\left(-\frac{-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}x}{c^{2/3}x^2-\sqrt[3]{c}x+1}dx-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}x+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{1}{2}\int\frac{2\sqrt[3]{c}x+1}{c^{2/3}x^2+\sqrt[3]{c}x+1}dx}{6c^{4/3}}+\frac{\operatorname{arctanh}(\cdots}{3c^{5/3}}\cdots\right.\right.$$

$$\frac{a+b\operatorname{arctanh}\left(cx^3\right)}{4x^4}$$

$$\Big\downarrow \; 1103$$

$$\frac{3}{4}bc\left(c^2\left(-\frac{\frac{\log\left(c^{2/3}x^2-\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}x}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}x+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x^2+\sqrt[3]{c}x+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\operatorname{arctanh}\left(\sqrt[3]{c}x\right)}{3c^{5/3}}\right)\cdots\right)$$

$$\frac{a+b\operatorname{arctanh}\left(cx^3\right)}{4x^4}$$

input
```
Int[(a + b*ArcTanh[c*x^3])/x^5,x]
```

output
```
-1/4*(a + b*ArcTanh[c*x^3])/x^4 + (3*b*c*(-x^(-1) + c^2*(ArcTanh[c^(1/3)*x
]/(3*c^(5/3)) - (-((Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*x)/Sqrt[3]])/c^(1/3)) +
Log[1 - c^(1/3)*x + c^(2/3)*x^2]/(2*c^(1/3)))/(6*c^(4/3)) - ((Sqrt[3]*ArcT
an[(1 + 2*c^(1/3)*x)/Sqrt[3]])/c^(1/3) - Log[1 + c^(1/3)*x + c^(2/3)*x^2]/
(2*c^(1/3)))/(6*c^(4/3)))))/4
```

3.115. $\int\frac{a+b\operatorname{arctanh}\left(cx^3\right)}{x^5}\,dx$

3.115.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

3.115. $\int \frac{a+b\text{arctanh}(cx^3)}{x^5}\,dx$

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.115.4 Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.99

method	result
default	$-\frac{a}{4x^4}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{4x^4}-\frac{3bc}{4x}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc}{}$
parts	$-\frac{a}{4x^4}-\frac{b\,\mathrm{arctanh}(c\,x^3)}{4x^4}-\frac{3bc}{4x}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc}{}$
risch	$-\frac{b\ln(c\,x^3+1)}{8x^4}-\frac{a}{4x^4}+\frac{b\ln(-c\,x^3+1)}{8x^4}-\frac{3bc}{4x}-\frac{bc\ln\left(x-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc\ln\left(x^2+\left(\frac{1}{c}\right)^{\frac{1}{3}}x+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{1}{c}\right)}\right.}{3}\right.}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}$

input
```
int((a+b*arctanh(c*x^3))/x^5,x,method=_RETURNVERBOSE)
```

3.115. $\int \frac{a+b\mathrm{arctanh}(cx^3)}{x^5}\,dx$

output
```
-1/4*a/x^4-1/4*b/x^4*arctanh(c*x^3)-3/4*b*c/x-1/8*b*c/(1/c)^(1/3)*ln(x-(1/
c)^(1/3))+1/16*b*c/(1/c)^(1/3)*ln(x^2+(1/c)^(1/3)*x+(1/c)^(2/3))-1/8*b*c*3
^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x+1))+1/8*b*c/(1/c)^(
1/3)*ln(x+(1/c)^(1/3))-1/16*b*c/(1/c)^(1/3)*ln(x^2-(1/c)^(1/3)*x+(1/c)^(2/
3))-1/8*b*c*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x-1))
```

3.115.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.13

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^5}\,dx =$$

$$-\frac{2\sqrt{3}b(-c)^{\frac{1}{3}}cx^4\arctan\left(\frac{2}{3}\sqrt{3}(-c)^{\frac{1}{3}}x-\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}bc^{\frac{4}{3}}x^4\arctan\left(\frac{2}{3}\sqrt{3}c^{\frac{1}{3}}x-\frac{1}{3}\sqrt{3}\right)+b(-c)^{\frac{1}{3}}cx^4\log$$

input
```
integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="fricas")
```

output
```
-1/16*(2*sqrt(3)*b*(-c)^(1/3)*c*x^4*arctan(2/3*sqrt(3)*(-c)^(1/3)*x - 1/3*
sqrt(3)) + 2*sqrt(3)*b*c^(4/3)*x^4*arctan(2/3*sqrt(3)*c^(1/3)*x - 1/3*sqrt
(3)) + b*(-c)^(1/3)*c*x^4*log(c*x^2 + (-c)^(2/3)*x - (-c)^(1/3)) + b*c^(4/
3)*x^4*log(c*x^2 - c^(2/3)*x + c^(1/3)) - 2*b*(-c)^(1/3)*c*x^4*log(c*x - (
-c)^(2/3)) - 2*b*c^(4/3)*x^4*log(c*x + c^(2/3)) + 12*b*c*x^3 + 2*b*log(-(c
*x^3 + 1)/(c*x^3 - 1)) + 4*a)/x^4
```

3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(cx^3)}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))/x**5,x)
```

output
```
Timed out
```

3.115.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^5}\,dx =$$

$$-\frac{1}{16}\left(\left(2\sqrt{3}c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x + c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right) + 2\sqrt{3}c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}x - c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right) - c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}x^2 + c^{\frac{1}{3}}\right.\right.\right.$$

$$-\frac{a}{4\,x^4}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="maxima")
```

output
```
-1/16*((2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x + c^(1/3))/c^(1/
3)) + 2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*x - c^(1/3))/c^(1/3)
) - c^(1/3)*log(c^(2/3)*x^2 + c^(1/3)*x + 1) + c^(1/3)*log(c^(2/3)*x^2 - c
^(1/3)*x + 1) - 2*c^(1/3)*log((c^(1/3)*x + 1)/c^(1/3)) + 2*c^(1/3)*log((c^
(1/3)*x - 1)/c^(1/3)) + 12/x)*c + 4*arctanh(c*x^3)/x^4)*b - 1/4*a/x^4
```

3.115.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.11

$$\int \frac{a + b\text{arctanh}(cx^3)}{x^5}\,dx = -\frac{\sqrt{3}bc^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x + \frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{8\,|c|^{\frac{5}{3}}} - \frac{\sqrt{3}bc^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2\,x - \frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{8\,|c|^{\frac{5}{3}}} + \frac{bc^3\log\left(x^2 + \frac{x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{16\,|c|^{\frac{5}{3}}} - \frac{bc^3\log\left(x^2 - \frac{x}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{16\,|c|^{\frac{5}{3}}} + \frac{bc^3\log\left(\left|x + \frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{8\,|c|^{\frac{5}{3}}} - \frac{bc^3\log\left(\left|x - \frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{8\,|c|^{\frac{5}{3}}} - \frac{b\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{8\,x^4} - \frac{3\,bcx^3 + a}{4\,x^4}$$

input
```
integrate((a+b*arctanh(c*x^3))/x^5,x, algorithm="giac")
```

output
```
-1/8*sqrt(3)*b*c^3*arctan(1/3*sqrt(3)*(2*x + 1/abs(c)^(1/3))*abs(c)^(1/3))
/abs(c)^(5/3) - 1/8*sqrt(3)*b*c^3*arctan(1/3*sqrt(3)*(2*x - 1/abs(c)^(1/3)
)*abs(c)^(1/3))/abs(c)^(5/3) + 1/16*b*c^3*log(x^2 + x/abs(c)^(1/3) + 1/abs
(c)^(2/3))/abs(c)^(5/3) - 1/16*b*c^3*log(x^2 - x/abs(c)^(1/3) + 1/abs(c)^(
2/3))/abs(c)^(5/3) + 1/8*b*c^3*log(abs(x + 1/abs(c)^(1/3)))/abs(c)^(5/3) -
 1/8*b*c^3*log(abs(x - 1/abs(c)^(1/3)))/abs(c)^(5/3) - 1/8*b*log(-(c*x^3 +
 1)/(c*x^3 - 1))/x^4 - 1/4*(3*b*c*x^3 + a)/x^4
```

3.115.9 Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.72

$$\int \frac{a + b\operatorname{arctanh}(cx^3)}{x^5}\,dx = \frac{b\,\ln\left(1 - c\,x^3\right)}{8\,x^4} - \frac{b\,c^{4/3}\left(-\frac{\operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right)}{2} + \frac{\operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)}{2} + \operatorname{atan}\left(c^{1/3}\,x\,1\mathrm{i}\right)\right)\,1\mathrm{i}}{4} - \frac{3\,b\,c}{4\,x} - \frac{b\,\ln\left(c\,x^3+1\right)}{8\,x^4} - \frac{a}{4\,x^4} - \frac{\sqrt{3}\,b\,c^{4/3}\left(\operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}-\mathrm{i}\right)}{2}\right) + \operatorname{atan}\left(\frac{c^{1/3}\,x\,\left(\sqrt{3}+1\mathrm{i}\right)}{2}\right)\right)}{8}$$

input
```
int((a + b*atanh(c*x^3))/x^5,x)
```

output
```
(b*log(1 - c*x^3))/(8*x^4) - (b*c^(4/3)*(atan((c^(1/3)*x*(3^(1/2) + 1i))/2
)/2 - atan((c^(1/3)*x*(3^(1/2) - 1i))/2)/2 + atan(c^(1/3)*x*1i))*1i)/4 - (
3*b*c)/(4*x) - (b*log(c*x^3 + 1))/(8*x^4) - a/(4*x^4) - (3^(1/2)*b*c^(4/3)
*(atan((c^(1/3)*x*(3^(1/2) - 1i))/2) + atan((c^(1/3)*x*(3^(1/2) + 1i))/2))
)/8
```

3.116 $\int x^{11}(a+b\text{arctanh}(cx^3))^2\,dx$

3.116.1 Optimal result

Integrand size = 16, antiderivative size = 125

$$\int x^{11}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx=\frac{abx^3}{6c^3}+\frac{b^2x^6}{36c^2}+\frac{b^2x^3\text{arctanh}(cx^3)}{6c^3}+\frac{bx^9(a+b\text{arctanh}(cx^3))}{18c}-\frac{(a+b\text{arctanh}(cx^3))^2}{12c^4}+\frac{1}{12}x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2+\frac{b^2\log\left(1-c^2x^6\right)}{9c^4}$$

output
```
1/6*a*b*x^3/c^3+1/36*b^2*x^6/c^2+1/6*b^2*x^3*arctanh(c*x^3)/c^3+1/18*b*x^9
*(a+b*arctanh(c*x^3))/c-1/12*(a+b*arctanh(c*x^3))^2/c^4+1/12*x^12*(a+b*arc
tanh(c*x^3))^2+1/9*b^2*ln(-c^2*x^6+1)/c^4
```

3.116.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.17

$$\int x^{11}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx=\frac{6abcx^3+b^2c^2x^6+2abc^3x^9+3a^2c^4x^{12}+2bcx^3(3ac^3x^9+b(3+c^2x^6))\,\text{arctanh}(cx^3)+3b^2(-1+c^4x^{12})\,\text{arc}}{36c^4}$$

input
```
Integrate[x^11*(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
(6*a*b*c*x^3 + b^2*c^2*x^6 + 2*a*b*c^3*x^9 + 3*a^2*c^4*x^12 + 2*b*c*x^3*(3
*a*c^3*x^9 + b*(3 + c^2*x^6))*ArcTanh[c*x^3] + 3*b^2*(-1 + c^4*x^12)*ArcTa
nh[c*x^3]^2 + b*(3*a + 4*b)*Log[1 - c*x^3] - 3*a*b*Log[1 + c*x^3] + 4*b^2*
Log[1 + c*x^3])/(36*c^4)
```

3.116.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6454, 6452, 6542, 6452, 243, 49, 2009, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{11}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{3}\int x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)^2dx^3$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{1}{2}bc\int \frac{x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3\right)$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\int \frac{x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3}{c^2}-\frac{\int x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\int \frac{x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{3}bc\int \frac{x^9}{1-c^2x^6}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\int \frac{x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{6}bc\int \frac{x^6}{1-c^2x^6}dx^6}{c^2}\right)\right)$$

$\downarrow$ 49

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\int \frac{x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{6}bc\int \left(-\frac{1}{c^2}-\frac{}{c^2(c^2}\right.}{c^2}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\int \frac{x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{6}bc\left(-\frac{x^6}{c^2}-\frac{\log(1-}{c^4}\right.}{c^2}\right.\right.$$

$\downarrow$ 6542

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{\int \frac{a+b\text{arctanh}\left(cx^3\right)}{1-c^2x^6}dx^3}{c^2}-\frac{\int \left(a+b\text{arctanh}\left(cx^3\right)\right)dx^3}{c^2}}{c^2}-\frac{\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)}{c}\right.\right.$$

$\downarrow$ 2009

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{\int \frac{a+b\text{arctanh}\left(cx^3\right)}{1-c^2x^6}dx^3}{c^2}-\frac{ax^3+bx^3\text{arctanh}\left(cx^3\right)+\frac{b\log\left(1-c^2x^6\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^9\left(a+b\text{arctan}\right.}{}\right.\right.$$

$\downarrow$ 6510

$$\frac{1}{3}\left(\frac{1}{4}x^{12}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{2bc^3}-\frac{ax^3+bx^3\text{arctanh}\left(cx^3\right)+\frac{b\log\left(1-c^2x^6\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^9\left(a+\text{barctanh}\right.}{}\right.\right.$$

input
```
Int[x^11*(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
((x^12*(a + b*ArcTanh[c*x^3])^2)/4 - (b*c*(-((((x^9*(a + b*ArcTanh[c*x^3]))
/3 - (b*c*(-(x^6/c^2) - Log[1 - c^2*x^6]/c^4))/6)/c^2) + ((a + b*ArcTanh[c
*x^3])^2/(2*b*c^3) - (a*x^3 + b*x^3*ArcTanh[c*x^3] + (b*Log[1 - c^2*x^6])/
(2*c))/c^2)/c^2))/2)/3
```

3.116. $\int x^{11}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx$

3.116.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.116.4 Maple [A] (verified)

Time = 1.77 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.30

method	result
parallelrisch	$\frac{3b^2 \operatorname{arctanh}(c\,x^3)^2 x^{12} c^4+6ab \operatorname{arctanh}(c\,x^3) x^{12} c^4+3a^2 c^4 x^{12}+2b^2 \operatorname{arctanh}(c\,x^3) x^9 c^3+2ab\, c^3 x^9+b^2 c^2 x^6+6b^2 \operatorname{arctanh}(c\,x^3) x^3}{36c^4}$
risch	$\frac{b^2 (x^{12} c^4-1) \ln(c\,x^3+1)^2}{48c^4}+\frac{b(-3x^{12} b \ln(-c\,x^3+1) c^4+6a\,c^4 x^{12}+2b\,c^3 x^9+6bc\,x^3+3b \ln(-c\,x^3+1)) \ln(c\,x^3+1)}{72c^4}+\frac{b^2 x^{12} \ln(}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^11*(a+b*arctanh(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*b^2*arctanh(c*x^3)^2*x^12*c^4+6*a*b*arctanh(c*x^3)*x^12*c^4+3*a^2*
c^4*x^12+2*b^2*arctanh(c*x^3)*x^9*c^3+2*a*b*c^3*x^9+b^2*c^2*x^6+6*b^2*arct
anh(c*x^3)*x^3*c+6*a*b*c*x^3-3*b^2*arctanh(c*x^3)^2+8*ln(c*x^3-1)*b^2-6*ar
ctanh(c*x^3)*a*b+8*arctanh(c*x^3)*b^2+b^2)/c^4
```

3.116.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.41

$$\int x^{11}(a+b\operatorname{arctanh}(cx^3))^2\,dx$$
$$=\frac{12\,a^2c^4x^{12}+8\,abc^3x^9+4\,b^2c^2x^6+24\,abcx^3+3\,(b^2c^4x^{12}-b^2)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2-4\,(3\,ab-4\,b^2)\log\left(cx^3+1\right.}{144\,c^4}$$

input
```
integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

output
```
1/144*(12*a^2*c^4*x^12 + 8*a*b*c^3*x^9 + 4*b^2*c^2*x^6 + 24*a*b*c*x^3 + 3*
(b^2*c^4*x^12 - b^2)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 - 4*(3*a*b - 4*b^2)*l
og(c*x^3 + 1) + 4*(3*a*b + 4*b^2)*log(c*x^3 - 1) + 4*(3*a*b*c^4*x^12 + b^2
*c^3*x^9 + 3*b^2*c*x^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)))/c^4
```

3.116.6 Sympy [F(-1)]

Timed out.

$$\int x^{11}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate(x**11*(a+b*atanh(c*x**3))**2,x)
```

output
```
Timed out
```

3.116.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.74

$$\int x^{11}\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx=\frac{1}{12}b^2x^{12}\,\text{artanh}\left(cx^3\right)^2+\frac{1}{12}a^2x^{12}+\frac{1}{36}\left(6\,x^{12}\,\text{artanh}\left(cx^3\right)+c\left(\frac{2\left(c^2x^9+3\,x^3\right)}{c^4}-\frac{3\,\log\left(cx^3+1\right)}{c^5}+\frac{3\,\log\left(cx^3-1\right)}{c^5}\right)\right)ab+\frac{1}{144}\left(4\,c\left(\frac{2\left(c^2x^9+3\,x^3\right)}{c^4}-\frac{3\,\log\left(cx^3+1\right)}{c^5}+\frac{3\,\log\left(cx^3-1\right)}{c^5}\right)\text{artanh}\left(cx^3\right)+\frac{4\,c^2x^6-2\left(3\,\log\left(cx^3-\right.\right.}{}\right.$$

input
```
integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/12*b^2*x^12*arctanh(c*x^3)^2 + 1/12*a^2*x^12 + 1/36*(6*x^12*arctanh(c*x^
3) + c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*log(c*x^3 + 1)/c^5 + 3*log(c*x^3 - 1)/
c^5))*a*b + 1/144*(4*c*(2*(c^2*x^9 + 3*x^3)/c^4 - 3*log(c*x^3 + 1)/c^5 + 3
*log(c*x^3 - 1)/c^5)*arctanh(c*x^3) + (4*c^2*x^6 - 2*(3*log(c*x^3 - 1) - 8
)*log(c*x^3 + 1) + 3*log(c*x^3 + 1)^2 + 3*log(c*x^3 - 1)^2 + 16*log(c*x^3
- 1))/c^4)*b^2
```

3.116.8 Giac [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.40

$$\int x^{11}(a+b\text{arctanh}(cx^3))^2\,dx = \frac{1}{12}\,a^2x^{12}+\frac{abx^9}{18\,c}+\frac{b^2x^6}{36\,c^2}+\frac{1}{48}\left(b^2x^{12}-\frac{b^2}{c^4}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2 + \frac{abx^3}{6\,c^3}+\frac{1}{36}\left(3\,abx^{12}+\frac{b^2x^9}{c}+\frac{3\,b^2x^3}{c^3}\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right) - \frac{(3\,ab-4\,b^2)\log(cx^3+1)}{36\,c^4}+\frac{(3\,ab+4\,b^2)\log(cx^3-1)}{36\,c^4}$$

input
```
integrate(x^11*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

output
```
1/12*a^2*x^12 + 1/18*a*b*x^9/c + 1/36*b^2*x^6/c^2 + 1/48*(b^2*x^12 - b^2/c
^4)*log(-(c*x^3 + 1)/(c*x^3 - 1))^2 + 1/6*a*b*x^3/c^3 + 1/36*(3*a*b*x^12 +
 b^2*x^9/c + 3*b^2*x^3/c^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)) - 1/36*(3*a*b -
4*b^2)*log(c*x^3 + 1)/c^4 + 1/36*(3*a*b + 4*b^2)*log(c*x^3 - 1)/c^4
```

3.116.9 Mupad [B] (verification not implemented)

Time = 4.39 (sec) , antiderivative size = 335, normalized size of antiderivative = 2.68

$$\int x^{11}(a+b\text{arctanh}(cx^3))^2\,dx = \frac{a^2\,x^{12}}{12}+\frac{b^2\,\ln(c\,x^3-1)}{9\,c^4}+\frac{b^2\,\ln(c\,x^3+1)}{9\,c^4} - \frac{b^2\ln(c\,x^3+1)^2}{48\,c^4}-\frac{b^2\ln(1-c\,x^3)^2}{48\,c^4}+\frac{b^2\,x^6}{36\,c^2} + \frac{b^2\,x^{12}\ln(c\,x^3+1)^2}{48}+\frac{b^2\,x^{12}\ln(1-c\,x^3)^2}{48} + \frac{b^2\,x^3\,\ln(c\,x^3+1)}{12\,c^3}-\frac{b^2\,x^3\,\ln(1-c\,x^3)}{12\,c^3} + \frac{b^2\,x^9\,\ln(c\,x^3+1)}{36\,c}-\frac{b^2\,x^9\,\ln(1-c\,x^3)}{36\,c} + \frac{a\,b\,\ln(c\,x^3-1)}{12\,c^4}-\frac{a\,b\,\ln(c\,x^3+1)}{12\,c^4}+\frac{a\,b\,x^{12}\,\ln(c\,x^3+1)}{12} - \frac{a\,b\,x^{12}\,\ln(1-c\,x^3)}{12}+\frac{b^2\,\ln(c\,x^3+1)\,\ln(1-c\,x^3)}{24\,c^4} + \frac{a\,b\,x^3}{6\,c^3}+\frac{a\,b\,x^9}{18\,c}-\frac{b^2\,x^{12}\,\ln(c\,x^3+1)\,\ln(1-c\,x^3)}{24}$$

input
```
int(x^11*(a + b*atanh(c*x^3))^2,x)
```

output
```
(a^2*x^12)/12 + (b^2*log(c*x^3 - 1))/(9*c^4) + (b^2*log(c*x^3 + 1))/(9*c^4
) - (b^2*log(c*x^3 + 1)^2)/(48*c^4) - (b^2*log(1 - c*x^3)^2)/(48*c^4) + (b
^2*x^6)/(36*c^2) + (b^2*x^12*log(c*x^3 + 1)^2)/48 + (b^2*x^12*log(1 - c*x^
3)^2)/48 + (b^2*x^3*log(c*x^3 + 1))/(12*c^3) - (b^2*x^3*log(1 - c*x^3))/(1
2*c^3) + (b^2*x^9*log(c*x^3 + 1))/(36*c) - (b^2*x^9*log(1 - c*x^3))/(36*c)
 + (a*b*log(c*x^3 - 1))/(12*c^4) - (a*b*log(c*x^3 + 1))/(12*c^4) + (a*b*x^
12*log(c*x^3 + 1))/12 - (a*b*x^12*log(1 - c*x^3))/12 + (b^2*log(c*x^3 + 1)
*log(1 - c*x^3))/(24*c^4) + (a*b*x^3)/(6*c^3) + (a*b*x^9)/(18*c) - (b^2*x^
12*log(c*x^3 + 1)*log(1 - c*x^3))/24
```

3.116. $\int x^{11}(a+b\text{arctanh}(cx^3))^2\,dx$

3.117 $\int x^8(a+b\text{arctanh}(cx^3))^2\,dx$

3.117.1 Optimal result

Integrand size = 16, antiderivative size = 146

$$\begin{aligned}\int x^8\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx=&\frac{b^2x^3}{9c^2}-\frac{b^2\text{arctanh}(cx^3)}{9c^3}+\frac{bx^6(a+b\text{arctanh}(cx^3))}{9c}\\&+\frac{(a+b\text{arctanh}(cx^3))^2}{9c^3}+\frac{1}{9}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\\&-\frac{2b(a+b\text{arctanh}(cx^3))\log\left(\frac{2}{1-cx^3}\right)}{9c^3}\\&-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{9c^3}\end{aligned}$$

output
```
1/9*b^2*x^3/c^2-1/9*b^2*arctanh(c*x^3)/c^3+1/9*b*x^6*(a+b*arctanh(c*x^3))/
c+1/9*(a+b*arctanh(c*x^3))^2/c^3+1/9*x^9*(a+b*arctanh(c*x^3))^2-2/9*b*(a+b
*arctanh(c*x^3))*ln(2/(-c*x^3+1))/c^3-1/9*b^2*polylog(2,1-2/(-c*x^3+1))/c^
3
```

3.117.2 Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.90

$$\int x^8\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx$$

$$=\frac{b^2cx^3+abc^2x^6+a^2c^3x^9+b^2(-1+c^3x^9)\,\text{arctanh}(cx^3)^2+b\text{arctanh}(cx^3)\left(-b+bc^2x^6+2ac^3x^9-2b\log\left(\right.\right.}{9c^3}$$

input
```
Integrate[x^8*(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
(b^2*c*x^3 + a*b*c^2*x^6 + a^2*c^3*x^9 + b^2*(-1 + c^3*x^9)*ArcTanh[c*x^3]
^2 + b*ArcTanh[c*x^3]*(-b + b*c^2*x^6 + 2*a*c^3*x^9 - 2*b*Log[1 + E^(-2*Ar
cTanh[c*x^3])]) + a*b*Log[-1 + c^2*x^6] + b^2*PolyLog[2, -E^(-2*ArcTanh[c*
x^3])])/(9*c^3)
```

3.117.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {6454, 6452, 6542, 6452, 262, 219, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+b\text{arctanh}\left(cx^3\right)\right)^2 dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{3}\int x^6 \left(a+b\text{arctanh}\left(cx^3\right)\right)^2 dx^3$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}\left(\frac{1}{3}x^9 \left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{2}{3}bc\int \frac{x^9 \left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3\right)$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{3}\left(\frac{1}{3}x^9 \left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\int \frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3}{c^2}-\frac{\int x^3 \left(a+b\text{arctanh}\left(cx^3\right)\right)dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}\left(\frac{1}{3}x^9 \left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\int \frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{2}x^6 \left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{2}bc\int \frac{x^6}{1-c^2x^6}dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\int\frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{\int\frac{1}{1-c^2x^6}dx^3}{c^2}-\right.}{c^2}\right.\right.$$

$\downarrow$ 219

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\int\frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{2}bc\left(\frac{\text{arctanh}\left(cx^3\right)}{c^3}\right.}{c^2}\right.\right.$$

$\downarrow$ 6546

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{\int\frac{a+b\text{arctanh}\left(cx^3\right)}{1-cx^3}dx^3}{c}-\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)-\frac{1}{2}b}{c^2}\right.\right.$$

$\downarrow$ 6470

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)}{c}-b\int\frac{\log\left(\frac{2}{1-cx^3}\right)}{1-c^2x^6}dx^3}{c}-\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^6(a}{}\right.\right.$$

$\downarrow$ 2849

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx^3}\right)}{1-\frac{2}{1-cx^3}}d\frac{1}{1-cx^3}}{c}+\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)}{c}}{c}-\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x}{}\right.\right.$$

$\downarrow$ 2752

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{2}{3}bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{2c}}{c}-\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{2bc^2}}{c^2}-\frac{\frac{1}{2}x^6}{}\right.\right.$$

input
```
Int[x^8*(a + b*ArcTanh[c*x^3])^2,x]
```

3.117. $\int x^8\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx$

output
```
((x^9*(a + b*ArcTanh[c*x^3])^2)/3 - (2*b*c*(-(((x^6*(a + b*ArcTanh[c*x^3])
)/2 - (b*c*(-(x^3/c^2) + ArcTanh[c*x^3]/c^3))/2)/c^2) + (-1/2*(a + b*ArcTa
nh[c*x^3])^2/(b*c^2) + (((a + b*ArcTanh[c*x^3])*Log[2/(1 - c*x^3)])/c + (b
*PolyLog[2, 1 - 2/(1 - c*x^3)])/(2*c))/c)/c^2))/3)/3
```

3.117.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.117.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.35 (sec) , antiderivative size = 2906, normalized size of antiderivative = 19.90

method	result	size
default	Expression too large to display	2906
parts	Expression too large to display	2906

input
```
int(x^8*(a+b*arctanh(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*a^2*x^9+b^2*(1/9*x^9*arctanh(c*x^3)^2-2/3*c*(-1/6*arctanh(c*x^3)/c^2*x
^6-1/6*arctanh(c*x^3)/c^4*ln(c^2*x^6-1)-1/2*c*(1/3/c^4*x^3-1/6/c^5*ln(c*x^
3+1)+1/6/c^5*ln(c*x^3-1)+1/c^4*(Sum(1/6*(ln(x-_alpha)*ln(c^2*x^6-1)-6*c^2*
(1/2/c*(1/3*ln(x-_alpha)*(ln((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=1)-x+_a
lpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,index=1))+ln((RootOf(_Z^2+_Z*_alpha+_
alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,index=2))+ln(1/2
*(x+_alpha)/_alpha))/c+1/3*(dilog((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=1)
-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,index=1))+dilog((RootOf(_Z^2+_Z*
_alpha+_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,index=2)
)+dilog(1/2*(x+_alpha)/_alpha))/c)+1/2/c*(1/6/_alpha^2/c*ln(x-_alpha)^2-1/
3*_alpha*ln(x-_alpha)*(2*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x
+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*RootOf(_Z^2+3*_Z*_al
pha+3*_alpha^2,index=2)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)+6*ln((
RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_al
pha+3*_alpha^2,index=1))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*_alph
a+3*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+
3*_Z*_alpha+3*_alpha^2,index=1))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=
1)*_alpha+9*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/Root
Of(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha^2+2*ln((RootOf(_Z^2+3*_Z*_
alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,...
```

3.117.5 Fricas [F]

$$\int x^8 \left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2 \, dx = \int \left(b\operatorname{artanh}\left(cx^3\right)+a\right)^2 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^8*arctanh(c*x^3)^2 + 2*a*b*x^8*arctanh(c*x^3) + a^2*x^8, x)
```

3.117.6 Sympy [F(-1)]

Timed out.

$$\int x^8 (a + b\operatorname{arctanh}(cx^3))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**8*(a+b*atanh(c*x**3))**2,x)
```

output
```
Timed out
```

3.117.7 Maxima [F]

$$\int x^8 (a + b\operatorname{arctanh}(cx^3))^2 \, dx = \int (b \operatorname{artanh}(cx^3) + a)^2 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/9*a^2*x^9 + 1/9*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)
*c)*a*b + 1/648*(18*x^9*log(-c*x^3 + 1)^2 - 2*c^4*(2*(c^2*x^9 + 3*x^3)/c^6
 - 3*log(c*x^3 + 1)/c^7 + 3*log(c*x^3 - 1)/c^7) + 3*(x^6/c^4 + log(c^2*x^6
 - 1)/c^6)*c^3 + 1944*c^3*integrate(1/9*x^11*log(c*x^3 + 1)/(c^4*x^6 - c^2
), x) - 9*c^2*(2*x^3/c^4 - log(c*x^3 + 1)/c^5 + log(c*x^3 - 1)/c^5) - 6*c*
((2*c^2*x^9 + 3*c*x^6 + 6*x^3)/c^3 + 6*log(c*x^3 - 1)/c^4)*log(-c*x^3 + 1)
 + 972*c*integrate(1/9*x^5*log(c*x^3 + 1)/(c^4*x^6 - c^2), x) + 6*(3*c^3*x
^9*log(c*x^3 + 1)^2 + (2*c^3*x^9 - 3*c^2*x^6 + 6*c*x^3 - 6*(c^3*x^9 + 1)*l
og(c*x^3 + 1))*log(-c*x^3 + 1))/c^3 + (4*c^3*x^9 + 15*c^2*x^6 + 66*c*x^3 +
 18*log(c*x^3 - 1)^2 + 66*log(c*x^3 - 1))/c^3 - 18*log(9*c^4*x^6 - 9*c^2)/
c^3 + 972*integrate(1/9*x^2*log(c*x^3 + 1)/(c^4*x^6 - c^2), x))*b^2
```

3.117.8 Giac [F]

$$\int x^8 (a + b\operatorname{arctanh}(cx^3))^2 \, dx = \int (b \operatorname{artanh}(cx^3) + a)^2 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^2*x^8, x)
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int x^8\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2\,dx=\int x^8\,\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)^2\,dx$$

input
```
int(x^8*(a + b*atanh(c*x^3))^2,x)
```

output
```
int(x^8*(a + b*atanh(c*x^3))^2, x)
```

3.118 $\int x^5(a+b\operatorname{arctanh}(cx^3))^2\,dx$

3.118.1 Optimal result

Integrand size = 16, antiderivative size = 91

$$\int x^5\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2\,dx=\frac{abx^3}{3c}+\frac{b^2x^3\operatorname{arctanh}(cx^3)}{3c}-\frac{(a+b\operatorname{arctanh}(cx^3))^2}{6c^2}+\frac{1}{6}x^6\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2+\frac{b^2\log\left(1-c^2x^6\right)}{6c^2}$$

output
```
1/3*a*b*x^3/c+1/3*b^2*x^3*arctanh(c*x^3)/c-1/6*(a+b*arctanh(c*x^3))^2/c^2+
1/6*x^6*(a+b*arctanh(c*x^3))^2+1/6*b^2*ln(-c^2*x^6+1)/c^2
```

3.118.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16

$$\int x^5\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2\,dx$$

$$=\frac{2abcx^3+a^2c^2x^6+2bcx^3(b+acx^3)\operatorname{arctanh}(cx^3)+b^2(-1+c^2x^6)\operatorname{arctanh}(cx^3)^2+b(a+b)\log\left(1-cx^3\right)-}{6c^2}$$

input
```
Integrate[x^5*(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
(2*a*b*c*x^3 + a^2*c^2*x^6 + 2*b*c*x^3*(b + a*c*x^3)*ArcTanh[c*x^3] + b^2*
(-1 + c^2*x^6)*ArcTanh[c*x^3]^2 + b*(a + b)*Log[1 - c*x^3] - a*b*Log[1 + c
*x^3] + b^2*Log[1 + c*x^3])/(6*c^2)
```

3.118.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6454, 6452, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 (a + b\text{arctanh}(cx^3))^2 \, dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{3} \int x^3 (a + b\text{arctanh}(cx^3))^2 \, dx^3$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 (a + b\text{arctanh}(cx^3))^2 - bc \int \frac{x^6 (a + b\text{arctanh}(cx^3))}{1 - c^2 x^6} dx^3 \right)$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 (a + b\text{arctanh}(cx^3))^2 - bc \left(\frac{\int \frac{a + b\text{arctanh}(cx^3)}{1 - c^2 x^6} dx^3}{c^2} - \frac{\int (a + b\text{arctanh}(cx^3)) \, dx^3}{c^2} \right) \right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 (a + b\text{arctanh}(cx^3))^2 - bc \left(\frac{\int \frac{a + b\text{arctanh}(cx^3)}{1 - c^2 x^6} dx^3}{c^2} - \frac{ax^3 + bx^3 \text{arctanh}(cx^3) + \frac{b \log(1 - c^2 x^6)}{2c}}{c^2} \right) \right)$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{3} \left(\frac{1}{2} x^6 (a + b\text{arctanh}(cx^3))^2 - bc \left(\frac{(a + b\text{arctanh}(cx^3))^2}{2bc^3} - \frac{ax^3 + bx^3 \text{arctanh}(cx^3) + \frac{b \log(1 - c^2 x^6)}{2c}}{c^2} \right) \right)$$

input
```
Int[x^5*(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
((x^6*(a + b*ArcTanh[c*x^3])^2)/2 - b*c*((a + b*ArcTanh[c*x^3])^2/(2*b*c^3
) - (a*x^3 + b*x^3*ArcTanh[c*x^3] + (b*Log[1 - c^2*x^6])/(2*c))/c^2))/3
```

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.118.4 Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

method	result
parallelrisch	$\frac{b^2 \operatorname{arctanh}(c x^3)^2 x^6 c^2+2x^6 \operatorname{arctanh}(c x^3) ab c^2+a^2c^2x^6+2b^2 \operatorname{arctanh}(c x^3) x^3 c+2abc x^3-b^2 \operatorname{arctanh}(c x^3)^2+2\ln(c x^3-1)b^2-}{6c^2}$
risch	$\frac{b^2(c^2x^6-1)\ln(c x^3+1)^2}{24c^2}+\frac{b(-2b x^6\ln(-c x^3+1)a c^2+4a^2c^2x^6+4abc x^3+2b\ln(-c x^3+1)a+b^2)\ln(c x^3+1)}{24a c^2}+\frac{b^2x^6\ln(-c}{24}$
default	Expression too large to display
parts	Expression too large to display

input
```
int(x^5*(a+b*arctanh(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(b^2*arctanh(c*x^3)^2*x^6*c^2+2*x^6*arctanh(c*x^3)*a*b*c^2+a^2*c^2*x^6
+2*b^2*arctanh(c*x^3)*x^3*c+2*a*b*c*x^3-b^2*arctanh(c*x^3)^2+2*ln(c*x^3-1)
*b^2-2*arctanh(c*x^3)*a*b+2*arctanh(c*x^3)*b^2)/c^2
```

3.118.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.52

$$\int x^5 \left(a + b\text{arctanh}\left(cx^3\right)\right)^2 dx$$
$$= \frac{4\,a^2c^2x^6 + 8\,abcx^3 + (b^2c^2x^6 - b^2)\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2 - 4\,(ab - b^2)\log\left(cx^3 + 1\right) + 4\,(ab + b^2)\log\left(cx^3 - 1\right) + 4}{24\,c^2}$$

input
```
integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

output
```
1/24*(4*a^2*c^2*x^6 + 8*a*b*c*x^3 + (b^2*c^2*x^6 - b^2)*log(-(c*x^3 + 1)/(
c*x^3 - 1))^2 - 4*(a*b - b^2)*log(c*x^3 + 1) + 4*(a*b + b^2)*log(c*x^3 - 1
) + 4*(a*b*c^2*x^6 + b^2*c*x^3)*log(-(c*x^3 + 1)/(c*x^3 - 1)))/c^2
```

3.118.6 Sympy [F(-1)]

Timed out.

$$\int x^5 \left(a + b\text{arctanh}\left(cx^3\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*atanh(c*x**3))**2,x)
```

output
```
Timed out
```

3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(81) = 162$.

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.04

$$\int x^5 \left(a + b\text{arctanh}\left(cx^3\right)\right)^2 \,dx = \frac{1}{6}\,b^2 x^6 \operatorname{artanh}\left(cx^3\right)^2 + \frac{1}{6}\,a^2 x^6 + \frac{1}{6}\left(2\,x^6 \operatorname{artanh}\left(cx^3\right) + c\left(\frac{2\,x^3}{c^2} - \frac{\log\left(cx^3+1\right)}{c^3} + \frac{\log\left(cx^3-1\right)}{c^3}\right)\right)ab + \frac{1}{24}\left(4\,c\left(\frac{2\,x^3}{c^2} - \frac{\log\left(cx^3+1\right)}{c^3} + \frac{\log\left(cx^3-1\right)}{c^3}\right)\operatorname{artanh}\left(cx^3\right) - \frac{2\left(\log\left(cx^3-1\right)-2\right)\log\left(cx^3+1\right) - \log\right.$$

input
```
integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/6*b^2*x^6*arctanh(c*x^3)^2 + 1/6*a^2*x^6 + 1/6*(2*x^6*arctanh(c*x^3) + c
*(2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3))*a*b + 1/24*(4*c*(2
*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3)*arctanh(c*x^3) - (2*(l
og(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - log(c*x^3 - 1)^2 -
4*log(c*x^3 - 1))/c^2)*b^2
```

3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(81) = 162$.

Time = 0.30 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.97

$$\int x^5 \left(a + b\text{arctanh}\left(cx^3\right)\right)^2 \,dx = \frac{1}{6}\left(\frac{\left(cx^3+1\right)b^2\log\left(-\frac{cx^3+1}{cx^3-1}\right)^2}{\left(cx^3-1\right)\left(\frac{\left(cx^3+1\right)^2c^3}{\left(cx^3-1\right)^2} - \frac{2\left(cx^3+1\right)c^3}{cx^3-1} + c^3\right)} + \frac{2\left(\frac{2\left(cx^3+1\right)ab}{cx^3-1} + \frac{\left(cx^3+1\right)b^2}{cx^3-1} - b^2\right)\log\left(-\frac{cx^3+1}{cx^3-1}\right)}{\frac{\left(cx^3+1\right)^2c^3}{\left(cx^3-1\right)^2} - \frac{2\left(cx^3+1\right)c^3}{cx^3-1} + c^3} + \frac{4\left(\frac{\left(cx^3+1\right)a^2}{cx^3-1}\right.}{\frac{\left(cx^3+1\right)^2c^3}{\left(cx^3-1\right)^2}}\right.$$

input
```
integrate(x^5*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

output
```
1/6*((c*x^3 + 1)*b^2*log(-(c*x^3 + 1)/(c*x^3 - 1))^2/((c*x^3 - 1)*((c*x^3
+ 1)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 - 1) + c^3)) + 2*(2*(c
*x^3 + 1)*a*b/(c*x^3 - 1) + (c*x^3 + 1)*b^2/(c*x^3 - 1) - b^2)*log(-(c*x^3
 + 1)/(c*x^3 - 1))/((c*x^3 + 1)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c
*x^3 - 1) + c^3) + 4*((c*x^3 + 1)*a^2/(c*x^3 - 1) + (c*x^3 + 1)*a*b/(c*x^3
 - 1) - a*b)/((c*x^3 + 1)^2*c^3/(c*x^3 - 1)^2 - 2*(c*x^3 + 1)*c^3/(c*x^3 -
 1) + c^3) - 2*b^2*log(-(c*x^3 + 1)/(c*x^3 - 1) + 1)/c^3 + 2*b^2*log(-(c*x
^3 + 1)/(c*x^3 - 1))/c^3)*c
```

3.118.9 Mupad [B] (verification not implemented)

Time = 3.67 (sec) , antiderivative size = 275, normalized size of antiderivative = 3.02

$$\int x^5 \left(a+b\text{arctanh}\left(cx^3\right)\right)^2 \, dx = \frac{a^2\,x^6}{6}+\frac{b^2\,\ln\left(c\,x^3-1\right)}{6\,c^2}+\frac{b^2\,\ln\left(c\,x^3+1\right)}{6\,c^2}-\frac{b^2\ln\left(c\,x^3+1\right)^2}{24\,c^2}-\frac{b^2\ln\left(1-c\,x^3\right)^2}{24\,c^2}+\frac{b^2\,x^6\ln\left(c\,x^3+1\right)^2}{24}+\frac{b^2\,x^6\ln\left(1-c\,x^3\right)^2}{24}+\frac{b^2\,x^3\,\ln\left(c\,x^3+1\right)}{6\,c}-\frac{b^2\,x^3\,\ln\left(1-c\,x^3\right)}{6\,c}+\frac{a\,b\,\ln\left(c\,x^3-1\right)}{6\,c^2}-\frac{a\,b\,\ln\left(c\,x^3+1\right)}{6\,c^2}+\frac{a\,b\,x^6\,\ln\left(c\,x^3+1\right)}{6}-\frac{a\,b\,x^6\,\ln\left(1-c\,x^3\right)}{6}+\frac{b^2\,\ln\left(c\,x^3+1\right)\,\ln\left(1-c\,x^3\right)}{12\,c^2}+\frac{a\,b\,x^3}{3\,c}-\frac{b^2\,x^6\,\ln\left(c\,x^3+1\right)\,\ln\left(1-c\,x^3\right)}{12}$$

input
```
int(x^5*(a + b*atanh(c*x^3))^2,x)
```

output
```
(a^2*x^6)/6 + (b^2*log(c*x^3 - 1))/(6*c^2) + (b^2*log(c*x^3 + 1))/(6*c^2)
- (b^2*log(c*x^3 + 1)^2)/(24*c^2) - (b^2*log(1 - c*x^3)^2)/(24*c^2) + (b^2
*x^6*log(c*x^3 + 1)^2)/24 + (b^2*x^6*log(1 - c*x^3)^2)/24 + (b^2*x^3*log(c
*x^3 + 1))/(6*c) - (b^2*x^3*log(1 - c*x^3))/(6*c) + (a*b*log(c*x^3 - 1))/(
6*c^2) - (a*b*log(c*x^3 + 1))/(6*c^2) + (a*b*x^6*log(c*x^3 + 1))/6 - (a*b*
x^6*log(1 - c*x^3))/6 + (b^2*log(c*x^3 + 1)*log(1 - c*x^3))/(12*c^2) + (a*
b*x^3)/(3*c) - (b^2*x^6*log(c*x^3 + 1)*log(1 - c*x^3))/12
```

3.119 $\int x^2(a+b\text{arctanh}(cx^3))^2\,dx$

3.119.1 Optimal result

Integrand size = 16, antiderivative size = 96

$$\int x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx=\frac{(a+b\text{arctanh}(cx^3))^2}{3c}+\frac{1}{3}x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{2b(a+b\text{arctanh}(cx^3))\log\left(\frac{2}{1-cx^3}\right)}{3c}-\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{3c}$$

output
```
1/3*(a+b*arctanh(c*x^3))^2/c+1/3*x^3*(a+b*arctanh(c*x^3))^2-2/3*b*(a+b*arc
tanh(c*x^3))*ln(2/(-c*x^3+1))/c-1/3*b^2*polylog(2,1-2/(-c*x^3+1))/c
```

3.119.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03

$$\int x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx$$
$$=\frac{b^2(-1+cx^3)\,\text{arctanh}(cx^3)^2+2b\text{arctanh}(cx^3)\left(acx^3-b\log\left(1+e^{-2\text{arctanh}(cx^3)}\right)\right)+a(acx^3+b\log(1-c\ldots}{3c}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
(b^2*(-1 + c*x^3)*ArcTanh[c*x^3]^2 + 2*b*ArcTanh[c*x^3]*(a*c*x^3 - b*Log[1
 + E^(-2*ArcTanh[c*x^3])]) + a*(a*c*x^3 + b*Log[1 - c^2*x^6]) + b^2*PolyLo
g[2, -E^(-2*ArcTanh[c*x^3])])/(3*c)
```

3.119.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6454, 6436, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 (a+b\text{arctanh}(cx^3))^2 \, dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{3}\int (a+b\text{arctanh}(cx^3))^2 dx^3$$

$$\Big\downarrow \text{6436}$$

$$\frac{1}{3}\left(x^3(a+b\text{arctanh}(cx^3))^2-2bc\int \frac{x^3(a+b\text{arctanh}(cx^3))}{1-c^2x^6}dx^3\right)$$

$$\Big\downarrow \text{6546}$$

$$\frac{1}{3}\left(x^3(a+b\text{arctanh}(cx^3))^2-2bc\left(\frac{\int \frac{a+b\text{arctanh}(cx^3)}{1-cx^3}dx^3}{c}-\frac{(a+b\text{arctanh}(cx^3))^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{6470}$$

$$\frac{1}{3}\left(x^3(a+b\text{arctanh}(cx^3))^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)(a+b\text{arctanh}(cx^3))}{c}-b\int \frac{\log\left(\frac{2}{1-cx^3}\right)}{1-c^2x^6}dx^3}{c}-\frac{(a+b\text{arctanh}(cx^3))^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{3}\left(x^3(a+b\text{arctanh}(cx^3))^2-2bc\left(\frac{\frac{b\int \frac{\log\left(\frac{2}{1-cx^3}\right)}{1-\frac{2}{1-cx^3}}d\frac{1}{1-cx^3}}{c}+\frac{\log\left(\frac{2}{1-cx^3}\right)(a+b\text{arctanh}(cx^3))}{c}}{c}-\frac{(a+b\text{arctanh}(cx^3))^2}{2bc^2}\right)\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{3}\left(x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{2c}}{c}-\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{2bc^2}\right)\right)$$

input
```
Int[x^2*(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
(x^3*(a + b*ArcTanh[c*x^3])^2 - 2*b*c*(-1/2*(a + b*ArcTanh[c*x^3])^2/(b*c^
2) + (((a + b*ArcTanh[c*x^3])*Log[2/(1 - c*x^3)])/c + (b*PolyLog[2, 1 - 2/
(1 - c*x^3)])/(2*c))/c))/3
```

3.119.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.119.4 Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{a^2cx^3+b^2\left(\operatorname{arctanh}(cx^3)^2(cx^3-1)+2\operatorname{arctanh}(cx^3)^2-2\operatorname{arctanh}(cx^3)\ln\left(1+\frac{(cx^3+1)^2}{-c^2x^6+1}\right)-\operatorname{polylog}\left(2,-\frac{(cx^3+1)^2}{-c^2x^6+1}\right)\right.}{3c}$
default	$\frac{a^2cx^3+b^2\left(\operatorname{arctanh}(cx^3)^2(cx^3-1)+2\operatorname{arctanh}(cx^3)^2-2\operatorname{arctanh}(cx^3)\ln\left(1+\frac{(cx^3+1)^2}{-c^2x^6+1}\right)-\operatorname{polylog}\left(2,-\frac{(cx^3+1)^2}{-c^2x^6+1}\right)\right.}{3c}$
parts	$\frac{a^2x^3}{3}+\frac{b^2\left(\operatorname{arctanh}(cx^3)^2(cx^3-1)+2\operatorname{arctanh}(cx^3)^2-2\operatorname{arctanh}(cx^3)\ln\left(1+\frac{(cx^3+1)^2}{-c^2x^6+1}\right)-\operatorname{polylog}\left(2,-\frac{(cx^3+1)^2}{-c^2x^6+1}\right.\right.}{3c}$
risch	$-\frac{b^2\operatorname{dilog}\left(\frac{cx^3}{2}+\frac{1}{2}\right)}{3c}-\frac{b^2\ln(cx^3-1)}{3c}+\frac{a^2x^3}{3}+\frac{\ln(-cx^3+1)^2b^2x^3}{12}-\frac{\ln(-cx^3+1)^2b^2}{12c}+\frac{\ln(-cx^3+1)b^2}{3c}+b^2$

input
```
int(x^2*(a+b*arctanh(c*x^3))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3/c*(a^2*c*x^3+b^2*(arctanh(c*x^3)^2*(c*x^3-1)+2*arctanh(c*x^3)^2-2*arct
anh(c*x^3)*ln(1+(c*x^3+1)^2/(-c^2*x^6+1))-polylog(2,-(c*x^3+1)^2/(-c^2*x^6
+1)))+2*a*b*c*x^3*arctanh(c*x^3)+a*b*ln(-c^2*x^6+1))
```

3.119.5 Fricas [F]

$$\int x^2 (a + b\text{arctanh}(cx^3))^2 \, dx = \int (b \text{ artanh} (cx^3) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctanh(c*x^3)^2 + 2*a*b*x^2*arctanh(c*x^3) + a^2*x^2, x)
```

3.119.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (a + b\text{arctanh}(cx^3))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atanh(c*x**3))**2,x)
```

output
```
Timed out
```

3.119.7 Maxima [F]

$$\int x^2 (a + b\text{arctanh}(cx^3))^2 \, dx = \int (b \text{ artanh} (cx^3) + a)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/12*(x^3*log(-c*x^3 + 1)^2 - c^2*(2*x^3/c^2 - log(c*x^3 + 1
)/c^3 + log(c*x^3 - 1)/c^3) - 2*(x^3/c + log(c*x^3 - 1)/c^2)*c*log(-c*x^3
+ 1) + 18*c*integrate(x^5*log(c*x^3 + 1)/(c^2*x^6 - 1), x) + (c*x^3*log(c*
x^3 + 1)^2 + 2*(c*x^3 - (c*x^3 + 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/c + (
2*c*x^3 + log(c*x^3 - 1)^2 + 2*log(c*x^3 - 1))/c - log(c^2*x^6 - 1)/c + 6*
integrate(x^2*log(c*x^3 + 1)/(c^2*x^6 - 1), x))*b^2 + 1/3*(2*c*x^3*arctanh
(c*x^3) + log(-c^2*x^6 + 1))*a*b/c
```

3.119.8 Giac [F]

$$\int x^2(a+b\text{arctanh}(cx^3))^2\,dx=\int (b\,\text{artanh}\,(cx^3)+a)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^2*x^2, x)
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\text{arctanh}(cx^3))^2\,dx=\int x^2\,\left(a+b\,\text{atanh}\left(c\,x^3\right)\right)^2\,dx$$

input
```
int(x^2*(a + b*atanh(c*x^3))^2,x)
```

output
```
int(x^2*(a + b*atanh(c*x^3))^2, x)
```

3.120 $\int \frac{\left(a+b\,\mathrm{arctanh}\left(cx^3\right)\right)^2}{x}\,dx$

3.120.1 Optimal result

Integrand size = 16, antiderivative size = 140

$$\begin{aligned}\int \frac{(a+b\,\mathrm{arctanh}(cx^3))^2}{x}\,dx =& \frac{2}{3}\left(a+b\,\mathrm{arctanh}\left(cx^3\right)\right)^2 \mathrm{arctanh}\left(1-\frac{2}{1-cx^3}\right)\\ &-\frac{1}{3}b\left(a+b\,\mathrm{arctanh}\left(cx^3\right)\right)\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\\ &+\frac{1}{3}b\left(a+b\,\mathrm{arctanh}\left(cx^3\right)\right)\mathrm{PolyLog}\left(2,-1+\frac{2}{1-cx^3}\right)\\ &+\frac{1}{6}b^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)\\ &-\frac{1}{6}b^2\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-cx^3}\right)\end{aligned}$$

output
```
-2/3*(a+b*arctanh(c*x^3))^2*arctanh(-1+2/(-c*x^3+1))-1/3*b*(a+b*arctanh(c*
x^3))*polylog(2,1-2/(-c*x^3+1))+1/3*b*(a+b*arctanh(c*x^3))*polylog(2,-1+2/
(-c*x^3+1))+1/6*b^2*polylog(3,1-2/(-c*x^3+1))-1/6*b^2*polylog(3,-1+2/(-c*x
^3+1))
```

3.120.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.29

$$\int \frac{(a+b\operatorname{arctanh}(cx^3))^2}{x}\,dx = a^2\log(x)+\frac{1}{3}ab\left(-\operatorname{PolyLog}\left(2,-cx^3\right)+\operatorname{PolyLog}\left(2,cx^3\right)\right)+\frac{1}{3}b^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\operatorname{arctanh}\left(cx^3\right)^3-\operatorname{arctanh}\left(cx^3\right)^2\log\left(1+e^{-2\operatorname{arctanh}(cx^3)}\right)+\operatorname{arctanh}\left(cx^3\right)^2\log\left(1-e^{2\operatorname{arctanh}(cx^3)}\right)+\operatorname{arctanh}\left(cx^3\right)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}(cx^3)}\right)+\operatorname{arctanh}\left(cx^3\right)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}(cx^3)}\right)+\frac{1}{2}\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}(cx^3)}\right)-\frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}(cx^3)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])^2/x,x]
```

output
```
a^2*Log[x] + (a*b*(-PolyLog[2, -(c*x^3)] + PolyLog[2, c*x^3]))/3 + (b^2*((
I/24)*Pi^3 - (2*ArcTanh[c*x^3]^3)/3 - ArcTanh[c*x^3]^2*Log[1 + E^(-2*ArcTa
nh[c*x^3])] + ArcTanh[c*x^3]^2*Log[1 - E^(2*ArcTanh[c*x^3])] + ArcTanh[c*x
^3]*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + ArcTanh[c*x^3]*PolyLog[2, E^(2*Ar
cTanh[c*x^3])] + PolyLog[3, -E^(-2*ArcTanh[c*x^3])]/2 - PolyLog[3, E^(2*Ar
cTanh[c*x^3])]/2))/3
```

3.120.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6450, 6448, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x}\,dx$$

$$\Big\downarrow \; 6450$$

$$\frac{1}{3}\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^3}dx^3$$

$$\Big\downarrow \; 6448$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)(a+b\text{arctanh}(cx^3))^2-4bc\int \frac{(a+b\text{arctanh}(cx^3))\,\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)}{1-c^2x^6}dx^3\right)$$

$$\Big\downarrow \; 6614$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)(a+b\text{arctanh}(cx^3))^2-4bc\left(\frac{1}{2}\int \frac{(a+b\text{arctanh}(cx^3))\log\left(2-\frac{2}{1-cx^3}\right)}{1-c^2x^6}dx^3-\frac{1}{2}\int -\right.\right.$$

$$\Big\downarrow \; 6620$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)(a+b\text{arctanh}(cx^3))^2-4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)(a+b\text{arctanh}(cx^3))}{2c}-\frac{1}{2}b\right.\right.\right.$$

$$\Big\downarrow \; 7164$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)(a+b\text{arctanh}(cx^3))^2-4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)(a+b\text{arctanh}(cx^3))}{2c}-\frac{b\,\text{P}}{}\right.\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x^3])^2/x,x]
```

output
```
(2*(a + b*ArcTanh[c*x^3])^2*ArcTanh[1 - 2/(1 - c*x^3)] - 4*b*c*((((a + b*A
rcTanh[c*x^3])*PolyLog[2, 1 - 2/(1 - c*x^3)])/(2*c) - (b*PolyLog[3, 1 - 2/
(1 - c*x^3)])/(4*c))/2 + (-1/2*((a + b*ArcTanh[c*x^3])*PolyLog[2, -1 + 2/(
1 - c*x^3)])/c + (b*PolyLog[3, -1 + 2/(1 - c*x^3)])/(4*c))/2))/3
```

3.120. $\int \frac{(a+b\text{arctanh}(cx^3))^2}{x}\,dx$

3.120.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.120.4 Maple [F]

$$\int \frac{(a + b \operatorname{arctanh}(c x^3))^2}{x} dx$$

input
```
int((a+b*arctanh(c*x^3))^2/x,x)
```

output
```
int((a+b*arctanh(c*x^3))^2/x,x)
```

3.120.5 Fricas [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx^3))^2}{x}\,dx = \int \frac{(b\operatorname{artanh}(cx^3)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x, x)
```

3.120.6 Sympy [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx^3))^2}{x}\,dx = \int \frac{(a+b\operatorname{atanh}(cx^3))^2}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x**3))**2/x,x)
```

output
```
Integral((a + b*atanh(c*x**3))**2/x, x)
```

3.120.7 Maxima [F]

$$\int \frac{(a+b\operatorname{arctanh}(cx^3))^2}{x}\,dx = \int \frac{(b\operatorname{artanh}(cx^3)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + integrate(1/4*b^2*(log(c*x^3 + 1) - log(-c*x^3 + 1))^2/x + a*
b*(log(c*x^3 + 1) - log(-c*x^3 + 1))/x, x)
```

3.120.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x} dx = \int \frac{(b \operatorname{artanh}(cx^3) + a)^2}{x} dx$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^2/x, x)
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x} dx = \int \frac{(a + b \operatorname{atanh}(c x^3))^2}{x} dx$$

input
```
int((a + b*atanh(c*x^3))^2/x,x)
```

output
```
int((a + b*atanh(c*x^3))^2/x, x)
```

3.121 $\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{x^4}\,dx$

3.121.1 Optimal result

Integrand size = 16, antiderivative size = 90

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^4}\,dx &= \frac{1}{3}c\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-\frac{(a+b\text{arctanh}(cx^3))^2}{3x^3}\\ &\quad+\frac{2}{3}bc\left(a+b\text{arctanh}\left(cx^3\right)\right)\log\left(2-\frac{2}{1+cx^3}\right)\\ &\quad-\frac{1}{3}b^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx^3}\right)\end{aligned}$$

output
```
1/3*c*(a+b*arctanh(c*x^3))^2-1/3*(a+b*arctanh(c*x^3))^2/x^3+2/3*b*c*(a+b*a
rctanh(c*x^3))*ln(2-2/(c*x^3+1))-1/3*b^2*c*polylog(2,-1+2/(c*x^3+1))
```

3.121.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.30

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^4}\,dx = \frac{b^2(-1+cx^3)\,\text{arctanh}(cx^3)^2+2b\text{arctanh}(cx^3)\left(-a+bcx^3\log\left(1-e^{-2\text{arctanh}(cx^3)}\right)\right)-a(a-2bcx^3\log(cx}{3x^3}$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])^2/x^4,x]
```

output
```
(b^2*(-1 + c*x^3)*ArcTanh[c*x^3]^2 + 2*b*ArcTanh[c*x^3]*(-a + b*c*x^3*Log[
1 - E^(-2*ArcTanh[c*x^3])]) - a*(a - 2*b*c*x^3*Log[c*x^3] + b*c*x^3*Log[1
- c^2*x^6]) - b^2*c*x^3*PolyLog[2, E^(-2*ArcTanh[c*x^3])])/(3*x^3)
```

3.121.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6454, 6452, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^4}dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{3}\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^6}dx^3$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}\left(2bc\int \frac{a+b\text{arctanh}(cx^3)}{x^3(1-c^2x^6)}dx^3-\frac{(a+b\text{arctanh}(cx^3))^2}{x^3}\right)$$

$$\Big\downarrow \text{6550}$$

$$\frac{1}{3}\left(2bc\left(\int \frac{a+b\text{arctanh}(cx^3)}{x^3(cx^3+1)}dx^3+\frac{(a+b\text{arctanh}(cx^3))^2}{2b}\right)-\frac{(a+b\text{arctanh}(cx^3))^2}{x^3}\right)$$

$$\Big\downarrow \text{6494}$$

$$\frac{1}{3}\left(2bc\left(-bc\int \frac{\log\left(2-\frac{2}{cx^3+1}\right)}{1-c^2x^6}dx^3+\frac{(a+b\text{arctanh}(cx^3))^2}{2b}+\log\left(2-\frac{2}{cx^3+1}\right)(a+b\text{arctanh}(cx^3))\right)-\frac{(a+\ldots}{}\right.$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{3}\left(2bc\left(\frac{(a+b\text{arctanh}(cx^3))^2}{2b}+\log\left(2-\frac{2}{cx^3+1}\right)(a+b\text{arctanh}(cx^3))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx^3+1}-1\right)\right)-\frac{(a\ldots}{}\right.$$

input
```
Int[(a + b*ArcTanh[c*x^3])^2/x^4,x]
```

3.121. $\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^4}dx$

output
```
(-((a + b*ArcTanh[c*x^3])^2/x^3) + 2*b*c*((a + b*ArcTanh[c*x^3])^2/(2*b) +
 (a + b*ArcTanh[c*x^3])*Log[2 - 2/(1 + c*x^3)] - (b*PolyLog[2, -1 + 2/(1 +
 c*x^3)])/2))/3
```

3.121.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.121. $\int \frac{\left(a+b\text{arctanh}(cx^3)\right)^2}{x^4} dx$

3.121.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.20 (sec) , antiderivative size = 2993, normalized size of antiderivative = 33.26

method	result	size
default	Expression too large to display	2993
parts	Expression too large to display	2993

input
```
int((a+b*arctanh(c*x^3))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+b^2*(-1/3/x^3*arctanh(c*x^3)^2+2*c*(arctanh(c*x^3)*ln(x)-1/6*
arctanh(c*x^3)*ln(c*x^3-1)-1/6*arctanh(c*x^3)*ln(c*x^3+1)-1/2*c*(Sum(1/6*(
ln(x-_alpha)*ln(c*x^3-1)-3*c*(1/6/_alpha^2/c*ln(x-_alpha)^2-1/3*_alpha*ln(
x-_alpha)*(2*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/Roo
tOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha
^2,index=2)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)+6*ln((RootOf(_Z^2+
3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha
^2,index=1))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*_alpha+3*ln((Root
Of(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+
3*_alpha^2,index=1))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*_alpha+9*
ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z
*_alpha+3*_alpha^2,index=1))*_alpha^2+2*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alp
ha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*RootO
f(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,
index=1)+3*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alpha)/RootO
f(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2
,index=2)*_alpha+6*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+_alph
a)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*
_alpha^2,index=1)*_alpha+9*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)
-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*_alpha^2)/(3*_a...
```

3.121.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^4}\,dx = \int \frac{(b\,\text{artanh}\,(cx^3)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x^4, x)
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))**2/x**4,x)
```

output
```
Timed out
```

3.121.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^4}\,dx = \int \frac{(b\,\text{artanh}\,(cx^3)+a)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*a*b - 1/12*b
^2*(log(-c*x^3 + 1)^2/x^3 + 3*integrate(-((c*x^3 - 1)*log(c*x^3 + 1)^2 + 2
*(c*x^3 - (c*x^3 - 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^7 - x^4), x))
- 1/3*a^2/x^3
```

3.121.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x^4} \, dx = \int \frac{(b\,\text{artanh}\,(cx^3) + a)^2}{x^4} \, dx$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^2/x^4, x)
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x^4} \, dx = \int \frac{(a + b\,\text{atanh}(c\,x^3))^2}{x^4} \, dx$$

input
```
int((a + b*atanh(c*x^3))^2/x^4,x)
```

output
```
int((a + b*atanh(c*x^3))^2/x^4, x)
```

3.122 $\int \frac{\left(a+b\text{arctanh}(cx^3)\right)^2}{x^7}\,dx$

3.122.1 Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^7}\,dx = -\frac{bc(a+b\text{arctanh}(cx^3))}{3x^3}+\frac{1}{6}c^2(a+b\text{arctanh}(cx^3))^2 - \frac{(a+b\text{arctanh}(cx^3))^2}{6x^6}+b^2c^2\log(x)-\frac{1}{6}b^2c^2\log\left(1-c^2x^6\right)$$

output
```
-1/3*b*c*(a+b*arctanh(c*x^3))/x^3+1/6*c^2*(a+b*arctanh(c*x^3))^2-1/6*(a+b*
arctanh(c*x^3))^2/x^6+b^2*c^2*ln(x)-1/6*b^2*c^2*ln(-c^2*x^6+1)
```

3.122.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.26

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^7}\,dx = \frac{1}{6}\left(-\frac{a^2}{x^6}-\frac{2abc}{x^3}-\frac{2b(a+bcx^3)\text{arctanh}(cx^3)}{x^6} + \frac{b^2(-1+c^2x^6)\text{arctanh}(cx^3)^2}{x^6}+6b^2c^2\log(x) - b(a+b)c^2\log\left(1-cx^3\right)+(a-b)bc^2\log\left(1+cx^3\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])^2/x^7,x]
```

output
```
(-(a^2/x^6) - (2*a*b*c)/x^3 - (2*b*(a + b*c*x^3)*ArcTanh[c*x^3])/x^6 + (b^
2*(-1 + c^2*x^6)*ArcTanh[c*x^3]^2)/x^6 + 6*b^2*c^2*Log[x] - b*(a + b)*c^2*
Log[1 - c*x^3] + (a - b)*b*c^2*Log[1 + c*x^3])/6
```

3.122.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6454, 6452, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^7}dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{3}\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^9}dx^3$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}\left(bc\int \frac{a+b\text{arctanh}(cx^3)}{x^6(1-c^2x^6)}dx^3-\frac{(a+b\text{arctanh}(cx^3))^2}{2x^6}\right)$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{3}\left(bc\left(c^2\int \frac{a+b\text{arctanh}(cx^3)}{1-c^2x^6}dx^3+\int \frac{a+b\text{arctanh}(cx^3)}{x^6}dx^3\right)-\frac{(a+b\text{arctanh}(cx^3))^2}{2x^6}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}\left(bc\left(c^2\int \frac{a+b\text{arctanh}(cx^3)}{1-c^2x^6}dx^3+bc\int \frac{1}{x^3(1-c^2x^6)}dx^3-\frac{a+b\text{arctanh}(cx^3)}{x^3}\right)-\frac{(a+b\text{arctanh}(cx^3))^2}{2x^6}\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}\left(bc\left(c^2\int \frac{a+b\text{arctanh}(cx^3)}{1-c^2x^6}dx^3+\frac{1}{2}bc\int \frac{1}{x^3(1-c^2x^6)}dx^6-\frac{a+b\text{arctanh}(cx^3)}{x^3}\right)-\frac{(a+b\text{arctanh}(cx^3))^2}{2x^6}\right)$$

$$\downarrow 47$$

$$\frac{1}{3}\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx^3)}{1-c^2x^6}dx^3+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^6}dx^6+\int\frac{1}{x^3}dx^6\right)-\frac{a+b\text{arctanh}(cx^3)}{x^3}\right)-\frac{(a+b\text{arcta}}{2x}\right.$$

$$\downarrow 14$$

$$\frac{1}{3}\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx^3)}{1-c^2x^6}dx^3+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^6}dx^6+\log\left(x^6\right)\right)-\frac{a+b\text{arctanh}(cx^3)}{x^3}\right)-\frac{(a+b\text{arctan}}{2x^6}\right.$$

$$\downarrow 16$$

$$\frac{1}{3}\left(bc\left(c^2\int\frac{a+b\text{arctanh}(cx^3)}{1-c^2x^6}dx^3-\frac{a+b\text{arctanh}(cx^3)}{x^3}+\frac{1}{2}bc\left(\log\left(x^6\right)-\log\left(1-c^2x^6\right)\right)\right)-\frac{(a+b\text{arctanh}(cx^3}{2x^6}\right.$$

$$\downarrow 6510$$

$$\frac{1}{3}\left(bc\left(\frac{c(a+b\text{arctanh}(cx^3))^2}{2b}-\frac{a+b\text{arctanh}(cx^3)}{x^3}+\frac{1}{2}bc\left(\log\left(x^6\right)-\log\left(1-c^2x^6\right)\right)\right)-\frac{(a+b\text{arctanh}(cx^3))^2}{2x^6}\right)$$

input
```
Int[(a + b*ArcTanh[c*x^3])^2/x^7,x]
```

output
```
(-1/2*(a + b*ArcTanh[c*x^3])^2/x^6 + b*c*(-((a + b*ArcTanh[c*x^3])/x^3) +
(c*(a + b*ArcTanh[c*x^3])^2)/(2*b) + (b*c*(Log[x^6] - Log[1 - c^2*x^6]))/2
))/3
```

3.122.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.122.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.73

method	result
parallelrisch	$\frac{b^2 \operatorname{arctanh}(c x^3)^2 x^6 c^2+6b^2c^2 \ln(x)x^6-2b^2 \ln(c x^3-1)c^2x^6+2x^6 \operatorname{arctanh}(c x^3)ab c^2-2 \operatorname{arctanh}(c x^3)b^2c^2x^6-a^2c^2x^6-2b^2 \operatorname{arc}}{6x^6}$
risch	$\frac{b^2(c^2x^6-1)\ln(c x^3+1)^2}{24x^6}-\frac{b(b c^2 \ln(-c x^3+1)x^6+2bc x^3-b\ln(-c x^3+1)+2a)\ln(c x^3+1)}{12x^6}+\frac{b^2c^2x^6\ln(-c x^3+1)^2+24b^2c^2}{}$
default	Expression too large to display
parts	Expression too large to display

input
```
int((a+b*arctanh(c*x^3))^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
1/6*(b^2*arctanh(c*x^3)^2*x^6*c^2+6*b^2*c^2*ln(x)*x^6-2*b^2*ln(c*x^3-1)*c^
2*x^6+2*x^6*arctanh(c*x^3)*a*b*c^2-2*arctanh(c*x^3)*b^2*c^2*x^6-a^2*c^2*x^
6-2*b^2*arctanh(c*x^3)*x^3*c-2*a*b*c*x^3-b^2*arctanh(c*x^3)^2-2*arctanh(c*
x^3)*a*b-a^2)/x^6
```

3.122.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.72

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^7}\,dx$$
$$= \frac{24\,b^2c^2x^6\log{(x)}+4\,(ab-b^2)c^2x^6\log{(cx^3+1)}-4\,(ab+b^2)c^2x^6\log{(cx^3-1)}-8\,abcx^3+(b^2c^2x^6-b^2)\,\mathrm{lo}}{24\,x^6}$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="fricas")
```

output
```
1/24*(24*b^2*c^2*x^6*log(x) + 4*(a*b - b^2)*c^2*x^6*log(c*x^3 + 1) - 4*(a*
b + b^2)*c^2*x^6*log(c*x^3 - 1) - 8*a*b*c*x^3 + (b^2*c^2*x^6 - b^2)*log(-(
c*x^3 + 1)/(c*x^3 - 1))^2 - 4*a^2 - 4*(b^2*c*x^3 + a*b)*log(-(c*x^3 + 1)/(
c*x^3 - 1)))/x^6
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))**2/x**7,x)
```

output
```
Timed out
```

3.122.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(80) = 160$.

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.99

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x^7}\,dx$$
$$= \frac{1}{6}\left(\left(c\log\left(cx^3+1\right) - c\log\left(cx^3-1\right) - \frac{2}{x^3}\right)c - \frac{2\,\text{artanh}\left(cx^3\right)}{x^6}\right)ab$$
$$+ \frac{1}{24}\left(\left(2\left(\log\left(cx^3-1\right)-2\right)\log\left(cx^3+1\right) - \log\left(cx^3+1\right)^2 - \log\left(cx^3-1\right)^2 - 4\log\left(cx^3-1\right) + 24\log\left(\right.\right.\right.$$
$$- \frac{b^2\,\text{artanh}\left(cx^3\right)^2}{6\,x^6} - \frac{a^2}{6\,x^6}$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="maxima")
```

output
```
1/6*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^
6)*a*b + 1/24*((2*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 -
 log(c*x^3 - 1)^2 - 4*log(c*x^3 - 1) + 24*log(x))*c^2 + 4*(c*log(c*x^3 + 1
) - c*log(c*x^3 - 1) - 2/x^3)*c*arctanh(c*x^3))*b^2 - 1/6*b^2*arctanh(c*x^
3)^2/x^6 - 1/6*a^2/x^6
```

3.122.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x^7}\,dx = \int \frac{\left(b\,\text{artanh}\left(cx^3\right)+a\right)^2}{x^7}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x^7,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^2/x^7, x)
```

3.122.9 Mupad [B] (verification not implemented)

Time = 3.96 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.16

$$\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^7}\,dx = \frac{b^2\,c^2\ln\left(c\,x^3+1\right)^2}{24} - \frac{b^2\,c^2\,\ln\left(c\,x^3-1\right)}{6} - \frac{b^2\,c^2\,\ln\left(c\,x^3+1\right)}{6} - \frac{a^2}{6\,x^6} + \frac{b^2\,c^2\ln\left(1-c\,x^3\right)^2}{24} - \frac{b^2\ln\left(c\,x^3+1\right)^2}{24\,x^6} - \frac{b^2\ln\left(1-c\,x^3\right)^2}{24\,x^6} + b^2\,c^2\,\ln\left(x\right) - \frac{a\,b\,c^2\,\ln\left(c\,x^3-1\right)}{6} + \frac{a\,b\,c^2\,\ln\left(c\,x^3+1\right)}{6} - \frac{a\,b\,c}{3\,x^3} - \frac{a\,b\,\ln\left(c\,x^3+1\right)}{6\,x^6} + \frac{a\,b\,\ln\left(1-c\,x^3\right)}{6\,x^6} - \frac{b^2\,c^2\,\ln\left(c\,x^3+1\right)\,\ln\left(1-c\,x^3\right)}{12} - \frac{b^2\,c\,\ln\left(c\,x^3+1\right)}{6\,x^3} + \frac{b^2\,c\,\ln\left(1-c\,x^3\right)}{6\,x^3} + \frac{b^2\,\ln\left(c\,x^3+1\right)\,\ln\left(1-c\,x^3\right)}{12\,x^6}$$

input
```
int((a + b*atanh(c*x^3))^2/x^7,x)
```

output
```
(b^2*c^2*log(c*x^3 + 1)^2)/24 - (b^2*c^2*log(c*x^3 - 1))/6 - (b^2*c^2*log(
c*x^3 + 1))/6 - a^2/(6*x^6) + (b^2*c^2*log(1 - c*x^3)^2)/24 - (b^2*log(c*x
^3 + 1)^2)/(24*x^6) - (b^2*log(1 - c*x^3)^2)/(24*x^6) + b^2*c^2*log(x) - (
a*b*c^2*log(c*x^3 - 1))/6 + (a*b*c^2*log(c*x^3 + 1))/6 - (a*b*c)/(3*x^3) -
 (a*b*log(c*x^3 + 1))/(6*x^6) + (a*b*log(1 - c*x^3))/(6*x^6) - (b^2*c^2*lo
g(c*x^3 + 1)*log(1 - c*x^3))/12 - (b^2*c*log(c*x^3 + 1))/(6*x^3) + (b^2*c*
log(1 - c*x^3))/(6*x^3) + (b^2*log(c*x^3 + 1)*log(1 - c*x^3))/(12*x^6)
```

3.123 $\int \frac{\left(a+b\,\text{arctanh}\left(cx^3\right)\right)^2}{x^{10}}\,dx$

3.123.1 Optimal result

Integrand size = 16, antiderivative size = 144

$$\begin{aligned}\int \frac{(a+b\,\text{arctanh}(cx^3))^2}{x^{10}}\,dx = & -\frac{b^2c^2}{9x^3}+\frac{1}{9}b^2c^3\text{arctanh}\left(cx^3\right)-\frac{bc(a+b\,\text{arctanh}(cx^3))}{9x^6}\\ & +\frac{1}{9}c^3\left(a+b\,\text{arctanh}\left(cx^3\right)\right)^2-\frac{(a+b\,\text{arctanh}(cx^3))^2}{9x^9}\\ & +\frac{2}{9}bc^3\left(a+b\,\text{arctanh}\left(cx^3\right)\right)\log\left(2-\frac{2}{1+cx^3}\right)\\ & -\frac{1}{9}b^2c^3\,\text{PolyLog}\left(2,-1+\frac{2}{1+cx^3}\right)\end{aligned}$$

output
```
-1/9*b^2*c^2/x^3+1/9*b^2*c^3*arctanh(c*x^3)-1/9*b*c*(a+b*arctanh(c*x^3))/x
^6+1/9*c^3*(a+b*arctanh(c*x^3))^2-1/9*(a+b*arctanh(c*x^3))^2/x^9+2/9*b*c^3
*(a+b*arctanh(c*x^3))*ln(2-2/(c*x^3+1))-1/9*b^2*c^3*polylog(2,-1+2/(c*x^3+
1))
```

3.123.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\operatorname{arctanh}(cx^3))^2}{x^{10}}\,dx =$$
$$-\frac{a^2+abcx^3+b^2c^2x^6+b^2(1-c^3x^9)\operatorname{arctanh}(cx^3)^2+b\operatorname{arctanh}(cx^3)\left(2a+bcx^3-bc^3x^9-2bc^3x^9\log\left(1\right.\right.}{9x^9}$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])^2/x^10,x]
```

output
```
-1/9*(a^2 + a*b*c*x^3 + b^2*c^2*x^6 + b^2*(1 - c^3*x^9)*ArcTanh[c*x^3]^2 +
 b*ArcTanh[c*x^3]*(2*a + b*c*x^3 - b*c^3*x^9 - 2*b*c^3*x^9*Log[1 - E^(-2*A
rcTanh[c*x^3])]) - 2*a*b*c^3*x^9*Log[c*x^3] + a*b*c^3*x^9*Log[1 - c^2*x^6]
 + b^2*c^3*x^9*PolyLog[2, E^(-2*ArcTanh[c*x^3])])/x^9
```

3.123.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6454, 6452, 6544, 6452, 264, 219, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\operatorname{arctanh}(cx^3))^2}{x^{10}}dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{3}\int \frac{(a+b\operatorname{arctanh}(cx^3))^2}{x^{12}}dx^3$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}\left(\frac{2}{3}bc\int \frac{a+b\operatorname{arctanh}(cx^3)}{x^9(1-c^2x^6)}dx^3-\frac{(a+b\operatorname{arctanh}(cx^3))^2}{3x^9}\right)$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\int \frac{a+b\operatorname{arctanh}(cx^3)}{x^3(1-c^2x^6)}dx^3+\int \frac{a+b\operatorname{arctanh}(cx^3)}{x^9}dx^3\right)-\frac{(a+b\operatorname{arctanh}(cx^3))^2}{3x^9}\right)$$

$$\Big\downarrow 6452$$

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\int\frac{a+b\text{arctanh}(cx^3)}{x^3(1-c^2x^6)}dx^3+\frac{1}{2}bc\int\frac{1}{x^6(1-c^2x^6)}dx^3-\frac{a+b\text{arctanh}(cx^3)}{2x^6}\right)-\frac{(a+b\text{arctanh}(cx^3))^2}{3x^9}\right)$$

$$\Big\downarrow 264$$

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\int\frac{a+b\text{arctanh}(cx^3)}{x^3(1-c^2x^6)}dx^3+\frac{1}{2}bc\left(c^2\int\frac{1}{1-c^2x^6}dx^3-\frac{1}{x^3}\right)-\frac{a+b\text{arctanh}(cx^3)}{2x^6}\right)-\frac{(a+b\text{arctanh}(c$$

$$\Big\downarrow 219$$

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\int\frac{a+b\text{arctanh}(cx^3)}{x^3(1-c^2x^6)}dx^3-\frac{a+b\text{arctanh}(cx^3)}{2x^6}+\frac{1}{2}bc\left(c\text{arctanh}(cx^3)-\frac{1}{x^3}\right)\right)-\frac{(a+b\text{arctanh}(cx^3)}{3x^9}$$

$$\Big\downarrow 6550$$

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\left(\int\frac{a+b\text{arctanh}(cx^3)}{x^3(cx^3+1)}dx^3+\frac{(a+b\text{arctanh}(cx^3))^2}{2b}\right)-\frac{a+b\text{arctanh}(cx^3)}{2x^6}+\frac{1}{2}bc\left(c\text{arctanh}(cx^3)-\frac{}{}\right.\right.\right.$$

$$\Big\downarrow 6494$$

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\left(-bc\int\frac{\log\left(2-\frac{2}{cx^3+1}\right)}{1-c^2x^6}dx^3+\frac{(a+b\text{arctanh}(cx^3))^2}{2b}+\log\left(2-\frac{2}{cx^3+1}\right)(a+b\text{arctanh}(cx^3))\right)-\right.\right.$$

$$\Big\downarrow 2897$$

$$\frac{1}{3}\left(\frac{2}{3}bc\left(c^2\left(\frac{(a+b\text{arctanh}(cx^3))^2}{2b}+\log\left(2-\frac{2}{cx^3+1}\right)(a+b\text{arctanh}(cx^3))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{cx^3+1}-1\right)\right)\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x^3])^2/x^10,x]
```

output
```
(-1/3*(a + b*ArcTanh[c*x^3])^2/x^9 + (2*b*c*(-1/2*(a + b*ArcTanh[c*x^3])/x
^6 + (b*c*(-x^(-3) + c*ArcTanh[c*x^3]))/2 + c^2*((a + b*ArcTanh[c*x^3])^2/
(2*b) + (a + b*ArcTanh[c*x^3])*Log[2 - 2/(1 + c*x^3)] - (b*PolyLog[2, -1 +
 2/(1 + c*x^3)])/2)))/3)/3
```

3.123. $\int\frac{(a+b\text{arctanh}(cx^3))^2}{x^{10}}\,dx$

3.123.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.123. $\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^{10}}\,dx$

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.123.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.44 (sec) , antiderivative size = 3062, normalized size of antiderivative = 21.26

method	result	size
default	Expression too large to display	3062
parts	Expression too large to display	3062

input
```
int((a+b*arctanh(c*x^3))^2/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/9*a^2/x^9+b^2*(-1/9/x^9*arctanh(c*x^3)^2+2/3*c*(-1/6/x^6*arctanh(c*x^3)
+arctanh(c*x^3)*c^2*ln(x)-1/6*arctanh(c*x^3)*c^2*ln(c*x^3-1)-1/6*arctanh(c
*x^3)*c^2*ln(c*x^3+1)-1/2*c*(1/3/x^3+1/6*c*ln(c*x^3-1)-1/6*c*ln(c*x^3+1)+c
^2*(Sum(1/6*(ln(x-_alpha)*ln(c*x^3-1)-3*c*(1/6/_alpha^2/c*ln(x-_alpha)^2-1
/3*_alpha*ln(x-_alpha)*(2*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-
x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*RootOf(_Z^2+3*_Z*_a
lpha+3*_alpha^2,index=2)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)+6*ln(
(RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+3*_Z*_a
lpha+3*_alpha^2,index=1))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*_alp
ha+3*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2
+3*_Z*_alpha+3*_alpha^2,index=1))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index
=1)*_alpha+9*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x+_alpha)/Roo
tOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha^2+2*ln((RootOf(_Z^2+3*_Z*
_alpha+3*_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,in
dex=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*RootOf(_Z^2+3*_Z*_alph
a+3*_alpha^2,index=1)+3*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x+
_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alp
ha+3*_alpha^2,index=2)*_alpha+6*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,ind
ex=2)-x+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*RootOf(_Z^2+3
*_Z*_alpha+3*_alpha^2,index=1)*_alpha+9*ln((RootOf(_Z^2+3*_Z*_alpha+3*_...
```

3.123.5 Fricas [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x^{10}}\,dx = \int \frac{(b\,\text{artanh}\,(cx^3) + a)^2}{x^{10}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)/x^10, x)
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x^{10}}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))**2/x**10,x)
```

output
```
Timed out
```

3.123.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x^{10}}\,dx = \int \frac{(b\,\text{artanh}\,(cx^3) + a)^2}{x^{10}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="maxima")
```

output
```
-1/9*((c^2*log(c^2*x^6 - 1) - c^2*log(x^6) + 1/x^6)*c + 2*arctanh(c*x^3)/x
^9)*a*b - 1/36*b^2*(log(-c*x^3 + 1)^2/x^9 + 9*integrate(-1/3*(3*(c*x^3 - 1
)*log(c*x^3 + 1)^2 + 2*(c*x^3 - 3*(c*x^3 - 1)*log(c*x^3 + 1))*log(-c*x^3 +
 1))/(c*x^13 - x^10), x)) - 1/9*a^2/x^9
```

3.123.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x^{10}} dx = \int \frac{(b\,\text{artanh}\,(cx^3) + a)^2}{x^{10}} dx$$

input
```
integrate((a+b*arctanh(c*x^3))^2/x^10,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^2/x^10, x)
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^3))^2}{x^{10}} dx = \int \frac{(a + b\,\text{atanh}(c\,x^3))^2}{x^{10}} dx$$

input
```
int((a + b*atanh(c*x^3))^2/x^10,x)
```

output
```
int((a + b*atanh(c*x^3))^2/x^10, x)
```

3.124 $\int x^8(a+b\text{arctanh}(cx^3))^3\,dx$

3.124.1 Optimal result

Integrand size = 16, antiderivative size = 231

$$\begin{aligned}\int x^8\left(a+b\text{arctanh}\left(cx^3\right)\right)^3\,dx=&\frac{ab^2x^3}{3c^2}+\frac{b^3x^3\text{arctanh}(cx^3)}{3c^2}-\frac{b(a+b\text{arctanh}(cx^3))^2}{6c^3}\\&+\frac{bx^6(a+b\text{arctanh}(cx^3))^2}{6c}+\frac{(a+b\text{arctanh}(cx^3))^3}{9c^3}\\&+\frac{1}{9}x^9\left(a+b\text{arctanh}\left(cx^3\right)\right)^3\\&-\frac{b(a+b\text{arctanh}(cx^3))^2\log\left(\frac{2}{1-cx^3}\right)}{3c^3}+\frac{b^3\log\left(1-c^2x^6\right)}{6c^3}\\&-\frac{b^2(a+b\text{arctanh}(cx^3))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{3c^3}\\&+\frac{b^3\,\text{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)}{6c^3}\end{aligned}$$

output
```
1/3*a*b^2*x^3/c^2+1/3*b^3*x^3*arctanh(c*x^3)/c^2-1/6*b*(a+b*arctanh(c*x^3)
)^2/c^3+1/6*b*x^6*(a+b*arctanh(c*x^3))^2/c+1/9*(a+b*arctanh(c*x^3))^3/c^3+
1/9*x^9*(a+b*arctanh(c*x^3))^3-1/3*b*(a+b*arctanh(c*x^3))^2*ln(2/(-c*x^3+1
))/c^3+1/6*b^3*ln(-c^2*x^6+1)/c^3-1/3*b^2*(a+b*arctanh(c*x^3))*polylog(2,1
-2/(-c*x^3+1))/c^3+1/6*b^3*polylog(3,1-2/(-c*x^3+1))/c^3
```

3.124.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.45

$$\int x^8 \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 \, dx$$
$$= \frac{6ab^2cx^3+3a^2bc^2x^6+2a^3c^3x^9-6ab^2\text{arctanh}(cx^3)+6b^3cx^3\text{arctanh}(cx^3)+6ab^2c^2x^6\text{arctanh}(cx^3)+6a^2bc}{}$$

input
```
Integrate[x^8*(a + b*ArcTanh[c*x^3])^3,x]
```

output
```
(6*a*b^2*c*x^3 + 3*a^2*b*c^2*x^6 + 2*a^3*c^3*x^9 - 6*a*b^2*ArcTanh[c*x^3]
+ 6*b^3*c*x^3*ArcTanh[c*x^3] + 6*a*b^2*c^2*x^6*ArcTanh[c*x^3] + 6*a^2*b*c^
3*x^9*ArcTanh[c*x^3] - 6*a*b^2*ArcTanh[c*x^3]^2 - 3*b^3*ArcTanh[c*x^3]^2 +
 3*b^3*c^2*x^6*ArcTanh[c*x^3]^2 + 6*a*b^2*c^3*x^9*ArcTanh[c*x^3]^2 - 2*b^3
*ArcTanh[c*x^3]^3 + 2*b^3*c^3*x^9*ArcTanh[c*x^3]^3 - 12*a*b^2*ArcTanh[c*x^
3]*Log[1 + E^(-2*ArcTanh[c*x^3])] - 6*b^3*ArcTanh[c*x^3]^2*Log[1 + E^(-2*A
rcTanh[c*x^3])] + 3*a^2*b*Log[1 - c^2*x^6] + 3*b^3*Log[1 - c^2*x^6] + 6*b^
2*(a + b*ArcTanh[c*x^3])*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + 3*b^3*PolyLo
g[3, -E^(-2*ArcTanh[c*x^3])])/(18*c^3)
```

3.124.3 Rubi [A] (verified)

Time = 1.74 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {6454, 6452, 6542, 6452, 6542, 2009, 6510, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8 \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 \, dx$$
$$\Big\downarrow \text{ 6454}$$
$$\frac{1}{3}\int x^6 \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 dx^3$$
$$\Big\downarrow \text{ 6452}$$
$$\frac{1}{3}\left(\frac{1}{3}x^9 \left(a+b\text{arctanh}\left(cx^3\right)\right)^3-bc\int \frac{x^9 \left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{1-c^2x^6}dx^3\right)$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}(cx^3)\right)^3-bc\left(\frac{\int\frac{x^3\left(a+b\text{arctanh}(cx^3)\right)^2}{1-c^2x^6}dx^3}{c^2}-\frac{\int x^3\left(a+b\text{arctanh}(cx^3)\right)^2dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}\Bigg(\frac{1}{3}x^9\left(a+b\text{arctanh}(cx^3)\right)^3-bc\Bigg(\frac{\int\frac{x^3\left(a+b\text{arctanh}(cx^3)\right)^2}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{2}x^6\left(a+b\text{arctanh}(cx^3)\right)^2-bc\int\frac{x^6(a+b\text{arctanh}}{1-c^2x^6}}{c^2}$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{3}\Bigg(\frac{1}{3}x^9\left(a+b\text{arctanh}(cx^3)\right)^3-bc\Bigg(\frac{\int\frac{x^3\left(a+b\text{arctanh}(cx^3)\right)^2}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{2}x^6\left(a+b\text{arctanh}(cx^3)\right)^2-bc\Bigg(\frac{\int\frac{a+b\text{arctanh}(c}{1-c^2x^6}}{c^2}}{c^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}\Bigg(\frac{1}{3}x^9\left(a+b\text{arctanh}(cx^3)\right)^3-bc\Bigg(\frac{\int\frac{x^3\left(a+b\text{arctanh}(cx^3)\right)^2}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{2}x^6\left(a+b\text{arctanh}(cx^3)\right)^2-bc\Bigg(\frac{\int\frac{a+b\text{arctanh}(c}{1-c^2x^6}}{c^2}}{c^2}$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{3}\Bigg(\frac{1}{3}x^9\left(a+b\text{arctanh}(cx^3)\right)^3-bc\Bigg(\frac{\int\frac{x^3\left(a+b\text{arctanh}(cx^3)\right)^2}{1-c^2x^6}dx^3}{c^2}-\frac{\frac{1}{2}x^6\left(a+b\text{arctanh}(cx^3)\right)^2-bc\Bigg(\frac{(a+b\text{arctanh}(cx}{2bc^3}}{c^2}$$

$$\Big\downarrow \text{6546}$$

3.124. $\int x^8\left(a+b\text{arctanh}(cx^3)\right)^3\,dx$

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}(cx^3)\right)^3-bc\left(\frac{\frac{\int\frac{\left(a+b\text{arctanh}(cx^3)\right)^2}{1-cx^3}dx^3}{c}-\frac{\left(a+b\text{arctanh}(cx^3)\right)^3}{3bc^2}}{c^2}-\frac{\frac{1}{2}x^6\left(a+b\text{arctanh}(cx^3)\right)^2-\ldots}{\ldots}\right.\right.$$

$\downarrow$ 6470

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}(cx^3)\right)^3-bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}(cx^3)\right)^2}{c}-2b\int\frac{\left(a+b\text{arctanh}(cx^3)\right)\log\left(\frac{2}{1-cx^3}\right)}{1-c^2x^6}dx^3}{c}-\frac{\left(a+b\text{arctan}\ldots\right.}{3bc^2\ldots}}{c^2}\right.\right.$$

$\downarrow$ 6620

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}(cx^3)\right)^3-bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}(cx^3)\right)^2}{c}-2b\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{1-c^2x^6}dx^3-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a+\ldots\right.}{2c}\right.}{c}}{c^2}\right.\right.$$

$\downarrow$ 7164

$$\frac{1}{3}\left(\frac{1}{3}x^9\left(a+b\text{arctanh}(cx^3)\right)^3-bc\left(\frac{\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}(cx^3)\right)^2}{c}-2b\left(\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)}{4c}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a+b\text{arcta}\ldots\right.}{2c}\right.}{c}}{c^2}\right.\right.$$

input
```
Int[x^8*(a + b*ArcTanh[c*x^3])^3,x]
```

output
```
((x^9*(a + b*ArcTanh[c*x^3])^3)/3 - b*c*(-(((x^6*(a + b*ArcTanh[c*x^3])^2)
/2 - b*c*((a + b*ArcTanh[c*x^3])^2/(2*b*c^3) - (a*x^3 + b*x^3*ArcTanh[c*x^
3] + (b*Log[1 - c^2*x^6])/(2*c))/c^2))/c^2) + (-1/3*(a + b*ArcTanh[c*x^3])
^3/(b*c^2) + (((a + b*ArcTanh[c*x^3])^2*Log[2/(1 - c*x^3)])/c - 2*b*(-1/2*
((a + b*ArcTanh[c*x^3])*PolyLog[2, 1 - 2/(1 - c*x^3)])/c + (b*PolyLog[3, 1
 - 2/(1 - c*x^3)])/(4*c)))/c)/c^2))/3
```

3.124. $\int x^8(a+b\text{arctanh}(cx^3))^3\,dx$

3.124.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.124.4 Maple [F]

$$\int x^8 \left(a+b \operatorname{arctanh}\left(c x^3\right)\right)^3 dx$$

input
```
int(x^8*(a+b*arctanh(c*x^3))^3,x)
```

output
```
int(x^8*(a+b*arctanh(c*x^3))^3,x)
```

3.124.5 Fricas [F]

$$\int x^8 (a+b\text{arctanh}(cx^3))^3 \, dx = \int \left(b \operatorname{artanh}\left(cx^3\right)+a\right)^3 x^8 \, dx$$

input
```
integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^8*arctanh(c*x^3)^3 + 3*a*b^2*x^8*arctanh(c*x^3)^2 + 3*a^2*b
*x^8*arctanh(c*x^3) + a^3*x^8, x)
```

3.124.6 Sympy [F(-1)]

Timed out.

$$\int x^8 \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 dx = \text{Timed out}$$

input
```
integrate(x**8*(a+b*atanh(c*x**3))**3,x)
```

output
```
Timed out
```

3.124.7 Maxima [F]

$$\int x^8 \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 dx = \int \left(b\,\text{artanh}\left(cx^3\right)+a\right)^3 x^8\,dx$$

input
```
integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

output
```
1/9*a^3*x^9 + 1/6*(2*x^9*arctanh(c*x^3) + (x^6/c^2 + log(c^2*x^6 - 1)/c^4)
*c)*a^2*b - 1/72*((b^3*c^3*x^9 - b^3)*log(-c*x^3 + 1)^3 - 3*(2*a*b^2*c^3*x
^9 + b^3*c^2*x^6 + (b^3*c^3*x^9 + b^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/
c^3 - integrate(-1/8*((b^3*c^3*x^11 - b^3*c^2*x^8)*log(c*x^3 + 1)^3 + 6*(a
*b^2*c^3*x^11 - a*b^2*c^2*x^8)*log(c*x^3 + 1)^2 - (4*a*b^2*c^3*x^11 + 2*b^
3*c^2*x^8 + 3*(b^3*c^3*x^11 - b^3*c^2*x^8)*log(c*x^3 + 1)^2 - 2*(6*a*b^2*c
^2*x^8 - (6*a*b^2*c^3 + b^3*c^3)*x^11 - b^3*x^2)*log(c*x^3 + 1))*log(-c*x^
3 + 1))/(c^3*x^3 - c^2), x)
```

3.124.8 Giac [F]

$$\int x^8 \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 dx = \int \left(b\,\text{artanh}\left(cx^3\right)+a\right)^3 x^8\,dx$$

input
```
integrate(x^8*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^3*x^8, x)
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int x^8 \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 \, dx=\int x^8\,{\left(a+b\,\mathrm{atanh}\left(c\,x^3\right)\right)}^3 \,d x$$

input
```
int(x^8*(a + b*atanh(c*x^3))^3,x)
```

output
```
int(x^8*(a + b*atanh(c*x^3))^3, x)
```

3.125 $\int x^5(a+b\text{arctanh}(cx^3))^3\,dx$

3.125.1 Optimal result

Integrand size = 16, antiderivative size = 139

$$\int x^5\left(a+b\text{arctanh}\left(cx^3\right)\right)^3\,dx=\frac{b(a+b\text{arctanh}(cx^3))^2}{2c^2}+\frac{bx^3(a+b\text{arctanh}(cx^3))^2}{2c}-\frac{(a+b\text{arctanh}(cx^3))^3}{6c^2}+\frac{1}{6}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-\frac{b^2(a+b\text{arctanh}(cx^3))\log\left(\frac{2}{1-cx^3}\right)}{c^2}-\frac{b^3\,\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{2c^2}$$

output
```
1/2*b*(a+b*arctanh(c*x^3))^2/c^2+1/2*b*x^3*(a+b*arctanh(c*x^3))^2/c-1/6*(a
+b*arctanh(c*x^3))^3/c^2+1/6*x^6*(a+b*arctanh(c*x^3))^3-b^2*(a+b*arctanh(c
*x^3))*ln(2/(-c*x^3+1))/c^2-1/2*b^3*polylog(2,1-2/(-c*x^3+1))/c^2
```

3.125.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.33

$$\int x^5\left(a+b\text{arctanh}\left(cx^3\right)\right)^3\,dx$$

$$=\frac{6b^2(-1+cx^3)\left(a+b+acx^3\right)\text{arctanh}(cx^3)^2+2b^3(-1+c^2x^6)\,\text{arctanh}(cx^3)^3+6b\text{arctanh}(cx^3)\left(acx^3(2b+\right.}{}$$

input
```
Integrate[x^5*(a + b*ArcTanh[c*x^3])^3,x]
```

output
```
(6*b^2*(-1 + c*x^3)*(a + b + a*c*x^3)*ArcTanh[c*x^3]^2 + 2*b^3*(-1 + c^2*x
^6)*ArcTanh[c*x^3]^3 + 6*b*ArcTanh[c*x^3]*(a*c*x^3*(2*b + a*c*x^3) - 2*b^2
*Log[1 + E^(-2*ArcTanh[c*x^3])]) + a*(6*a*b*c*x^3 + 2*a^2*c^2*x^6 + 3*a*b*
Log[1 - c*x^3] - 3*a*b*Log[1 + c*x^3] + 6*b^2*Log[1 - c^2*x^6]) + 6*b^3*Po
lyLog[2, -E^(-2*ArcTanh[c*x^3])])/(12*c^2)
```

3.125.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.14, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6454, 6452, 6542, 6436, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(a+b\text{arctanh}\left(cx^3\right)\right)^3 dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{1}{3}\int x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^3 dx^3$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-\frac{3}{2}bc\int\frac{x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{1-c^2x^6}dx^3\right)$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{\int\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{1-c^2x^6}dx^3}{c^2}-\frac{\int\left(a+b\text{arctanh}\left(cx^3\right)\right)^2dx^3}{c^2}\right)\right)$$

$$\Big\downarrow \text{6436}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{\int\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{1-c^2x^6}dx^3}{c^2}-\frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-2bc\int\frac{x^3\left(a+b\text{arctanh}\left(\cdots\right)\right)}{1-c^2x^6}}{c^2}\right)\right)$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{3bc^3}-\frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-2bc\int\frac{x^3\left(a+b\text{arctanh}\left(cx^3\right.\right.}{1-c^2x^6}}{c^2}\right.\right.$$

$$\Big\downarrow \text{ 6546}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{3bc^3}-\frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-2bc\left(\frac{\int\frac{a+b\text{arctanh}\left(cx^3\right)}{1-cx^3}d}{c}\right.}{c^2}\right.\right.$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{3bc^3}-\frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arct}\right.}{c}}{}\right.}{c}\right.\right.$$

$$\Big\downarrow \text{ 2849}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{3bc^3}-\frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-2bc\left(\frac{\frac{b\int\frac{\log\left(\frac{2}{1-cx^3}\right)}{1-\frac{2}{1-cx^3}}d\frac{1}{1-cx^3}}{c}}{}\right.}{}\right.\right.$$

$$\Big\downarrow \text{ 2752}$$

$$\frac{1}{3}\left(\frac{1}{2}x^6\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{3bc^3}-\frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arct}\right.}{c}}{}\right.}{}\right.\right.$$

3.125. $\int x^5\left(a+b\text{arctanh}\left(cx^3\right)\right)^3\,dx$

input
```
Int[x^5*(a + b*ArcTanh[c*x^3])^3,x]
```

output
```
((x^6*(a + b*ArcTanh[c*x^3])^3)/2 - (3*b*c*((a + b*ArcTanh[c*x^3])^3/(3*b*
c^3) - (x^3*(a + b*ArcTanh[c*x^3])^2 - 2*b*c*(-1/2*(a + b*ArcTanh[c*x^3])^
2/(b*c^2) + (((a + b*ArcTanh[c*x^3])*Log[2/(1 - c*x^3)])/c + (b*PolyLog[2,
 1 - 2/(1 - c*x^3)])/(2*c))/c))/c^2))/2)/3
```

3.125.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.125.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.80 (sec) , antiderivative size = 800, normalized size of antiderivative = 5.76

method	result	size
risch	Expression too large to display	800

input
```
int(x^5*(a+b*arctanh(c*x^3))^3,x,method=_RETURNVERBOSE)
```

output
```
1/48*b^3*(c^2*x^6-1)/c^2*ln(c*x^3+1)^3+1/16*b^2*(-b*c^2*ln(-c*x^3+1)*x^6+2
*a*c^2*x^6+2*b*c*x^3+b*ln(-c*x^3+1)-2*a+2*b)/c^2*ln(c*x^3+1)^2+(1/16*b^3*(
c^2*x^6-1)/c^2*ln(-c*x^3+1)^2-1/16*b^2*(2*a*c*x^3+b)^2/a/c^2*ln(-c*x^3+1)-
1/16*b*(-4*a^3*c^2*x^6-8*a^2*b*c*x^3-4*ln(-c*x^3+1)*a^2*b-4*ln(-c*x^3+1)*a
*b^2-ln(-c*x^3+1)*b^3-4*a*b^2)/a/c^2)*ln(c*x^3+1)+1/6*a^3*x^6+3/4/c*b^2*Su
m(-2/3*(ln(x-_alpha)*ln(-c*x^3+1)+3*c*(-1/3*ln(x-_alpha)*(ln((RootOf(_Z^2+
_Z*_alpha+_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,index
=1))+ln((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=2)-x+_alpha)/RootOf(_Z^2+_Z*
_alpha+_alpha^2,index=2))+ln(1/2*(x+_alpha)/_alpha))/c-1/3*(dilog((RootOf(
_Z^2+_Z*_alpha+_alpha^2,index=1)-x+_alpha)/RootOf(_Z^2+_Z*_alpha+_alpha^2,
index=1))+dilog((RootOf(_Z^2+_Z*_alpha+_alpha^2,index=2)-x+_alpha)/RootOf(
_Z^2+_Z*_alpha+_alpha^2,index=2))+dilog(1/2*(x+_alpha)/_alpha))/c))*b/c,_a
lpha=RootOf(_Z^3*c+1))-1/48*b^3*x^6*ln(-c*x^3+1)^3-1/8*b^3/c^2*ln(-c*x^3+1
)^2+1/48*b^3/c^2*ln(-c*x^3+1)^3-1/4*b/c^2*ln(c*x^3+1)*a^2+1/2*b^2/c^2*ln(c
*x^3+1)*a-1/2/c*a*b^2*x^3*ln(-c*x^3+1)+1/4/c^2*b^3*ln(-c*x^3+1)-1/4/c^2*b^
3*ln(c*x^3-1)-1/4/c^2*b^3*ln(c*x^3+1)-1/8*b^3/c^2+1/8/c^2*b^2*a*ln(c*x^3-1
)+1/8/c*b^3*x^3*ln(-c*x^3+1)^2-1/4*a^2*b*x^6*ln(-c*x^3+1)+1/4*a^2*b/c^2*ln
(c*x^3-1)+1/8*a*b^2*x^6*ln(-c*x^3+1)^2+3/8/c^2*a*b^2*ln(-c*x^3+1)-1/8/c^2*
a*b^2*ln(-c*x^3+1)^2+1/2/c*a^2*b*x^3
```

3.125.5 Fricas [F]

$$\int x^5 \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 \, dx = \int \left(b\,\text{artanh}\left(cx^3\right)+a\right)^3 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^5*arctanh(c*x^3)^3 + 3*a*b^2*x^5*arctanh(c*x^3)^2 + 3*a^2*b
*x^5*arctanh(c*x^3) + a^3*x^5, x)
```

3.125.6 Sympy [F(-1)]

Timed out.

$$\int x^5 \left(a + b\text{arctanh}\left(cx^3\right)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(x**5*(a+b*atanh(c*x**3))**3,x)
```

output
```
Timed out
```

3.125.7 Maxima [F]

$$\int x^5 \left(a + b\text{arctanh}\left(cx^3\right)\right)^3 \, dx = \int \left(b\,\text{artanh}\left(cx^3\right) + a\right)^3 x^5 \, dx$$

input
```
integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

output
```
1/2*a*b^2*x^6*arctanh(c*x^3)^2 + 1/6*a^3*x^6 + 1/4*(2*x^6*arctanh(c*x^3) +
 c*(2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3))*a^2*b + 1/8*(4*c
*(2*x^3/c^2 - log(c*x^3 + 1)/c^3 + log(c*x^3 - 1)/c^3)*arctanh(c*x^3) - (2
*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2 - log(c*x^3 - 1)^2
 - 4*log(c*x^3 - 1))/c^2)*a*b^2 - 1/192*(4*x^6*log(-c*x^3 + 1)^3 + 3*(x^6/
c^3 + log(c^2*x^6 - 1)/c^5)*c^3 - 6*c*((c*x^6 + 2*x^3)/c^2 + 2*log(c*x^3 -
 1)/c^3)*log(-c*x^3 + 1)^2 + 21*c^2*(2*x^3/c^3 - log(c*x^3 + 1)/c^4 + log(
c*x^3 - 1)/c^4) + c*(6*(c^2*x^6 + 6*c*x^3 + 2*log(c*x^3 - 1)^2 + 6*log(c*x
^3 - 1))*log(-c*x^3 + 1)/c^3 - (3*c^2*x^6 + 42*c*x^3 + 4*log(c*x^3 - 1)^3
+ 18*log(c*x^3 - 1)^2 + 42*log(c*x^3 - 1))/c^3) - 1728*c*integrate(1/4*x^5
*log(c*x^3 + 1)/(c^3*x^6 - c), x) - 2*(12*c*x^3*log(c*x^3 + 1)^2 + 2*(c^2*
x^6 - 1)*log(c*x^3 + 1)^3 - 3*(c^2*x^6 - 2*c*x^3 - 2*(c^2*x^6 - 1)*log(c*x
^3 + 1) + 1)*log(-c*x^3 + 1)^2 + 3*(c^2*x^6 + 6*c*x^3 - 2*(c^2*x^6 - 1)*lo
g(c*x^3 + 1)^2 - 8*(c*x^3 + 1)*log(c*x^3 + 1))*log(-c*x^3 + 1))/c^2 + 18*l
og(4*c^3*x^6 - 4*c)/c^2 - 576*integrate(1/4*x^2*log(c*x^3 + 1)/(c^3*x^6 -
c), x))*b^3
```

3.125.8 Giac [F]

$$\int x^5(a+b\text{arctanh}(cx^3))^3\,dx=\int (b\,\text{artanh}\left(cx^3\right)+a)^3x^5\,dx$$

input
```
integrate(x^5*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^3*x^5, x)
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int x^5(a+b\text{arctanh}(cx^3))^3\,dx=\int x^5\,\left(a+b\,\text{atanh}\left(c\,x^3\right)\right)^3\,dx$$

input
```
int(x^5*(a + b*atanh(c*x^3))^3,x)
```

output
```
int(x^5*(a + b*atanh(c*x^3))^3, x)
```

3.126 $\int x^2(a+b\text{arctanh}(cx^3))^3\,dx$

3.126.1 Optimal result

Integrand size = 16, antiderivative size = 130

$$\int x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)^3\,dx=\frac{(a+b\text{arctanh}(cx^3))^3}{3c}+\frac{1}{3}x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-\frac{b(a+b\text{arctanh}(cx^3))^2\log\left(\frac{2}{1-cx^3}\right)}{c}-\frac{b^2(a+b\text{arctanh}(cx^3))\,\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{c}+\frac{b^3\,\text{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)}{2c}$$

output
```
1/3*(a+b*arctanh(c*x^3))^3/c+1/3*x^3*(a+b*arctanh(c*x^3))^3-b*(a+b*arctanh
(c*x^3))^2*ln(2/(-c*x^3+1))/c-b^2*(a+b*arctanh(c*x^3))*polylog(2,1-2/(-c*x
^3+1))/c+1/2*b^3*polylog(3,1-2/(-c*x^3+1))/c
```

3.126.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.47

$$\int x^2\left(a+b\text{arctanh}\left(cx^3\right)\right)^3\,dx$$
$$=\frac{2a^3cx^3+6a^2bcx^3\text{arctanh}(cx^3)+3a^2b\log\left(1-c^2x^6\right)+6ab^2\Big(\text{arctanh}(cx^3)\Big(\left(-1+cx^3\right)\text{arctanh}(cx^3)-21}{}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x^3])^3,x]
```

output
```
(2*a^3*c*x^3 + 6*a^2*b*c*x^3*ArcTanh[c*x^3] + 3*a^2*b*Log[1 - c^2*x^6] + 6
*a*b^2*(ArcTanh[c*x^3]*((-1 + c*x^3)*ArcTanh[c*x^3] - 2*Log[1 + E^(-2*ArcT
anh[c*x^3])]) + PolyLog[2, -E^(-2*ArcTanh[c*x^3])]) + b^3*(2*ArcTanh[c*x^3
]^2*((-1 + c*x^3)*ArcTanh[c*x^3] - 3*Log[1 + E^(-2*ArcTanh[c*x^3])]) + 6*A
rcTanh[c*x^3]*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + 3*PolyLog[3, -E^(-2*Arc
Tanh[c*x^3])]))/(6*c)
```

3.126.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6454, 6436, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 \, dx$$

$$\Big\downarrow \text{ 6454}$$

$$\frac{1}{3}\int \left(a+b\text{arctanh}\left(cx^3\right)\right)^3dx^3$$

$$\Big\downarrow \text{ 6436}$$

$$\frac{1}{3}\left(x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-3bc\int \frac{x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{1-c^2x^6}dx^3\right)$$

$$\Big\downarrow \text{ 6546}$$

$$\frac{1}{3}\left(x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-3bc\left(\frac{\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{1-cx^3}dx^3}{c}-\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{3bc^2}\right)\right)$$

$$\Big\downarrow \text{ 6470}$$

$$\frac{1}{3}\left(x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-3bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{c}-2b\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)\log\left(\frac{2}{1-cx^3}\right)}{1-c^2x^6}dx^3}{c}-\frac{(a+b}{}\right.\right.$$

$\downarrow$ 6620

$$\frac{1}{3}\left(x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-3bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{c}-2b\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)}{1-c^2x^6}dx^3-\frac{\text{PolyLog}\left(2,1-\frac{}{}\right.}{}\right.}{c}\right.\right.$$

$\downarrow$ 7164

$$\frac{1}{3}\left(x^3\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-3bc\left(\frac{\frac{\log\left(\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{c}-2b\left(\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)}{4c}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a\right.}{2c}\right.}{c}\right.\right.$$

input
```
Int[x^2*(a + b*ArcTanh[c*x^3])^3,x]
```

output
```
(x^3*(a + b*ArcTanh[c*x^3])^3 - 3*b*c*(-1/3*(a + b*ArcTanh[c*x^3])^3/(b*c^
2) + (((a + b*ArcTanh[c*x^3])^2*Log[2/(1 - c*x^3)])/c - 2*b*(-1/2*((a + b*
ArcTanh[c*x^3])*PolyLog[2, 1 - 2/(1 - c*x^3)])/c + (b*PolyLog[3, 1 - 2/(1
- c*x^3)])/(4*c)))/c))/3
```

3.126.3.1 Defintions of rubi rules used

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(124) = 248$.

Time = 2.92 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.04

method	result
derivativedivides	$\frac{a^3 c x^3 + b^3 \left(\operatorname{arctanh}(c x^3)^3 (c x^3-1)+2 \operatorname{arctanh}(c x^3)^3-3 \operatorname{arctanh}(c x^3)^2 \ln\left(1+\frac{(c x^3+1)^2}{-c^2 x^6+1}\right)-3 \operatorname{arctanh}(c x^3) \operatorname{polylog}(\right.}{}$
default	$\frac{a^3 c x^3 + b^3 \left(\operatorname{arctanh}(c x^3)^3 (c x^3-1)+2 \operatorname{arctanh}(c x^3)^3-3 \operatorname{arctanh}(c x^3)^2 \ln\left(1+\frac{(c x^3+1)^2}{-c^2 x^6+1}\right)-3 \operatorname{arctanh}(c x^3) \operatorname{polylog}(\right.}{}$
parts	$\frac{a^3 x^3}{3} + \frac{b^3 \left(\operatorname{arctanh}(c x^3)^3 (c x^3-1)+2 \operatorname{arctanh}(c x^3)^3-3 \operatorname{arctanh}(c x^3)^2 \ln\left(1+\frac{(c x^3+1)^2}{-c^2 x^6+1}\right)-3 \operatorname{arctanh}(c x^3) \operatorname{polylo}\right.}{3c}$

input
```
int(x^2*(a+b*arctanh(c*x^3))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/c*(a^3*c*x^3+b^3*(arctanh(c*x^3)^3*(c*x^3-1)+2*arctanh(c*x^3)^3-3*arct
anh(c*x^3)^2*ln(1+(c*x^3+1)^2/(-c^2*x^6+1))-3*arctanh(c*x^3)*polylog(2,-(c
*x^3+1)^2/(-c^2*x^6+1))+3/2*polylog(3,-(c*x^3+1)^2/(-c^2*x^6+1)))+3*a*b^2*
(arctanh(c*x^3)^2*(c*x^3-1)+2*arctanh(c*x^3)^2-2*arctanh(c*x^3)*ln(1+(c*x^
3+1)^2/(-c^2*x^6+1))-polylog(2,-(c*x^3+1)^2/(-c^2*x^6+1)))+3*a^2*b*(c*x^3*
arctanh(c*x^3)+1/2*ln(-c^2*x^6+1)))
```

3.126.5 Fricas [F]

$$\int x^2 (a + b\text{arctanh}(cx^3))^3 \, dx = \int (b \operatorname{artanh}(cx^3) + a)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arctanh(c*x^3)^3 + 3*a*b^2*x^2*arctanh(c*x^3)^2 + 3*a^2*b
*x^2*arctanh(c*x^3) + a^3*x^2, x)
```

3.126.6 Sympy [F(-1)]

Timed out.

$$\int x^2 \left(a + b\operatorname{arctanh}\left(cx^3\right)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atanh(c*x**3))**3,x)
```

output
```
Timed out
```

3.126.7 Maxima [F]

$$\int x^2 \left(a + b\operatorname{arctanh}\left(cx^3\right)\right)^3 \, dx = \int \left(b \operatorname{artanh}\left(cx^3\right) + a\right)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*x^3 + 1/2*(2*c*x^3*arctanh(c*x^3) + log(-c^2*x^6 + 1))*a^2*b/c - 1
/24*((b^3*c*x^3 - b^3)*log(-c*x^3 + 1)^3 - 3*(2*a*b^2*c*x^3 + (b^3*c*x^3 +
 b^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/c - integrate(-1/8*((b^3*c*x^5 -
b^3*x^2)*log(c*x^3 + 1)^3 + 6*(a*b^2*c*x^5 - a*b^2*x^2)*log(c*x^3 + 1)^2 -
 3*(4*a*b^2*c*x^5 + (b^3*c*x^5 - b^3*x^2)*log(c*x^3 + 1)^2 + 2*((2*a*b^2*c
 + b^3*c)*x^5 - (2*a*b^2 - b^3)*x^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x
^3 - 1), x)
```

3.126.8 Giac [F]

$$\int x^2 \left(a + b\operatorname{arctanh}\left(cx^3\right)\right)^3 \, dx = \int \left(b \operatorname{artanh}\left(cx^3\right) + a\right)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^3*x^2, x)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\operatorname{arctanh}(cx^3))^3\,dx=\int x^2\,\left(a+b\,\operatorname{atanh}\left(c\,x^3\right)\right)^3\,dx$$

input
```
int(x^2*(a + b*atanh(c*x^3))^3,x)
```

output
```
int(x^2*(a + b*atanh(c*x^3))^3, x)
```

3.127 $\int \frac{(a+b\text{arctanh}(cx^3))^3}{x}\,dx$

3.127.1 Optimal result

Integrand size = 16, antiderivative size = 210

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(cx^3))^3}{x}\,dx &= \frac{2}{3}(a+b\text{arctanh}(cx^3))^3 \text{arctanh}\left(1-\frac{2}{1-cx^3}\right)\\
&-\frac{1}{2}b(a+b\text{arctanh}(cx^3))^2 \text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\\
&+\frac{1}{2}b(a+b\text{arctanh}(cx^3))^2 \text{PolyLog}\left(2,-1+\frac{2}{1-cx^3}\right)\\
&+\frac{1}{2}b^2(a+b\text{arctanh}(cx^3)) \text{PolyLog}\left(3,1-\frac{2}{1-cx^3}\right)\\
&-\frac{1}{2}b^2(a+b\text{arctanh}(cx^3)) \text{PolyLog}\left(3,-1+\frac{2}{1-cx^3}\right)\\
&-\frac{1}{4}b^3 \text{PolyLog}\left(4,1-\frac{2}{1-cx^3}\right)\\
&+\frac{1}{4}b^3 \text{PolyLog}\left(4,-1+\frac{2}{1-cx^3}\right)\end{aligned}$$

output
```
-2/3*(a+b*arctanh(c*x^3))^3*arctanh(-1+2/(-c*x^3+1))-1/2*b*(a+b*arctanh(c*
x^3))^2*polylog(2,1-2/(-c*x^3+1))+1/2*b*(a+b*arctanh(c*x^3))^2*polylog(2,-
1+2/(-c*x^3+1))+1/2*b^2*(a+b*arctanh(c*x^3))*polylog(3,1-2/(-c*x^3+1))-1/2
*b^2*(a+b*arctanh(c*x^3))*polylog(3,-1+2/(-c*x^3+1))-1/4*b^3*polylog(4,1-2
/(-c*x^3+1))+1/4*b^3*polylog(4,-1+2/(-c*x^3+1))
```

3.127.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.75

$$\int \frac{(a+b\text{arctanh}(cx^3))^3}{x}\,dx = a^3\log(x)+\frac{1}{2}a^2b(-\text{PolyLog}\left(2,-cx^3\right)+\text{PolyLog}\left(2,cx^3\right)) + ab^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\text{arctanh}\left(cx^3\right)^3 - \text{arctanh}\left(cx^3\right)^2\log\left(1+e^{-2\text{arctanh}(cx^3)}\right) + \text{arctanh}\left(cx^3\right)^2\log\left(1-e^{2\text{arctanh}(cx^3)}\right) + \text{arctanh}\left(cx^3\right)\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx^3)}\right) + \text{arctanh}\left(cx^3\right)\text{PolyLog}\left(2,e^{2\text{arctanh}(cx^3)}\right) + \frac{1}{2}\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx^3)}\right) - \frac{1}{2}\text{PolyLog}\left(3,e^{2\text{arctanh}(cx^3)}\right)\right) + \frac{1}{192}b^3\left(\pi^4 - 32\text{arctanh}\left(cx^3\right)^4 - 64\text{arctanh}\left(cx^3\right)^3\log\left(1+e^{-2\text{arctanh}(cx^3)}\right) + 64\text{arctanh}\left(cx^3\right)^3\log\left(1-e^{2\text{arctanh}(cx^3)}\right) + 96\text{arctanh}\left(cx^3\right)^2\text{PolyLog}\left(2,-e^{-2\text{arctanh}(cx^3)}\right) + 96\text{arctanh}\left(cx^3\right)^2\text{PolyLog}\left(2,e^{2\text{arctanh}(cx^3)}\right) + 96\text{arctanh}\left(cx^3\right)\text{PolyLog}\left(3,-e^{-2\text{arctanh}(cx^3)}\right) - 96\text{arctanh}\left(cx^3\right)\text{PolyLog}\left(3,e^{2\text{arctanh}(cx^3)}\right) + 48\,\text{PolyLog}\left(4,-e^{-2\text{arctanh}(cx^3)}\right) + 48\,\text{PolyLog}\left(4,e^{2\text{arctanh}(cx^3)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])^3/x,x]
```

output
```
a^3*Log[x] + (a^2*b*(-PolyLog[2, -(c*x^3)] + PolyLog[2, c*x^3]))/2 + a*b^2
*((I/24)*Pi^3 - (2*ArcTanh[c*x^3]^3)/3 - ArcTanh[c*x^3]^2*Log[1 + E^(-2*Ar
cTanh[c*x^3])] + ArcTanh[c*x^3]^2*Log[1 - E^(2*ArcTanh[c*x^3])] + ArcTanh[
c*x^3]*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + ArcTanh[c*x^3]*PolyLog[2, E^(2
*ArcTanh[c*x^3])] + PolyLog[3, -E^(-2*ArcTanh[c*x^3])]/2 - PolyLog[3, E^(2
*ArcTanh[c*x^3])]/2) + (b^3*(Pi^4 - 32*ArcTanh[c*x^3]^4 - 64*ArcTanh[c*x^3
]^3*Log[1 + E^(-2*ArcTanh[c*x^3])] + 64*ArcTanh[c*x^3]^3*Log[1 - E^(2*ArcT
anh[c*x^3])] + 96*ArcTanh[c*x^3]^2*PolyLog[2, -E^(-2*ArcTanh[c*x^3])] + 96
*ArcTanh[c*x^3]^2*PolyLog[2, E^(2*ArcTanh[c*x^3])] + 96*ArcTanh[c*x^3]*Pol
yLog[3, -E^(-2*ArcTanh[c*x^3])] - 96*ArcTanh[c*x^3]*PolyLog[3, E^(2*ArcTan
h[c*x^3])] + 48*PolyLog[4, -E^(-2*ArcTanh[c*x^3])] + 48*PolyLog[4, E^(2*Ar
cTanh[c*x^3])]))/192
```

3.127.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6450, 6448, 6614, 6620, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{x}dx$$

$$\Big\downarrow \text{ 6450}$$

$$\frac{1}{3}\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{x^3}dx^3$$

$$\Big\downarrow \text{ 6448}$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-6bc\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)}{1-c^2x^6}dx^3\right)$$

$$\Big\downarrow \text{ 6614}$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-6bc\left(\frac{1}{2}\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2\log\left(2-\frac{2}{1-cx^3}\right)}{1-c^2x^6}dx^3-\frac{1}{2}\int\right.\right.$$

$$\Big\downarrow \text{ 6620}$$

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{2c}-b\int\right.\right.\right.$$

$\downarrow$ 6624

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{2c}-b\left(\right.\right.\right.\right.$$

$\downarrow$ 7164

$$\frac{1}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^3-6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^3}\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{2c}-b\left(\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x^3])^3/x,x]
```

output
```
(2*(a + b*ArcTanh[c*x^3])^3*ArcTanh[1 - 2/(1 - c*x^3)] - 6*b*c*((((a + b*A
rcTanh[c*x^3])^2*PolyLog[2, 1 - 2/(1 - c*x^3)])/(2*c) - b*(((a + b*ArcTanh
[c*x^3])*PolyLog[3, 1 - 2/(1 - c*x^3)])/(2*c) - (b*PolyLog[4, 1 - 2/(1 - c
*x^3)])/(4*c)))/2 + (-1/2*((a + b*ArcTanh[c*x^3])^2*PolyLog[2, -1 + 2/(1 -
 c*x^3)])/c + b*(((a + b*ArcTanh[c*x^3])*PolyLog[3, -1 + 2/(1 - c*x^3)])/(
2*c) - (b*PolyLog[4, -1 + 2/(1 - c*x^3)])/(4*c)))/2))/3
```

3.127.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.127. $\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{x}\,dx$

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.127.4 Maple [F]

$$\int \frac{(a + b \operatorname{arctanh}(c x^3))^3}{x} dx$$

input
```
int((a+b*arctanh(c*x^3))^3/x,x)
```

output
```
int((a+b*arctanh(c*x^3))^3/x,x)
```

3.127.5 Fricas [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^3}{x} \, dx = \int \frac{(b \operatorname{artanh}(cx^3) + a)^3}{x} \, dx$$

input
```
integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctan
h(c*x^3) + a^3)/x, x)
```

3.127.6 Sympy [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^3}{x} \, dx = \int \frac{(a + b \operatorname{atanh}(cx^3))^3}{x} \, dx$$

input
```
integrate((a+b*atanh(c*x**3))**3/x,x)
```

output
```
Integral((a + b*atanh(c*x**3))**3/x, x)
```

3.127.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^3}{x} \, dx = \int \frac{(b \operatorname{artanh}(cx^3) + a)^3}{x} \, dx$$

input
```
integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + integrate(1/8*b^3*(log(c*x^3 + 1) - log(-c*x^3 + 1))^3/x + 3/
4*a*b^2*(log(c*x^3 + 1) - log(-c*x^3 + 1))^2/x + 3/2*a^2*b*(log(c*x^3 + 1)
 - log(-c*x^3 + 1))/x, x)
```

3.127.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^3}{x}\,dx = \int \frac{(b\,\text{artanh}\,(cx^3) + a)^3}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^3/x, x)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^3))^3}{x}\,dx = \int \frac{(a + b\,\text{atanh}(c\,x^3))^3}{x}\,dx$$

input
```
int((a + b*atanh(c*x^3))^3/x,x)
```

output
```
int((a + b*atanh(c*x^3))^3/x, x)
```

3.128 $\int \frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^3}{x^4}\,dx$

3.128.1 Optimal result

Integrand size = 16, antiderivative size = 120

$$\begin{aligned}\int \frac{(a+b\operatorname{arctanh}(cx^3))^3}{x^4}\,dx = {} & \frac{1}{3}c\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^3-\frac{(a+b\operatorname{arctanh}(cx^3))^3}{3x^3}\\ & +bc\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2\log\left(2-\frac{2}{1+cx^3}\right)\\ & -b^2c\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\operatorname{PolyLog}\left(2,-1+\frac{2}{1+cx^3}\right)\\ & -\frac{1}{2}b^3c\operatorname{PolyLog}\left(3,-1+\frac{2}{1+cx^3}\right)\end{aligned}$$

output
```
1/3*c*(a+b*arctanh(c*x^3))^3-1/3*(a+b*arctanh(c*x^3))^3/x^3+b*c*(a+b*arcta
nh(c*x^3))^2*ln(2-2/(c*x^3+1))-b^2*c*(a+b*arctanh(c*x^3))*polylog(2,-1+2/(
c*x^3+1))-1/2*b^3*c*polylog(3,-1+2/(c*x^3+1))
```

3.128.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.86

$$\int \frac{(a+b\text{arctanh}(cx^3))^3}{x^4}\,dx = -\frac{a^3}{3x^3} - \frac{a^2b\text{arctanh}(cx^3)}{x^3} + 3a^2bc\log(x) - \frac{1}{2}a^2bc\log\left(1-c^2x^6\right) + ab^2c\left(\text{arctanh}\left(cx^3\right)\left(\left(1-\frac{1}{cx^3}\right)\text{arctanh}\left(cx^3\right) + 2\log\left(1-e^{-2\text{arctanh}(cx^3)}\right)\right) - \text{PolyLog}\left(2,e^{-2\text{arctanh}(cx^3)}\right)\right) + \frac{1}{3}b^3c\left(\frac{i\pi^3}{8} - \text{arctanh}\left(cx^3\right)^3 - \frac{\text{arctanh}(cx^3)^3}{cx^3} + 3\text{arctanh}\left(cx^3\right)^2\log\left(1-e^{2\text{arctanh}(cx^3)}\right) + 3\text{arctanh}\left(cx^3\right)\text{PolyLog}\left(2,e^{2\text{arctanh}(cx^3)}\right) - \frac{3}{2}\text{PolyLog}\left(3,e^{2\text{arctanh}(cx^3)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])^3/x^4,x]
```

output
```
-1/3*a^3/x^3 - (a^2*b*ArcTanh[c*x^3])/x^3 + 3*a^2*b*c*Log[x] - (a^2*b*c*Lo
g[1 - c^2*x^6])/2 + a*b^2*c*(ArcTanh[c*x^3]*((1 - 1/(c*x^3))*ArcTanh[c*x^3
] + 2*Log[1 - E^(-2*ArcTanh[c*x^3])]) - PolyLog[2, E^(-2*ArcTanh[c*x^3])])
 + (b^3*c*((I/8)*Pi^3 - ArcTanh[c*x^3]^3 - ArcTanh[c*x^3]^3/(c*x^3) + 3*Ar
cTanh[c*x^3]^2*Log[1 - E^(2*ArcTanh[c*x^3])] + 3*ArcTanh[c*x^3]*PolyLog[2,
 E^(2*ArcTanh[c*x^3])] - (3*PolyLog[3, E^(2*ArcTanh[c*x^3])])/2))/3
```

3.128.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6454, 6452, 6550, 6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{x^4}dx$$

$$\Big\downarrow \; 6454$$

$$\frac{1}{3}\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{x^6}dx^3$$

$$\Big\downarrow \; 6452$$

$$\frac{1}{3}\left(3bc\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{x^3\left(1-c^2x^6\right)}dx^3-\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{x^3}\right)$$

$$\Big\downarrow \; 6550$$

$$\frac{1}{3}\left(3bc\left(\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^2}{x^3\left(cx^3+1\right)}dx^3+\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{3b}\right)-\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{x^3}\right)$$

$$\Big\downarrow \; 6494$$

$$\frac{1}{3}\left(3bc\left(-2bc\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)\log\left(2-\frac{2}{cx^3+1}\right)}{1-c^2x^6}dx^3+\frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{3b}+\log\left(2-\frac{2}{cx^3+1}\right)\left(a+bar\right.\right.\right.$$

$$\Big\downarrow \; 6618$$

$$\frac{1}{3}\left(3bc\left(-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx^3+1}-1\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)}{2c}-\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,\frac{2}{cx^3+1}-1\right)}{1-c^2x^6}dx^3\right)+\frac{\left(a+b\text{arcta}\right.}{3}\right.\right.$$

$$\Big\downarrow \; 7164$$

$$\frac{1}{3}\left(3bc\left(-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{cx^3+1}-1\right)\left(a+b\text{arctanh}\left(cx^3\right)\right)}{2c}+\frac{b\,\text{PolyLog}\left(3,\frac{2}{cx^3+1}-1\right)}{4c}\right)+\frac{\left(a+b\text{arctanh}\left(cx^3\right.\right.}{3b}\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x^3])^3/x^4,x]
```

output
```
(-((a + b*ArcTanh[c*x^3])^3/x^3) + 3*b*c*((a + b*ArcTanh[c*x^3])^3/(3*b) +
 (a + b*ArcTanh[c*x^3])^2*Log[2 - 2/(1 + c*x^3)] - 2*b*c*(((a + b*ArcTanh[
c*x^3])*PolyLog[2, -1 + 2/(1 + c*x^3)])/(2*c) + (b*PolyLog[3, -1 + 2/(1 +
c*x^3)])/(4*c))))/3
```

3.128. $\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{x^4}dx$

3.128.3.1 Defintions of rubi rules used

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.128. $\int \frac{\left(a+b\text{arctanh}\left(cx^3\right)\right)^3}{x^4}\,dx$

3.128.4 Maple [F]

$$\int \frac{(a+b\,\mathrm{arctanh}\,(c\,x^3))^3}{x^4}dx$$

input
```
int((a+b*arctanh(c*x^3))^3/x^4,x)
```

output
```
int((a+b*arctanh(c*x^3))^3/x^4,x)
```

3.128.5 Fricas [F]

$$\int \frac{(a+b\mathrm{arctanh}(cx^3))^3}{x^4}\,dx = \int \frac{(b\,\mathrm{artanh}\,(cx^3)+a)^3}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctan
h(c*x^3) + a^3)/x^4, x)
```

3.128.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+b\mathrm{arctanh}(cx^3))^3}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))**3/x**4,x)
```

output
```
Timed out
```

3.128.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^3}{x^4} dx = \int \frac{(b \operatorname{artanh}(cx^3) + a)^3}{x^4} dx$$

input
```
integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="maxima")
```

output
```
-1/2*(c*(log(c^2*x^6 - 1) - log(x^6)) + 2*arctanh(c*x^3)/x^3)*a^2*b - 1/3*
a^3/x^3 - 1/24*((b^3*c*x^3 - b^3)*log(-c*x^3 + 1)^3 + 3*(2*a*b^2 + (b^3*c*
x^3 + b^3)*log(c*x^3 + 1))*log(-c*x^3 + 1)^2)/x^3 - integrate(-1/8*((b^3*c
*x^3 - b^3)*log(c*x^3 + 1)^3 + 6*(a*b^2*c*x^3 - a*b^2)*log(c*x^3 + 1)^2 +
3*(4*a*b^2*c*x^3 - (b^3*c*x^3 - b^3)*log(c*x^3 + 1)^2 + 2*(b^3*c^2*x^6 - (
2*a*b^2*c - b^3*c)*x^3 + 2*a*b^2)*log(c*x^3 + 1))*log(-c*x^3 + 1))/(c*x^7
- x^4), x)
```

3.128.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^3))^3}{x^4} dx = \int \frac{(b \operatorname{artanh}(cx^3) + a)^3}{x^4} dx$$

input
```
integrate((a+b*arctanh(c*x^3))^3/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^3/x^4, x)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^3))^3}{x^4} dx = \int \frac{(a + b \operatorname{atanh}(c x^3))^3}{x^4} dx$$

input
```
int((a + b*atanh(c*x^3))^3/x^4,x)
```

output
```
int((a + b*atanh(c*x^3))^3/x^4, x)
```

3.129 $\int \frac{\left(a+b\,\mathrm{arctanh}\left(cx^3\right)\right)^3}{x^7}\,dx$

3.129.1 Optimal result

Integrand size = 16, antiderivative size = 136

$$\begin{aligned}\int \frac{(a+b\,\mathrm{arctanh}(cx^3))^3}{x^7}\,dx &= \frac{1}{2}bc^2\left(a+b\,\mathrm{arctanh}\left(cx^3\right)\right)^2-\frac{bc(a+b\,\mathrm{arctanh}(cx^3))^2}{2x^3}\\&+\frac{1}{6}c^2\left(a+b\,\mathrm{arctanh}\left(cx^3\right)\right)^3-\frac{(a+b\,\mathrm{arctanh}(cx^3))^3}{6x^6}\\&+b^2c^2\left(a+b\,\mathrm{arctanh}\left(cx^3\right)\right)\log\left(2-\frac{2}{1+cx^3}\right)\\&-\frac{1}{2}b^3c^2\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+cx^3}\right)\end{aligned}$$

output
```
1/2*b*c^2*(a+b*arctanh(c*x^3))^2-1/2*b*c*(a+b*arctanh(c*x^3))^2/x^3+1/6*c^
2*(a+b*arctanh(c*x^3))^3-1/6*(a+b*arctanh(c*x^3))^3/x^6+b^2*c^2*(a+b*arcta
nh(c*x^3))*ln(2-2/(c*x^3+1))-1/2*b^3*c^2*polylog(2,-1+2/(c*x^3+1))
```

3.129.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.60

$$\int \frac{(a+b\text{arctanh}(cx^3))^3}{x^7}\,dx = \frac{6b^2(-1+cx^3)\,(a+acx^3+bcx^3)\,\text{arctanh}(cx^3)^2+2b^3(-1+c^2x^6)\,\text{arctanh}(cx^3)^3-6b\text{arctanh}(cx^3)\left(a^2+2\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x^3])^3/x^7,x]
```

output
```
(6*b^2*(-1 + c*x^3)*(a + a*c*x^3 + b*c*x^3)*ArcTanh[c*x^3]^2 + 2*b^3*(-1 +
 c^2*x^6)*ArcTanh[c*x^3]^3 - 6*b*ArcTanh[c*x^3]*(a^2 + 2*a*b*c*x^3 - 2*b^2
*c^2*x^6*Log[1 - E^(-2*ArcTanh[c*x^3])]) + a*(-2*a^2 - 6*a*b*c*x^3 - 3*a*b
*c^2*x^6*Log[1 - c*x^3] + 3*a*b*c^2*x^6*Log[1 + c*x^3] + 12*b^2*c^2*x^6*Lo
g[(c*x^3)/Sqrt[1 - c^2*x^6]]) - 6*b^3*c^2*x^6*PolyLog[2, E^(-2*ArcTanh[c*x
^3])])/(12*x^6)
```

3.129.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6454, 6452, 6544, 6452, 6510, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^3))^3}{x^7}\,dx$$

$$\Big\downarrow \; 6454$$

$$\frac{1}{3}\int \frac{(a+b\text{arctanh}(cx^3))^3}{x^9}dx^3$$

$$\Big\downarrow \; 6452$$

$$\frac{1}{3}\left(\frac{3}{2}bc\int \frac{(a+b\text{arctanh}(cx^3))^2}{x^6\,(1-c^2x^6)}dx^3-\frac{(a+b\text{arctanh}(cx^3))^3}{2x^6}\right)$$

$$\Big\downarrow \; 6544$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(c^2\int \frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2}{1-c^2x^6}dx^3+\int \frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2}{x^6}dx^3\right)-\frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^3}{2x^6}\right)$$

$$\Big\downarrow \; 6452$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(c^2\int \frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2}{1-c^2x^6}dx^3+2bc\int \frac{a+b\operatorname{arctanh}\left(cx^3\right)}{x^3\left(1-c^2x^6\right)}dx^3-\frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2}{x^3}\right)-\frac{\left(a+b\operatorname{arcta}\right.}{2x}$$

$$\Big\downarrow \; 6510$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(2bc\int \frac{a+b\operatorname{arctanh}\left(cx^3\right)}{x^3\left(1-c^2x^6\right)}dx^3+\frac{c\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^3}{3b}-\frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2}{x^3}\right)-\frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right.}{2x^6}\right.$$

$$\Big\downarrow \; 6550$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(2bc\left(\int \frac{a+b\operatorname{arctanh}\left(cx^3\right)}{x^3\left(cx^3+1\right)}dx^3+\frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2}{2b}\right)+\frac{c\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^3}{3b}-\frac{\left(a+b\operatorname{arctanh}\left(cx^3\right.\right.}{x^3}\right.\right.$$

$$\Big\downarrow \; 6494$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(2bc\left(-bc\int \frac{\log\left(2-\frac{2}{cx^3+1}\right)}{1-c^2x^6}dx^3+\frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2}{2b}+\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)\right)+\right.\right.$$

$$\Big\downarrow \; 2897$$

$$\frac{1}{3}\left(\frac{3}{2}bc\left(2bc\left(\frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^2}{2b}+\log\left(2-\frac{2}{cx^3+1}\right)\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)-\frac{1}{2}b\operatorname{PolyLog}\left(2,\frac{2}{cx^3+1}-1\right)\right)\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x^3])^3/x^7,x]
```

output
```
(-1/2*(a + b*ArcTanh[c*x^3])^3/x^6 + (3*b*c*(-((a + b*ArcTanh[c*x^3])^2/x^
3) + (c*(a + b*ArcTanh[c*x^3])^3)/(3*b) + 2*b*c*((a + b*ArcTanh[c*x^3])^2/
(2*b) + (a + b*ArcTanh[c*x^3])*Log[2 - 2/(1 + c*x^3)] - (b*PolyLog[2, -1 +
 2/(1 + c*x^3)])/2)))/2)/3
```

3.129. $\int \frac{\left(a+b\operatorname{arctanh}\left(cx^3\right)\right)^3}{x^7}\,dx$

3.129.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.129. $\int \frac{(a+b\text{arctanh}(cx^3))^3}{x^7}\,dx$

3.129.4 Maple [F]

$$\int \frac{(a + b\,\mathrm{arctanh}\,(c\,x^3))^3}{x^7}\,dx$$

input
```
int((a+b*arctanh(c*x^3))^3/x^7,x)
```

output
```
int((a+b*arctanh(c*x^3))^3/x^7,x)
```

3.129.5 Fricas [F]

$$\int \frac{(a + b\mathrm{arctanh}(cx^3))^3}{x^7}\,dx = \int \frac{(b\,\mathrm{artanh}\,(cx^3) + a)^3}{x^7}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctan
h(c*x^3) + a^3)/x^7, x)
```

3.129.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a + b\mathrm{arctanh}(cx^3))^3}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**3))**3/x**7,x)
```

output
```
Timed out
```

3.129.7 Maxima [F]

$$\int \frac{(a+b\text{arctanh}(cx^3))^3}{x^7}\,dx = \int \frac{(b\,\text{artanh}\,(cx^3)+a)^3}{x^7}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="maxima")
```

output
```
1/4*((c*log(c*x^3 + 1) - c*log(c*x^3 - 1) - 2/x^3)*c - 2*arctanh(c*x^3)/x^
6)*a^2*b + 1/8*((2*(log(c*x^3 - 1) - 2)*log(c*x^3 + 1) - log(c*x^3 + 1)^2
- log(c*x^3 - 1)^2 - 4*log(c*x^3 - 1) + 24*log(x))*c^2 + 4*(c*log(c*x^3 +
1) - c*log(c*x^3 - 1) - 2/x^3)*c*arctanh(c*x^3))*a*b^2 - 1/48*b^3*(((c^2*x
^6 - 1)*log(-c*x^3 + 1)^3 + 3*(2*c*x^3 - (c^2*x^6 - 1)*log(c*x^3 + 1))*log
(-c*x^3 + 1)^2)/x^6 + 6*integrate(-((c*x^3 - 1)*log(c*x^3 + 1)^3 + 3*(2*c^
2*x^6 - (c*x^3 - 1)*log(c*x^3 + 1)^2 - (c^3*x^9 - c*x^3)*log(c*x^3 + 1))*l
og(-c*x^3 + 1))/(c*x^10 - x^7), x)) - 1/2*a*b^2*arctanh(c*x^3)^2/x^6 - 1/6
*a^3/x^6
```

3.129.8 Giac [F]

$$\int \frac{(a+b\text{arctanh}(cx^3))^3}{x^7}\,dx = \int \frac{(b\,\text{artanh}\,(cx^3)+a)^3}{x^7}\,dx$$

input
```
integrate((a+b*arctanh(c*x^3))^3/x^7,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^3/x^7, x)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\text{arctanh}(cx^3))^3}{x^7}\,dx = \int \frac{(a+b\,\text{atanh}(c\,x^3))^3}{x^7}\,dx$$

input
```
int((a + b*atanh(c*x^3))^3/x^7,x)
```

output
```
int((a + b*atanh(c*x^3))^3/x^7, x)
```

3.130 $\int (dx)^m \left(a+b\,\text{arctanh}(cx^3)\right)^3 \, dx$

3.130.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(cx^3\right)\right)^3 \, dx = \text{Int}\left((dx)^m \left(a+b\,\text{arctanh}\left(cx^3\right)\right)^3, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c*x^3))^3,x)
```

3.130.2 Mathematica [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(cx^3\right)\right)^3 \, dx = \int (dx)^m \left(a+b\,\text{arctanh}\left(cx^3\right)\right)^3 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^3, x]
```

3.130.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{arctanh}(cx^3)\right)^3 \, dx$$

$$\Big\downarrow \; 6468$$

$$\int (dx)^m \left(a+b\text{arctanh}(cx^3)\right)^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x^3])^3,x]
```

output
```
$Aborted
```

3.130.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.130.4 Maple [N/A] (verified)

Not integrable

Time = 0.15 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b \,\text{arctanh}\left(c\,x^3\right)\right)^3 dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x^3))^3,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x^3))^3,x)
```

3.130.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 \, dx = \int \left(b \operatorname{artanh}\left(cx^3\right)+a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*x^3)^3 + 3*a*b^2*arctanh(c*x^3)^2 + 3*a^2*b*arctan
h(c*x^3) + a^3)*(d*x)^m, x)
```

3.130.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x**3))**3,x)
```

output
```
Timed out
```

3.130.7 Maxima [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 416, normalized size of antiderivative = 23.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right)^3 \, dx = \int \left(b \operatorname{artanh}\left(cx^3\right)+a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="maxima")
```

output
```
-1/8*b^3*d^m*x*x^m*log(-c*x^3 + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1
)) + integrate(1/8*((b^3*c*d^m*(m + 1)*x^3 - b^3*d^m*(m + 1))*x^m*log(c*x^
3 + 1)^3 + 6*(a*b^2*c*d^m*(m + 1)*x^3 - a*b^2*d^m*(m + 1))*x^m*log(c*x^3 +
 1)^2 + 12*(a^2*b*c*d^m*(m + 1)*x^3 - a^2*b*d^m*(m + 1))*x^m*log(c*x^3 + 1
) + 3*((b^3*c*d^m*(m + 1)*x^3 - b^3*d^m*(m + 1))*x^m*log(c*x^3 + 1) - (2*a
*b^2*d^m*(m + 1) - (2*a*b^2*c*d^m*(m + 1) + 3*b^3*c*d^m)*x^3)*x^m)*log(-c*
x^3 + 1)^2 - 3*((b^3*c*d^m*(m + 1)*x^3 - b^3*d^m*(m + 1))*x^m*log(c*x^3 +
1)^2 + 4*(a*b^2*c*d^m*(m + 1)*x^3 - a*b^2*d^m*(m + 1))*x^m*log(c*x^3 + 1)
+ 4*(a^2*b*c*d^m*(m + 1)*x^3 - a^2*b*d^m*(m + 1))*x^m)*log(-c*x^3 + 1))/(c
*(m + 1)*x^3 - m - 1), x)
```

3.130.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a + b\text{arctanh}\left(cx^3\right)\right)^3 \, dx = \int \left(b\,\text{artanh}\left(cx^3\right) + a\right)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^3))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^3*(d*x)^m, x)
```

3.130.9 Mupad [N/A]

Not integrable

Time = 3.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a + b\text{arctanh}\left(cx^3\right)\right)^3 \, dx = \int (d\,x)^m \left(a + b\,\text{atanh}\left(c\,x^3\right)\right)^3 dx$$

input
```
int((d*x)^m*(a + b*atanh(c*x^3))^3,x)
```

output
```
int((d*x)^m*(a + b*atanh(c*x^3))^3, x)
```

3.131 $\int (dx)^m \left(a + b\text{arctanh}(cx^3)\right)^2 dx$

3.131.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a + b\text{arctanh}(cx^3)\right)^2 dx = \text{Int}\left((dx)^m \left(a + b\text{arctanh}(cx^3)\right)^2, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

3.131.2 Mathematica [N/A]

Not integrable

Time = 0.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a + b\text{arctanh}(cx^3)\right)^2 dx = \int (dx)^m \left(a + b\text{arctanh}(cx^3)\right)^2 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3])^2, x]
```

3.131.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{arctanh}(cx^3)\right)^2 dx$$
$$\Big\downarrow \text{ 6468}$$
$$\int (dx)^m \left(a+b\text{arctanh}(cx^3)\right)^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
$Aborted
```

3.131.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.131.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(c\,x^3\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x^3))^2,x)
```

3.131.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(cx^3\right)+a\right)^2 (dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2)*(d*x)^m, x)
```

3.131.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x**3))**2,x)
```

output
```
Timed out
```

3.131.7 Maxima [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 239, normalized size of antiderivative = 13.28

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(cx^3\right)+a\right)^2 (dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*d^m*x*x^m*log(-c*x^3 + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1)
) - integrate(-1/4*((b^2*c*d^m*(m + 1)*x^3 - b^2*d^m*(m + 1))*x^m*log(c*x^
3 + 1)^2 + 4*(a*b*c*d^m*(m + 1)*x^3 - a*b*d^m*(m + 1))*x^m*log(c*x^3 + 1)
- 2*((b^2*c*d^m*(m + 1)*x^3 - b^2*d^m*(m + 1))*x^m*log(c*x^3 + 1) - (2*a*b
*d^m*(m + 1) - (2*a*b*c*d^m*(m + 1) + 3*b^2*c*d^m)*x^3)*x^m)*log(-c*x^3 +
1))/(c*(m + 1)*x^3 - m - 1), x)
```

3.131.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx=\int \left(b\,\text{artanh}\left(cx^3\right)+a\right)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)^2*(d*x)^m, x)
```

3.131.9 Mupad [N/A]

Not integrable

Time = 3.12 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right)^2\,dx=\int (d\,x)^m\,\left(a+b\,\text{atanh}\left(c\,x^3\right)\right)^2\,dx$$

input
```
int((d*x)^m*(a + b*atanh(c*x^3))^2,x)
```

output
```
int((d*x)^m*(a + b*atanh(c*x^3))^2, x)
```

3.132 $\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right) dx$

3.132.1 Optimal result

Integrand size = 16, antiderivative size = 74

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right) dx = \frac{(dx)^{1+m}\left(a+b\text{arctanh}\left(cx^3\right)\right)}{d(1+m)} - \frac{3bc(dx)^{4+m}\text{Hypergeometric2F1}\left(1,\frac{4+m}{6},\frac{10+m}{6},c^2x^6\right)}{d^4(1+m)(4+m)}$$

output
```
(d*x)^(1+m)*(a+b*arctanh(c*x^3))/d/(1+m)-3*b*c*(d*x)^(4+m)*hypergeom([1, 2
/3+1/6*m],[5/3+1/6*m],c^2*x^6)/d^4/(1+m)/(4+m)
```

3.132.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right) dx = -\frac{x(dx)^m\left(-\left((4+m)\left(a+b\text{arctanh}\left(cx^3\right)\right)\right)+3bcx^3\text{Hypergeometric2F1}\left(1,\frac{4+m}{6},\frac{10+m}{6},c^2x^6\right)\right)}{(1+m)(4+m)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^3]),x]
```

output
```
-((x*(d*x)^m*(-((4 + m)*(a + b*ArcTanh[c*x^3])) + 3*b*c*x^3*Hypergeometric
2F1[1, (4 + m)/6, (10 + m)/6, c^2*x^6]))/((1 + m)*(4 + m)))
```

3.132.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6464, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\operatorname{arctanh}\left(cx^3\right)\right) \, dx$$

$$\Big\downarrow \text{ 6464}$$

$$\frac{(dx)^{m+1} \left(a+b\operatorname{arctanh}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc\int \frac{(dx)^{m+3}}{1-c^2x^6}dx}{d^3(m+1)}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(dx)^{m+1} \left(a+b\operatorname{arctanh}\left(cx^3\right)\right)}{d(m+1)}-\frac{3bc(dx)^{m+4} \text{Hypergeometric2F1}\left(1,\frac{m+4}{6},\frac{m+10}{6},c^2x^6\right)}{d^4(m+1)(m+4)}$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x^3]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcTanh[c*x^3]))/(d*(1 + m)) - (3*b*c*(d*x)^(4 + m)*
Hypergeometric2F1[1, (4 + m)/6, (10 + m)/6, c^2*x^6])/(d^4*(1 + m)*(4 + m)
)
```

3.132.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.132.4 Maple [F]

$$\int (dx)^m \left(a+b\,\operatorname{arctanh}\left(c\,x^3\right)\right) dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x^3)),x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x^3)),x)
```

3.132.5 Fricas [F]

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx = \int \left(b\,\text{artanh}\left(cx^3\right)+a\right)(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x^3) + a)*(d*x)^m, x)
```

3.132.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x**3)),x)
```

output
```
Timed out
```

3.132.7 Maxima [F]

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right) \, dx = \int \left(b\,\text{artanh}\left(cx^3\right)+a\right)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

output
```
1/2*(6*c*d^m*integrate(x^3*x^m/(c^2*(m + 1)*x^6 - m - 1), x) + (d^m*x*x^m*
log(c*x^3 + 1) - d^m*x*x^m*log(-c*x^3 + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(
d*(m + 1))
```

3.132.8 Giac [F]

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right) \, dx = \int \left(b\,\text{artanh}\left(cx^3\right)+a\right)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^3) + a)*(d*x)^m, x)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\text{arctanh}\left(cx^3\right)\right) \, dx = \int (d\,x)^m \left(a+b\,\text{atanh}\left(c\,x^3\right)\right) \, dx$$

input
```
int((d*x)^m*(a + b*atanh(c*x^3)),x)
```

output
```
int((d*x)^m*(a + b*atanh(c*x^3)), x)
```

3.133 $\int \frac{(dx)^m}{a+b\textbf{arctanh}(cx^3)}\,dx$

3.133.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{a+b\operatorname{arctanh}(cx^3)}\,dx=\operatorname{Int}\left(\frac{(dx)^m}{a+b\operatorname{arctanh}(cx^3)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctanh(c*x^3)),x)
```

3.133.2 Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\operatorname{arctanh}(cx^3)}\,dx=\int \frac{(dx)^m}{a+b\operatorname{arctanh}(cx^3)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3]), x]
```

3.133.3 Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^3)} \, dx$$
$$\Big\downarrow \; 6468$$
$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^3)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTanh[c*x^3]),x]
```

output
```
$Aborted
```

3.133.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.133.4 Maple [N/A] (verified)

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\,\text{arctanh}\left(c\,x^3\right)}dx$$

input
```
int((d*x)^m/(a+b*arctanh(c*x^3)),x)
```

output
```
int((d*x)^m/(a+b*arctanh(c*x^3)),x)
```

3.133.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^3)}\,dx=\int \frac{(dx)^m}{b\,\text{artanh}(cx^3)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^3)}\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atanh(c*x**3)),x)
```

output
```
Timed out
```

3.133.7 Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^3)}\,dx=\int \frac{(dx)^m}{b\,\text{artanh}(cx^3)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

3.133.8 Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\text{arctanh}(cx^3)}\,dx = \int \frac{(dx)^m}{b\,\text{artanh}(cx^3) + a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^3)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x^3) + a), x)
```

3.133.9 Mupad [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\text{arctanh}(cx^3)}\,dx = \int \frac{(d\,x)^m}{a + b\,\text{atanh}(c\,x^3)}\,dx$$

input
```
int((d*x)^m/(a + b*atanh(c*x^3)),x)
```

output
```
int((d*x)^m/(a + b*atanh(c*x^3)), x)
```

3.134 $\int \frac{(dx)^m}{\left(a+b\,\mathrm{arctanh}(cx^3)\right)^2}\,dx$

3.134.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{(a+b\operatorname{arctanh}(cx^3))^2}\,dx=\operatorname{Int}\left(\frac{(dx)^m}{(a+b\operatorname{arctanh}(cx^3))^2},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

3.134.2 Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\operatorname{arctanh}(cx^3))^2}\,dx=\int \frac{(dx)^m}{(a+b\operatorname{arctanh}(cx^3))^2}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^3])^2, x]
```

3.134.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a + b\text{arctanh}(cx^3))^2} \, dx$$

$$\Big\downarrow \; 6468$$

$$\int \frac{(dx)^m}{(a + b\text{arctanh}(cx^3))^2} dx$$

input
```
Int[(d*x)^m/(a + b*ArcTanh[c*x^3])^2,x]
```

output
```
$Aborted
```

3.134.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.134.4 Maple [N/A] (verified)

Not integrable

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a + b \operatorname{arctanh}(c x^3))^2} dx$$

input
```
int((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

output
```
int((d*x)^m/(a+b*arctanh(c*x^3))^2,x)
```

3.134.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^3))^2}\,dx=\int \frac{(dx)^m}{(b\,\text{artanh}(cx^3)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arctanh(c*x^3)^2 + 2*a*b*arctanh(c*x^3) + a^2), x)
```

3.134.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^3))^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atanh(c*x**3))**2,x)
```

output
```
Timed out
```

3.134.7 Maxima [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 141, normalized size of antiderivative = 7.83

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^3))^2}\,dx=\int \frac{(dx)^m}{(b\,\text{artanh}(cx^3)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="maxima")
```

output
```
2/3*(c^2*d^m*x^6 - d^m)*x^m/(b^2*c*x^2*log(c*x^3 + 1) - b^2*c*x^2*log(-c*x
^3 + 1) + 2*a*b*c*x^2) + integrate(-2/3*(c^2*d^m*(m + 4)*x^6 - d^m*(m - 2)
)*x^m/(b^2*c*x^3*log(c*x^3 + 1) - b^2*c*x^3*log(-c*x^3 + 1) + 2*a*b*c*x^3)
, x)
```

3.134.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\text{arctanh}\,(cx^3))^2}\,dx = \int \frac{(dx)^m}{(b\,\text{artanh}\,(cx^3)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^3))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x^3) + a)^2, x)
```

3.134.9 Mupad [N/A]

Not integrable

Time = 3.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\text{arctanh}\,(cx^3))^2}\,dx = \int \frac{(d\,x)^m}{(a+b\,\text{atanh}\,(c\,x^3))^2}\,dx$$

input
```
int((d*x)^m/(a + b*atanh(c*x^3))^2,x)
```

output
```
int((d*x)^m/(a + b*atanh(c*x^3))^2, x)
```

3.135 $\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx$

3.135.1 Optimal result

Integrand size = 14, antiderivative size = 50

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)-\frac{1}{4}bc^4\text{arctanh}\left(\frac{x}{c}\right)$$

output
```
1/4*b*c^3*x+1/12*b*c*x^3+1/4*x^4*(a+b*arctanh(c/x))-1/4*b*c^4*arctanh(x/c)
```

3.135.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.34

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{4}bc^3x+\frac{1}{12}bcx^3+\frac{ax^4}{4}+\frac{1}{4}bx^4\text{arctanh}\left(\frac{c}{x}\right)+\frac{1}{8}bc^4\log(-c+x)-\frac{1}{8}bc^4\log(c+x)$$

input
```
Integrate[x^3*(a + b*ArcTanh[c/x]),x]
```

output
```
(b*c^3*x)/4 + (b*c*x^3)/12 + (a*x^4)/4 + (b*x^4*ArcTanh[c/x])/4 + (b*c^4*L
og[-c + x])/8 - (b*c^4*Log[c + x])/8
```

3.135.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 795, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{4}bc\int \frac{x^2}{1-\frac{c^2}{x^2}}dx+\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 795}$$

$$\frac{1}{4}bc\int \frac{x^4}{x^2-c^2}dx+\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 254}$$

$$\frac{1}{4}bc\int \left(\frac{c^4}{x^2-c^2}+c^2+x^2\right)dx+\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{4}bc\left(c^3\left(-\text{arctanh}\left(\frac{x}{c}\right)\right)+c^2x+\frac{x^3}{3}\right)$$

input
```
Int[x^3*(a + b*ArcTanh[c/x]),x]
```

output
```
(x^4*(a + b*ArcTanh[c/x]))/4 + (b*c*(c^2*x + x^3/3 - c^3*ArcTanh[x/c]))/4
```

3.135.3.1 Defintions of rubi rules used

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.135.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{x^4 \operatorname{arctanh}\left(\frac{c}{x}\right) b}{4}-\frac{\operatorname{arctanh}\left(\frac{c}{x}\right) b\, c^4}{4}+\frac{a\, x^4}{4}+\frac{b c\, x^3}{12}+\frac{b\, c^3 x}{4}$
parts	$\frac{a\, x^4}{4}-b\, c^4\left(-\frac{x^4 \operatorname{arctanh}\left(\frac{c}{x}\right)}{4 c^4}+\frac{\ln\left(1+\frac{c}{x}\right)}{8}-\frac{x^3}{12 c^3}-\frac{x}{4 c}-\frac{\ln\left(\frac{c}{x}-1\right)}{8}\right)$
derivativedivides	$-c^4\left(-\frac{a\, x^4}{4 c^4}+b\left(-\frac{x^4 \operatorname{arctanh}\left(\frac{c}{x}\right)}{4 c^4}+\frac{\ln\left(1+\frac{c}{x}\right)}{8}-\frac{x^3}{12 c^3}-\frac{x}{4 c}-\frac{\ln\left(\frac{c}{x}-1\right)}{8}\right)\right)$
default	$-c^4\left(-\frac{a\, x^4}{4 c^4}+b\left(-\frac{x^4 \operatorname{arctanh}\left(\frac{c}{x}\right)}{4 c^4}+\frac{\ln\left(1+\frac{c}{x}\right)}{8}-\frac{x^3}{12 c^3}-\frac{x}{4 c}-\frac{\ln\left(\frac{c}{x}-1\right)}{8}\right)\right)$
risch	$\frac{b\, x^4 \ln(x+c)}{8}-\frac{b\, x^4 \ln(c-x)}{8}-\frac{i\pi b\, x^4 \operatorname{csgn}\left(\frac{i}{x}\right) \operatorname{csgn}\left(\frac{i(c-x)}{x}\right)^2}{16}-\frac{i\pi b\, x^4 \operatorname{csgn}\left(\frac{i(c-x)}{x}\right)^3}{16}+\frac{i\pi b\, x^4 \operatorname{csgn}\left(\frac{i}{x}\right) \operatorname{csgn}(i(c}{16}$

input
```
int(x^3*(a+b*arctanh(c/x)),x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arctanh(c/x)*b-1/4*arctanh(c/x)*b*c^4+1/4*a*x^4+1/12*b*c*x^3+1/4*b
*c^3*x
```

3.135.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96

$$\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{4}\,bc^3x+\frac{1}{12}\,bcx^3+\frac{1}{4}\,ax^4-\frac{1}{8}\,(bc^4-bx^4)\log\left(-\frac{c+x}{c-x}\right)$$

input
```
integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

output
```
1/4*b*c^3*x + 1/12*b*c*x^3 + 1/4*a*x^4 - 1/8*(b*c^4 - b*x^4)*log(-(c + x)/
(c - x))
```

3.135.6 Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^4}{4}-\frac{bc^4\,\text{atanh}\left(\frac{c}{x}\right)}{4}+\frac{bc^3x}{4}+\frac{bcx^3}{12}+\frac{bx^4\,\text{atanh}\left(\frac{c}{x}\right)}{4}$$

input
```
integrate(x**3*(a+b*atanh(c/x)),x)
```

output
```
a*x**4/4 - b*c**4*atanh(c/x)/4 + b*c**3*x/4 + b*c*x**3/12 + b*x**4*atanh(c
/x)/4
```

3.135.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.14

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx$$
$$=\frac{1}{4}\,ax^4+\frac{1}{24}\left(6\,x^4\,\text{artanh}\left(\frac{c}{x}\right)-\left(3\,c^3\log\left(c+x\right)-3\,c^3\log\left(-c+x\right)-6\,c^2x-2\,x^3\right)c\right)b$$

input
```
integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/24*(6*x^4*arctanh(c/x) - (3*c^3*log(c + x) - 3*c^3*log(-c +
x) - 6*c^2*x - 2*x^3)*c)*b
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(42) = 84$.

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 5.24

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx = -\frac{\frac{3\left(\frac{b(c+x)^3c^5}{(c-x)^3}+\frac{b(c+x)c^5}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^4}{(c-x)^4}+\frac{4(c+x)^3}{(c-x)^3}+\frac{6(c+x)^2}{(c-x)^2}+\frac{4(c+x)}{c-x}+1}+\frac{2bc^5+\frac{6a(c+x)^3c^5}{(c-x)^3}+\frac{3b(c+x)^3c^5}{(c-x)^3}+\frac{6b(c+x)^2c^5}{(c-x)^2}+\frac{6a(c+x)c^5}{c-x}+\frac{5b(c+x)c^5}{c-x}}{\frac{(c+x)^4}{(c-x)^4}+\frac{4(c+x)^3}{(c-x)^3}+\frac{6(c+x)^2}{(c-x)^2}+\frac{4(c+x)}{c-x}+1}}{3c}$$

input
```
integrate(x^3*(a+b*arctanh(c/x)),x, algorithm="giac")
```

output
```
-1/3*(3*(b*(c + x)^3*c^5/(c - x)^3 + b*(c + x)*c^5/(c - x))*log(-(c + x)/(
c - x))/((c + x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3 + 6*(c + x)^2/(c - x)
^2 + 4*(c + x)/(c - x) + 1) + (2*b*c^5 + 6*a*(c + x)^3*c^5/(c - x)^3 + 3*b
*(c + x)^3*c^5/(c - x)^3 + 6*b*(c + x)^2*c^5/(c - x)^2 + 6*a*(c + x)*c^5/(
c - x) + 5*b*(c + x)*c^5/(c - x))/((c + x)^4/(c - x)^4 + 4*(c + x)^3/(c -
x)^3 + 6*(c + x)^2/(c - x)^2 + 4*(c + x)/(c - x) + 1))/c
```

3.135.9 Mupad [B] (verification not implemented)

Time = 3.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.90

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx = \frac{a\,x^4}{4}-\frac{b\,c^4\,\text{atanh}\left(\frac{c}{x}\right)}{4}+\frac{b\,x^4\,\text{atanh}\left(\frac{c}{x}\right)}{4}+\frac{b\,c\,x^3}{12}+\frac{b\,c^3\,x}{4}$$

input
```
int(x^3*(a + b*atanh(c/x)),x)
```

output
```
(a*x^4)/4 - (b*c^4*atanh(c/x))/4 + (b*x^4*atanh(c/x))/4 + (b*c*x^3)/12 + (
b*c^3*x)/4
```

3.136 $\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx$

3.136.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{6}bcx^2+\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc^3\log\left(c^2-x^2\right)$$

output
```
1/6*b*c*x^2+1/3*x^3*(a+b*arctanh(c/x))+1/6*b*c^3*ln(c^2-x^2)
```

3.136.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{6}bcx^2+\frac{ax^3}{3}+\frac{1}{3}bx^3\text{arctanh}\left(\frac{c}{x}\right)+\frac{1}{6}bc^3\log\left(-c^2+x^2\right)$$

input
```
Integrate[x^2*(a + b*ArcTanh[c/x]),x]
```

output
```
(b*c*x^2)/6 + (a*x^3)/3 + (b*x^3*ArcTanh[c/x])/3 + (b*c^3*Log[-c^2 + x^2])
/6
```

3.136.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6452, 795, 243, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}bc\int \frac{x}{1-\frac{c^2}{x^2}}dx+\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{795}$$

$$\frac{1}{3}bc\int \frac{x^3}{x^2-c^2}dx+\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{6}bc\int -\frac{x^2}{c^2-x^2}dx^2+\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc\int \frac{x^2}{c^2-x^2}dx^2$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)-\frac{1}{6}bc\int \left(\frac{c^2}{c^2-x^2}-1\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc(c^2\log\left(c^2-x^2\right)+x^2)$$

input
```
Int[x^2*(a + b*ArcTanh[c/x]),x]
```

output
```
(x^3*(a + b*ArcTanh[c/x]))/3 + (b*c*(x^2 + c^2*Log[c^2 - x^2]))/6
```

3.136.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.136.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

method	result
parallelrisch	$\frac{\ln(x-c)b\,c^3}{3}+\frac{b\,x^3\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{3}+\frac{\mathrm{arctanh}\left(\frac{c}{x}\right)b\,c^3}{3}+\frac{a\,x^3}{3}+\frac{bc\,x^2}{6}+\frac{b\,c^3}{6}$
parts	$\frac{a\,x^3}{3}-b\,c^3\left(-\frac{x^3\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{3c^3}-\frac{\ln\left(\frac{c}{x}-1\right)}{6}-\frac{\ln\left(1+\frac{c}{x}\right)}{6}-\frac{x^2}{6c^2}+\frac{\ln\left(\frac{c}{x}\right)}{3}\right)$
derivativedivides	$-c^3\left(-\frac{a\,x^3}{3c^3}+b\left(-\frac{x^3\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{3c^3}-\frac{\ln\left(\frac{c}{x}-1\right)}{6}-\frac{\ln\left(1+\frac{c}{x}\right)}{6}-\frac{x^2}{6c^2}+\frac{\ln\left(\frac{c}{x}\right)}{3}\right)\right)$
default	$-c^3\left(-\frac{a\,x^3}{3c^3}+b\left(-\frac{x^3\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{3c^3}-\frac{\ln\left(\frac{c}{x}-1\right)}{6}-\frac{\ln\left(1+\frac{c}{x}\right)}{6}-\frac{x^2}{6c^2}+\frac{\ln\left(\frac{c}{x}\right)}{3}\right)\right)$
risch	$\frac{b\,x^3\ln(x+c)}{6}-\frac{b\,x^3\ln(c-x)}{6}+\frac{i\pi b\,x^3\,\mathrm{csgn}(i(x+c))\,\mathrm{csgn}\left(\frac{i(x+c)}{x}\right)^2}{12}-\frac{i\pi b\,x^3\,\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}(i(x+c))\,\mathrm{csgn}\left(\frac{i(x+c)}{x}\right)}{12}-$

input
```
int(x^2*(a+b*arctanh(c/x)),x,method=_RETURNVERBOSE)
```

output
```
1/3*ln(x-c)*b*c^3+1/3*b*x^3*arctanh(c/x)+1/3*arctanh(c/x)*b*c^3+1/3*a*x^3+
1/6*b*c*x^2+1/6*b*c^3
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x^2\left(a+b\mathrm{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{6}\,bc^3\log\left(-c^2+x^2\right)+\frac{1}{6}\,bx^3\log\left(-\frac{c+x}{c-x}\right)+\frac{1}{6}\,bcx^2+\frac{1}{3}\,ax^3$$

input
```
integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

output
```
1/6*b*c^3*log(-c^2 + x^2) + 1/6*b*x^3*log(-(c + x)/(c - x)) + 1/6*b*c*x^2
+ 1/3*a*x^3
```

3.136.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int x^2\left(a+b\mathrm{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^3}{3}+\frac{bc^3\log\left(-c+x\right)}{3}+\frac{bc^3\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3}+\frac{bcx^2}{6}+\frac{bx^3\,\mathrm{atanh}\left(\frac{c}{x}\right)}{3}$$

input
```
integrate(x**2*(a+b*atanh(c/x)),x)
```

output
```
a*x**3/3 + b*c**3*log(-c + x)/3 + b*c**3*atanh(c/x)/3 + b*c*x**2/6 + b*x**
3*atanh(c/x)/3
```

3.136.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{3}\,ax^3+\frac{1}{6}\left(2\,x^3\,\text{artanh}\left(\frac{c}{x}\right)+\left(c^2\log\left(-c^2+x^2\right)+x^2\right)c\right)b$$

input
```
integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*b
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(39) = 78.

Time = 0.26 (sec) , antiderivative size = 227, normalized size of antiderivative = 5.04

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=$$
$$-\frac{bc^4\log\left(-\frac{c+x}{c-x}-1\right)-bc^4\log\left(-\frac{c+x}{c-x}\right)+\frac{\left(bc^4+\frac{3\,b(c+x)^2c^4}{(c-x)^2}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^3}{(c-x)^3}+\frac{3\,(c+x)^2}{(c-x)^2}+\frac{3\,(c+x)}{c-x}+1}+\frac{2\left(ac^4+\frac{3\,a(c+x)^2c^4}{(c-x)^2}+\frac{b(c+x)^2c^4}{(c-x)^2}+\frac{b(c+x)c^4}{c-x}\right)}{\frac{(c+x)^3}{(c-x)^3}+\frac{3\,(c+x)^2}{(c-x)^2}+\frac{3\,(c+x)}{c-x}+1}}{3\,c}$$

input
```
integrate(x^2*(a+b*arctanh(c/x)),x, algorithm="giac")
```

output
```
-1/3*(b*c^4*log(-(c + x)/(c - x) - 1) - b*c^4*log(-(c + x)/(c - x)) + (b*c
^4 + 3*b*(c + x)^2*c^4/(c - x)^2)*log(-(c + x)/(c - x))/((c + x)^3/(c - x)
^3 + 3*(c + x)^2/(c - x)^2 + 3*(c + x)/(c - x) + 1) + 2*(a*c^4 + 3*a*(c +
x)^2*c^4/(c - x)^2 + b*(c + x)^2*c^4/(c - x)^2 + b*(c + x)*c^4/(c - x))/((
c + x)^3/(c - x)^3 + 3*(c + x)^2/(c - x)^2 + 3*(c + x)/(c - x) + 1))/c
```

3.136.9 Mupad [B] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{a\,x^3}{3}+\frac{b\,c^3\,\ln\left(x^2-c^2\right)}{6}+\frac{b\,x^3\,\operatorname{atanh}\left(\frac{c}{x}\right)}{3}+\frac{b\,c\,x^2}{6}$$

input
```
int(x^2*(a + b*atanh(c/x)),x)
```

output
```
(a*x^3)/3 + (b*c^3*log(x^2 - c^2))/6 + (b*x^3*atanh(c/x))/3 + (b*c*x^2)/6
```

3.137 $\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx$

3.137.1 Optimal result

Integrand size = 12, antiderivative size = 39

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{bcx}{2}+\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)-\frac{1}{2}bc^2\text{arctanh}\left(\frac{x}{c}\right)$$

output
```
1/2*b*c*x+1/2*x^2*(a+b*arctanh(c/x))-1/2*b*c^2*arctanh(x/c)
```

3.137.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{bcx}{2}+\frac{ax^2}{2}+\frac{1}{2}bx^2\text{arctanh}\left(\frac{c}{x}\right)+\frac{1}{4}bc^2\log(-c+x)-\frac{1}{4}bc^2\log(c+x)$$

input
```
Integrate[x*(a + b*ArcTanh[c/x]),x]
```

output
```
(b*c*x)/2 + (a*x^2)/2 + (b*x^2*ArcTanh[c/x])/2 + (b*c^2*Log[-c + x])/4 - (
b*c^2*Log[c + x])/4
```

3.137.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {6452, 772, 262, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}bc\int \frac{1}{1-\frac{c^2}{x^2}}dx+\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{772}$$

$$\frac{1}{2}bc\int \frac{x^2}{x^2-c^2}dx+\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{2}bc\left(c^2\int \frac{1}{x^2-c^2}dx+x\right)+\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(x-c\text{arctanh}\left(\frac{x}{c}\right)\right)$$

input
```
Int[x*(a + b*ArcTanh[c/x]),x]
```

output
```
(x^2*(a + b*ArcTanh[c/x]))/2 + (b*c*(x - c*ArcTanh[x/c]))/2
```

3.137.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.137.4 Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.10

method	result
parallelrisch	$\frac{\operatorname{arctanh}\left(\frac{c}{x}\right) b\,x^{2}}{2}-\frac{\operatorname{arctanh}\left(\frac{c}{x}\right) b\,c^{2}}{2}+\frac{a\,x^{2}}{2}+\frac{b c x}{2}+\frac{a\,c^{2}}{2}$
parts	$\frac{a\,x^{2}}{2}-b\,c^{2}\left(-\frac{x^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)}{2 c^{2}}-\frac{x}{2 c}+\frac{\ln\left(1+\frac{c}{x}\right)}{4}-\frac{\ln\left(\frac{c}{x}-1\right)}{4}\right)$
derivativedivides	$-c^{2}\left(-\frac{a\,x^{2}}{2 c^{2}}+b\left(-\frac{x^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)}{2 c^{2}}-\frac{x}{2 c}+\frac{\ln\left(1+\frac{c}{x}\right)}{4}-\frac{\ln\left(\frac{c}{x}-1\right)}{4}\right)\right)$
default	$-c^{2}\left(-\frac{a\,x^{2}}{2 c^{2}}+b\left(-\frac{x^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)}{2 c^{2}}-\frac{x}{2 c}+\frac{\ln\left(1+\frac{c}{x}\right)}{4}-\frac{\ln\left(\frac{c}{x}-1\right)}{4}\right)\right)$
risch	$\frac{b\,x^{2} \ln\left(x+c\right)}{4}-\frac{b\,x^{2} \ln\left(c-x\right)}{4}+\frac{i\pi b\,x^{2} \operatorname{csgn}\left(\frac{i\left(c-x\right)}{x}\right)^{2}}{4}-\frac{i\pi b\,x^{2} \operatorname{csgn}\left(\frac{i\left(c-x\right)}{x}\right)^{3}}{8}-\frac{i\pi b\,x^{2} \operatorname{csgn}\left(\frac{i\left(x+c\right)}{x}\right)^{3}}{8}-\frac{i\pi b\,x^{2}}{4}$

input
```
int(x*(a+b*arctanh(c/x)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh(c/x)*b*x^2-1/2*arctanh(c/x)*b*c^2+1/2*a*x^2+1/2*b*c*x+1/2*a*c^
2
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}\,bcx+\frac{1}{2}\,ax^2-\frac{1}{4}\,\left(bc^2-bx^2\right)\log\left(-\frac{c+x}{c-x}\right)$$

input
```
integrate(x*(a+b*arctanh(c/x)),x, algorithm="fricas")
```

output
```
1/2*b*c*x + 1/2*a*x^2 - 1/4*(b*c^2 - b*x^2)*log(-(c + x)/(c - x))
```

3.137.6 Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{ax^2}{2}-\frac{bc^2\operatorname{atanh}\left(\frac{c}{x}\right)}{2}+\frac{bcx}{2}+\frac{bx^2\operatorname{atanh}\left(\frac{c}{x}\right)}{2}$$

input
```
integrate(x*(a+b*atanh(c/x)),x)
```

output
```
a*x**2/2 - b*c**2*atanh(c/x)/2 + b*c*x/2 + b*x**2*atanh(c/x)/2
```

3.137.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)\,dx$$
$$=\frac{1}{2}\,ax^2+\frac{1}{4}\,\left(2\,x^2\operatorname{artanh}\left(\frac{c}{x}\right)-\left(c\log\left(c+x\right)-c\log\left(-c+x\right)-2\,x\right)c\right)b$$

input
```
integrate(x*(a+b*arctanh(c/x)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/4*(2*x^2*arctanh(c/x) - (c*log(c + x) - c*log(-c + x) - 2*x)
*c)*b
```

3.137.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(33) = 66.

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 3.33

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=-\frac{\frac{b(c+x)c^3\log\left(-\frac{c+x}{c-x}\right)}{(c-x)\left(\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1\right)}+\frac{bc^3+\frac{2\,a(c+x)c^3}{c-x}+\frac{b(c+x)c^3}{c-x}}{\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1}}{c}$$

input
```
integrate(x*(a+b*arctanh(c/x)),x, algorithm="giac")
```

output
```
-(b*(c + x)*c^3*log(-(c + x)/(c - x))/((c - x)*((c + x)^2/(c - x)^2 + 2*(c
 + x)/(c - x) + 1)) + (b*c^3 + 2*a*(c + x)*c^3/(c - x) + b*(c + x)*c^3/(c
- x))/((c + x)^2/(c - x)^2 + 2*(c + x)/(c - x) + 1))/c
```

3.137.9 Mupad [B] (verification not implemented)

Time = 3.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{a\,x^2}{2}-\frac{b\,c^2\,\text{atanh}\left(\frac{c}{x}\right)}{2}+\frac{b\,x^2\,\text{atanh}\left(\frac{c}{x}\right)}{2}+\frac{b\,c\,x}{2}$$

input
```
int(x*(a + b*atanh(c/x)),x)
```

output
```
(a*x^2)/2 - (b*c^2*atanh(c/x))/2 + (b*x^2*atanh(c/x))/2 + (b*c*x)/2
```

3.138 $\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx$

3.138.1 Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=ax+bx\text{arctanh}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2-x^2\right)$$

output
```
a*x+b*x*arctanh(c/x)+1/2*b*c*ln(c^2-x^2)
```

3.138.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=ax+bx\text{arctanh}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2-x^2\right)$$

input
```
Integrate[a + b*ArcTanh[c/x],x]
```

output
```
a*x + b*x*ArcTanh[c/x] + (b*c*Log[c^2 - x^2])/2
```

3.138.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+bx\text{arctanh}\left(\frac{c}{x}\right)+\frac{1}{2}bc\log\left(c^2-x^2\right)$$

input
```
Int[a + b*ArcTanh[c/x],x]
```

output
```
a*x + b*x*ArcTanh[c/x] + (b*c*Log[c^2 - x^2])/2
```

3.138.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.138.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.10

method	result
parallelrisch	$b\left(\ln\left(x-c\right)c+x\ \text{arctanh}\left(\frac{c}{x}\right)+\text{arctanh}\left(\frac{c}{x}\right)c\right)+ax$
default	$ax-bc\left(-\frac{x\ \text{arctanh}\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c}{x}\right)}{2}+\ln\left(\frac{c}{x}\right)-\frac{\ln\left(\frac{c}{x}-1\right)}{2}\right)$
parts	$ax-bc\left(-\frac{x\ \text{arctanh}\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c}{x}\right)}{2}+\ln\left(\frac{c}{x}\right)-\frac{\ln\left(\frac{c}{x}-1\right)}{2}\right)$
derivativedivides	$-c\left(-\frac{ax}{c}+b\left(-\frac{x\ \text{arctanh}\left(\frac{c}{x}\right)}{c}-\frac{\ln\left(1+\frac{c}{x}\right)}{2}+\ln\left(\frac{c}{x}\right)-\frac{\ln\left(\frac{c}{x}-1\right)}{2}\right)\right)$
risch	$ax+\frac{bx\ln(x+c)}{2}-\frac{b\ln(c-x)x}{2}+\frac{ib\pi\,\text{csgn}\left(\frac{i(c-x)}{x}\right)^2x}{2}-\frac{ib\pi\ \text{csgn}\left(\frac{i}{x}\right)\text{csgn}(i(x+c))\,\text{csgn}\left(\frac{i(x+c)}{x}\right)x}{4}+\frac{ib\pi\ \text{csgn}(}{}$

input
```
int(a+b*arctanh(c/x),x,method=_RETURNVERBOSE)
```

output
```
b*(ln(x-c)*c+x*arctanh(c/x)+arctanh(c/x)*c)+a*x
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.21

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}\,bc\log\left(-c^2+x^2\right)+\frac{1}{2}\,bx\log\left(-\frac{c+x}{c-x}\right)+ax$$

input
```
integrate(a+b*arctanh(c/x),x, algorithm="fricas")
```

output
```
1/2*b*c*log(-c^2 + x^2) + 1/2*b*x*log(-(c + x)/(c - x)) + a*x
```

3.138.6 Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=ax+b\left(c\log\left(-c+x\right)+c\,\text{atanh}\left(\frac{c}{x}\right)+x\,\text{atanh}\left(\frac{c}{x}\right)\right)$$

input
```
integrate(a+b*atanh(c/x),x)
```

output
```
a*x + b*(c*log(-c + x) + c*atanh(c/x) + x*atanh(c/x))
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\,dx=\frac{1}{2}\left(2\,x\,\text{artanh}\left(\frac{c}{x}\right)+c\log\left(-c^2+x^2\right)\right)b+ax$$

input
```
integrate(a+b*arctanh(c/x),x, algorithm="maxima")
```

output
```
1/2*(2*x*arctanh(c/x) + c*log(-c^2 + x^2))*b + a*x
```

3.138.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(27) = 54$.

Time = 0.26 (sec) , antiderivative size = 150, normalized size of antiderivative = 5.17

$$\int \left(a + b\operatorname{arctanh}\left(\frac{c}{x}\right)\right) dx = ax + \frac{\left(c^2\left(\log\left(\frac{|-c-x|}{|c-x|}\right) - \log\left(\left|-\frac{c+x}{c-x} - 1\right|\right)\right) - \frac{c^2 \log\left(-\frac{\frac{c\left(\frac{c+x}{(c-x)c} + \frac{1}{c}\right)}{\frac{c+x}{c-x} - 1} + 1}{\frac{c\left(\frac{c+x}{(c-x)c} + \frac{1}{c}\right)}{\frac{c+x}{c-x} - 1} - 1}\right)}{\frac{c+x}{c-x} + 1}\right) b}{c}$$

input
```
integrate(a+b*arctanh(c/x),x, algorithm="giac")
```

output
```
a*x + (c^2*(log(abs(-c - x)/abs(c - x)) - log(abs(-(c + x)/(c - x) - 1)))
- c^2*log(-(c*((c + x)/((c - x)*c) + 1/c)/((c + x)/(c - x) - 1) + 1)/(c*((
c + x)/((c - x)*c) + 1/c)/((c + x)/(c - x) - 1) - 1))/((c + x)/(c - x) + 1
))*b/c
```

3.138.9 Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93

$$\int \left(a + b\operatorname{arctanh}\left(\frac{c}{x}\right)\right) dx = a\,x + b\,x\,\operatorname{atanh}\left(\frac{c}{x}\right) + \frac{b\,c\,\ln\left(x^2 - c^2\right)}{2}$$

input
```
int(a + b*atanh(c/x),x)
```

output
```
a*x + b*x*atanh(c/x) + (b*c*log(x^2 - c^2))/2
```

3.139 $\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x}\,dx$

3.139.1 Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x}\,dx = a\log(x)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{c}{x}\right)$$

output
```
a*ln(x)+1/2*b*polylog(2,-c/x)-1/2*b*polylog(2,c/x)
```

3.139.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x}\,dx = a\log(x)+\frac{1}{2}b\left(\text{PolyLog}\left(2,-\frac{c}{x}\right)-\text{PolyLog}\left(2,\frac{c}{x}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c/x])/x,x]
```

output
```
a*Log[x] + (b*(PolyLog[2, -(c/x)] - PolyLog[2, c/x]))/2
```

3.139.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6450, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x}dx$$

$$\Big\downarrow \text{6450}$$

$$-\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)d\frac{1}{x}$$

$$\Big\downarrow \text{6446}$$

$$-a\log\left(\frac{1}{x}\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{c}{x}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{c}{x}\right)$$

input
```
Int[(a + b*ArcTanh[c/x])/x,x]
```

output
```
-(a*Log[x^(-1)]) + (b*PolyLog[2, -(c/x)])/2 - (b*PolyLog[2, c/x])/2
```

3.139.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.139.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.68 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.90

method	result
parts	$a\ln\left(x\right)+b\left(-\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)+\frac{\operatorname{dilog}\left(1+\frac{c}{x}\right)}{2}+\frac{\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}+\frac{\operatorname{dilog}\left(\frac{c}{x}\right)}{2}\right)$
derivativedivides	$-a\ln\left(\frac{c}{x}\right)-b\left(\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)-\frac{\operatorname{dilog}\left(1+\frac{c}{x}\right)}{2}-\frac{\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{c}{x}\right)}{2}\right)$
default	$-a\ln\left(\frac{c}{x}\right)-b\left(\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)-\frac{\operatorname{dilog}\left(1+\frac{c}{x}\right)}{2}-\frac{\ln\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}-\frac{\operatorname{dilog}\left(\frac{c}{x}\right)}{2}\right)$
risch	$\frac{b\ln(x)\ln(x+c)}{2}-\frac{\left(-2ib\pi\operatorname{csgn}\left(\frac{i(c-x)}{x}\right)^{2}-ib\pi\operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}(i(c-x))\operatorname{csgn}\left(\frac{i(c-x)}{x}\right)+ib\pi\operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}(i(x+c))\operatorname{csgn}\left(\right.\right.}{}$

input
```
int((a+b*arctanh(c/x))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(-ln(c/x)*arctanh(c/x)+1/2*dilog(1+c/x)+1/2*ln(c/x)*ln(1+c/x)+1/
2*dilog(c/x))
```

3.139.5 Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}\left(\frac{c}{x}\right)}{x}\,dx=\int \frac{b\operatorname{artanh}\left(\frac{c}{x}\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x))/x,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c/x) + a)/x, x)
```

3.139.6 Sympy [F]

$$\int \frac{a + b\text{arctanh}\left(\frac{c}{x}\right)}{x} \, dx = \int \frac{a + b \,\text{atanh}\left(\frac{c}{x}\right)}{x} \, dx$$

input
```
integrate((a+b*atanh(c/x))/x,x)
```

output
```
Integral((a + b*atanh(c/x))/x, x)
```

3.139.7 Maxima [F]

$$\int \frac{a + b\text{arctanh}\left(\frac{c}{x}\right)}{x} \, dx = \int \frac{b \,\text{artanh}\left(\frac{c}{x}\right) + a}{x} \, dx$$

input
```
integrate((a+b*arctanh(c/x))/x,x, algorithm="maxima")
```

output
```
1/2*b*integrate((log(c/x + 1) - log(-c/x + 1))/x, x) + a*log(x)
```

3.139.8 Giac [F]

$$\int \frac{a + b\text{arctanh}\left(\frac{c}{x}\right)}{x} \, dx = \int \frac{b \,\text{artanh}\left(\frac{c}{x}\right) + a}{x} \, dx$$

input
```
integrate((a+b*arctanh(c/x))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)/x, x)
```

3.139.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x}\right)}{x}\,dx = \int \frac{a + b\operatorname{atanh}\left(\frac{c}{x}\right)}{x}\,dx$$

input
```
int((a + b*atanh(c/x))/x,x)
```

output
```
int((a + b*atanh(c/x))/x, x)
```

3.140 $\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^2} dx$

3.140.1 Optimal result

Integrand size = 14, antiderivative size = 35

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^2} dx = -\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x} - \frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

output
```
(-a-b*arctanh(c/x))/x-1/2*b*ln(1-c^2/x^2)/c
```

3.140.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^2} dx = -\frac{a}{x} - \frac{b\text{arctanh}\left(\frac{c}{x}\right)}{x} - \frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

input
```
Integrate[(a + b*ArcTanh[c/x])/x^2,x]
```

output
```
-(a/x) - (b*ArcTanh[c/x])/x - (b*Log[1 - c^2/x^2])/(2*c)
```

3.140.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6452, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^2}\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$-bc\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^3}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x}$$

$$\Big\downarrow \text{ 792}$$

$$-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x}-\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$

input
```
Int[(a + b*ArcTanh[c/x])/x^2,x]
```

output
```
-((a + b*ArcTanh[c/x])/x) - (b*Log[1 - c^2/x^2])/(2*c)
```

3.140.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.140.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result
parts	$-\frac{a}{x}-\frac{b\,\operatorname{arctanh}\left(\frac{c}{x}\right)}{x}-\frac{b\ln\left(1-\frac{c^{2}}{x^{2}}\right)}{2c}$
derivativedivides	$-\frac{\frac{ca}{x}+b\left(\frac{c\,\operatorname{arctanh}\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1-\frac{c^{2}}{x^{2}}\right)}{2}\right)}{c}$
default	$-\frac{\frac{ca}{x}+b\left(\frac{c\,\operatorname{arctanh}\left(\frac{c}{x}\right)}{x}+\frac{\ln\left(1-\frac{c^{2}}{x^{2}}\right)}{2}\right)}{c}$
parallelrisch	$\frac{b\ln(x)x-\ln(x-c)xb-bx\,\operatorname{arctanh}\left(\frac{c}{x}\right)-\operatorname{arctanh}\left(\frac{c}{x}\right)bc-ac}{cx}$
risch	$-\frac{b\ln(x+c)}{2x}+\frac{2i\pi bc+i\pi bc\,\operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}(i(x+c))\operatorname{csgn}\left(\frac{i(x+c)}{x}\right)-i\pi bc\,\operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}(i(c-x))\operatorname{csgn}\left(\frac{i(c-x)}{x}\right)-2i\pi bc\,\operatorname{csg}}{}$

input
```
int((a+b*arctanh(c/x))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-b/x*arctanh(c/x)-1/2*b*ln(1-c^2/x^2)/c
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^2}\,dx=-\frac{bx\log\left(-c^2+x^2\right)-2\,bx\log\left(x\right)+bc\log\left(-\frac{c+x}{c-x}\right)+2\,ac}{2\,cx}$$

input
```
integrate((a+b*arctanh(c/x))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(b*x*log(-c^2 + x^2) - 2*b*x*log(x) + b*c*log(-(c + x)/(c - x)) + 2*a
*c)/(c*x)
```

3.140.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.11

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x}\right)}{x^2}\,dx = \begin{cases} -\frac{a}{x} - \frac{b\operatorname{atanh}\left(\frac{c}{x}\right)}{x} + \frac{b\log\left(x\right)}{c} - \frac{b\log\left(-c+x\right)}{c} - \frac{b\operatorname{atanh}\left(\frac{c}{x}\right)}{c} & \text{for}\ c \neq 0 \\ -\frac{a}{x} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atanh(c/x))/x**2,x)
```

output
```
Piecewise((-a/x - b*atanh(c/x)/x + b*log(x)/c - b*log(-c + x)/c - b*atanh(
c/x)/c, Ne(c, 0)), (-a/x, True))
```

3.140.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x}\right)}{x^2}\,dx = -\frac{b\left(\frac{2\,c\operatorname{artanh}\left(\frac{c}{x}\right)}{x} + \log\left(-\frac{c^2}{x^2} + 1\right)\right)}{2\,c} - \frac{a}{x}$$

input
```
integrate((a+b*arctanh(c/x))/x^2,x, algorithm="maxima")
```

output
```
-1/2*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a/x
```

3.140.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(33) = 66.

Time = 0.28 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.49

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x}\right)}{x^2}\,dx = \frac{b\log\left(-\frac{c+x}{c-x} + 1\right) - b\log\left(-\frac{c+x}{c-x}\right) - \frac{b\log\left(-\frac{c+x}{c-x}\right)}{\frac{c+x}{c-x}-1} - \frac{2\,a}{\frac{c+x}{c-x}-1}}{c}$$

input
```
integrate((a+b*arctanh(c/x))/x^2,x, algorithm="giac")
```

output
```
(b*log(-(c + x)/(c - x) + 1) - b*log(-(c + x)/(c - x)) - b*log(-(c + x)/(c
 - x))/((c + x)/(c - x) - 1) - 2*a/((c + x)/(c - x) - 1))/c
```

3.140.9 Mupad [B] (verification not implemented)

Time = 3.46 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.23

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^2}\,dx = \frac{b\,x\,\ln\left(x\right)-\frac{b\,x\,\ln\left(x^2-c^2\right)}{2}}{c\,x}-\frac{a+b\,\text{atanh}\left(\frac{c}{x}\right)}{x}$$

input
```
int((a + b*atanh(c/x))/x^2,x)
```

output
```
(b*x*log(x) - (b*x*log(x^2 - c^2))/2)/(c*x) - (a + b*atanh(c/x))/x
```

3.141 $\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^3}\,dx$

3.141.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^3}\,dx=-\frac{b}{2cx}-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{2x^2}+\frac{b\text{arctanh}\left(\frac{x}{c}\right)}{2c^2}$$

output
```
-1/2*b/c/x+1/2*(-a-b*arctanh(c/x))/x^2+1/2*b*arctanh(x/c)/c^2
```

3.141.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{b}{2cx}-\frac{b\text{arctanh}\left(\frac{c}{x}\right)}{2x^2}-\frac{b\log(-c+x)}{4c^2}+\frac{b\log(c+x)}{4c^2}$$

input
```
Integrate[(a + b*ArcTanh[c/x])/x^3,x]
```

output
```
-1/2*a/x^2 - b/(2*c*x) - (b*ArcTanh[c/x])/(2*x^2) - (b*Log[-c + x])/(4*c^2
) + (b*Log[c + x])/(4*c^2)
```

3.141.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {6452, 795, 264, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^3}dx$$

$$\Big\downarrow \text{6452}$$

$$-\frac{1}{2}bc\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^4}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{795}$$

$$-\frac{1}{2}bc\int \frac{1}{x^2\left(x^2-c^2\right)}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{2}bc\left(\frac{\int \frac{1}{x^2-c^2}dx}{c^2}+\frac{1}{c^2x}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{2x^2}$$

$$\Big\downarrow \text{220}$$

$$-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{2x^2}-\frac{1}{2}bc\left(\frac{1}{c^2x}-\frac{\text{arctanh}\left(\frac{x}{c}\right)}{c^3}\right)$$

input
```
Int[(a + b*ArcTanh[c/x])/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c/x])/x^2 - (b*c*(1/(c^2*x) - ArcTanh[x/c]/c^3))/2
```

3.141.3.1 Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.141.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

method	result
parallelrisch	$-\frac{-\operatorname{arctanh}\left(\frac{c}{x}\right) b\,x^{2}+\operatorname{arctanh}\left(\frac{c}{x}\right) b\,c^{2}+b c x+a\,c^{2}}{2 x^{2} c^{2}}$
parts	$-\frac{a}{2 x^{2}}-\frac{b \left(\frac{c^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)}{2 x^{2}}+\frac{c}{2 x}+\frac{\ln\left(\frac{c}{x}-1\right)}{4}-\frac{\ln\left(1+\frac{c}{x}\right)}{4}\right)}{c^{2}}$
derivativedivides	$-\frac{\frac{a\,c^{2}}{2 x^{2}}+b \left(\frac{c^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)}{2 x^{2}}+\frac{c}{2 x}+\frac{\ln\left(\frac{c}{x}-1\right)}{4}-\frac{\ln\left(1+\frac{c}{x}\right)}{4}\right)}{c^{2}}$
default	$-\frac{\frac{a\,c^{2}}{2 x^{2}}+b \left(\frac{c^{2} \operatorname{arctanh}\left(\frac{c}{x}\right)}{2 x^{2}}+\frac{c}{2 x}+\frac{\ln\left(\frac{c}{x}-1\right)}{4}-\frac{\ln\left(1+\frac{c}{x}\right)}{4}\right)}{c^{2}}$
risch	$-\frac{b \ln\left(x+c\right)}{4 x^{2}}-\frac{i \pi b\,c^{2} \operatorname{csgn}\left(\frac{i}{x}\right) \operatorname{csgn}\left(\frac{i \left(x+c\right)}{x}\right)^{2}-i \pi b\,c^{2} \operatorname{csgn}\left(\frac{i \left(x+c\right)}{x}\right)^{3}+i \pi b\,c^{2} \operatorname{csgn}\left(\frac{i}{x}\right) \operatorname{csgn}\left(i \left(c-x\right)\right) \operatorname{csgn}\left(\frac{i \left(c-x\right)}{x}\right)+2 i \pi \cdots}{}$

input
```
int((a+b*arctanh(c/x))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-arctanh(c/x)*b*x^2+arctanh(c/x)*b*c^2+b*c*x+a*c^2)/x^2/c^2
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^3}\,dx = -\frac{2\,ac^2+2\,bcx+(bc^2-bx^2)\log\left(-\frac{c+x}{c-x}\right)}{4\,c^2x^2}$$

input
```
integrate((a+b*arctanh(c/x))/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*c^2 + 2*b*c*x + (b*c^2 - b*x^2)*log(-(c + x)/(c - x)))/(c^2*x^2)
```

3.141.6 Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^3}\,dx = \begin{cases} -\frac{a}{2x^2}-\frac{b\,\text{atanh}\left(\frac{c}{x}\right)}{2x^2}-\frac{b}{2cx}+\frac{b\,\text{atanh}\left(\frac{c}{x}\right)}{2c^2} & \text{for } c\neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atanh(c/x))/x**3,x)
```

output
```
Piecewise((-a/(2*x**2) - b*atanh(c/x)/(2*x**2) - b/(2*c*x) + b*atanh(c/x)/
(2*c**2), Ne(c, 0)), (-a/(2*x**2), True))
```

3.141.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^3}\,dx = \frac{1}{4}\left(c\left(\frac{\log(c+x)}{c^3}-\frac{\log(-c+x)}{c^3}-\frac{2}{c^2x}\right)-\frac{2\,\text{artanh}\left(\frac{c}{x}\right)}{x^2}\right)b-\frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctanh(c/x))/x^3,x, algorithm="maxima")
```

output
```
1/4*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2
)*b - 1/2*a/x^2
```

3.141.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. 2(37) = 74.

Time = 0.28 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.86

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^3}\,dx = -\frac{\frac{b(c+x)\log\left(-\frac{c+x}{c-x}\right)}{\left(\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c\right)(c-x)}-\frac{b-\frac{2\,a(c+x)}{c-x}-\frac{b(c+x)}{c-x}}{\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c}}{c}$$

input
```
integrate((a+b*arctanh(c/x))/x^3,x, algorithm="giac")
```

output
```
-(b*(c + x)*log(-(c + x)/(c - x))/(((c + x)^2*c/(c - x)^2 - 2*(c + x)*c/(c
 - x) + c)*(c - x)) - (b - 2*a*(c + x)/(c - x) - b*(c + x)/(c - x))/((c +
x)^2*c/(c - x)^2 - 2*(c + x)*c/(c - x) + c))/c
```

3.141.9 Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^3}\,dx = \frac{b\,c\,\text{atan}\left(\frac{x}{\sqrt{-c^2}}\right)}{2\,(-c^2)^{3/2}}-\frac{b}{2\,c\,x}-\frac{b\,\text{atanh}\left(\frac{c}{x}\right)}{2\,x^2}-\frac{a}{2\,x^2}$$

input
```
int((a + b*atanh(c/x))/x^3,x)
```

output
```
(b*c*atan(x/(-c^2)^(1/2)))/(2*(-c^2)^(3/2)) - b/(2*c*x) - (b*atanh(c/x))/(
2*x^2) - a/(2*x^2)
```

3.142 $\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^4}\,dx$

3.142.1 Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^4}\,dx=-\frac{b}{6cx^2}-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^2\right)}{6c^3}$$

output
```
-1/6*b/c/x^2+1/3*(-a-b*arctanh(c/x))/x^3+1/3*b*ln(x)/c^3-1/6*b*ln(c^2-x^2)
/c^3
```

3.142.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^4}\,dx=-\frac{a}{3x^3}-\frac{b}{6cx^2}-\frac{b\text{arctanh}\left(\frac{c}{x}\right)}{3x^3}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(-c^2+x^2\right)}{6c^3}$$

input
```
Integrate[(a + b*ArcTanh[c/x])/x^4,x]
```

output
```
-1/3*a/x^3 - b/(6*c*x^2) - (b*ArcTanh[c/x])/(3*x^3) + (b*Log[x])/(3*c^3) -
 (b*Log[-c^2 + x^2])/(6*c^3)
```

3.142.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6452, 795, 243, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^4}dx$$

$$\Big\downarrow \text{6452}$$

$$-\frac{1}{3}bc\int \frac{1}{\left(1-\frac{c^2}{x^2}\right)x^5}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{795}$$

$$-\frac{1}{3}bc\int \frac{1}{x^3\left(x^2-c^2\right)}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{243}$$

$$-\frac{1}{6}bc\int -\frac{1}{x^4\left(c^2-x^2\right)}dx^2-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}bc\int \frac{1}{x^4\left(c^2-x^2\right)}dx^2-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{54}$$

$$\frac{1}{6}bc\int \left(\frac{1}{c^4x^2}+\frac{1}{c^2x^4}+\frac{1}{c^4\left(c^2-x^2\right)}\right)dx^2-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{3x^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{3x^3}-\frac{1}{6}bc\left(-\frac{\log\left(x^2\right)}{c^4}+\frac{1}{c^2x^2}+\frac{\log\left(c^2-x^2\right)}{c^4}\right)$$

input
```
Int[(a + b*ArcTanh[c/x])/x^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c/x])/x^3 - (b*c*(1/(c^2*x^2) - Log[x^2]/c^4 + Log[c^2
 - x^2]/c^4))/6
```

3.142.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.142.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

method	result
parts	$-\frac{a}{3x^3}-\frac{b\left(\frac{c^3\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{3x^3}+\frac{c^2}{6x^2}+\frac{\ln\left(\frac{c}{x}-1\right)}{6}+\frac{\ln\left(1+\frac{c}{x}\right)}{6}\right)}{c^3}$
derivativedivides	$-\frac{\frac{a\,c^3}{3x^3}+b\left(\frac{c^3\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{3x^3}+\frac{c^2}{6x^2}+\frac{\ln\left(\frac{c}{x}-1\right)}{6}+\frac{\ln\left(1+\frac{c}{x}\right)}{6}\right)}{c^3}$
default	$-\frac{\frac{a\,c^3}{3x^3}+b\left(\frac{c^3\,\mathrm{arctanh}\left(\frac{c}{x}\right)}{3x^3}+\frac{c^2}{6x^2}+\frac{\ln\left(\frac{c}{x}-1\right)}{6}+\frac{\ln\left(1+\frac{c}{x}\right)}{6}\right)}{c^3}$
parallelrisch	$\frac{2b\ln(x)x^3-2\ln(x-c)x^3b-2b\,x^3\,\mathrm{arctanh}\left(\frac{c}{x}\right)-2\,\mathrm{arctanh}\left(\frac{c}{x}\right)b\,c^3-b\,c^2x-2a\,c^3}{6x^3c^3}$
risch	$-\frac{b\ln(x+c)}{6x^3}-\frac{i\pi b\,c^3\,\mathrm{csgn}\left(\frac{i}{x}\right)\mathrm{csgn}\left(\frac{i(x+c)}{x}\right)^2+i\pi b\,c^3\,\mathrm{csgn}(i(x+c))\,\mathrm{csgn}\left(\frac{i(x+c)}{x}\right)^2-i\pi b\,c^3\,\mathrm{csgn}(i(c-x))\,\mathrm{csgn}\left(\frac{i(c-x)}{x}\right)}{}$

input
```
int((a+b*arctanh(c/x))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3-b/c^3*(1/3*c^3/x^3*arctanh(c/x)+1/6*c^2/x^2+1/6*ln(c/x-1)+1/6*l
n(1+c/x))
```

3.142.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{a+b\mathrm{arctanh}\left(\frac{c}{x}\right)}{x^4}\,dx=-\frac{bx^3\log\left(-c^2+x^2\right)-2\,bx^3\log\left(x\right)+bc^3\log\left(-\frac{c+x}{c-x}\right)+2\,ac^3+bc^2x}{6\,c^3x^3}$$

input
```
integrate((a+b*arctanh(c/x))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(b*x^3*log(-c^2 + x^2) - 2*b*x^3*log(x) + b*c^3*log(-(c + x)/(c - x))
 + 2*a*c^3 + b*c^2*x)/(c^3*x^3)
```

3.142.6 Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^4}\,dx = \begin{cases} -\frac{a}{3x^3}-\frac{b\,\text{atanh}\left(\frac{c}{x}\right)}{3x^3}-\frac{b}{6cx^2}+\frac{b\log(x)}{3c^3}-\frac{b\log(-c+x)}{3c^3}-\frac{b\,\text{atanh}\left(\frac{c}{x}\right)}{3c^3} & \text{for } c\neq 0 \\ -\frac{a}{3x^3} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atanh(c/x))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3) - b*atanh(c/x)/(3*x**3) - b/(6*c*x**2) + b*log(x)/(
3*c**3) - b*log(-c + x)/(3*c**3) - b*atanh(c/x)/(3*c**3), Ne(c, 0)), (-a/(
3*x**3), True))
```

3.142.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{1}{6}\left(c\left(\frac{\log\left(-c^2+x^2\right)}{c^4}-\frac{\log\left(x^2\right)}{c^4}+\frac{1}{c^2x^2}\right)+\frac{2\,\text{artanh}\left(\frac{c}{x}\right)}{x^3}\right)b - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctanh(c/x))/x^4,x, algorithm="maxima")
```

output
```
-1/6*(c*(log(-c^2 + x^2)/c^4 - log(x^2)/c^4 + 1/(c^2*x^2)) + 2*arctanh(c/x
)/x^3)*b - 1/3*a/x^3
```

3.142.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(49) = 98$.

Time = 0.27 (sec) , antiderivative size = 234, normalized size of antiderivative = 4.11

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{\frac{\left(b+\frac{3\,b(c+x)^2}{(c-x)^2}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^3c^2}{(c-x)^3}-\frac{3\,(c+x)^2c^2}{(c-x)^2}+\frac{3\,(c+x)c^2}{c-x}-c^2}+\frac{2\left(a+\frac{3\,a(c+x)^2}{(c-x)^2}+\frac{b(c+x)^2}{(c-x)^2}-\frac{b(c+x)}{c-x}\right)}{\frac{(c+x)^3c^2}{(c-x)^3}-\frac{3\,(c+x)^2c^2}{(c-x)^2}+\frac{3\,(c+x)c^2}{c-x}-c^2}-\frac{b\log\left(-\frac{c+x}{c-x}+1\right)}{c^2}+\frac{b\log\left(-\frac{c+x}{c-x}\right)}{c^2}}{3\,c}$$

input
```
integrate((a+b*arctanh(c/x))/x^4,x, algorithm="giac")
```

output
```
-1/3*((b + 3*b*(c + x)^2/(c - x)^2)*log(-(c + x)/(c - x))/((c + x)^3*c^2/(
c - x)^3 - 3*(c + x)^2*c^2/(c - x)^2 + 3*(c + x)*c^2/(c - x) - c^2) + 2*(a
 + 3*a*(c + x)^2/(c - x)^2 + b*(c + x)^2/(c - x)^2 - b*(c + x)/(c - x))/((
c + x)^3*c^2/(c - x)^3 - 3*(c + x)^2*c^2/(c - x)^2 + 3*(c + x)*c^2/(c - x)
 - c^2) - b*log(-(c + x)/(c - x) + 1)/c^2 + b*log(-(c + x)/(c - x))/c^2)/c
```

3.142.9 Mupad [B] (verification not implemented)

Time = 3.40 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{a + b\text{arctanh}\left(\frac{c}{x}\right)}{x^4}\,dx = -\frac{\frac{a}{3} + \frac{b\,\text{atanh}\left(\frac{c}{x}\right)}{3}}{x^3} - \frac{\frac{b\,x^3\,\ln\left(x^2-c^2\right)}{6} - \frac{b\,x^3\,\ln(x)}{3} + \frac{b\,c^2\,x}{6}}{c^3\,x^3}$$

input
```
int((a + b*atanh(c/x))/x^4,x)
```

output
```
- (a/3 + (b*atanh(c/x))/3)/x^3 - ((b*x^3*log(x^2 - c^2))/6 - (b*x^3*log(x)
)/3 + (b*c^2*x)/6)/(c^3*x^3)
```

3.143 $\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx$

3.143.1 Optimal result

Integrand size = 16, antiderivative size = 123

$$\begin{aligned}\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx={}&\frac{1}{12}b^2c^2x^2+\frac{1}{2}bc^3x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\\&+\frac{1}{6}bcx^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\\&-\frac{1}{4}c^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\\&+\frac{1}{3}b^2c^4\log\left(1-\frac{c^2}{x^2}\right)+\frac{2}{3}b^2c^4\log(x)\end{aligned}$$

output
```
1/12*b^2*c^2*x^2+1/2*b*c^3*x*(a+b*arccoth(x/c))+1/6*b*c*x^3*(a+b*arccoth(x
/c))-1/4*c^4*(a+b*arccoth(x/c))^2+1/4*x^4*(a+b*arccoth(x/c))^2+1/3*b^2*c^4
*ln(1-c^2/x^2)+2/3*b^2*c^4*ln(x)
```

3.143.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.07

$$\begin{aligned}\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx={}&\frac{1}{12}\Bigg(6abc^3x+b^2c^2x^2+2abcx^3+3a^2x^4\\&+2bx\left(3ax^3+bc\left(3c^2+x^2\right)\right)\text{arctanh}\left(\frac{c}{x}\right)\\&+3b^2\left(-c^4+x^4\right)\text{arctanh}\left(\frac{c}{x}\right)^2+b(3a+4b)c^4\log(-c+x)\\&-3abc^4\log(c+x)+4b^2c^4\log(c+x)\Bigg)\end{aligned}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c/x])^2,x]
```

output
```
(6*a*b*c^3*x + b^2*c^2*x^2 + 2*a*b*c*x^3 + 3*a^2*x^4 + 2*b*x*(3*a*x^3 + b*
c*(3*c^2 + x^2))*ArcTanh[c/x] + 3*b^2*(-c^4 + x^4)*ArcTanh[c/x]^2 + b*(3*a
 + 4*b)*c^4*Log[-c + x] - 3*a*b*c^4*Log[c + x] + 4*b^2*c^4*Log[c + x])/12
```

3.143.3 Rubi [A] (warning: unable to verify)

Time = 1.01 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {6454, 6452, 6544, 6452, 243, 54, 2009, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2 dx$$

$$\Big\downarrow \text{ 6454}$$

$$-\int x^5 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2 d\frac{1}{x}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{4}x^4 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\int \frac{x^4 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6544}$$

$$\frac{1}{4}x^4 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(c^2\int \frac{x^2 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\int x^4 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{4}x^4 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(c^2\int \frac{x^2 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{3}bc\int \frac{x^3}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{1}{3}x^3 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{ 243}$$

$$\frac{1}{4}x^4 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(c^2\int \frac{x^2 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{6}bc\int \frac{x^2}{1-\frac{c^2}{x^2}}d\frac{1}{x^2}-\frac{1}{3}x^3 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \; 54$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(c^2\int\frac{x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{6}bc\int\left(-\frac{c^4}{\frac{c^2}{x^2}-1}+xc^2+x^2\right)d\frac{1}{x^2}-\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(c^2\int\frac{x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc\left(c^2\left(-\log\left(1-\frac{c^2}{x^2}\right)\right)+c^2\log\left(\frac{1}{x^2}\right)-x\right)\right.$$

$$\Big\downarrow \; 6544$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(c^2\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)d\frac{1}{x}\right)-\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc\left(c^2\left(-\log\right.\right.\right.$$

$$\Big\downarrow \; 6452$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(c^2\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\int\frac{x}{1-\frac{c^2}{x^2}}d\frac{1}{x}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{6}bc\right.$$

$$\Big\downarrow \; 243$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(c^2\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\int\frac{x}{1-\frac{c^2}{x^2}}d\frac{1}{x^2}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{6}\right.$$

$$\Big\downarrow \; 47$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(c^2\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\left(c^2\int\frac{1}{1-\frac{c^2}{x^2}}d\frac{1}{x^2}+\int xd\frac{1}{x^2}\right)-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\left(a+b\text{a}\right.\right.$$

$$\Big\downarrow \; 14$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{1}{2}bc\left(c^2\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\left(c^2\int\frac{1}{1-\frac{c^2}{x^2}}d\frac{1}{x^2}+\log\left(\frac{1}{x^2}\right)\right)-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)-\frac{1}{3}x^3\left(a+\right.\right.$$

$$\Big\downarrow \; 16$$

3.143. $\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{1}{2}bc\left(c^2\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(1-\frac{c^2}{x^2}\right)\right)\right)-\frac{1}{3}x^3\left(a+\text{bar}\right.\right.$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-$$
$$\frac{1}{2}bc\left(c^2\left(\frac{c\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(1-\frac{c^2}{x^2}\right)\right)\right)-\frac{1}{3}x^3\left(a+\text{barctan}\right.\right.$$

input
```
Int[x^3*(a + b*ArcTanh[c/x])^2,x]
```

output
```
(x^4*(a + b*ArcTanh[c/x])^2)/4 - (b*c*(-1/3*(x^3*(a + b*ArcTanh[c/x])) + (
b*c*(-x - c^2*Log[1 - c^2/x^2] + c^2*Log[x^(-2)]))/6 + c^2*(-(x*(a + b*Arc
Tanh[c/x])) + (c*(a + b*ArcTanh[c/x])^2)/(2*b) + (b*c*(-Log[1 - c^2/x^2] +
 Log[x^(-2)]))/2)))/2
```

3.143.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.143.4 Maple [A] (verified)

Time = 3.58 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.31

method	result
parallelrisch	$\frac{x^4 \operatorname{arctanh}\left(\frac{c}{x}\right)^2 b^2}{4} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)^2 b^2 c^4}{4} + \frac{2b^2 c^4 \ln(x-c)}{3} + \frac{x^4 \operatorname{arctanh}\left(\frac{c}{x}\right) ab}{2} + \frac{x^3 \operatorname{arctanh}\left(\frac{c}{x}\right) b^2 c}{6} + \frac{x \operatorname{arctanh}(}{2}$
parts	$\frac{a^2 x^4}{4} - b^2 c^4 \left(-\frac{x^4 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{4c^4} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right) \ln\left(\frac{c}{x}-1\right)}{4} + \frac{\operatorname{arctanh}\left(\frac{c}{x}\right) \ln\left(1+\frac{c}{x}\right)}{4} - \frac{x^3 \operatorname{arctanh}\left(\frac{c}{x}\right)}{6c^3} - \frac{x \operatorname{arcta}}{2} \right.$
derivativedivides	$-c^4 \left(-\frac{a^2 x^4}{4c^4} + b^2 \left(-\frac{x^4 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{4c^4} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right) \ln\left(\frac{c}{x}-1\right)}{4} + \frac{\operatorname{arctanh}\left(\frac{c}{x}\right) \ln\left(1+\frac{c}{x}\right)}{4} - \frac{x^3 \operatorname{arctanh}\left(\frac{c}{x}\right)}{6c^3} - \right.\right.$
default	$-c^4 \left(-\frac{a^2 x^4}{4c^4} + b^2 \left(-\frac{x^4 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{4c^4} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right) \ln\left(\frac{c}{x}-1\right)}{4} + \frac{\operatorname{arctanh}\left(\frac{c}{x}\right) \ln\left(1+\frac{c}{x}\right)}{4} - \frac{x^3 \operatorname{arctanh}\left(\frac{c}{x}\right)}{6c^3} - \right.\right.$
risch	Expression too large to display

input
```
int(x^3*(a+b*arctanh(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*x^4*arctanh(c/x)^2*b^2-1/4*arctanh(c/x)^2*b^2*c^4+2/3*b^2*c^4*ln(x-c)+
1/2*x^4*arctanh(c/x)*a*b+1/6*x^3*arctanh(c/x)*b^2*c+1/2*x*arctanh(c/x)*b^2
*c^3-1/2*arctanh(c/x)*a*b*c^4+2/3*arctanh(c/x)*b^2*c^4+1/4*a^2*x^4+1/6*a*b
*c*x^3+1/12*b^2*c^2*x^2+1/2*a*b*c^3*x+1/12*b^2*c^4
```

3.143.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.21

$$\begin{aligned}\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx &= \frac{1}{2}\,abc^3x+\frac{1}{12}\,b^2c^2x^2+\frac{1}{6}\,abcx^3+\frac{1}{4}\,a^2x^4\\ &\quad-\frac{1}{12}\left(3\,ab-4\,b^2\right)c^4\log\left(c+x\right)\\ &\quad+\frac{1}{12}\left(3\,ab+4\,b^2\right)c^4\log\left(-c+x\right)\\ &\quad-\frac{1}{16}\left(b^2c^4-b^2x^4\right)\log\left(-\frac{c+x}{c-x}\right)^2\\ &\quad+\frac{1}{12}\left(3\,b^2c^3x+b^2cx^3+3\,abx^4\right)\log\left(-\frac{c+x}{c-x}\right)\end{aligned}$$

input
```
integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

output
```
1/2*a*b*c^3*x + 1/12*b^2*c^2*x^2 + 1/6*a*b*c*x^3 + 1/4*a^2*x^4 - 1/12*(3*a
*b - 4*b^2)*c^4*log(c + x) + 1/12*(3*a*b + 4*b^2)*c^4*log(-c + x) - 1/16*(
b^2*c^4 - b^2*x^4)*log(-(c + x)/(c - x))^2 + 1/12*(3*b^2*c^3*x + b^2*c*x^3
 + 3*a*b*x^4)*log(-(c + x)/(c - x))
```

3.143.6 Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.28

$$\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx = \frac{a^2x^4}{4} - \frac{abc^4\operatorname{atanh}\left(\frac{c}{x}\right)}{2} + \frac{abc^3x}{2} + \frac{abcx^3}{6} + \frac{abx^4\operatorname{atanh}\left(\frac{c}{x}\right)}{2} + \frac{2b^2c^4\log\left(-c+x\right)}{3} - \frac{b^2c^4\operatorname{atanh}^2\left(\frac{c}{x}\right)}{4} + \frac{2b^2c^4\operatorname{atanh}\left(\frac{c}{x}\right)}{3} + \frac{b^2c^3x\operatorname{atanh}\left(\frac{c}{x}\right)}{2} + \frac{b^2c^2x^2}{12} + \frac{b^2cx^3\operatorname{atanh}\left(\frac{c}{x}\right)}{6} + \frac{b^2x^4\operatorname{atanh}^2\left(\frac{c}{x}\right)}{4}$$

input
```
integrate(x**3*(a+b*atanh(c/x))**2,x)
```

output
```
a**2*x**4/4 - a*b*c**4*atanh(c/x)/2 + a*b*c**3*x/2 + a*b*c*x**3/6 + a*b*x*
*4*atanh(c/x)/2 + 2*b**2*c**4*log(-c + x)/3 - b**2*c**4*atanh(c/x)**2/4 +
2*b**2*c**4*atanh(c/x)/3 + b**2*c**3*x*atanh(c/x)/2 + b**2*c**2*x**2/12 +
b**2*c*x**3*atanh(c/x)/6 + b**2*x**4*atanh(c/x)**2/4
```

3.143.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.54

$$\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx = \frac{1}{4}\,b^2x^4\operatorname{artanh}\left(\frac{c}{x}\right)^2 + \frac{1}{4}\,a^2x^4 + \frac{1}{12}\left(6\,x^4\operatorname{artanh}\left(\frac{c}{x}\right) - \left(3\,c^3\log\left(c+x\right) - 3\,c^3\log\left(-c+x\right) - 6\,c^2x - 2\,x^3\right)c\right)ab + \frac{1}{48}\left(\left(3\,c^2\log\left(c+x\right)^2 + 3\,c^2\log\left(-c+x\right)^2 + 16\,c^2\log\left(c+x\right) + 4\,x^2 - 2\left(3\,c^2\log\left(c+x\right) - 8\,c^2\right)\log\left(-c\right.\right.\right.$$

input
```
integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctanh(c/x)^2 + 1/4*a^2*x^4 + 1/12*(6*x^4*arctanh(c/x) - (3*c
^3*log(c + x) - 3*c^3*log(-c + x) - 6*c^2*x - 2*x^3)*c)*a*b + 1/48*((3*c^2
*log(c + x)^2 + 3*c^2*log(-c + x)^2 + 16*c^2*log(c + x) + 4*x^2 - 2*(3*c^2
*log(c + x) - 8*c^2)*log(-c + x))*c^2 - 4*(3*c^3*log(c + x) - 3*c^3*log(-c
 + x) - 6*c^2*x - 2*x^3)*c*arctanh(c/x))*b^2
```

3.143.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 552 vs. $2(109) = 218$.

Time = 0.27 (sec) , antiderivative size = 552, normalized size of antiderivative = 4.49

$$\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx = -\frac{4\,b^2c^5\log\left(-\frac{c+x}{c-x}-1\right)-4\,b^2c^5\log\left(-\frac{c+x}{c-x}\right)+\frac{3\left(\frac{b^2(c+x)^3c^5}{(c-x)^3}+\frac{b^2(c+x)c^5}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)^2}{\frac{(c+x)^4}{(c-x)^4}+\frac{4\,(c+x)^3}{(c-x)^3}+\frac{6\,(c+x)^2}{(c-x)^2}+\frac{4\,(c+x)}{c-x}+1}+\frac{2\left(2\,b^2c^5+\frac{6\,ab(c+x)^3c^5}{(c-x)^3}+\frac{3\,b^2(c+x)\cdots}{(c-x)\cdots}\cdots\right.}{\frac{(c+x)^4}{(c-x)^4}\cdots}\cdots}{\cdots}$$

input
```
integrate(x^3*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

output
```
-1/6*(4*b^2*c^5*log(-(c + x)/(c - x) - 1) - 4*b^2*c^5*log(-(c + x)/(c - x)
) + 3*(b^2*(c + x)^3*c^5/(c - x)^3 + b^2*(c + x)*c^5/(c - x))*log(-(c + x)
/(c - x))^2/((c + x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3 + 6*(c + x)^2/(c
- x)^2 + 4*(c + x)/(c - x) + 1) + 2*(2*b^2*c^5 + 6*a*b*(c + x)^3*c^5/(c -
x)^3 + 3*b^2*(c + x)^3*c^5/(c - x)^3 + 6*b^2*(c + x)^2*c^5/(c - x)^2 + 6*a
*b*(c + x)*c^5/(c - x) + 5*b^2*(c + x)*c^5/(c - x))*log(-(c + x)/(c - x))/
((c + x)^4/(c - x)^4 + 4*(c + x)^3/(c - x)^3 + 6*(c + x)^2/(c - x)^2 + 4*(
c + x)/(c - x) + 1) + 2*(4*a*b*c^5 + 6*a^2*(c + x)^3*c^5/(c - x)^3 + 6*a*b
*(c + x)^3*c^5/(c - x)^3 + b^2*(c + x)^3*c^5/(c - x)^3 + 12*a*b*(c + x)^2*
c^5/(c - x)^2 + 2*b^2*(c + x)^2*c^5/(c - x)^2 + 6*a^2*(c + x)*c^5/(c - x)
+ 10*a*b*(c + x)*c^5/(c - x) + b^2*(c + x)*c^5/(c - x))/((c + x)^4/(c - x)
^4 + 4*(c + x)^3/(c - x)^3 + 6*(c + x)^2/(c - x)^2 + 4*(c + x)/(c - x) + 1
))/c
```

3.143.9 Mupad [B] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.15

$$\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx = \frac{a^2\,x^4}{4}-\frac{b^2\,c^4\,\mathrm{atanh}\left(\frac{c}{x}\right)^2}{4}+\frac{b^2\,x^4\,\mathrm{atanh}\left(\frac{c}{x}\right)^2}{4}+\frac{b^2\,c^4\,\ln\left(x^2-c^2\right)}{3}+\frac{b^2\,c^2\,x^2}{12}+\frac{b^2\,c\,x^3\,\mathrm{atanh}\left(\frac{c}{x}\right)}{6}+\frac{b^2\,c^3\,x\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{a\,b\,c\,x^3}{6}+\frac{a\,b\,c^3\,x}{2}-\frac{a\,b\,c^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}+\frac{a\,b\,x^4\,\mathrm{atanh}\left(\frac{c}{x}\right)}{2}$$

input
```
int(x^3*(a + b*atanh(c/x))^2,x)
```

output
```
(a^2*x^4)/4 - (b^2*c^4*atanh(c/x)^2)/4 + (b^2*x^4*atanh(c/x)^2)/4 + (b^2*c
^4*log(x^2 - c^2))/3 + (b^2*c^2*x^2)/12 + (b^2*c*x^3*atanh(c/x))/6 + (b^2*
c^3*x*atanh(c/x))/2 + (a*b*c*x^3)/6 + (a*b*c^3*x)/2 - (a*b*c^4*atanh(c/x))
/2 + (a*b*x^4*atanh(c/x))/2
```

3.143. $\int x^3 \left(a + b\text{arctanh}\left(\frac{c}{x}\right)\right)^2 \, dx$

3.144 $\int x^2 \left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2 \, dx$

3.144.1 Optimal result

Integrand size = 16, antiderivative size = 142

$$\begin{aligned}\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2 dx &= \frac{1}{3}b^2c^2x-\frac{1}{3}b^2c^3\coth^{-1}\left(\frac{x}{c}\right)+\frac{1}{3}bcx^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\\ &\quad-\frac{1}{3}c^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{3}x^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\\ &\quad-\frac{2}{3}bc^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1+\frac{c}{x}}\right)\\ &\quad+\frac{1}{3}b^2c^3\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{c}{x}}\right)\end{aligned}$$

output
```
1/3*b^2*c^2*x-1/3*b^2*c^3*arccoth(x/c)+1/3*b*c*x^2*(a+b*arccoth(x/c))-1/3*
c^3*(a+b*arccoth(x/c))^2+1/3*x^3*(a+b*arccoth(x/c))^2-2/3*b*c^3*(a+b*arcco
th(x/c))*ln(2-2/(1+c/x))+1/3*b^2*c^3*polylog(2,-1+2/(1+c/x))
```

3.144.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2 dx &= \frac{1}{3}\Bigg(b^2c^2x+abcx^2+a^2x^3+b^2\left(-c^3+x^3\right)\operatorname{arctanh}\left(\frac{c}{x}\right)^2\\ &\quad+b\operatorname{arctanh}\left(\frac{c}{x}\right)\left(-bc^3+bcx^2+2ax^3\right.\\ &\quad\left.-2bc^3\log\left(1-e^{-2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right)\right)+abc^3\log\left(1-\frac{c^2}{x^2}\right)\\ &\quad-2abc^3\log\left(\frac{c}{x}\right)+b^2c^3\operatorname{PolyLog}\left(2,e^{-2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right)\Bigg)\end{aligned}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c/x])^2,x]
```

output
```
(b^2*c^2*x + a*b*c*x^2 + a^2*x^3 + b^2*(-c^3 + x^3)*ArcTanh[c/x]^2 + b*Arc
Tanh[c/x]*(-(b*c^3) + b*c*x^2 + 2*a*x^3 - 2*b*c^3*Log[1 - E^(-2*ArcTanh[c/
x])]) + a*b*c^3*Log[1 - c^2/x^2] - 2*a*b*c^3*Log[c/x] + b^2*c^3*PolyLog[2,
 E^(-2*ArcTanh[c/x])])/3
```

3.144.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {6454, 6452, 6544, 6452, 264, 219, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2dx$$

$$\Big\downarrow \text{6454}$$

$$-\int x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\int \frac{x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(c^2\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(c^2\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\int \frac{x^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(c^2\int\frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\left(c^2\int\frac{1}{1-\frac{c^2}{x^2}}d\frac{1}{x}-x\right)-\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)$$

$\downarrow$ 219

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(c^2\int\frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(c\text{arctanh}\left(\frac{c}{x}\right)-x\right)\right)$$

$\downarrow$ 6550

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(c^2\left(\int\frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{\frac{c}{x}+1}d\frac{1}{x}+\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}\right)-\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(c\text{arctanh}\left(\frac{c}{x}\right)-x\right.\right.$$

$\downarrow$ 6494

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(c^2\left(-bc\int\frac{\log\left(2-\frac{2}{\frac{c}{x}+1}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}+\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)-\frac{1}{2}x^2\left(a+\right.\right.$$

$\downarrow$ 2897

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-\frac{2}{3}bc\left(c^2\left(\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}+\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{\frac{c}{x}+1}-1\right)\right)-\frac{1}{2}x^2\left(a\right.\right.$$

input
```
Int[x^2*(a + b*ArcTanh[c/x])^2,x]
```

output
```
(x^3*(a + b*ArcTanh[c/x])^2)/3 - (2*b*c*(-1/2*(x^2*(a + b*ArcTanh[c/x])) +
 (b*c*(-x + c*ArcTanh[c/x]))/2 + c^2*((a + b*ArcTanh[c/x])^2/(2*b) + (a +
b*ArcTanh[c/x])*Log[2 - 2/(1 + c/x)] - (b*PolyLog[2, -1 + 2/(1 + c/x)])/2)
))/3
```

3.144. $\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx$

3.144.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.144.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(128) = 256$.

Time = 3.53 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.10

method	result
parts	$\frac{a^2x^3}{3} - b^2c^3\left(-\frac{x^3 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{3c^3} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{3} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{3} - \frac{x^2 \operatorname{arctanh}\left(\frac{c}{x}\right)}{3c^2} + \frac{2\ln\left(\frac{c}{x}\right)\right.$
derivativedivides	$-c^3\left(-\frac{a^2x^3}{3c^3} + b^2\left(-\frac{x^3 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{3c^3} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{3} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{3} - \frac{x^2 \operatorname{arctanh}\left(\frac{c}{x}\right)}{3c^2} + \right.\right.$
default	$-c^3\left(-\frac{a^2x^3}{3c^3} + b^2\left(-\frac{x^3 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{3c^3} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{3} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{3} - \frac{x^2 \operatorname{arctanh}\left(\frac{c}{x}\right)}{3c^2} + \right.\right.$
risch	Expression too large to display

input
```
int(x^2*(a+b*arctanh(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3-b^2*c^3*(-1/3/c^3*x^3*arctanh(c/x)^2-1/3*arctanh(c/x)*ln(c/x-1
)-1/3*arctanh(c/x)*ln(1+c/x)-1/3/c^2*x^2*arctanh(c/x)+2/3*ln(c/x)*arctanh(
c/x)-1/6*ln(c/x-1)+1/6*ln(1+c/x)-1/3/c*x-1/12*ln(c/x-1)^2+1/3*dilog(1/2*c/
x+1/2)+1/6*ln(c/x-1)*ln(1/2*c/x+1/2)-1/6*(ln(1+c/x)-ln(1/2*c/x+1/2))*ln(-1
/2*c/x+1/2)+1/12*ln(1+c/x)^2-1/3*dilog(c/x)-1/3*dilog(1+c/x)-1/3*ln(c/x)*l
n(1+c/x))-2*a*b*c^3*(-1/3/c^3*x^3*arctanh(c/x)-1/6*ln(c/x-1)-1/6*ln(1+c/x)
-1/6/c^2*x^2+1/3*ln(c/x))
```

3.144.5 Fricas [F]

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctanh(c/x)^2 + 2*a*b*x^2*arctanh(c/x) + a^2*x^2, x)
```

3.144.6 Sympy [F]

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx=\int x^2\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^2\,dx$$

input
```
integrate(x**2*(a+b*atanh(c/x))**2,x)
```

output
```
Integral(x**2*(a + b*atanh(c/x))**2, x)
```

3.144.7 Maxima [F]

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx=\int\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*a*b
 + 1/12*(6*c^4*integrate(-1/3*log(c + x)/(c^2 - x^2), x) + x^3*log(c + x)^
2 + 6*c^3*integrate(-1/3*x*log(c + x)/(c^2 - x^2), x) - (c*log(c + x) - c*
log(-c + x) - 2*x)*c^2 - (c^3 - x^3)*log(-c + x)^2 + (c^2*log(-c^2 + x^2)
+ x^2)*c + 12*c*integrate(-1/3*x^3*log(c + x)/(c^2 - x^2), x) - 2*(c*x^2 +
 (c^3 + x^3)*log(c + x))*log(-c + x))*b^2
```

3.144.8 Giac [F]

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx=\int \left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^2*x^2, x)
```

3.144.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx=\int x^2\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^2\,dx$$

input
```
int(x^2*(a + b*atanh(c/x))^2,x)
```

output
```
int(x^2*(a + b*atanh(c/x))^2, x)
```

3.145 $\int x\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx$

3.145.1 Optimal result

Integrand size = 14, antiderivative size = 83

$$\int x\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx=bcx\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{2}c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{1}{2}b^2c^2\log\left(1-\frac{c^2}{x^2}\right)+b^2c^2\log(x)$$

output
```
b*c*x*(a+b*arccoth(x/c))-1/2*c^2*(a+b*arccoth(x/c))^2+1/2*x^2*(a+b*arccoth
(x/c))^2+1/2*b^2*c^2*ln(1-c^2/x^2)+b^2*c^2*ln(x)
```

3.145.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.11

$$\int x\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx=\frac{1}{2}\left(2abcx+a^2x^2+2bx(bc+ax)\text{arctanh}\left(\frac{c}{x}\right)+b^2\left(-c^2+x^2\right)\text{arctanh}\left(\frac{c}{x}\right)^2+b(a+b)c^2\log(-c+x)-abc^2\log(c+x)+b^2c^2\log(c+x)\right)$$

input
```
Integrate[x*(a + b*ArcTanh[c/x])^2,x]
```

output
```
(2*a*b*c*x + a^2*x^2 + 2*b*x*(b*c + a*x)*ArcTanh[c/x] + b^2*(-c^2 + x^2)*A
rcTanh[c/x]^2 + b*(a + b)*c^2*Log[-c + x] - a*b*c^2*Log[c + x] + b^2*c^2*L
og[c + x])/2
```

3.145.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6454, 6452, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2dx$$

$$\Big\downarrow \text{6454}$$

$$-\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-bc\int \frac{x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)d\frac{1}{x}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\int \frac{x}{1-\frac{c^2}{x^2}}d\frac{1}{x}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-bc\left(c^2\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\int \frac{x}{1-\frac{c^2}{x^2}}d\frac{1}{x^2}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{47}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-$$
$$bc\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\left(c^2\int\frac{1}{1-\frac{c^2}{x^2}}d\frac{1}{x^2}+\int xd\frac{1}{x^2}\right)-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{ 14}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-$$
$$bc\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\left(c^2\int\frac{1}{1-\frac{c^2}{x^2}}d\frac{1}{x^2}+\log\left(\frac{1}{x^2}\right)\right)-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)$$

$$\Big\downarrow \text{ 16}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-$$
$$bc\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(1-\frac{c^2}{x^2}\right)\right)\right)$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2-$$
$$bc\left(\frac{c\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(1-\frac{c^2}{x^2}\right)\right)\right)$$

input
```
Int[x*(a + b*ArcTanh[c/x])^2,x]
```

output
```
(x^2*(a + b*ArcTanh[c/x])^2)/2 - b*c*(-(x*(a + b*ArcTanh[c/x])) + (c*(a +
b*ArcTanh[c/x])^2)/(2*b) + (b*c*(-Log[1 - c^2/x^2] + Log[x^(-2)]))/2)
```

3.145.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.145.4 Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.42

method	result
parallelrisch	$\frac{x^2 \operatorname{arctanh}\left(\frac{c}{x}\right)^2 b^2}{2} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)^2 b^2 c^2}{2} + b^2c^2 \ln(x-c) + x^2 \operatorname{arctanh}\left(\frac{c}{x}\right) ab + x \operatorname{arctanh}\left(\frac{c}{x}\right) b^2 c$
parts	$\frac{a^2x^2}{2} - b^2c^2\left(-\frac{x^2 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{2c^2} - \frac{x \operatorname{arctanh}\left(\frac{c}{x}\right)}{c} + \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2} - \frac{\ln\left(\frac{c}{x}-1\right.}{8}\right.$
derivativedivides	$-c^2\left(-\frac{a^2x^2}{2c^2} + b^2\left(-\frac{x^2 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{2c^2} - \frac{x \operatorname{arctanh}\left(\frac{c}{x}\right)}{c} + \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2} - \right.\right.$
default	$-c^2\left(-\frac{a^2x^2}{2c^2} + b^2\left(-\frac{x^2 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{2c^2} - \frac{x \operatorname{arctanh}\left(\frac{c}{x}\right)}{c} + \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2} - \frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2} - \right.\right.$
risch	Expression too large to display

input
```
int(x*(a+b*arctanh(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctanh(c/x)^2*b^2-1/2*arctanh(c/x)^2*b^2*c^2+b^2*c^2*ln(x-c)+x^2*
arctanh(c/x)*a*b+x*arctanh(c/x)*b^2*c-arctanh(c/x)*a*b*c^2+arctanh(c/x)*b^
2*c^2+1/2*a^2*x^2+a*b*c*x+1/2*a^2*c^2
```

3.145.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.34

$$
\begin{aligned}
\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2 dx = abcx &+ \frac{1}{2}a^2x^2 - \frac{1}{2}\left(ab-b^2\right)c^2\log(c+x) \\
&+ \frac{1}{2}\left(ab+b^2\right)c^2\log(-c+x) \\
&- \frac{1}{8}\left(b^2c^2-b^2x^2\right)\log\left(-\frac{c+x}{c-x}\right)^2 \\
&+ \frac{1}{2}\left(b^2cx+abx^2\right)\log\left(-\frac{c+x}{c-x}\right)
\end{aligned}
$$

input
```
integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

output
```
a*b*c*x + 1/2*a^2*x^2 - 1/2*(a*b - b^2)*c^2*log(c + x) + 1/2*(a*b + b^2)*c
^2*log(-c + x) - 1/8*(b^2*c^2 - b^2*x^2)*log(-(c + x)/(c - x))^2 + 1/2*(b^
2*c*x + a*b*x^2)*log(-(c + x)/(c - x))
```

3.145.6 Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx = \frac{a^2x^2}{2} - abc^2\,\text{atanh}\left(\frac{c}{x}\right) + abcx + abx^2\,\text{atanh}\left(\frac{c}{x}\right) + b^2c^2\log\left(-c+x\right) - \frac{b^2c^2\,\text{atanh}^2\left(\frac{c}{x}\right)}{2} + b^2c^2\,\text{atanh}\left(\frac{c}{x}\right) + b^2cx\,\text{atanh}\left(\frac{c}{x}\right) + \frac{b^2x^2\,\text{atanh}^2\left(\frac{c}{x}\right)}{2}$$

input
```
integrate(x*(a+b*atanh(c/x))**2,x)
```

output
```
a**2*x**2/2 - a*b*c**2*atanh(c/x) + a*b*c*x + a*b*x**2*atanh(c/x) + b**2*c
**2*log(-c + x) - b**2*c**2*atanh(c/x)**2/2 + b**2*c**2*atanh(c/x) + b**2*
c*x*atanh(c/x) + b**2*x**2*atanh(c/x)**2/2
```

3.145.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.64

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx = \frac{1}{2}\,b^2x^2\,\text{artanh}\left(\frac{c}{x}\right)^2 + \frac{1}{2}\,a^2x^2 + \frac{1}{2}\left(2\,x^2\,\text{artanh}\left(\frac{c}{x}\right) - (c\log\left(c+x\right) - c\log\left(-c+x\right) - 2\,x)c\right)ab + \frac{1}{8}\left(\left(\log\left(c+x\right)^2 - 2\left(\log\left(c+x\right) - 2\right)\log\left(-c+x\right) + \log\left(-c+x\right)^2 + 4\,\log\left(c+x\right)\right)c^2 - 4\left(c\log\left(c+x\right)\right.\right.$$

input
```
integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arctanh(c/x)^2 + 1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c/x) - (c*lo
g(c + x) - c*log(-c + x) - 2*x)*c)*a*b + 1/8*((log(c + x)^2 - 2*(log(c + x
) - 2)*log(-c + x) + log(-c + x)^2 + 4*log(c + x))*c^2 - 4*(c*log(c + x) -
 c*log(-c + x) - 2*x)*c*arctanh(c/x))*b^2
```

3.145.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(77) = 154$.

Time = 0.29 (sec) , antiderivative size = 268, normalized size of antiderivative = 3.23

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx = -\frac{2\,b^2c^3\log\left(-\frac{c+x}{c-x}-1\right)-2\,b^2c^3\log\left(-\frac{c+x}{c-x}\right)+\frac{b^2(c+x)c^3\log\left(-\frac{c+x}{c-x}\right)^2}{(c-x)\left(\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1\right)}+\frac{2\left(b^2c^3+\frac{2\,ab(c+x)c^3}{c-x}+\frac{b^2(c+x)c^3}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^2}{(c-x)^2}+\frac{2\,(c+x)}{c-x}+1}}{2\,c}$$

input
```
integrate(x*(a+b*arctanh(c/x))^2,x, algorithm="giac")
```

output
```
-1/2*(2*b^2*c^3*log(-(c + x)/(c - x) - 1) - 2*b^2*c^3*log(-(c + x)/(c - x)
) + b^2*(c + x)*c^3*log(-(c + x)/(c - x))^2/((c - x)*((c + x)^2/(c - x)^2
+ 2*(c + x)/(c - x) + 1)) + 2*(b^2*c^3 + 2*a*b*(c + x)*c^3/(c - x) + b^2*(
c + x)*c^3/(c - x))*log(-(c + x)/(c - x))/((c + x)^2/(c - x)^2 + 2*(c + x)
/(c - x) + 1) + 4*(a*b*c^3 + a^2*(c + x)*c^3/(c - x) + a*b*(c + x)*c^3/(c
- x))/((c + x)^2/(c - x)^2 + 2*(c + x)/(c - x) + 1))/c
```

3.145.9 Mupad [B] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2\,dx = \frac{a^2\,x^2}{2}-\frac{b^2\,c^2\,\mathrm{atanh}\left(\frac{c}{x}\right)^2}{2}+\frac{b^2\,x^2\,\mathrm{atanh}\left(\frac{c}{x}\right)^2}{2} + \frac{b^2\,c^2\,\ln\left(x^2-c^2\right)}{2}-a\,b\,c^2\,\mathrm{atanh}\left(\frac{c}{x}\right) + a\,b\,x^2\,\mathrm{atanh}\left(\frac{c}{x}\right)+b^2\,c\,x\,\mathrm{atanh}\left(\frac{c}{x}\right)+a\,b\,c\,x$$

input
```
int(x*(a + b*atanh(c/x))^2,x)
```

output
```
(a^2*x^2)/2 - (b^2*c^2*atanh(c/x)^2)/2 + (b^2*x^2*atanh(c/x)^2)/2 + (b^2*c
^2*log(x^2 - c^2))/2 - a*b*c^2*atanh(c/x) + a*b*x^2*atanh(c/x) + b^2*c*x*a
tanh(c/x) + a*b*c*x
```

3.146 $\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2 dx$

3.146.1 Optimal result

Integrand size = 12, antiderivative size = 74

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2 dx = c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2 - 2bc\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c-x}\right) - b^2c\,\text{PolyLog}\left(2,-\frac{c+x}{c-x}\right)$$

output
```
c*(a+b*arccoth(x/c))^2+x*(a+b*arccoth(x/c))^2-2*b*c*(a+b*arccoth(x/c))*ln(
2*c/(c-x))-b^2*c*polylog(2,(-c-x)/(c-x))
```

3.146.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.31

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2 dx = b^2(-c+x)\text{arctanh}\left(\frac{c}{x}\right)^2 + 2b\text{arctanh}\left(\frac{c}{x}\right)\left(ax-bc\log\left(1-e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right) + a\left(ax+bc\log\left(1-\frac{c^2}{x^2}\right)-2bc\log\left(\frac{c}{x}\right)\right) + b^2c\,\text{PolyLog}\left(2,e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)$$

input
```
Integrate[(a + b*ArcTanh[c/x])^2,x]
```

output
```
b^2*(-c + x)*ArcTanh[c/x]^2 + 2*b*ArcTanh[c/x]*(a*x - b*c*Log[1 - E^(-2*Ar
cTanh[c/x])]) + a*(a*x + b*c*Log[1 - c^2/x^2] - 2*b*c*Log[c/x]) + b^2*c*Po
lyLog[2, E^(-2*ArcTanh[c/x])]
```

3.146.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6440, 6437, 27, 6547, 27, 6471, 27, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2 dx$$

$$\Big\downarrow \text{6440}$$

$$\int \left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2 dx$$

$$\Big\downarrow \text{6437}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-\frac{2b\int \frac{c^2x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c^2-x^2}dx}{c}$$

$$\Big\downarrow \text{27}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\int \frac{x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c^2-x^2}dx$$

$$\Big\downarrow \text{6547}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\left(\frac{\int \frac{c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{c-x}dx}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}\right)$$

$$\Big\downarrow \text{27}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-2bc\left(\int \frac{a+b\coth^{-1}\left(\frac{x}{c}\right)}{c-x}dx-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}\right)$$

$$\Big\downarrow \text{6471}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-$$
$$2bc\left(-\frac{b\int\frac{c^2\log\left(\frac{2c}{c-x}\right)}{c^2-x^2}dx}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}+\log\left(\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\right)$$

$\Big\downarrow$ 27

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-$$
$$2bc\left(-bc\int\frac{\log\left(\frac{2c}{c-x}\right)}{c^2-x^2}dx-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}+\log\left(\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\right)$$

$\Big\downarrow$ 2849

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-$$
$$2bc\left(bc\int\frac{\log\left(\frac{2c}{c-x}\right)}{1-\frac{2c}{c-x}}d\frac{1}{c-x}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}+\log\left(\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\right)$$

$\Big\downarrow$ 2752

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2-$$
$$2bc\left(-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2b}+\log\left(\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,1-\frac{2c}{c-x}\right)\right)$$

input
```
Int[(a + b*ArcTanh[c/x])^2,x]
```

output
```
x*(a + b*ArcCoth[x/c])^2 - 2*b*c*(-1/2*(a + b*ArcCoth[x/c])^2/b + (a + b*A
rcCoth[x/c])*Log[(2*c)/(c - x)] + (b*PolyLog[2, 1 - (2*c)/(c - x)])/2)
```

3.146.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

3.146. $\int\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2dx$

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6440
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*
ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0
]
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.146.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(77) = 154$.

Time = 1.30 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.18

method	result
parts	$a^2x-b^2c\left(-\frac{x\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{c}+2\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)-\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)-\operatorname{arctanh}\left(\frac{c}{x}\right.\right.$
derivativedivides	$-c\left(-\frac{a^2x}{c}+b^2\left(-\frac{x\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{c}+2\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)-\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)-\operatorname{arctan}\right.\right.$
default	$-c\left(-\frac{a^2x}{c}+b^2\left(-\frac{x\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{c}+2\ln\left(\frac{c}{x}\right)\operatorname{arctanh}\left(\frac{c}{x}\right)-\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)-\operatorname{arctan}\right.\right.$
risch	Expression too large to display

input
```
int((a+b*arctanh(c/x))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x-b^2*c*(-1/c*x*arctanh(c/x)^2+2*ln(c/x)*arctanh(c/x)-arctanh(c/x)*ln(
1+c/x)-arctanh(c/x)*ln(c/x-1)-1/4*ln(c/x-1)^2+dilog(1/2*c/x+1/2)+1/2*ln(c/
x-1)*ln(1/2*c/x+1/2)-1/2*(ln(1+c/x)-ln(1/2*c/x+1/2))*ln(-1/2*c/x+1/2)+1/4*
ln(1+c/x)^2-dilog(c/x)-dilog(1+c/x)-ln(c/x)*ln(1+c/x))-2*a*b*c*(-1/c*x*arc
tanh(c/x)-1/2*ln(1+c/x)+ln(c/x)-1/2*ln(c/x-1))
```

3.146.5 Fricas [F]

$$\int\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2 dx=\int\left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^2 dx$$

input
```
integrate((a+b*arctanh(c/x))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2, x)
```

3.146.6 Sympy [F]

$$\int\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2 dx=\int\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^2 dx$$

input
```
integrate((a+b*atanh(c/x))**2,x)
```

output
```
Integral((a + b*atanh(c/x))**2, x)
```

3.146.7 Maxima [F]

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2 dx$$

input
```
integrate((a+b*arctanh(c/x))^2,x, algorithm="maxima")
```

output
```
(2*x*arctanh(c/x) + c*log(-c^2 + x^2))*a*b + 1/4*(x*log(c + x)^2 - 2*(c +
x)*log(c + x)*log(-c + x) - (c - x)*log(-c + x)^2 + integrate(-2*(c^2 + 3*
c*x)*log(c + x)/(c^2 - x^2), x))*b^2 + a^2*x
```

3.146.8 Giac [F]

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2 dx$$

input
```
integrate((a+b*arctanh(c/x))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^2, x)
```

3.146.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2 dx = \int \left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^2 dx$$

input
```
int((a + b*atanh(c/x))^2,x)
```

output
```
int((a + b*atanh(c/x))^2, x)
```

3.147 $\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

3.147.1 Optimal result

Integrand size = 16, antiderivative size = 133

$$\begin{aligned}\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx = & -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\mathrm{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\\ & +b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\mathrm{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\\ & -b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\mathrm{PolyLog}\left(2,-1+\frac{2}{1-\frac{c}{x}}\right)\\ & -\frac{1}{2}b^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)+\frac{1}{2}b^2\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-\frac{c}{x}}\right)\end{aligned}$$

output
```
2*(a+b*arccoth(x/c))^2*arctanh(-1+2/(1-c/x))+b*(a+b*arccoth(x/c))*polylog(
2,1-2/(1-c/x))-b*(a+b*arccoth(x/c))*polylog(2,-1+2/(1-c/x))-1/2*b^2*polylo
g(3,1-2/(1-c/x))+1/2*b^2*polylog(3,-1+2/(1-c/x))
```

3.147.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx = a^2\log(x)+ab\left(\text{PolyLog}\left(2,-\frac{c}{x}\right)-\text{PolyLog}\left(2,\frac{c}{x}\right)\right)+b^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}\text{arctanh}\left(\frac{c}{x}\right)^3+\text{arctanh}\left(\frac{c}{x}\right)^2\log\left(1+e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)-\text{arctanh}\left(\frac{c}{x}\right)^2\log\left(1-e^{2\text{arctanh}\left(\frac{c}{x}\right)}\right)-\text{arctanh}\left(\frac{c}{x}\right)\text{PolyLog}\left(2,-e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)-\text{arctanh}\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{2\text{arctanh}\left(\frac{c}{x}\right)}\right)-\frac{1}{2}\text{PolyLog}\left(3,-e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)+\frac{1}{2}\text{PolyLog}\left(3,e^{2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c/x])^2/x,x]
```

output
```
a^2*Log[x] + a*b*(PolyLog[2, -(c/x)] - PolyLog[2, c/x]) + b^2*((-1/24*I)*P
i^3 + (2*ArcTanh[c/x]^3)/3 + ArcTanh[c/x]^2*Log[1 + E^(-2*ArcTanh[c/x])] -
 ArcTanh[c/x]^2*Log[1 - E^(2*ArcTanh[c/x])] - ArcTanh[c/x]*PolyLog[2, -E^(
-2*ArcTanh[c/x])] - ArcTanh[c/x]*PolyLog[2, E^(2*ArcTanh[c/x])] - PolyLog[
3, -E^(-2*ArcTanh[c/x])]/2 + PolyLog[3, E^(2*ArcTanh[c/x])]/2)
```

3.147.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6450, 6448, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{ 6450}$$

$$-\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}$$

$$\Big\downarrow \text{6448}$$

$$4bc\int \frac{\operatorname{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-2\operatorname{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2$$

$$\Big\downarrow \text{6614}$$

$$4bc\left(\frac{1}{2}\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)\log\left(2-\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{1}{2}\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)\log\left(\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}\right)-2\operatorname{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2$$

$$\Big\downarrow \text{6620}$$

$$4bc\left(\frac{1}{2}\left(\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)}{2c}-\frac{1}{2}b\int \frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}\right)+\frac{1}{2}\left(\frac{1}{2}b\int \frac{\operatorname{PolyLog}\left(2,\frac{2}{1-\frac{c}{x}}\right.}{1-\frac{c^2}{x^2}}\right.\right.$$

$$2\operatorname{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2$$

$$\Big\downarrow \text{7164}$$

$$4bc\left(\frac{1}{2}\left(\frac{\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)}{2c}-\frac{b\operatorname{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)}{4c}\right)+\frac{1}{2}\left(\frac{b\operatorname{PolyLog}\left(3,\frac{2}{1-\frac{c}{x}}-1\right)}{4c}-\right.\right.$$

$$2\operatorname{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2$$

input
```
Int[(a + b*ArcTanh[c/x])^2/x,x]
```

output
```
-2*ArcTanh[1 - 2/(1 - c/x)]*(a + b*ArcTanh[c/x])^2 + 4*b*c*((((a + b*ArcTa
nh[c/x])*PolyLog[2, 1 - 2/(1 - c/x)])/(2*c) - (b*PolyLog[3, 1 - 2/(1 - c/x
)])/(4*c))/2 + (-1/2*((a + b*ArcTanh[c/x])*PolyLog[2, -1 + 2/(1 - c/x)])/c
 + (b*PolyLog[3, -1 + 2/(1 - c/x)])/(4*c))/2)
```

3.147. $\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}dx$

3.147.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.147. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$

3.147.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 56.96 (sec) , antiderivative size = 704, normalized size of antiderivative = 5.29

method	result
parts	$a^2 \ln(x) + b^2 \left(-\ln\left(\frac{c}{x}\right) \operatorname{arctanh}\left(\frac{c}{x}\right)^2 + \operatorname{arctanh}\left(\frac{c}{x}\right) \operatorname{polylog}\left(2, -\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right) - \frac{\operatorname{polylog}\left(3, -\right.}{2}\right.$
derivativedivides	$-a^2 \ln\left(\frac{c}{x}\right) - b^2 \left(\ln\left(\frac{c}{x}\right) \operatorname{arctanh}\left(\frac{c}{x}\right)^2 - \operatorname{arctanh}\left(\frac{c}{x}\right) \operatorname{polylog}\left(2, -\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right) + \frac{\operatorname{polylog}\left(3, -\right.}{2}\right.$
default	$-a^2 \ln\left(\frac{c}{x}\right) - b^2 \left(\ln\left(\frac{c}{x}\right) \operatorname{arctanh}\left(\frac{c}{x}\right)^2 - \operatorname{arctanh}\left(\frac{c}{x}\right) \operatorname{polylog}\left(2, -\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right) + \frac{\operatorname{polylog}\left(3, -\right.}{2}\right.$

input
```
int((a+b*arctanh(c/x))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2*(-ln(c/x)*arctanh(c/x)^2+arctanh(c/x)*polylog(2,-(1+c/x)^2/(
1-c^2/x^2))-1/2*polylog(3,-(1+c/x)^2/(1-c^2/x^2))+arctanh(c/x)^2*ln((1+c/x
)^2/(1-c^2/x^2)-1)-arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))-2*arctan
h(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+2*polylog(3,-(1+c/x)/(1-c^2/x
^2)^(1/2))-arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2))-2*arctanh(c/x)*p
olylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))+2*polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))
-1/2*I*Pi*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1)/(1-(1+c/x)^2/(c^2/x^2-1)))*(cs
gn(I*(-(1+c/x)^2/(c^2/x^2-1)-1))*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))-csgn(I*
(-(1+c/x)^2/(c^2/x^2-1)-1))*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1)/(1-(1+c/x)^2
/(c^2/x^2-1)))-csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))*csgn(I*(-(1+c/x)^2/(c^2/x
^2-1)-1)/(1-(1+c/x)^2/(c^2/x^2-1)))+csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1)/(1-(
1+c/x)^2/(c^2/x^2-1)))^2)*arctanh(c/x)^2)+2*a*b*(-ln(c/x)*arctanh(c/x)+1/2
*dilog(1+c/x)+1/2*ln(c/x)*ln(1+c/x)+1/2*dilog(c/x))
```

3.147.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2)/x, x)
```

3.147.6 Sympy [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

input
```
integrate((a+b*atanh(c/x))**2/x,x)
```

output
```
Integral((a + b*atanh(c/x))**2/x, x)
```

3.147.7 Maxima [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + integrate(1/4*b^2*(log(c/x + 1) - log(-c/x + 1))^2/x + a*b*(l
og(c/x + 1) - log(-c/x + 1))/x, x)
```

3.147.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^2/x, x)
```

3.147.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*atanh(c/x))^2/x,x)
```

output
```
int((a + b*atanh(c/x))^2/x, x)
```

3.148 $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$

3.148.1 Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = -\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c} - \frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{x} + \frac{2b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1-\frac{c}{x}}\right)}{c} + \frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{c}$$

output
```
-(a+b*arccoth(x/c))^2/c-(a+b*arccoth(x/c))^2/x+2*b*(a+b*arccoth(x/c))*ln(2
/(1-c/x))/c+b^2*polylog(2,1-2/(1-c/x))/c
```

3.148.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = \frac{b^2(-c+x)\text{arctanh}\left(\frac{c}{x}\right)^2 + 2b\text{arctanh}\left(\frac{c}{x}\right)\left(-ac+bx\log\left(1+e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right) + a\left(-ac+2bx\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^2}}}\right.\right.}{cx}$$

input
```
Integrate[(a + b*ArcTanh[c/x])^2/x^2,x]
```

output
```
(b^2*(-c + x)*ArcTanh[c/x]^2 + 2*b*ArcTanh[c/x]*(-(a*c) + b*x*Log[1 + E^(-
2*ArcTanh[c/x])]) + a*(-(a*c) + 2*b*x*Log[1/Sqrt[1 - c^2/x^2]]) - b^2*x*Po
lyLog[2, -E^(-2*ArcTanh[c/x])])/(c*x)
```

3.148.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6454, 6436, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^2}dx$$

$$\Big\downarrow \text{6454}$$

$$-\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}$$

$$\Big\downarrow \text{6436}$$

$$2bc\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{\left(1-\frac{c^2}{x^2}\right)x}d\frac{1}{x}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}$$

$$\Big\downarrow \text{6546}$$

$$2bc\left(\frac{\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c}{x}}d\frac{1}{x}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}$$

$$\Big\downarrow \text{6470}$$

$$2bc\left(\frac{\frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{c}-b\int \frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}$$

$$\Big\downarrow \text{2849}$$

$$2bc\left(\frac{\frac{b\int \frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{2}{1-\frac{c}{x}}}d\frac{1}{1-\frac{c}{x}}}{c}+\frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{c}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}$$

$$\Big\downarrow \text{2752}$$

$$2bc\left(\frac{\frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{2c}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}$$

input
```
Int[(a + b*ArcTanh[c/x])^2/x^2,x]
```

output
```
-((a + b*ArcTanh[c/x])^2/x) + 2*b*c*(-1/2*(a + b*ArcTanh[c/x])^2/(b*c^2) +
 (((a + b*ArcTanh[c/x])*Log[2/(1 - c/x)])/c + (b*PolyLog[2, 1 - 2/(1 - c/x
)])/(2*c))/c)
```

3.148.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.148.4 Maple [A] (verified)

Time = 1.97 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.54

method	result
derivativedivides	$-\frac{\frac{c a^2}{x}+b^2\left(\operatorname{arctanh}\left(\frac{c}{x}\right)^2\left(\frac{c}{x}-1\right)+2\operatorname{arctanh}\left(\frac{c}{x}\right)^2-2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}+1\right)-\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)\right)+\frac{2abc\,\mathrm{arc}\cdots}{\cdots}}{c}$
default	$-\frac{\frac{c a^2}{x}+b^2\left(\operatorname{arctanh}\left(\frac{c}{x}\right)^2\left(\frac{c}{x}-1\right)+2\operatorname{arctanh}\left(\frac{c}{x}\right)^2-2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}+1\right)-\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)\right)+\frac{2abc\,\mathrm{arc}\cdots}{\cdots}}{c}$
parts	$-\frac{a^2}{x}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{x}-\frac{b^2\operatorname{arctanh}\left(\frac{c}{x}\right)^2}{c}+\frac{2b^2\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}+1\right)}{c}+\frac{b^2\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right)}{c}-\cdots$

input
```
int((a+b*arctanh(c/x))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c*(c/x*a^2+b^2*(arctanh(c/x)^2*(c/x-1)+2*arctanh(c/x)^2-2*arctanh(c/x)*
ln((1+c/x)^2/(1-c^2/x^2)+1)-polylog(2,-(1+c/x)^2/(1-c^2/x^2)))+2*a*b*c/x*a
rctanh(c/x)+a*b*ln(1-c^2/x^2))
```

3.148.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c/x)^2 + 2*a*b*arctanh(c/x) + a^2)/x^2, x)
```

3.148.6 Sympy [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*atanh(c/x))**2/x**2,x)
```

output
```
Integral((a + b*atanh(c/x))**2/x**2, x)
```

3.148.7 Maxima [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="maxima")
```

output
```
1/4*(c^3*integrate(-log(x)^2/(c^3*x^2 - c*x^4), x) + c^2*(log(-c^2 + x^2)/
c^3 - log(x^2)/c^3) - 4*c^2*integrate(-x*log(c + x)/(c^3*x^2 - c*x^4), x)
+ 2*c^2*integrate(-x*log(x)/(c^3*x^2 - c*x^4), x) + 2*c*(log(-c + x)/c^2 -
 log(x)/c^2 + 1/(c*x))*log(-c/x + 1) - c*(log(c + x)/c^2 - log(-c + x)/c^2
) - c*integrate(-x^2*log(x)^2/(c^3*x^2 - c*x^4), x) - 2*c*integrate(-x^2*l
og(c + x)/(c^3*x^2 - c*x^4), x) + 4*c*integrate(-x^2*log(x)/(c^3*x^2 - c*x
^4), x) - log(-c/x + 1)^2/x - (c*log(c + x)^2 - 2*((c + x)*log(c + x) - (c
 + x)*log(x) - c)*log(-c + x))/(c*x) - (x*log(-c + x)^2 + x*log(x)^2 - 2*(
x*log(x) - x)*log(-c + x) - 2*x*log(x) + 2*c)/(c*x) - 2*integrate(-x^3*log
(c + x)/(c^3*x^2 - c*x^4), x) + 2*integrate(-x^3*log(x)/(c^3*x^2 - c*x^4),
 x))*b^2 - a*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a^2/x
```

3.148.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = \int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^2/x^2, x)
```

3.148.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx = \int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^2}{x^2}\,dx$$

input
```
int((a + b*atanh(c/x))^2/x^2,x)
```

output
```
int((a + b*atanh(c/x))^2/x^2, x)
```

3.149 $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

3.149.1 Optimal result

Integrand size = 16, antiderivative size = 87

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = -\frac{ab}{cx}-\frac{b^2\coth^{-1}\left(\frac{x}{c}\right)}{cx}+\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2x^2}-\frac{b^2\log\left(1-\frac{c^2}{x^2}\right)}{2c^2}$$

output
```
-a*b/c/x-b^2*arccoth(x/c)/c/x+1/2*(a+b*arccoth(x/c))^2/c^2-1/2*(a+b*arccot
h(x/c))^2/x^2-1/2*b^2*ln(1-c^2/x^2)/c^2
```

3.149.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = -\frac{a^2c^2+2abcx+2bc(ac+bx)\text{arctanh}\left(\frac{c}{x}\right)+b^2(c^2-x^2)\,\text{arctanh}\left(\frac{c}{x}\right)^2-2b^2x^2\log(x)+abx^2\log(-c+x)}{2c^2x^2}$$

input
```
Integrate[(a + b*ArcTanh[c/x])^2/x^3,x]
```

output
```
-1/2*(a^2*c^2 + 2*a*b*c*x + 2*b*c*(a*c + b*x)*ArcTanh[c/x] + b^2*(c^2 - x^
2)*ArcTanh[c/x]^2 - 2*b^2*x^2*Log[x] + a*b*x^2*Log[-c + x] + b^2*x^2*Log[-
c + x] - a*b*x^2*Log[c + x] + b^2*x^2*Log[c + x])/(c^2*x^2)
```

3.149.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6454, 6452, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{6454}$$

$$-\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{6452}$$

$$bc\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{\left(1-\frac{c^2}{x^2}\right)x^2}d\frac{1}{x}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

$$\Big\downarrow \text{6542}$$

$$bc\left(\frac{\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}}{c^2}-\frac{\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

$$\Big\downarrow \text{2009}$$

$$bc\left(\frac{\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}}{c^2}-\frac{\frac{a}{x}+\frac{b\text{arctanh}\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}}{c^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

$$\Big\downarrow \text{6510}$$

$$bc\left(\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2bc^3}-\frac{\frac{a}{x}+\frac{b\text{arctanh}\left(\frac{c}{x}\right)}{x}+\frac{b\log\left(1-\frac{c^2}{x^2}\right)}{2c}}{c^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2x^2}$$

input
```
Int[(a + b*ArcTanh[c/x])^2/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c/x])^2/x^2 + b*c*((a + b*ArcTanh[c/x])^2/(2*b*c^3) -
(a/x + (b*ArcTanh[c/x])/x + (b*Log[1 - c^2/x^2])/(2*c))/c^2)
```

3.149.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.149.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.52

method	result
parallelrisch	$\frac{x^2 \operatorname{arctanh}\left(\frac{c}{x}\right)^2 b^2-\operatorname{arctanh}\left(\frac{c}{x}\right)^2 b^2 c^2+2b^2 \ln(x)x^2-2\ln(x-c)x^2b^2+2x^2 \operatorname{arctanh}\left(\frac{c}{x}\right)ab-2x^2 \operatorname{arctanh}\left(\frac{c}{x}\right)b^2-2x \operatorname{arctanh}}{2x^2c^2}$
parts	$-\frac{a^2}{2x^2}-\frac{b^2\left(\frac{c^2 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{2x^2}+\frac{c \operatorname{arctanh}\left(\frac{c}{x}\right)}{x}+\frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}+\frac{\ln\left(\frac{c}{x}-1\right)^2}{8}-\frac{\ln\left(\frac{c}{x}-1\right)\ln\left(\frac{c}{2x}\right.}{4}\right.}{c^2}$
derivativedivides	$-\frac{\frac{a^2c^2}{2x^2}+b^2\left(\frac{c^2 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{2x^2}+\frac{c \operatorname{arctanh}\left(\frac{c}{x}\right)}{x}+\frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}+\frac{\ln\left(\frac{c}{x}-1\right)^2}{8}-\frac{\ln\left(\frac{c}{x}-1\right)\ln\left(\frac{c}{2x}+\right.}{4}\right.}{}$
default	$-\frac{\frac{a^2c^2}{2x^2}+b^2\left(\frac{c^2 \operatorname{arctanh}\left(\frac{c}{x}\right)^2}{2x^2}+\frac{c \operatorname{arctanh}\left(\frac{c}{x}\right)}{x}+\frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(\frac{c}{x}-1\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{c}{x}\right)\ln\left(1+\frac{c}{x}\right)}{2}+\frac{\ln\left(\frac{c}{x}-1\right)^2}{8}-\frac{\ln\left(\frac{c}{x}-1\right)\ln\left(\frac{c}{2x}+\right.}{4}\right.}{}$
risch	Expression too large to display

input
```
int((a+b*arctanh(c/x))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2*arctanh(c/x)^2*b^2-arctanh(c/x)^2*b^2*c^2+2*b^2*ln(x)*x^2-2*ln(x-
c)*x^2*b^2+2*x^2*arctanh(c/x)*a*b-2*x^2*arctanh(c/x)*b^2-2*x*arctanh(c/x)*
b^2*c-2*arctanh(c/x)*a*b*c^2-2*a*b*c*x-a^2*c^2)/x^2/c^2
```

3.149.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.49

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$=\frac{8\,b^2x^2\log\left(x\right)-4\,a^2c^2-8\,abcx+4\,(ab-b^2)x^2\log\left(c+x\right)-4\,(ab+b^2)x^2\log\left(-c+x\right)-(b^2c^2-b^2x^2)\log}{8\,c^2x^2}$$

input
```
integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="fricas")
```

output
```
1/8*(8*b^2*x^2*log(x) - 4*a^2*c^2 - 8*a*b*c*x + 4*(a*b - b^2)*x^2*log(c +
x) - 4*(a*b + b^2)*x^2*log(-c + x) - (b^2*c^2 - b^2*x^2)*log(-(c + x)/(c -
 x))^2 - 4*(a*b*c^2 + b^2*c*x)*log(-(c + x)/(c - x)))/(c^2*x^2)
```

3.149.6 Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.43

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$$
$$= \begin{cases} -\frac{a^2}{2x^2} - \frac{ab\,\text{atanh}\left(\frac{c}{x}\right)}{x^2} - \frac{ab}{cx} + \frac{ab\,\text{atanh}\left(\frac{c}{x}\right)}{c^2} - \frac{b^2\,\text{atanh}^2\left(\frac{c}{x}\right)}{2x^2} - \frac{b^2\,\text{atanh}\left(\frac{c}{x}\right)}{cx} + \frac{b^2\log(x)}{c^2} - \frac{b^2\log(-c+x)}{c^2} + \frac{b^2\,\text{atanh}^2\left(\frac{c}{x}\right)}{2c^2} - \\ -\frac{a^2}{2x^2} \end{cases}$$

input
```
integrate((a+b*atanh(c/x))**2/x**3,x)
```

output
```
Piecewise((-a**2/(2*x**2) - a*b*atanh(c/x)/x**2 - a*b/(c*x) + a*b*atanh(c/
x)/c**2 - b**2*atanh(c/x)**2/(2*x**2) - b**2*atanh(c/x)/(c*x) + b**2*log(x
)/c**2 - b**2*log(-c + x)/c**2 + b**2*atanh(c/x)**2/(2*c**2) - b**2*atanh(
c/x)/c**2, Ne(c, 0)), (-a**2/(2*x**2), True))
```

3.149.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(81) = 162.

Time = 0.20 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.90

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = \frac{1}{2}\left(c\left(\frac{\log(c+x)}{c^3} - \frac{\log(-c+x)}{c^3} - \frac{2}{c^2x}\right) - \frac{2\,\text{artanh}\left(\frac{c}{x}\right)}{x^2}\right)ab$$
$$-\frac{1}{8}\left(c^2\left(\frac{\log(c+x)^2 - 2(\log(c+x)-2)\log(-c+x) + \log(-c+x)^2 + 4\log(c+x)}{c^4} - \frac{8\log(x)}{c^4}\right) - \right.$$
$$-\frac{b^2\,\text{artanh}\left(\frac{c}{x}\right)^2}{2x^2} - \frac{a^2}{2x^2}$$

input
```
integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="maxima")
```

output
```
1/2*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2
)*a*b - 1/8*(c^2*((log(c + x)^2 - 2*(log(c + x) - 2)*log(-c + x) + log(-c
+ x)^2 + 4*log(c + x))/c^4 - 8*log(x)/c^4) - 4*c*(log(c + x)/c^3 - log(-c
+ x)/c^3 - 2/(c^2*x))*arctanh(c/x))*b^2 - 1/2*b^2*arctanh(c/x)^2/x^2 - 1/2
*a^2/x^2
```

3.149.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(81) = 162$.

Time = 0.29 (sec) , antiderivative size = 255, normalized size of antiderivative = 2.93

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = -\frac{\frac{b^2(c+x)\log\left(-\frac{c+x}{c-x}\right)^2}{\left(\frac{(c+x)^2c}{(c-x)^2}-\frac{2(c+x)c}{c-x}+c\right)(c-x)} - \frac{2\,b^2\log\left(-\frac{c+x}{c-x}+1\right)}{c} + \frac{2\,b^2\log\left(-\frac{c+x}{c-x}\right)}{c} - \frac{2\left(b^2-\frac{2\,ab(c+x)}{c-x}-\frac{b^2(c+x)}{c-x}\right)\log\left(-\frac{c+x}{c-x}\right)}{\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c} - \frac{4\left(ab-\frac{a^2(c+x)}{c-x}-\frac{ab(c+x)}{c-x}\right)}{\frac{(c+x)^2c}{(c-x)^2}-\frac{2\,(c+x)c}{c-x}+c}}{2\,c}$$

input
```
integrate((a+b*arctanh(c/x))^2/x^3,x, algorithm="giac")
```

output
```
-1/2*(b^2*(c + x)*log(-(c + x)/(c - x))^2/(((c + x)^2*c/(c - x)^2 - 2*(c +
 x)*c/(c - x) + c)*(c - x)) - 2*b^2*log(-(c + x)/(c - x) + 1)/c + 2*b^2*lo
g(-(c + x)/(c - x))/c - 2*(b^2 - 2*a*b*(c + x)/(c - x) - b^2*(c + x)/(c -
x))*log(-(c + x)/(c - x))/((c + x)^2*c/(c - x)^2 - 2*(c + x)*c/(c - x) + c
) - 4*(a*b - a^2*(c + x)/(c - x) - a*b*(c + x)/(c - x))/((c + x)^2*c/(c -
x)^2 - 2*(c + x)*c/(c - x) + c))/c
```

3.149.9 Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.70

$$\begin{aligned}\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx = {} & \ln\left(1-\frac{c}{x}\right)\left(\frac{a\,b}{2\,x^2}-\ln\left(\frac{c}{x}+1\right)\left(\frac{b^2}{4\,c^2}-\frac{b^2}{4\,x^2}\right)+\frac{b^2\,(2\,c\,x-c^2)}{8\,c^2\,x^2}\right.\\ & \left.+\frac{b^2\,(2\,c^2+4\,x\,c)}{16\,c^2\,x^2}\right)-\frac{\frac{a^2}{2}+\frac{a\,b\,x}{c}}{x^2}+\ln\left(\frac{c}{x}+1\right)^2\left(\frac{b^2}{8\,c^2}-\frac{b^2}{8\,x^2}\right)\\ & +\ln\left(1-\frac{c}{x}\right)^2\left(\frac{b^2}{8\,c^2}-\frac{b^2}{8\,x^2}\right)-\frac{\ln(x-c)\,(b^2+a\,b)}{2\,c^2}\\ & +\frac{\ln(c+x)\,(a\,b-b^2)}{2\,c^2}-\frac{\ln\left(\frac{c}{x}+1\right)\left(\frac{a\,b}{2}+\frac{b^2\,x}{2\,c}\right)}{x^2}+\frac{b^2\,\ln(x)}{c^2}\end{aligned}$$

input
```
int((a + b*atanh(c/x))^2/x^3,x)
```

output
```
log(1 - c/x)*((a*b)/(2*x^2) - log(c/x + 1)*(b^2/(4*c^2) - b^2/(4*x^2)) + (
b^2*(2*c*x - c^2))/(8*c^2*x^2) + (b^2*(4*c*x + 2*c^2))/(16*c^2*x^2)) - (a^
2/2 + (a*b*x)/c)/x^2 + log(c/x + 1)^2*(b^2/(8*c^2) - b^2/(8*x^2)) + log(1
- c/x)^2*(b^2/(8*c^2) - b^2/(8*x^2)) - (log(x - c)*(a*b + b^2))/(2*c^2) +
(log(c + x)*(a*b - b^2))/(2*c^2) - (log(c/x + 1)*((a*b)/2 + (b^2*x)/(2*c))
)/x^2 + (b^2*log(x))/c^2
```

3.149. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x^3}\,dx$

3.150 $\int x^3 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx$

3.150.1 Optimal result

Integrand size = 16, antiderivative size = 203

$$\begin{aligned}\int x^3 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx = {} & \frac{1}{4}b^3c^3x-\frac{1}{4}b^3c^4\coth^{-1}\left(\frac{x}{c}\right)+\frac{1}{4}b^2c^2x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right) \\ & -bc^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{4}bc^3x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2 \\ & +\frac{1}{4}bcx^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2 \\ & -\frac{1}{4}c^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3 \\ & -2b^2c^4\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1+\frac{c}{x}}\right) \\ & +b^3c^4\operatorname{PolyLog}\left(2,-1+\frac{2}{1+\frac{c}{x}}\right)\end{aligned}$$

output
```
1/4*b^3*c^3*x-1/4*b^3*c^4*arccoth(x/c)+1/4*b^2*c^2*x^2*(a+b*arccoth(x/c))-
b*c^4*(a+b*arccoth(x/c))^2+3/4*b*c^3*x*(a+b*arccoth(x/c))^2+1/4*b*c*x^3*(a
+b*arccoth(x/c))^2-1/4*c^4*(a+b*arccoth(x/c))^3+1/4*x^4*(a+b*arccoth(x/c))
^3-2*b^2*c^4*(a+b*arccoth(x/c))*ln(2-2/(1+c/x))+b^3*c^4*polylog(2,-1+2/(1+
c/x))
```

3.150.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.41

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\frac{1}{8}\left(-2ab^2c^4+6a^2bc^3x+2b^3c^3x+2ab^2c^2x^2+2a^2bcx^3+2a^3x^4+2b^2\left(bc\left(-4c^3+3c^2x+x^3\right)+3a\left(-c^4+x^4\right)\right)\text{arctanh}\left(\frac{c}{x}\right)^2+2b^3\left(-c^4+x^4\right)\text{arctanh}\left(\frac{c}{x}\right)^3+2b\text{arctanh}\left(\frac{c}{x}\right)\left(3a^2x^4+2abcx\left(3c^2+x^2\right)+b^2\left(-c^4+c^2x^2\right)-8b^2c^4\log\left(1-e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)+3a^2bc^4\log\left(1-\frac{c}{x}\right)-16ab^2c^4\log\left(\frac{c}{\sqrt{1-\frac{c^2}{x^2}}x}\right)-3a^2bc^4\log\left(\frac{c+x}{x}\right)+8b^3c^4\,\text{PolyLog}\left(2,e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[x^3*(a + b*ArcTanh[c/x])^3,x]
```

output
```
(-2*a*b^2*c^4 + 6*a^2*b*c^3*x + 2*b^3*c^3*x + 2*a*b^2*c^2*x^2 + 2*a^2*b*c*
x^3 + 2*a^3*x^4 + 2*b^2*(b*c*(-4*c^3 + 3*c^2*x + x^3) + 3*a*(-c^4 + x^4))*
ArcTanh[c/x]^2 + 2*b^3*(-c^4 + x^4)*ArcTanh[c/x]^3 + 2*b*ArcTanh[c/x]*(3*a
^2*x^4 + 2*a*b*c*x*(3*c^2 + x^2) + b^2*(-c^4 + c^2*x^2) - 8*b^2*c^4*Log[1
- E^(-2*ArcTanh[c/x])]) + 3*a^2*b*c^4*Log[1 - c/x] - 16*a*b^2*c^4*Log[c/(S
qrt[1 - c^2/x^2]*x)] - 3*a^2*b*c^4*Log[(c + x)/x] + 8*b^3*c^4*PolyLog[2, E
^(-2*ArcTanh[c/x])])/8
```

3.150.3 Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6454, 6452, 6544, 6452, 6544, 6452, 264, 219, 6510, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx$$

$$\Big\downarrow \text{6454}$$

$$-\int x^5 \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 d\frac{1}{x}$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-\frac{3}{4}bc\int \frac{x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-\frac{3}{4}bc\left(c^2\int \frac{x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\int x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(c^2\int \frac{x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{2}{3}bc\int \frac{x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(c^2\left(c^2\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}\right)+\frac{2}{3}bc\left(c^2\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+ \right.\right.$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\int \frac{x^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)+c^2\left(c^2\int \frac{\left(a+b\text{arctan}\right.}{1-\frac{c^2}{x^2}}\right.\right.$$

$$\Big\downarrow \text{264}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\left(c^2\int \frac{1}{1-\frac{c^2}{x^2}}d\frac{1}{x}-x\right)-\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)+c^2\left(c^2\int \frac{(a}{}\right.\right.$$

$$\Big\downarrow \text{219}$$

3.150. $\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\int\frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(c\text{arctanh}\left(\frac{c}{x}\right)-x\right)\right)+c^2\left(c^2\int\frac{(a+\right.\right.$$

$\downarrow$ 6510

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(c^2\left(2bc\int\frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{c\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3b}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\right)+\frac{2}{3}bc\left(c^2\int\frac{x(a+ba}{1}\right.\right.$$

$\downarrow$ 6550

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\left(\int\frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{\frac{c}{x}+1}d\frac{1}{x}+\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}\right)-\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(c\text{arctanh}\left(\frac{c}{x}\right.\right.\right.\right.$$

$\downarrow$ 6494

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(c^2\left(2bc\left(-bc\int\frac{\log\left(2-\frac{2}{\frac{c}{x}+1}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}+\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)+\frac{c(a}{}\right.\right.\right.$$

$\downarrow$ 2897

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\left(\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}+\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{\frac{c}{x}+1}-1\right)\right)-\frac{1}{2}\right.\right.$$

input
```
Int[x^3*(a + b*ArcTanh[c/x])^3,x]
```

output
```
(x^4*(a + b*ArcTanh[c/x])^3)/4 - (3*b*c*(-1/3*(x^3*(a + b*ArcTanh[c/x])^2)
 + c^2*(-(x*(a + b*ArcTanh[c/x])^2) + (c*(a + b*ArcTanh[c/x])^3)/(3*b) + 2
*b*c*((a + b*ArcTanh[c/x])^2/(2*b) + (a + b*ArcTanh[c/x])*Log[2 - 2/(1 + c
/x)] - (b*PolyLog[2, -1 + 2/(1 + c/x)])/2)) + (2*b*c*(-1/2*(x^2*(a + b*Arc
Tanh[c/x])) + (b*c*(-x + c*ArcTanh[c/x]))/2 + c^2*((a + b*ArcTanh[c/x])^2/
(2*b) + (a + b*ArcTanh[c/x])*Log[2 - 2/(1 + c/x)] - (b*PolyLog[2, -1 + 2/(
1 + c/x)])/2)))/3))/4
```

3.150. $\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx$

3.150.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

3.150. $\int x^3 \left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3 dx$

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.150.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 88.03 (sec) , antiderivative size = 1267, normalized size of antiderivative = 6.24

method	result	size
derivativedivides	Expression too large to display	1267
default	Expression too large to display	1267
parts	Expression too large to display	1322
risch	Expression too large to display	44754

input
```
int(x^3*(a+b*arctanh(c/x))^3,x,method=_RETURNVERBOSE)
```

output
```
-c^4*(-3/16*I*b^3*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))^2*csgn(I*(1+c/x)^2/
(c^2/x^2-1))*arctanh(c/x)^2-3/8*I*b^3*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))
*csgn(I*(1+c/x)^2/(c^2/x^2-1))^2*arctanh(c/x)^2-3/16*I*b^3*Pi*csgn(I/(1-(1
+c/x)^2/(c^2/x^2-1)))*csgn(I*(1+c/x)^2/(c^2/x^2-1)/(1-(1+c/x)^2/(c^2/x^2-1
)))^2*arctanh(c/x)^2+3/8*I*b^3*Pi*arctanh(c/x)^2-1/4*b^3*arctanh(c/x)^2/c^
3*x^3-3/4*b^3*arctanh(c/x)^2/c*x-1/4*b^3/c^4*x^4*arctanh(c/x)^3-1/4*b^3*ar
ctanh(c/x)/c^2*x^2+3/8*I*b^3*Pi*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))^3*arctan
h(c/x)^2-3/16*I*b^3*Pi*csgn(I*(1+c/x)^2/(c^2/x^2-1))^3*arctanh(c/x)^2-3/16
*I*b^3*Pi*csgn(I*(1+c/x)^2/(c^2/x^2-1)/(1-(1+c/x)^2/(c^2/x^2-1)))^3*arctan
h(c/x)^2-3/8*I*b^3*Pi*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))^2*arctanh(c/x)^2+3
/16*I*b^3*Pi*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))*csgn(I*(1+c/x)^2/(c^2/x^2-1
))*csgn(I*(1+c/x)^2/(c^2/x^2-1)/(1-(1+c/x)^2/(c^2/x^2-1)))*arctanh(c/x)^2-
1/4*a^3/c^4*x^4+1/4*b^3/((1-c^2/x^2)^(1/2)+c/x+1)*(1-c^2/x^2)^(1/2)+2*b^3*
arctanh(c/x)*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))-3/4*b^3*arctanh(c/x)^2*ln((1+
c/x)/(1-c^2/x^2)^(1/2))-3/8*b^3*arctanh(c/x)^2*ln(c/x-1)+3/8*b^3*arctanh(c
/x)^2*ln(1+c/x)-1/4*b^3/(c/x+1-(1-c^2/x^2)^(1/2))*(1-c^2/x^2)^(1/2)+3*a^2*
b*(-1/4/c^4*x^4*arctanh(c/x)+1/8*ln(1+c/x)-1/12/c^3*x^3-1/4/c*x-1/8*ln(c/x
-1))+3*a*b^2*(-1/4/c^4*x^4*arctanh(c/x)^2-1/4*arctanh(c/x)*ln(c/x-1)+1/4*a
rctanh(c/x)*ln(1+c/x)-1/6/c^3*x^3*arctanh(c/x)-1/2/c*x*arctanh(c/x)+1/8*ln
(c/x-1)*ln(1/2*c/x+1/2)-1/16*ln(c/x-1)^2+1/8*(ln(1+c/x)-ln(1/2*c/x+1/2)...
```

3.150.5 Fricas [F]

$$\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arctanh(c/x)^3 + 3*a*b^2*x^3*arctanh(c/x)^2 + 3*a^2*b*x^3
*arctanh(c/x) + a^3*x^3, x)
```

3.150.6 Sympy [F]

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int x^3\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
integrate(x**3*(a+b*atanh(c/x))**3,x)
```

output
```
Integral(x**3*(a + b*atanh(c/x))**3, x)
```

3.150.7 Maxima [F]

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

output
```
3/4*a*b^2*x^4*arctanh(c/x)^2 + 1/4*a^3*x^4 + 1/8*(6*x^4*arctanh(c/x) - (3*
c^3*log(c + x) - 3*c^3*log(-c + x) - 6*c^2*x - 2*x^3)*c)*a^2*b + 1/16*((3*
c^2*log(c + x)^2 + 3*c^2*log(-c + x)^2 + 16*c^2*log(c + x) + 4*x^2 - 2*(3*
c^2*log(c + x) - 8*c^2)*log(-c + x))*c^2 - 4*(3*c^3*log(c + x) - 3*c^3*log
(-c + x) - 6*c^2*x - 2*x^3)*c*arctanh(c/x))*a*b^2 + 1/32*(16*c^5*integrate
(-log(c + x)/(c^2 - x^2), x) + 40*c^4*integrate(-x*log(c + x)/(c^2 - x^2),
 x) - 2*(c*log(c + x) - c*log(-c + x) - 2*x)*c^3 - (c^4 - x^4)*log(c + x)^
3 + (c^4 - x^4)*log(-c + x)^3 + 2*(c^2*log(-c^2 + x^2) + x^2)*c^2 + 8*c^2*
integrate(-x^3*log(c + x)/(c^2 - x^2), x) + 2*(3*c^3*x + c*x^3)*log(c + x)
^2 - (8*c^4 - 6*c^3*x - 2*c*x^3 + 3*(c^4 - x^4)*log(c + x))*log(-c + x)^2
- (4*c^2*x^2 - 3*(c^4 - x^4)*log(c + x)^2 + 4*(4*c^4 + 3*c^3*x + c*x^3)*lo
g(c + x))*log(-c + x))*b^3
```

3.150.8 Giac [F]

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3x^3\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^3*x^3, x)
```

3.150.9 Mupad [F(-1)]

Timed out.

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int x^3\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
int(x^3*(a + b*atanh(c/x))^3,x)
```

output
```
int(x^3*(a + b*atanh(c/x))^3, x)
```

3.151 $\int x^2 \left(a + b\,\mathrm{arctanh}\left(\frac{c}{x}\right)\right)^3 \, dx$

3.151.1 Optimal result

Integrand size = 16, antiderivative size = 217

$$\begin{aligned}\int x^2\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x}\right)\right)^3dx &= b^2c^2x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)-\frac{1}{2}bc^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\\ &+\frac{1}{2}bcx^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\\ &-\frac{1}{3}c^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{3}x^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\\ &-bc^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(2-\frac{2}{1+\frac{c}{x}}\right)\\ &+\frac{1}{2}b^3c^3\log\left(1-\frac{c^2}{x^2}\right)+b^3c^3\log(x)\\ &+b^2c^3\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\mathrm{PolyLog}\left(2,-1+\frac{2}{1+\frac{c}{x}}\right)\\ &+\frac{1}{2}b^3c^3\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1+\frac{c}{x}}\right)\end{aligned}$$

output
```
b^2*c^2*x*(a+b*arccoth(x/c))-1/2*b*c^3*(a+b*arccoth(x/c))^2+1/2*b*c*x^2*(a
+b*arccoth(x/c))^2-1/3*c^3*(a+b*arccoth(x/c))^3+1/3*x^3*(a+b*arccoth(x/c))
^3-b*c^3*(a+b*arccoth(x/c))^2*ln(2-2/(1+c/x))+1/2*b^3*c^3*ln(1-c^2/x^2)+b^
3*c^3*ln(x)+b^2*c^3*(a+b*arccoth(x/c))*polylog(2,-1+2/(1+c/x))+1/2*b^3*c^3
*polylog(3,-1+2/(1+c/x))
```

3.151.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.48

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx = \frac{1}{6}\left(3a^2bcx^2+2a^3x^3+6a^2bx^3\text{arctanh}\left(\frac{c}{x}\right)+3a^2bc^3\log\left(-c^2+x^2\right)+6ab^2\left(c^2x+\left(-c^3+x^3\right)\text{arctanh}\left(\frac{c}{x}\right)^2+c\,\text{arctanh}\left(\frac{c}{x}\right)\left(-c^2+x^2-2c^2\log\left(1-e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)+c^3\,\text{PolyLog}\left(2,e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)+\frac{1}{4}b^3\left(-ic^3\pi^3+24c^2x\text{arctanh}\left(\frac{c}{x}\right)-12c^3\text{arctanh}\left(\frac{c}{x}\right)^2+12cx^2\text{arctanh}\left(\frac{c}{x}\right)^2+8c^3\text{arctanh}\left(\frac{c}{x}\right)^3+8x^3\text{arctanh}\left(\frac{c}{x}\right)^3-24c^3\text{arctanh}\left(\frac{c}{x}\right)^2\log\left(1-e^{2\text{arctanh}\left(\frac{c}{x}\right)}\right)-24c^3\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^2}}}\right)-24c^3\log\left(\frac{c}{x}\right)-24c^3\text{arctanh}\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{2\text{arctanh}\left(\frac{c}{x}\right)}\right)+12c^3\,\text{PolyLog}\left(3,e^{2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)\right)$$

input
```
Integrate[x^2*(a + b*ArcTanh[c/x])^3,x]
```

output
```
(3*a^2*b*c*x^2 + 2*a^3*x^3 + 6*a^2*b*x^3*ArcTanh[c/x] + 3*a^2*b*c^3*Log[-c
^2 + x^2] + 6*a*b^2*(c^2*x + (-c^3 + x^3)*ArcTanh[c/x]^2 + c*ArcTanh[c/x]*
(-c^2 + x^2 - 2*c^2*Log[1 - E^(-2*ArcTanh[c/x])]) + c^3*PolyLog[2, E^(-2*A
rcTanh[c/x])]) + (b^3*((-I)*c^3*Pi^3 + 24*c^2*x*ArcTanh[c/x] - 12*c^3*ArcT
anh[c/x]^2 + 12*c*x^2*ArcTanh[c/x]^2 + 8*c^3*ArcTanh[c/x]^3 + 8*x^3*ArcTan
h[c/x]^3 - 24*c^3*ArcTanh[c/x]^2*Log[1 - E^(2*ArcTanh[c/x])] - 24*c^3*Log[
1/Sqrt[1 - c^2/x^2]] - 24*c^3*Log[c/x] - 24*c^3*ArcTanh[c/x]*PolyLog[2, E^
(2*ArcTanh[c/x])] + 12*c^3*PolyLog[3, E^(2*ArcTanh[c/x])]))/4)/6
```

3.151.3 Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {6454, 6452, 6544, 6452, 6544, 6452, 243, 47, 14, 16, 6510, 6550, 6494, 6618, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

$$\Big\downarrow \text{6454}$$

$$-\int x^4\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^3 d\frac{1}{x}$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^3-bc\int \frac{x^3\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{3}x^3\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^3-bc\left(c^2\int \frac{x\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\int x^3\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^2 d\frac{1}{x}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\int \frac{x\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\int \frac{x^2\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{1}{2}x^2\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{3}x^3\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\int \frac{x\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\left(c^2\int \frac{a+b\,\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\int x^2\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)d\frac{1}{x}\right)-\frac{1}{2}x^2\left(a+b\right.\right.$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\int \frac{x\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\left(c^2\int \frac{a+b\,\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\int \frac{x}{1-\frac{c^2}{x^2}}d\frac{1}{x}-x\left(a+b\,\text{arctanh}\left(\frac{c}{x}\right)\right)\right)-\right.$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\left(c^2\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\int \frac{x}{1-\frac{c^2}{x^2}}d\frac{1}{x^2}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right)\right.$$

$\downarrow$ 47

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\left(c^2\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\left(c^2\int \frac{1}{1-\frac{c^2}{x^2}}d\frac{1}{x^2}+\int xd\frac{1}{x^2}\right)-x\left(a+\right.\right.\right.$$

$\downarrow$ 14

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\left(c^2\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{1}{2}bc\left(c^2\int \frac{1}{1-\frac{c^2}{x^2}}d\frac{1}{x^2}+\log\left(\frac{1}{x^2}\right)\right)-x\left(a+\right.\right.\right.$$

$\downarrow$ 16

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\left(c^2\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\text{lo}\right.\right.\right.$$

$\downarrow$ 6510

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+bc\left(\frac{c\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^2}\right)-\log\left(1\right.\right.\right.\right.$$

$\downarrow$ 6550

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(\int \frac{x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{\frac{c}{x}+1}d\frac{1}{x}+\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3b}\right)+bc\left(\frac{c\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}-x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\right.\right.$$

$\downarrow$ 6494

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-2bc\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\log\left(2-\frac{2}{\frac{c}{x}+1}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3b}+\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\text{arctanh}\left(\right.\right.\right.\right.$$

$\downarrow$ 6618

3.151. $\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{\frac{c}{x}+1}-1\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{2c}-\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,\frac{2}{\frac{c}{x}+1}-1\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}\right)+\frac{(a+b\text{arctanh}(\frac{c}{x}$$

$\downarrow$ 7164

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$bc\left(c^2\left(-2bc\left(\frac{\text{PolyLog}\left(2,\frac{2}{\frac{c}{x}+1}-1\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{2c}+\frac{b\,\text{PolyLog}\left(3,\frac{2}{\frac{c}{x}+1}-1\right)}{4c}\right)+\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3b}+\text{lo}\right.\right.$$

input
```
Int[x^2*(a + b*ArcTanh[c/x])^3,x]
```

output
```
(x^3*(a + b*ArcTanh[c/x])^3)/3 - b*c*(-1/2*(x^2*(a + b*ArcTanh[c/x])^2) +
b*c*(-(x*(a + b*ArcTanh[c/x])) + (c*(a + b*ArcTanh[c/x])^2)/(2*b) + (b*c*(
-Log[1 - c^2/x^2] + Log[x^(-2)]))/2) + c^2*((a + b*ArcTanh[c/x])^3/(3*b) +
 (a + b*ArcTanh[c/x])^2*Log[2 - 2/(1 + c/x)] - 2*b*c*(((a + b*ArcTanh[c/x]
)*PolyLog[2, -1 + 2/(1 + c/x)])/(2*c) + (b*PolyLog[3, -1 + 2/(1 + c/x)])/(
4*c))))
```

3.151.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^(m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

rule 6618
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[2, 1 - u]/(2*c*d)), x
] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(d +
 e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d +
e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 + c*x))^2, 0]
```

3.151. $\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx$

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.151.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 303.08 (sec) , antiderivative size = 1773, normalized size of antiderivative = 8.17

method	result	size
derivativedivides	Expression too large to display	1773
default	Expression too large to display	1773
parts	Expression too large to display	1775

input
```
int(x^2*(a+b*arctanh(c/x))^3,x,method=_RETURNVERBOSE)
```

output
```
-c^3*(-1/3*a^3/c^3*x^3+b^3*(-1/3/c^3*x^3*arctanh(c/x)^3-1/2*arctanh(c/x)^2
*ln(1+c/x)-1/2*arctanh(c/x)^2*ln(c/x-1)-1/2/c^2*x^2*arctanh(c/x)^2+ln(c/x)
*arctanh(c/x)^2+arctanh(c/x)^2*ln((1+c/x)/(1-c^2/x^2)^(1/2))-arctanh(c/x)^
2*ln((1+c/x)^2/(1-c^2/x^2)-1)+arctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2
))+2*arctanh(c/x)*polylog(2,(1+c/x)/(1-c^2/x^2)^(1/2))-2*polylog(3,(1+c/x)
/(1-c^2/x^2)^(1/2))+arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))+2*arcta
nh(c/x)*polylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))-2*polylog(3,-(1+c/x)/(1-c^2/
x^2)^(1/2))-1/12*arctanh(c/x)*(-3*I*arctanh(c/x)*csgn(I*(1+c/x)^2/(c^2/x^2
-1)/(1-(1+c/x)^2/(c^2/x^2-1)))^3*Pi*c/x-6*I*arctanh(c/x)*csgn(I*(1+c/x)^2/
(c^2/x^2-1))^2*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))*Pi*c/x-6*I*arctanh(c/x)*P
i*c/x+3*I*arctanh(c/x)*csgn(I*(1+c/x)^2/(c^2/x^2-1))*csgn(I*(1+c/x)^2/(c^2
/x^2-1)/(1-(1+c/x)^2/(c^2/x^2-1)))*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))*Pi*c/
x+6*I*arctanh(c/x)*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1)/(1-(1+c/x)^2/(c^2/x^2
-1)))^2*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))*Pi*c/x-6*I*arctanh(c/x)*csgn(I*(
-(1+c/x)^2/(c^2/x^2-1)-1)/(1-(1+c/x)^2/(c^2/x^2-1)))^3*Pi*c/x-3*I*arctanh(
c/x)*csgn(I*(1+c/x)^2/(c^2/x^2-1))*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))^2*Pi*
c/x-6*I*arctanh(c/x)*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1))*csgn(I*(-(1+c/x)^2
/(c^2/x^2-1)-1)/(1-(1+c/x)^2/(c^2/x^2-1)))*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)
))*Pi*c/x-3*I*arctanh(c/x)*csgn(I*(1+c/x)^2/(c^2/x^2-1)/(1-(1+c/x)^2/(c^2/
x^2-1)))^2*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))*Pi*c/x-3*I*arctanh(c/x)*cs...
```

3.151.5 Fricas [F]

$$\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arctanh(c/x)^3 + 3*a*b^2*x^2*arctanh(c/x)^2 + 3*a^2*b*x^2
*arctanh(c/x) + a^3*x^2, x)
```

3.151.6 Sympy [F]

$$\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int x^2\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
integrate(x**2*(a+b*atanh(c/x))**3,x)
```

output
```
Integral(x**2*(a + b*atanh(c/x))**3, x)
```

3.151.7 Maxima [F]

$$\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*x^3 + 1/2*(2*x^3*arctanh(c/x) + (c^2*log(-c^2 + x^2) + x^2)*c)*a^2
*b + 1/24*(b^3*c^3 - b^3*x^3)*log(-c + x)^3 + 1/8*(b^3*c*x^2 + 2*a*b^2*x^3
 + (b^3*c^3 + b^3*x^3)*log(c + x))*log(-c + x)^2 - integrate(-1/8*((b^3*c*
x^2 - b^3*x^3)*log(c + x)^3 + 6*(a*b^2*c*x^2 - a*b^2*x^3)*log(c + x)^2 + (
2*b^3*c*x^2 + 4*a*b^2*x^3 - 3*(b^3*c*x^2 - b^3*x^3)*log(c + x)^2 + 2*(b^3*
c^3 - 6*a*b^2*c*x^2 + (6*a*b^2 + b^3)*x^3)*log(c + x))*log(-c + x))/(c - x
), x)
```

3.151.8 Giac [F]

$$\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3 x^2\, dx$$

input
```
integrate(x^2*(a+b*arctanh(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^3*x^2, x)
```

3.151.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3 dx = \int x^2\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^3 dx$$

input
```
int(x^2*(a + b*atanh(c/x))^3,x)
```

output
```
int(x^2*(a + b*atanh(c/x))^3, x)
```

3.152 $\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx$

3.152.1 Optimal result

Integrand size = 14, antiderivative size = 135

$$\begin{aligned}\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx &= -\frac{3}{2}bc^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2+\frac{3}{2}bcx\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\\&\quad-\frac{1}{2}c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\\&\quad-3b^2c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\log\left(2-\frac{2}{1+\frac{c}{x}}\right)\\&\quad+\frac{3}{2}b^3c^2\,\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{c}{x}}\right)\end{aligned}$$

output
```
-3/2*b*c^2*(a+b*arccoth(x/c))^2+3/2*b*c*x*(a+b*arccoth(x/c))^2-1/2*c^2*(a+
b*arccoth(x/c))^3+1/2*x^2*(a+b*arccoth(x/c))^3-3*b^2*c^2*(a+b*arccoth(x/c)
)*ln(2-2/(1+c/x))+3/2*b^3*c^2*polylog(2,-1+2/(1+c/x))
```

3.152.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.43

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx = \frac{1}{4}\left(6b^2(-c+x)(bc+a(c+x))\text{arctanh}\left(\frac{c}{x}\right)^2 + 2b^3\left(-c^2+x^2\right)\text{arctanh}\left(\frac{c}{x}\right)^3 + 6b\text{arctanh}\left(\frac{c}{x}\right)\left(ax(2bc+ax) - 2b^2c^2\log\left(1-e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right) + a\left(3abc^2\log\left(1-\frac{c}{x}\right) - 12b^2c^2\log\left(\frac{c}{\sqrt{1-\frac{c^2}{x^2}}x}\right) + a\left(6bcx+2ax^2-3bc^2\log\left(\frac{c+x}{x}\right)\right)\right) + 6b^3c^2\text{PolyLog}\left(2,e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[x*(a + b*ArcTanh[c/x])^3,x]
```

output
```
(6*b^2*(-c + x)*(b*c + a*(c + x))*ArcTanh[c/x]^2 + 2*b^3*(-c^2 + x^2)*ArcT
anh[c/x]^3 + 6*b*ArcTanh[c/x]*(a*x*(2*b*c + a*x) - 2*b^2*c^2*Log[1 - E^(-2
*ArcTanh[c/x])]) + a*(3*a*b*c^2*Log[1 - c/x] - 12*b^2*c^2*Log[c/(Sqrt[1 -
c^2/x^2]*x)] + a*(6*b*c*x + 2*a*x^2 - 3*b*c^2*Log[(c + x)/x])) + 6*b^3*c^2
*PolyLog[2, E^(-2*ArcTanh[c/x])])/4
```

3.152.3 Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {6454, 6452, 6544, 6452, 6510, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3dx$$

$$\Big\downarrow \text{6454}$$

$$-\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\int \frac{x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{2}x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3-\frac{3}{2}bc\left(c^2\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{2}bc\left(c^2\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}+2bc\int \frac{x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{2}x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{2}bc\left(2bc\int \frac{x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{c\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3}{3b}-x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{6550}$$

$$\frac{1}{2}x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{2}bc\left(2bc\left(\int \frac{x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)}{\frac{c}{x}+1}d\frac{1}{x}+\frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}\right)+\frac{c\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3}{3b}-x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2\right)$$

$$\Big\downarrow \text{6494}$$

$$\frac{1}{2}x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{2}bc\left(2bc\left(-bc\int \frac{\log\left(2-\frac{2}{\frac{c}{x}+1}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}+\frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}+\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)\right)+\frac{c(a+ba}{}\right.$$

$$\Big\downarrow \text{2897}$$

3.152. $\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3-$$
$$\frac{3}{2}bc\left(2bc\left(\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2b}+\log\left(2-\frac{2}{\frac{c}{x}+1}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{\frac{c}{x}+1}-1\right)\right)+\frac{c(a+$$

input
```
Int[x*(a + b*ArcTanh[c/x])^3,x]
```

output
```
(x^2*(a + b*ArcTanh[c/x])^3)/2 - (3*b*c*(-(x*(a + b*ArcTanh[c/x])^2) + (c*
(a + b*ArcTanh[c/x])^3)/(3*b) + 2*b*c*((a + b*ArcTanh[c/x])^2/(2*b) + (a +
 b*ArcTanh[c/x])*Log[2 - 2/(1 + c/x)] - (b*PolyLog[2, -1 + 2/(1 + c/x)])/2
)))/2
```

3.152.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.152.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.14 (sec) , antiderivative size = 5036, normalized size of antiderivative = 37.30

output too large to display

input
```
int(x*(a+b*arctanh(c/x))^3,x)
```

output
```
result too large to display
```

3.152.5 Fricas [F]

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arctanh(c/x)^3 + 3*a*b^2*x*arctanh(c/x)^2 + 3*a^2*b*x*arcta
nh(c/x) + a^3*x, x)
```

3.152.6 Sympy [F]

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int x\left(a+b\operatorname{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
integrate(x*(a+b*atanh(c/x))**3,x)
```

output
```
Integral(x*(a + b*atanh(c/x))**3, x)
```

3.152.7 Maxima [F]

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

output
```
3/2*a*b^2*x^2*arctanh(c/x)^2 + 1/2*a^3*x^2 + 3/4*(2*x^2*arctanh(c/x) - (c*
log(c + x) - c*log(-c + x) - 2*x)*c)*a^2*b + 3/8*((log(c + x)^2 - 2*(log(c
 + x) - 2)*log(-c + x) + log(-c + x)^2 + 4*log(c + x))*c^2 - 4*(c*log(c +
x) - c*log(-c + x) - 2*x)*c*arctanh(c/x))*a*b^2 + 1/16*(6*c*x*log(c + x)^2
 - (c^2 - x^2)*log(c + x)^3 + (c^2 - x^2)*log(-c + x)^3 - 3*(2*c^2 - 2*c*x
 + (c^2 - x^2)*log(c + x))*log(-c + x)^2 + 3*((c^2 - x^2)*log(c + x)^2 - 4
*(c^2 + c*x)*log(c + x))*log(-c + x) + 2*integrate(-6*(c^3 + 3*c^2*x)*log(
c + x)/(c^2 - x^2), x))*b^3
```

3.152.8 Giac [F]

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int\left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctanh(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^3*x, x)
```

3.152.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx=\int x\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3\,dx$$

input
```
int(x*(a + b*atanh(c/x))^3,x)
```

output
```
int(x*(a + b*atanh(c/x))^3, x)
```

3.153 $\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx$

3.153.1 Optimal result

Integrand size = 12, antiderivative size = 108

$$\begin{aligned}\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx =& c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3+x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\\ &-3bc\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2c}{c-x}\right)\\ &-3b^2c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2c}{c-x}\right)\\ &+\frac{3}{2}b^3c\,\text{PolyLog}\left(3,1-\frac{2c}{c-x}\right)\end{aligned}$$

output
```
c*(a+b*arccoth(x/c))^3+x*(a+b*arccoth(x/c))^3-3*b*c*(a+b*arccoth(x/c))^2*l
n(2*c/(c-x))-3*b^2*c*(a+b*arccoth(x/c))*polylog(2,1-2*c/(c-x))+3/2*b^3*c*p
olylog(3,1-2*c/(c-x))
```

3.153.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.83

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx = a^3x+3a^2bx\text{arctanh}\left(\frac{c}{x}\right)+\frac{3}{2}a^2bc\log\left(-c^2+x^2\right) - 3ab^2\left(\text{arctanh}\left(\frac{c}{x}\right)\left((c-x)\text{arctanh}\left(\frac{c}{x}\right) + 2c\log\left(1-e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)-c\,\text{PolyLog}\left(2,e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right) + \frac{1}{8}b^3\left(-ic\pi^3+8c\text{arctanh}\left(\frac{c}{x}\right)^3+8x\text{arctanh}\left(\frac{c}{x}\right)^3 - 24c\text{arctanh}\left(\frac{c}{x}\right)^2\log\left(1-e^{2\text{arctanh}\left(\frac{c}{x}\right)}\right) - 24c\text{arctanh}\left(\frac{c}{x}\right)\text{PolyLog}\left(2,e^{2\text{arctanh}\left(\frac{c}{x}\right)}\right) + 12c\,\text{PolyLog}\left(3,e^{2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c/x])^3,x]
```

output
```
a^3*x + 3*a^2*b*x*ArcTanh[c/x] + (3*a^2*b*c*Log[-c^2 + x^2])/2 - 3*a*b^2*(
ArcTanh[c/x]*((c - x)*ArcTanh[c/x] + 2*c*Log[1 - E^(-2*ArcTanh[c/x])]) - c
*PolyLog[2, E^(-2*ArcTanh[c/x])]) + (b^3*((-I)*c*Pi^3 + 8*c*ArcTanh[c/x]^3
 + 8*x*ArcTanh[c/x]^3 - 24*c*ArcTanh[c/x]^2*Log[1 - E^(2*ArcTanh[c/x])] -
24*c*ArcTanh[c/x]*PolyLog[2, E^(2*ArcTanh[c/x])] + 12*c*PolyLog[3, E^(2*Ar
cTanh[c/x])]))/8
```

3.153.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6440, 6437, 27, 6547, 27, 6471, 27, 6621, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx$$

$$\Big\downarrow \text{6440}$$

$$\int \left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3 dx$$

$$\Big\downarrow \text{6437}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-\frac{3b\int\frac{c^2x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c^2-x^2}dx}{c}$$

$$\Big\downarrow \text{27}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-3bc\int\frac{x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c^2-x^2}dx$$

$$\Big\downarrow \text{6547}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-3bc\left(\frac{\int\frac{c\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c-x}dx}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}\right)$$

$$\Big\downarrow \text{27}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-3bc\left(\int\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{c-x}dx-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}\right)$$

$$\Big\downarrow \text{6471}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-$$
$$3bc\left(-\frac{2b\int\frac{c^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c-x}\right)}{c^2-x^2}dx}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}+\log\left(\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\right)$$

$$\Big\downarrow \text{27}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-$$
$$3bc\left(-2bc\int\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2c}{c-x}\right)}{c^2-x^2}dx-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}+\log\left(\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\right)$$

$$\Big\downarrow \text{6621}$$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-$$
$$3bc\left(-2bc\left(\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2c}{c-x}\right)}{c^2-x^2}dx-\frac{\text{PolyLog}\left(2,1-\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{2c}\right)-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}+\right.$$

$$\Big\downarrow \text{7164}$$

3.153. $\int\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3\,dx$

$$x\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3-$$
$$3bc\left(-2bc\left(\frac{b\,\text{PolyLog}\left(3,1-\frac{2c}{c-x}\right)}{4c}-\frac{\text{PolyLog}\left(2,1-\frac{2c}{c-x}\right)\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)}{2c}\right)-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{3b}+\log\left(\right.\right.$$

input
```
Int[(a + b*ArcTanh[c/x])^3,x]
```

output
```
x*(a + b*ArcCoth[x/c])^3 - 3*b*c*(-1/3*(a + b*ArcCoth[x/c])^3/b + (a + b*A
rcCoth[x/c])^2*Log[(2*c)/(c - x)] - 2*b*c*(-1/2*((a + b*ArcCoth[x/c])*Poly
Log[2, 1 - (2*c)/(c - x)])/c + (b*PolyLog[3, 1 - (2*c)/(c - x)])/(4*c)))
```

3.153.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 6437
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcCoth[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcCoth[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6440
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*
ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0
]
```

rule 6471
```
Int[((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6547
```
Int[(((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcCoth[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcCoth[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6621
```
Int[(Log[u_]*((a_.) + ArcCoth[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcCoth[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcCoth[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.153.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 338.37 (sec) , antiderivative size = 1475, normalized size of antiderivative = 13.66

method	result	size
parts	Expression too large to display	1475
derivativedivides	Expression too large to display	1478
default	Expression too large to display	1478

input
```
int((a+b*arctanh(c/x))^3,x,method=_RETURNVERBOSE)
```

output
```
a^3*x-b^3*c*(-1/c*x*arctanh(c/x)^3-3/2*arctanh(c/x)^2*ln(1+c/x)-3/2*arctan
h(c/x)^2*ln(c/x-1)+3*ln(c/x)*arctanh(c/x)^2+3*arctanh(c/x)^2*ln((1+c/x)/(1
-c^2/x^2)^(1/2))-arctanh(c/x)^3-3*arctanh(c/x)^2*ln((1+c/x)^2/(1-c^2/x^2)-
1)+3*arctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))+6*arctanh(c/x)*polylog
(2,-(1+c/x)/(1-c^2/x^2)^(1/2))-6*polylog(3,-(1+c/x)/(1-c^2/x^2)^(1/2))+3*a
rctanh(c/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2))+6*arctanh(c/x)*polylog(2,(1+
c/x)/(1-c^2/x^2)^(1/2))-6*polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))+3/4*(-2*I*P
i*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1)/(1-(
1+c/x)^2/(c^2/x^2-1)))^2+I*Pi*csgn(I*(1+c/x)^2/(c^2/x^2-1))^3+I*Pi*csgn(I/
(1-(1+c/x)^2/(c^2/x^2-1)))*csgn(I*(1+c/x)^2/(c^2/x^2-1)/(1-(1+c/x)^2/(c^2/
x^2-1)))^2+2*I*Pi*csgn(I*(1+c/x)/(1-c^2/x^2)^(1/2))*csgn(I*(1+c/x)^2/(c^2/
x^2-1))^2+2*I*Pi*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1)/(1-(1+c/x)^2/(c^2/x^2-1
)))^3+2*I*Pi*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))^3+I*Pi*csgn(I*(1+c/x)/(1-c^
2/x^2)^(1/2))^2*csgn(I*(1+c/x)^2/(c^2/x^2-1))+2*I*Pi-2*I*Pi*csgn(I/(1-(1+c
/x)^2/(c^2/x^2-1)))^2-I*Pi*csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))*csgn(I*(1+c/x
)^2/(c^2/x^2-1))*csgn(I*(1+c/x)^2/(c^2/x^2-1)/(1-(1+c/x)^2/(c^2/x^2-1)))-2
*I*Pi*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1))*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1)
/(1-(1+c/x)^2/(c^2/x^2-1)))^2+I*Pi*csgn(I*(1+c/x)^2/(c^2/x^2-1)/(1-(1+c/x)
^2/(c^2/x^2-1)))^3+2*I*Pi*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1))*csgn(I/(1-(1+
c/x)^2/(c^2/x^2-1)))*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1)/(1-(1+c/x)^2/(c^...
```

3.153.5 Fricas [F]

$$\int \left(a + b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x}\right) + a\right)^3 dx$$

input
```
integrate((a+b*arctanh(c/x))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/x
) + a^3, x)
```

3.153.6 Sympy [F]

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3 dx$$

input
```
integrate((a+b*atanh(c/x))**3,x)
```

output
```
Integral((a + b*atanh(c/x))**3, x)
```

3.153.7 Maxima [F]

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3 dx$$

input
```
integrate((a+b*arctanh(c/x))^3,x, algorithm="maxima")
```

output
```
3/2*(2*x*arctanh(c/x) + c*log(-c^2 + x^2))*a^2*b + a^3*x + 1/8*(b^3*c - b^
3*x)*log(-c + x)^3 + 3/8*(2*a*b^2*x + (b^3*c + b^3*x)*log(c + x))*log(-c +
 x)^2 - integrate(-1/8*((b^3*c - b^3*x)*log(c + x)^3 + 6*(a*b^2*c - a*b^2*
x)*log(c + x)^2 + 3*(4*a*b^2*x - (b^3*c - b^3*x)*log(c + x)^2 - 2*(2*a*b^2
*c - b^3*c - (2*a*b^2 + b^3)*x)*log(c + x))*log(-c + x))/(c - x), x)
```

3.153.8 Giac [F]

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3 dx$$

input
```
integrate((a+b*arctanh(c/x))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^3, x)
```

3.153.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3 \,dx=\int \left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3 \,dx$$

input
```
int((a + b*atanh(c/x))^3,x)
```

output
```
int((a + b*atanh(c/x))^3, x)
```

3.154 $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

3.154.1 Optimal result

Integrand size = 16, antiderivative size = 208

$$\begin{aligned}\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx = & -2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3\text{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right) \\ & +\frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right) \\ & -\frac{3}{2}b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{c}{x}}\right) \\ & -\frac{3}{2}b^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right) \\ & +\frac{3}{2}b^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(3,-1+\frac{2}{1-\frac{c}{x}}\right) \\ & +\frac{3}{4}b^3\text{PolyLog}\left(4,1-\frac{2}{1-\frac{c}{x}}\right)-\frac{3}{4}b^3\text{PolyLog}\left(4,-1+\frac{2}{1-\frac{c}{x}}\right)\end{aligned}$$

output
```
2*(a+b*arccoth(x/c))^3*arctanh(-1+2/(1-c/x))+3/2*b*(a+b*arccoth(x/c))^2*po
lylog(2,1-2/(1-c/x))-3/2*b*(a+b*arccoth(x/c))^2*polylog(2,-1+2/(1-c/x))-3/
2*b^2*(a+b*arccoth(x/c))*polylog(3,1-2/(1-c/x))+3/2*b^2*(a+b*arccoth(x/c))
*polylog(3,-1+2/(1-c/x))+3/4*b^3*polylog(4,1-2/(1-c/x))-3/4*b^3*polylog(4,
-1+2/(1-c/x))
```

3.154.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.79

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx = a^3\log(x)+\frac{3}{2}a^2b\left(\operatorname{PolyLog}\left(2,-\frac{c}{x}\right)-\operatorname{PolyLog}\left(2,\frac{c}{x}\right)\right) + 3ab^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}\operatorname{arctanh}\left(\frac{c}{x}\right)^3 + \operatorname{arctanh}\left(\frac{c}{x}\right)^2\log\left(1+e^{-2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) - \operatorname{arctanh}\left(\frac{c}{x}\right)^2\log\left(1-e^{2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) - \operatorname{arctanh}\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) - \operatorname{arctanh}\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right)+\frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right)\right) + \frac{1}{64}b^3\left(-\pi^4+32\operatorname{arctanh}\left(\frac{c}{x}\right)^4 + 64\operatorname{arctanh}\left(\frac{c}{x}\right)^3\log\left(1+e^{-2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) - 64\operatorname{arctanh}\left(\frac{c}{x}\right)^3\log\left(1-e^{2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) - 96\operatorname{arctanh}\left(\frac{c}{x}\right)^2\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) - 96\operatorname{arctanh}\left(\frac{c}{x}\right)^2\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) - 96\operatorname{arctanh}\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) + 96\operatorname{arctanh}\left(\frac{c}{x}\right)\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) - 48\operatorname{PolyLog}\left(4,-e^{-2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right) - 48\operatorname{PolyLog}\left(4,e^{2\operatorname{arctanh}\left(\frac{c}{x}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c/x])^3/x,x]
```

output
```
a^3*Log[x] + (3*a^2*b*(PolyLog[2, -(c/x)] - PolyLog[2, c/x]))/2 + 3*a*b^2*
((-1/24*I)*Pi^3 + (2*ArcTanh[c/x]^3)/3 + ArcTanh[c/x]^2*Log[1 + E^(-2*ArcT
anh[c/x])] - ArcTanh[c/x]^2*Log[1 - E^(2*ArcTanh[c/x])] - ArcTanh[c/x]*Pol
yLog[2, -E^(-2*ArcTanh[c/x])] - ArcTanh[c/x]*PolyLog[2, E^(2*ArcTanh[c/x])
] - PolyLog[3, -E^(-2*ArcTanh[c/x])]/2 + PolyLog[3, E^(2*ArcTanh[c/x])]/2)
 + (b^3*(-Pi^4 + 32*ArcTanh[c/x]^4 + 64*ArcTanh[c/x]^3*Log[1 + E^(-2*ArcTa
nh[c/x])] - 64*ArcTanh[c/x]^3*Log[1 - E^(2*ArcTanh[c/x])] - 96*ArcTanh[c/x
]^2*PolyLog[2, -E^(-2*ArcTanh[c/x])] - 96*ArcTanh[c/x]^2*PolyLog[2, E^(2*A
rcTanh[c/x])] - 96*ArcTanh[c/x]*PolyLog[3, -E^(-2*ArcTanh[c/x])] + 96*ArcT
anh[c/x]*PolyLog[3, E^(2*ArcTanh[c/x])] - 48*PolyLog[4, -E^(-2*ArcTanh[c/x
])] - 48*PolyLog[4, E^(2*ArcTanh[c/x])]))/64
```

3.154.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6450, 6448, 6614, 6620, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}dx$$

$$\Big\downarrow \text{ 6450}$$

$$-\int x\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$

$$\Big\downarrow \text{ 6448}$$

$$6bc\int \frac{\text{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}-2\text{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3$$

$$\Big\downarrow \text{ 6614}$$

$$6bc\left(\frac{1}{2}\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\log\left(2-\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{1}{2}\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2\log\left(\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}\right)-2\text{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3$$

$$\Big\downarrow \text{ 6620}$$

$$6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2c}-b\int\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}\right)+\frac{1}{2}\left(b\right.\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3$$

$\big\downarrow$ 6624

$$6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2c}-b\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{2c}-\frac{1}{2}b\int\frac{\text{Poly}}{}\right.\right.\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3$$

$\big\downarrow$ 7164

$$6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2c}-b\left(\frac{\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{2c}-\frac{b\,\text{PolyLog}}{}\right.\right.\right.$$
$$2\text{arctanh}\left(1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3$$

input
```
Int[(a + b*ArcTanh[c/x])^3/x,x]
```

output
```
-2*ArcTanh[1 - 2/(1 - c/x)]*(a + b*ArcTanh[c/x])^3 + 6*b*c*((((a + b*ArcTa
nh[c/x])^2*PolyLog[2, 1 - 2/(1 - c/x)])/(2*c) - b*(((a + b*ArcTanh[c/x])*P
olyLog[3, 1 - 2/(1 - c/x)])/(2*c) - (b*PolyLog[4, 1 - 2/(1 - c/x)])/(4*c))
)/2 + (-1/2*((a + b*ArcTanh[c/x])^2*PolyLog[2, -1 + 2/(1 - c/x)])/c + b*((
(a + b*ArcTanh[c/x])*PolyLog[3, -1 + 2/(1 - c/x)])/(2*c) - (b*PolyLog[4, -
1 + 2/(1 - c/x)])/(4*c)))/2)
```

3.154.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

3.154. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.154.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 246.33 (sec) , antiderivative size = 1452, normalized size of antiderivative = 6.98

method	result	size
parts	Expression too large to display	1452
derivativedivides	Expression too large to display	1454
default	Expression too large to display	1454

input
```
int((a+b*arctanh(c/x))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(-ln(c/x)*arctanh(c/x)^3+arctanh(c/x)^3*ln((1+c/x)^2/(1-c^2/
x^2)-1)-arctanh(c/x)^3*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))-3*arctanh(c/x)^2*po
lylog(2,-(1+c/x)/(1-c^2/x^2)^(1/2))+6*arctanh(c/x)*polylog(3,-(1+c/x)/(1-c
^2/x^2)^(1/2))-6*polylog(4,-(1+c/x)/(1-c^2/x^2)^(1/2))-arctanh(c/x)^3*ln(1
-(1+c/x)/(1-c^2/x^2)^(1/2))-3*arctanh(c/x)^2*polylog(2,(1+c/x)/(1-c^2/x^2)
^(1/2))+6*arctanh(c/x)*polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))-6*polylog(4,(1
+c/x)/(1-c^2/x^2)^(1/2))-1/2*I*Pi*csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1)/(1-(1+
c/x)^2/(c^2/x^2-1)))*(csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1))*csgn(I/(1-(1+c/x)
^2/(c^2/x^2-1)))-csgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1))*csgn(I*(-(1+c/x)^2/(c^
2/x^2-1)-1)/(1-(1+c/x)^2/(c^2/x^2-1)))-csgn(I/(1-(1+c/x)^2/(c^2/x^2-1)))*c
sgn(I*(-(1+c/x)^2/(c^2/x^2-1)-1)/(1-(1+c/x)^2/(c^2/x^2-1)))+csgn(I*(-(1+c/
x)^2/(c^2/x^2-1)-1)/(1-(1+c/x)^2/(c^2/x^2-1)))^2)*arctanh(c/x)^3+3/2*arcta
nh(c/x)^2*polylog(2,-(1+c/x)^2/(1-c^2/x^2))-3/2*arctanh(c/x)*polylog(3,-(1
+c/x)^2/(1-c^2/x^2))+3/4*polylog(4,-(1+c/x)^2/(1-c^2/x^2)))+3*a*b^2*(-ln(c
/x)*arctanh(c/x)^2+arctanh(c/x)*polylog(2,-(1+c/x)^2/(1-c^2/x^2))-1/2*poly
log(3,-(1+c/x)^2/(1-c^2/x^2))+arctanh(c/x)^2*ln((1+c/x)^2/(1-c^2/x^2)-1)-a
rctanh(c/x)^2*ln(1+(1+c/x)/(1-c^2/x^2)^(1/2))-2*arctanh(c/x)*polylog(2,-(1
+c/x)/(1-c^2/x^2)^(1/2))+2*polylog(3,-(1+c/x)/(1-c^2/x^2)^(1/2))-arctanh(c
/x)^2*ln(1-(1+c/x)/(1-c^2/x^2)^(1/2))-2*arctanh(c/x)*polylog(2,(1+c/x)/(1-
c^2/x^2)^(1/2))+2*polylog(3,(1+c/x)/(1-c^2/x^2)^(1/2))-1/2*I*Pi*csgn(I*...
```

3.154.5 Fricas [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/
x) + a^3)/x, x)
```

3.154.6 Sympy [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*atanh(c/x))**3/x,x)
```

output
```
Integral((a + b*atanh(c/x))**3/x, x)
```

3.154.7 Maxima [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^3/x,x, algorithm="maxima")
```

output
```
a^3*log(x) + integrate(1/8*b^3*(log(c/x + 1) - log(-c/x + 1))^3/x + 3/4*a*
b^2*(log(c/x + 1) - log(-c/x + 1))^2/x + 3/2*a^2*b*(log(c/x + 1) - log(-c/
x + 1))/x, x)
```

3.154.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^3/x, x)
```

3.154.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3}{x}\,dx$$

input
```
int((a + b*atanh(c/x))^3/x,x)
```

output
```
int((a + b*atanh(c/x))^3/x, x)
```

3.155 $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

3.155.1 Optimal result

Integrand size = 16, antiderivative size = 126

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = -\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{c}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{x}+\frac{3b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2\log\left(\frac{2}{1-\frac{c}{x}}\right)}{c}+\frac{3b^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{c}-\frac{3b^3\,\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)}{2c}$$

output
```
-(a+b*arccoth(x/c))^3/c-(a+b*arccoth(x/c))^3/x+3*b*(a+b*arccoth(x/c))^2*ln
(2/(1-c/x))/c+3*b^2*(a+b*arccoth(x/c))*polylog(2,1-2/(1-c/x))/c-3/2*b^3*po
lylog(3,1-2/(1-c/x))/c
```

3.155.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.63

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = -\frac{a^3}{x}-\frac{3a^2b\text{arctanh}\left(\frac{c}{x}\right)}{x}-\frac{3a^2b\log\left(1-\frac{c^2}{x^2}\right)}{2c}$$
$$-\frac{3ab^2\left(\text{arctanh}\left(\frac{c}{x}\right)\left(-\text{arctanh}\left(\frac{c}{x}\right)+\frac{c\text{arctanh}\left(\frac{c}{x}\right)}{x}-2\log\left(1+e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)}{c}$$
$$-\frac{b^3\left(\text{arctanh}\left(\frac{c}{x}\right)^2\left(-\text{arctanh}\left(\frac{c}{x}\right)+\frac{c\text{arctanh}\left(\frac{c}{x}\right)}{x}-3\log\left(1+e^{-2\text{arctanh}\left(\frac{c}{x}\right)}\right)\right)+3\text{arctanh}\left(\frac{c}{x}\right)\text{PolyLog}\left(2\ldots\right.\right.}{c}$$

input
```
Integrate[(a + b*ArcTanh[c/x])^3/x^2,x]
```

output
```
-(a^3/x) - (3*a^2*b*ArcTanh[c/x])/x - (3*a^2*b*Log[1 - c^2/x^2])/(2*c) - (
3*a*b^2*(ArcTanh[c/x]*(-ArcTanh[c/x] + (c*ArcTanh[c/x])/x - 2*Log[1 + E^(-
2*ArcTanh[c/x])]) + PolyLog[2, -E^(-2*ArcTanh[c/x])]))/c - (b^3*(ArcTanh[c
/x]^2*(-ArcTanh[c/x] + (c*ArcTanh[c/x])/x - 3*Log[1 + E^(-2*ArcTanh[c/x])]
) + 3*ArcTanh[c/x]*PolyLog[2, -E^(-2*ArcTanh[c/x])] + (3*PolyLog[3, -E^(-2
*ArcTanh[c/x])])/2))/c
```

3.155.3 Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6454, 6436, 6546, 6470, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}dx$$
$$\Big\downarrow \text{6454}$$
$$-\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3d\frac{1}{x}$$
$$\Big\downarrow \text{6436}$$

$$3bc\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{\left(1-\frac{c^2}{x^2}\right)x}d\frac{1}{x}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}$$

$$\Big\downarrow \text{6546}$$

$$3bc\left(\frac{\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c}{x}}d\frac{1}{x}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3bc^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}$$

$$\Big\downarrow \text{6470}$$

$$3bc\left(\frac{\frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{c}-2b\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)\log\left(\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3bc^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}$$

$$\Big\downarrow \text{6620}$$

$$3bc\left(\frac{\frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{c}-2b\left(\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{2c}\right)}{c}-\frac{(a+ba}{}\right.$$

$$\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}$$

$$\Big\downarrow \text{7164}$$

$$3bc\left(\frac{\frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{c}-2b\left(\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x}}\right)}{4c}-\frac{\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{2c}\right)}{c}-\frac{(a+b\text{arctanh}}{3bc^2}\right.$$

$$\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}$$

input
```
Int[(a + b*ArcTanh[c/x])^3/x^2,x]
```

3.155. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

output
```
-((a + b*ArcTanh[c/x])^3/x) + 3*b*c*(-1/3*(a + b*ArcTanh[c/x])^3/(b*c^2) +
 (((a + b*ArcTanh[c/x])^2*Log[2/(1 - c/x)])/c - 2*b*(-1/2*((a + b*ArcTanh[
c/x])*PolyLog[2, 1 - 2/(1 - c/x)])/c + (b*PolyLog[3, 1 - 2/(1 - c/x)])/(4*
c)))/c)
```

3.155.3.1 Defintions of rubi rules used

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.155. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$

3.155.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(124) = 248$.

Time = 3.64 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.10

method	result
derivativedivides	$-\frac{\frac{c a^3}{x}+b^3\left(\operatorname{arctanh}\left(\frac{c}{x}\right)^3\left(\frac{c}{x}-1\right)+2\operatorname{arctanh}\left(\frac{c}{x}\right)^3-3\operatorname{arctanh}\left(\frac{c}{x}\right)^2\ln\left(\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}+1\right)-3\operatorname{arctanh}\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right.\right.}{}$
default	$-\frac{\frac{c a^3}{x}+b^3\left(\operatorname{arctanh}\left(\frac{c}{x}\right)^3\left(\frac{c}{x}-1\right)+2\operatorname{arctanh}\left(\frac{c}{x}\right)^3-3\operatorname{arctanh}\left(\frac{c}{x}\right)^2\ln\left(\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}+1\right)-3\operatorname{arctanh}\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right.\right.}{}$
parts	$-\frac{a^3}{x}-\frac{b^3\left(\operatorname{arctanh}\left(\frac{c}{x}\right)^3\left(\frac{c}{x}-1\right)+2\operatorname{arctanh}\left(\frac{c}{x}\right)^3-3\operatorname{arctanh}\left(\frac{c}{x}\right)^2\ln\left(\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}+1\right)-3\operatorname{arctanh}\left(\frac{c}{x}\right)\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x}\right)^2}{1-\frac{c^2}{x^2}}\right.\right.}{c}$

input
```
int((a+b*arctanh(c/x))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c*(c/x*a^3+b^3*(arctanh(c/x)^3*(c/x-1)+2*arctanh(c/x)^3-3*arctanh(c/x)^
2*ln((1+c/x)^2/(1-c^2/x^2)+1)-3*arctanh(c/x)*polylog(2,-(1+c/x)^2/(1-c^2/x
^2))+3/2*polylog(3,-(1+c/x)^2/(1-c^2/x^2)))+3*a*b^2*(arctanh(c/x)^2*(c/x-1
)+2*arctanh(c/x)^2-2*arctanh(c/x)*ln((1+c/x)^2/(1-c^2/x^2)+1)-polylog(2,-(
1+c/x)^2/(1-c^2/x^2)))+3*a^2*b*(c/x*arctanh(c/x)+1/2*ln(1-c^2/x^2)))
```

3.155.5 Fricas [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\operatorname{artanh}\left(\frac{c}{x}\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/
x) + a^3)/x^2, x)
```

3.155.6 Sympy [F]

$$\int \frac{\left(a + b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = \int \frac{\left(a + b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*atanh(c/x))**3/x**2,x)
```

output
```
Integral((a + b*atanh(c/x))**3/x**2, x)
```

3.155.7 Maxima [F]

$$\int \frac{\left(a + b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = \int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right) + a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="maxima")
```

output
```
-3/2*a^2*b*(2*c*arctanh(c/x)/x + log(-c^2/x^2 + 1))/c - a^3/x + 1/8*((b^3*
c - b^3*x)*log(-c + x)^3 - 3*(2*a*b^2*c + (b^3*c + b^3*x)*log(c + x))*log(
-c + x)^2)/(c*x) - integrate(-1/8*((b^3*c^2 - b^3*c*x)*log(c + x)^3 + 6*(a
*b^2*c^2 - a*b^2*c*x)*log(c + x)^2 - 3*(4*a*b^2*c*x + (b^3*c^2 - b^3*c*x)*
log(c + x)^2 + 2*(2*a*b^2*c^2 + b^3*x^2 - (2*a*b^2*c - b^3*c)*x)*log(c + x
))*log(-c + x))/(c^2*x^2 - c*x^3), x)
```

3.155.8 Giac [F]

$$\int \frac{\left(a + b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx = \int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right) + a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^3/x^2, x)
```

3.155.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3}{x^2}\,dx$$

input
```
int((a + b*atanh(c/x))^3/x^2,x)
```

output
```
int((a + b*atanh(c/x))^3/x^2, x)
```

3.156 $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

3.156.1 Optimal result

Integrand size = 16, antiderivative size = 139

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx = -\frac{3b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2c^2}-\frac{3b\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^2}{2cx}+\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2c^2}-\frac{\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)^3}{2x^2}+\frac{3b^2\left(a+b\coth^{-1}\left(\frac{x}{c}\right)\right)\log\left(\frac{2}{1-\frac{c}{x}}\right)}{c^2}+\frac{3b^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{2c^2}$$

output
```
-3/2*b*(a+b*arccoth(x/c))^2/c^2-3/2*b*(a+b*arccoth(x/c))^2/c/x+1/2*(a+b*ar
ccoth(x/c))^3/c^2-1/2*(a+b*arccoth(x/c))^3/x^2+3*b^2*(a+b*arccoth(x/c))*ln
(2/(1-c/x))/c^2+3/2*b^3*polylog(2,1-2/(1-c/x))/c^2
```

3.156.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.40

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx = \frac{6b^2(-c+x)(bx+a(c+x))\text{arctanh}\left(\frac{c}{x}\right)^2+2b^3\left(-c^2+x^2\right)\text{arctanh}\left(\frac{c}{x}\right)^3+6b\text{arctanh}\left(\frac{c}{x}\right)\left(-ac(ac+2bx)+\cdots\right.}{\cdots}$$

input
```
Integrate[(a + b*ArcTanh[c/x])^3/x^3,x]
```

output
```
(6*b^2*(-c + x)*(b*x + a*(c + x))*ArcTanh[c/x]^2 + 2*b^3*(-c^2 + x^2)*ArcT
anh[c/x]^3 + 6*b*ArcTanh[c/x]*(-(a*c*(a*c + 2*b*x)) + 2*b^2*x^2*Log[1 + E^
(-2*ArcTanh[c/x])]) + a*(12*b^2*x^2*Log[1/Sqrt[1 - c^2/x^2]] - a*(2*a*c^2
+ 6*b*c*x + 3*b*x^2*Log[1 - c/x] - 3*b*x^2*Log[(c + x)/x])) - 6*b^3*x^2*Po
lyLog[2, -E^(-2*ArcTanh[c/x])])/(4*c^2*x^2)
```

3.156.3 Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6454, 6452, 6542, 6436, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}dx$$

$$\Big\downarrow \text{ 6454}$$

$$-\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x}d\frac{1}{x}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{3}{2}bc\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{\left(1-\frac{c^2}{x^2}\right)x^2}d\frac{1}{x}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \text{ 6542}$$

$$\frac{3}{2}bc\left(\frac{\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}}{c^2}-\frac{\int \left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \text{ 6436}$$

$$\frac{3}{2}bc\left(\frac{\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{1-\frac{c^2}{x^2}}d\frac{1}{x}}{c^2}-\frac{\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}-2bc\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{\left(1-\frac{c^2}{x^2}\right)x}d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \text{6510}$$

$$\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3bc^3}-\frac{\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}-2bc\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{\left(1-\frac{c^2}{x^2}\right)x}d\frac{1}{x}}{c^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \text{6546}$$

$$\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3bc^3}-\frac{\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(\frac{\int \frac{a+b\text{arctanh}\left(\frac{c}{x}\right)}{1-\frac{c}{x}}d\frac{1}{x}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{2bc^2}\right)}{c^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \text{6470}$$

$$\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3bc^3}-\frac{\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(\frac{\frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{c}-b\int \frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{c^2}{x^2}}d\frac{1}{x}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right.}{2bc^2}\right.}{c^2}\right.$$

$$-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$$\Big\downarrow \text{2849}$$

3.156. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}dx$

$$\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3bc^3}-\frac{\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(\frac{\frac{b\int\frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)}{1-\frac{2}{1-\frac{c}{x}}}d\frac{1}{1-\frac{c}{x}}}{c}+\frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{c}}{c}-\frac{\left(a+b\text{arctanh}\right.}{2bc^2}\right.}{c^2}\right.$$

$$\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

$\downarrow$ 2752

$$\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{3bc^3}-\frac{\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^2}{x}-2bc\left(\frac{\frac{\log\left(\frac{2}{1-\frac{c}{x}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x}}\right)}{2c}}{c}-\frac{\left(a+b\text{arctanh}\right.}{2bc^2}\right.}{c^2}\right.$$

$$\frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{2x^2}$$

input
```
Int[(a + b*ArcTanh[c/x])^3/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c/x])^3/x^2 + (3*b*c*((a + b*ArcTanh[c/x])^3/(3*b*c^3)
 - ((a + b*ArcTanh[c/x])^2/x - 2*b*c*(-1/2*(a + b*ArcTanh[c/x])^2/(b*c^2)
+ (((a + b*ArcTanh[c/x])*Log[2/(1 - c/x)])/c + (b*PolyLog[2, 1 - 2/(1 - c/
x)])/(2*c))/c))/c^2))/2
```

3.156.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

3.156. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.156. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$

3.156.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.70 (sec) , antiderivative size = 6081, normalized size of antiderivative = 43.75

output too large to display

input
```
int((a+b*arctanh(c/x))^3/x^3,x)
```

output
```
result too large to display
```

3.156.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c/x)^3 + 3*a*b^2*arctanh(c/x)^2 + 3*a^2*b*arctanh(c/
x) + a^3)/x^3, x)
```

3.156.6 Sympy [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*atanh(c/x))**3/x**3,x)
```

output
```
Integral((a + b*atanh(c/x))**3/x**3, x)
```

3.156.7 Maxima [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="maxima")
```

output
```
3/4*(c*(log(c + x)/c^3 - log(-c + x)/c^3 - 2/(c^2*x)) - 2*arctanh(c/x)/x^2
)*a^2*b - 3/8*(c^2*((log(c + x)^2 - 2*(log(c + x) - 2)*log(-c + x) + log(-
c + x)^2 + 4*log(c + x))/c^4 - 8*log(x)/c^4) - 4*c*(log(c + x)/c^3 - log(-
c + x)/c^3 - 2/(c^2*x))*arctanh(c/x))*a*b^2 + 1/64*(32*c^4*integrate(-1/4*
log(x)^3/(c^4*x^3 - c^2*x^5), x) - 3*c^3*(log(c + x)/c^5 - log(-c + x)/c^5
 - 2/(c^4*x)) + 48*c^3*integrate(-1/4*x*log(x)^2/(c^4*x^3 - c^2*x^5), x) +
 48*c^3*integrate(-1/4*x*log(x)/(c^4*x^3 - c^2*x^5), x) - 6*c*(2*log(-c +
x)/c^3 - 2*log(x)/c^3 + (c + 2*x)/(c^2*x^2))*log(-c/x + 1)^2 + 21*c^2*(log
(c + x)/c^4 + log(-c + x)/c^4 - 2*log(x)/c^4) - 32*c^2*integrate(-1/4*x^2*
log(x)^3/(c^4*x^3 - c^2*x^5), x) + 48*c^2*integrate(-1/4*x^2*log(x)^2/(c^4
*x^3 - c^2*x^5), x) - 384*c^2*integrate(-1/4*x^2*log(c + x)/(c^4*x^3 - c^2
*x^5), x) + 144*c^2*integrate(-1/4*x^2*log(x)/(c^4*x^3 - c^2*x^5), x) - 18
*c*(log(c + x)/c^3 - log(-c + x)/c^3) + c*(6*(2*x^2*log(-c + x)^2 + 2*x^2*
log(x)^2 - 6*x^2*log(x) + c^2 + 6*c*x - 2*(2*x^2*log(x) - 3*x^2)*log(-c +
x))*log(-c/x + 1)/(c^3*x^2) - (4*x^2*log(-c + x)^3 - 4*x^2*log(x)^3 + 18*x
^2*log(x)^2 - 6*(2*x^2*log(x) - 3*x^2)*log(-c + x)^2 - 42*x^2*log(x) + 3*c
^2 + 42*c*x + 6*(2*x^2*log(x)^2 - 6*x^2*log(x) + 7*x^2)*log(-c + x))/(c^3*
x^2)) - 48*c*integrate(-1/4*x^3*log(x)^2/(c^4*x^3 - c^2*x^5), x) - 192*c*i
ntegrate(-1/4*x^3*log(c + x)/(c^4*x^3 - c^2*x^5), x) + 336*c*integrate(-1/
4*x^3*log(x)/(c^4*x^3 - c^2*x^5), x) + 4*log(-c/x + 1)^3/x^2 - 2*(12*c*...
```

3.156.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c/x))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x) + a)^3/x^3, x)
```

3.156.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x}\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*atanh(c/x))^3/x^3,x)
```

output
```
int((a + b*atanh(c/x))^3/x^3, x)
```

3.157 $\int x^7\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$

3.157.1 Optimal result

Integrand size = 14, antiderivative size = 54

$$\int x^7\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{1}{8}x^8\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)-\frac{1}{8}bc^4\text{arctanh}\left(\frac{x^2}{c}\right)$$

output
```
1/8*b*c^3*x^2+1/24*b*c*x^6+1/8*x^8*(a+b*arctanh(c/x^2))-1/8*b*c^4*arctanh(
x^2/c)
```

3.157.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int x^7\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{8}bc^3x^2+\frac{1}{24}bcx^6+\frac{ax^8}{8}+\frac{1}{8}bx^8\text{arctanh}\left(\frac{c}{x^2}\right)+\frac{1}{16}bc^4\log\left(-c+x^2\right)-\frac{1}{16}bc^4\log\left(c+x^2\right)$$

input
```
Integrate[x^7*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(b*c^3*x^2)/8 + (b*c*x^6)/24 + (a*x^8)/8 + (b*x^8*ArcTanh[c/x^2])/8 + (b*c
^4*Log[-c + x^2])/16 - (b*c^4*Log[c + x^2])/16
```

3.157.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6452, 795, 807, 25, 254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^7 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{4} b c \int \frac{x^5}{1-\frac{c^2}{x^4}}dx+\frac{1}{8} x^8 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{795}$$

$$\frac{1}{4} b c \int \frac{x^9}{x^4-c^2}dx+\frac{1}{8} x^8 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{8} b c \int -\frac{x^8}{c^2-x^4}dx^2+\frac{1}{8} x^8 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{8} x^8 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)-\frac{1}{8} b c \int \frac{x^8}{c^2-x^4}dx^2$$

$$\Big\downarrow \text{254}$$

$$\frac{1}{8} x^8 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)-\frac{1}{8} b c \int \left(\frac{c^4}{c^2-x^4}-c^2-x^4\right)dx^2$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8} x^8 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)+\frac{1}{8} b c \left(-c^3 \text{arctanh}\left(\frac{x^2}{c}\right)+c^2 x^2+\frac{x^6}{3}\right)$$

input
```
Int[x^7*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(x^8*(a + b*ArcTanh[c/x^2]))/8 + (b*c*(c^2*x^2 + x^6/3 - c^3*ArcTanh[x^2/c
]))/8
```

3.157.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.157.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

method	result
parallelrisch	$\frac{x^8 \operatorname{arctanh}\left(\frac{c}{x^2}\right) b}{8}+\frac{x^8 a}{8}+\frac{b c\, x^6}{24}+\frac{b\, c^3 x^2}{8}-\frac{\operatorname{arctanh}\left(\frac{c}{x^2}\right) b\, c^4}{8}$
parts	$\frac{x^8 a}{8}+b\left(\frac{x^8 \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{8}-\frac{c\left(-\frac{x^6}{6}-\frac{c^2 x^2}{2}+\frac{c^3 \ln\left(1+\frac{c}{x^2}\right)}{4}-\frac{c^3 \ln\left(\frac{c}{x^2}-1\right)}{4}\right)}{4}\right)$
derivativedivides	$\frac{x^8 a}{8}-b\left(-\frac{x^8 \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{8}+\frac{c\left(-\frac{x^6}{6}-\frac{c^2 x^2}{2}+\frac{c^3 \ln\left(1+\frac{c}{x^2}\right)}{4}-\frac{c^3 \ln\left(\frac{c}{x^2}-1\right)}{4}\right)}{4}\right)$
default	$\frac{x^8 a}{8}-b\left(-\frac{x^8 \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{8}+\frac{c\left(-\frac{x^6}{6}-\frac{c^2 x^2}{2}+\frac{c^3 \ln\left(1+\frac{c}{x^2}\right)}{4}-\frac{c^3 \ln\left(\frac{c}{x^2}-1\right)}{4}\right)}{4}\right)$
risch	$\frac{x^8 b \ln(x^2+c)}{16}-\frac{x^8 b \ln(-x^2+c)}{16}-\frac{i\pi b\, x^8 \operatorname{csgn}\left(\frac{i}{x^2}\right) \operatorname{csgn}\left(i(x^2+c)\right) \operatorname{csgn}\left(\frac{i(x^2+c)}{x^2}\right)}{32}-\frac{i\pi b\, x^8 \operatorname{csgn}\left(i(-x^2+c)\right) \operatorname{csgn}}{32}$

input
```
int(x^7*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/8*x^8*arctanh(c/x^2)*b+1/8*x^8*a+1/24*b*c*x^6+1/8*b*c^3*x^2-1/8*arctanh(
c/x^2)*b*c^4
```

3.157.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int x^7 \left(a+b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right) dx=\frac{1}{8} a x^8+\frac{1}{24} b c x^6+\frac{1}{8} b c^3 x^2+\frac{1}{16} \left(b x^8-b c^4\right) \log\left(\frac{x^2+c}{x^2-c}\right)$$

input
```
integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

output
```
1/8*a*x^8 + 1/24*b*c*x^6 + 1/8*b*c^3*x^2 + 1/16*(b*x^8 - b*c^4)*log((x^2 +
 c)/(x^2 - c))
```

3.157.6 Sympy [A] (verification not implemented)

Time = 3.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int x^7\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{ax^8}{8}-\frac{bc^4\,\text{atanh}\left(\frac{c}{x^2}\right)}{8}+\frac{bc^3x^2}{8}+\frac{bcx^6}{24}+\frac{bx^8\,\text{atanh}\left(\frac{c}{x^2}\right)}{8}$$

input
```
integrate(x**7*(a+b*atanh(c/x**2)),x)
```

output
```
a*x**8/8 - b*c**4*atanh(c/x**2)/8 + b*c**3*x**2/8 + b*c*x**6/24 + b*x**8*a
tanh(c/x**2)/8
```

3.157.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int x^7\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$$
$$=\frac{1}{8}\,ax^8+\frac{1}{48}\left(6\,x^8\,\text{artanh}\left(\frac{c}{x^2}\right)+\left(2\,x^6+6\,c^2x^2-3\,c^3\log\left(x^2+c\right)+3\,c^3\log\left(x^2-c\right)\right)c\right)b$$

input
```
integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

output
```
1/8*a*x^8 + 1/48*(6*x^8*arctanh(c/x^2) + (2*x^6 + 6*c^2*x^2 - 3*c^3*log(x^
2 + c) + 3*c^3*log(x^2 - c))*c)*b
```

3.157.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.31

$$\int x^7\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{16}\,bx^8\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{8}\,ax^8+\frac{1}{24}\,bcx^6+\frac{1}{8}\,bc^3x^2$$
$$-\frac{1}{16}\,bc^4\log\left(x^2+c\right)+\frac{1}{16}\,bc^4\log\left(-x^2+c\right)$$

input
```
integrate(x^7*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

output
```
1/16*b*x^8*log((x^2 + c)/(x^2 - c)) + 1/8*a*x^8 + 1/24*b*c*x^6 + 1/8*b*c^3
*x^2 - 1/16*b*c^4*log(x^2 + c) + 1/16*b*c^4*log(-x^2 + c)
```

3.157.9 Mupad [B] (verification not implemented)

Time = 3.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int x^7\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{a\,x^8}{8}+\frac{b\,c^3\,x^2}{8}+\frac{b\,x^8\,\ln\left(x^2+c\right)}{16}+\frac{b\,c\,x^6}{24}-\frac{b\,x^8\,\ln\left(x^2-c\right)}{16}+\frac{b\,c^4\,\mathrm{atan}\left(\frac{x^2\,1\mathrm{i}}{c}\right)\,1\mathrm{i}}{8}$$

input
```
int(x^7*(a + b*atanh(c/x^2)),x)
```

output
```
(a*x^8)/8 + (b*c^3*x^2)/8 + (b*x^8*log(c + x^2))/16 + (b*c^4*atan((x^2*1i)
/c)*1i)/8 + (b*c*x^6)/24 - (b*x^8*log(x^2 - c))/16
```

3.158 $\int x^5 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx$

3.158.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int x^5 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx=\frac{1}{12}bcx^4+\frac{1}{6}x^6\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc^3\log\left(c^2-x^4\right)$$

output
```
1/12*b*c*x^4+1/6*x^6*(a+b*arctanh(c/x^2))+1/12*b*c^3*ln(-x^4+c^2)
```

3.158.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int x^5 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx=\frac{1}{12}bcx^4+\frac{ax^6}{6}+\frac{1}{6}bx^6\text{arctanh}\left(\frac{c}{x^2}\right)+\frac{1}{12}bc^3\log\left(-c^2+x^4\right)$$

input
```
Integrate[x^5*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(b*c*x^4)/12 + (a*x^6)/6 + (b*x^6*ArcTanh[c/x^2])/6 + (b*c^3*Log[-c^2 + x^
4])/12
```

3.158.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6452, 795, 798, 25, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}bc\int \frac{x^3}{1-\frac{c^2}{x^4}}dx+\frac{1}{6}x^6\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{795}$$

$$\frac{1}{3}bc\int \frac{x^7}{x^4-c^2}dx+\frac{1}{6}x^6\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{798}$$

$$\frac{1}{12}bc\int -\frac{x^4}{c^2-x^4}dx^4+\frac{1}{6}x^6\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{6}x^6\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)-\frac{1}{12}bc\int \frac{x^4}{c^2-x^4}dx^4$$

$$\Big\downarrow \text{49}$$

$$\frac{1}{6}x^6\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)-\frac{1}{12}bc\int \left(\frac{c^2}{c^2-x^4}-1\right)dx^4$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}x^6\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)+\frac{1}{12}bc\left(c^2\log\left(c^2-x^4\right)+x^4\right)$$

input
```
Int[x^5*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(x^6*(a + b*ArcTanh[c/x^2]))/6 + (b*c*(x^4 + c^2*Log[c^2 - x^4]))/12
```

3.158.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.158.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

method	result
parallelrisch	$\frac{x^{6}\operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b}{6}+\frac{a\,x^{6}}{6}+\frac{b c\,x^{4}}{12}+\frac{\ln\left(x^{2}-c\right) b\,c^{3}}{6}+\frac{\operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b\,c^{3}}{6}+\frac{b\,c^{3}}{12}$
parts	$\frac{a\,x^{6}}{6}+b\left(\frac{x^{6}\operatorname{arctanh}\left(\frac{c}{x^{2}}\right)}{6}-\frac{c\left(-\frac{c^{2}\ln\left(1+\frac{c}{x^{2}}\right)}{4}-\frac{x^{4}}{4}+c^{2}\ln\left(\frac{1}{x}\right)-\frac{c^{2}\ln\left(\frac{c}{x^{2}}-1\right)}{4}\right)}{3}\right)$
derivativedivides	$\frac{a\,x^{6}}{6}-b\left(-\frac{x^{6}\operatorname{arctanh}\left(\frac{c}{x^{2}}\right)}{6}+\frac{c\left(-\frac{c^{2}\ln\left(1+\frac{c}{x^{2}}\right)}{4}-\frac{x^{4}}{4}+c^{2}\ln\left(\frac{1}{x}\right)-\frac{c^{2}\ln\left(\frac{c}{x^{2}}-1\right)}{4}\right)}{3}\right)$
default	$\frac{a\,x^{6}}{6}-b\left(-\frac{x^{6}\operatorname{arctanh}\left(\frac{c}{x^{2}}\right)}{6}+\frac{c\left(-\frac{c^{2}\ln\left(1+\frac{c}{x^{2}}\right)}{4}-\frac{x^{4}}{4}+c^{2}\ln\left(\frac{1}{x}\right)-\frac{c^{2}\ln\left(\frac{c}{x^{2}}-1\right)}{4}\right)}{3}\right)$
risch	$\frac{b\,x^{6}\ln\left(x^{2}+c\right)}{12}-\frac{b\,x^{6}\ln\left(-x^{2}+c\right)}{12}-\frac{i\pi b\,x^{6}\operatorname{csgn}\left(\frac{i}{x^{2}}\right)\operatorname{csgn}\left(i\left(x^{2}+c\right)\right)\operatorname{csgn}\left(\frac{i\left(x^{2}+c\right)}{x^{2}}\right)}{24}-\frac{i\pi b\,x^{6}\operatorname{csgn}\left(\frac{i}{x^{2}}\right)\operatorname{csgn}\left(\frac{i(-}{}\right.}{24}$

input
```
int(x^5*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/6*x^6*arctanh(c/x^2)*b+1/6*a*x^6+1/12*b*c*x^4+1/6*ln(x^2-c)*b*c^3+1/6*ar
ctanh(c/x^2)*b*c^3+1/12*b*c^3
```

3.158.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int x^5\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{12}\,bx^{6}\log\left(\frac{x^{2}+c}{x^{2}-c}\right)+\frac{1}{6}\,ax^{6}+\frac{1}{12}\,bcx^{4}+\frac{1}{12}\,bc^{3}\log\left(x^{4}-c^{2}\right)$$

input
```
integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

output
```
1/12*b*x^6*log((x^2 + c)/(x^2 - c)) + 1/6*a*x^6 + 1/12*b*c*x^4 + 1/12*b*c^
3*log(x^4 - c^2)
```

3.158.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. 2(37) = 74.

Time = 2.57 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.67

$$\int x^5\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{ax^6}{6}+\frac{bc^3\log\left(x-\sqrt{-c}\right)}{6}+\frac{bc^3\log\left(x+\sqrt{-c}\right)}{6}-\frac{bc^3\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6}+\frac{bcx^4}{12}+\frac{bx^6\operatorname{atanh}\left(\frac{c}{x^2}\right)}{6}$$

input
```
integrate(x**5*(a+b*atanh(c/x**2)),x)
```

output
```
a*x**6/6 + b*c**3*log(x - sqrt(-c))/6 + b*c**3*log(x + sqrt(-c))/6 - b*c**
3*atanh(c/x**2)/6 + b*c*x**4/12 + b*x**6*atanh(c/x**2)/6
```

3.158.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x^5\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{6}\,ax^6+\frac{1}{12}\left(2\,x^6\operatorname{artanh}\left(\frac{c}{x^2}\right)+\left(x^4+c^2\log\left(x^4-c^2\right)\right)c\right)b$$

input
```
integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

output
```
1/6*a*x^6 + 1/12*(2*x^6*arctanh(c/x^2) + (x^4 + c^2*log(x^4 - c^2))*c)*b
```

3.158.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int x^5\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{12}\,bx^6\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{6}\,ax^6+\frac{1}{12}\,bcx^4+\frac{1}{12}\,bc^3\log\left(x^4-c^2\right)$$

input
```
integrate(x^5*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

output
```
1/12*b*x^6*log((x^2 + c)/(x^2 - c)) + 1/6*a*x^6 + 1/12*b*c*x^4 + 1/12*b*c^
3*log(x^4 - c^2)
```

3.158.9 Mupad [B] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int x^5\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{a\,x^6}{6}+\frac{b\,c^3\,\ln\left(x^4-c^2\right)}{12}+\frac{b\,x^6\,\ln\left(x^2+c\right)}{12}+\frac{b\,c\,x^4}{12}-\frac{b\,x^6\,\ln\left(x^2-c\right)}{12}$$

input
```
int(x^5*(a + b*atanh(c/x^2)),x)
```

output
```
(a*x^6)/6 + (b*c^3*log(x^4 - c^2))/12 + (b*x^6*log(c + x^2))/12 + (b*c*x^4
)/12 - (b*x^6*log(x^2 - c))/12
```

3.159 $\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$

3.159.1 Optimal result

Integrand size = 14, antiderivative size = 43

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{4}bcx^2+\frac{1}{4}x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)-\frac{1}{4}bc^2\text{arctanh}\left(\frac{x^2}{c}\right)$$

output
```
1/4*b*c*x^2+1/4*x^4*(a+b*arctanh(c/x^2))-1/4*b*c^2*arctanh(x^2/c)
```

3.159.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.44

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{4}bcx^2+\frac{ax^4}{4}+\frac{1}{4}bx^4\text{arctanh}\left(\frac{c}{x^2}\right)+\frac{1}{8}bc^2\log\left(-c+x^2\right)-\frac{1}{8}bc^2\log\left(c+x^2\right)$$

input
```
Integrate[x^3*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(b*c*x^2)/4 + (a*x^4)/4 + (b*x^4*ArcTanh[c/x^2])/4 + (b*c^2*Log[-c + x^2])
/8 - (b*c^2*Log[c + x^2])/8
```

3.159.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6452, 795, 807, 25, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2} b c \int \frac{x}{1-\frac{c^2}{x^4}}dx+\frac{1}{4} x^4 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{795}$$

$$\frac{1}{2} b c \int \frac{x^5}{x^4-c^2}dx+\frac{1}{4} x^4 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{4} b c \int -\frac{x^4}{c^2-x^4}dx^2+\frac{1}{4} x^4 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4} x^4 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)-\frac{1}{4} b c \int \frac{x^4}{c^2-x^4}dx^2$$

$$\Big\downarrow \text{262}$$

$$\frac{1}{4} b c \left(x^2-c^2 \int \frac{1}{c^2-x^4}dx^2\right)+\frac{1}{4} x^4 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4} x^4 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4} b c \left(x^2-c\text{arctanh}\left(\frac{x^2}{c}\right)\right)$$

input
```
Int[x^3*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(x^4*(a + b*ArcTanh[c/x^2]))/4 + (b*c*(x^2 - c*ArcTanh[x^2/c]))/4
```

3.159.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.159.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$\frac{\operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b\,x^{4}}{4}+\frac{a\,x^{4}}{4}+\frac{bc\,x^{2}}{4}-\frac{\operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b\,c^{2}}{4}+\frac{a\,c^{2}}{4}$	45
derivativedivides	$\frac{a\,x^{4}}{4}+\frac{\operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b\,x^{4}}{4}+\frac{bc\,x^{2}}{4}+\frac{b\,c^{2}\ln\left(\frac{c}{x^{2}}-1\right)}{8}-\frac{b\,c^{2}\ln\left(1+\frac{c}{x^{2}}\right)}{8}$	55
default	$\frac{a\,x^{4}}{4}+\frac{\operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b\,x^{4}}{4}+\frac{bc\,x^{2}}{4}+\frac{b\,c^{2}\ln\left(\frac{c}{x^{2}}-1\right)}{8}-\frac{b\,c^{2}\ln\left(1+\frac{c}{x^{2}}\right)}{8}$	55
parts	$\frac{a\,x^{4}}{4}+\frac{\operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b\,x^{4}}{4}+\frac{bc\,x^{2}}{4}+\frac{b\,c^{2}\ln\left(\frac{c}{x^{2}}-1\right)}{8}-\frac{b\,c^{2}\ln\left(1+\frac{c}{x^{2}}\right)}{8}$	55
risch	Expression too large to display	4322

input
```
int(x^3*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/4*arctanh(c/x^2)*b*x^4+1/4*a*x^4+1/4*b*c*x^2-1/4*arctanh(c/x^2)*b*c^2+1/
4*a*c^2
```

3.159.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int x^{3}\left(a+b\operatorname{arctanh}\left(\frac{c}{x^{2}}\right)\right)\,dx=\frac{1}{4}\,ax^{4}+\frac{1}{4}\,bcx^{2}+\frac{1}{8}\,\left(bx^{4}-bc^{2}\right)\log\left(\frac{x^{2}+c}{x^{2}-c}\right)$$

input
```
integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

output
```
1/4*a*x^4 + 1/4*b*c*x^2 + 1/8*(b*x^4 - b*c^2)*log((x^2 + c)/(x^2 - c))
```

3.159.6 Sympy [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{ax^4}{4}-\frac{bc^2\operatorname{atanh}\left(\frac{c}{x^2}\right)}{4}+\frac{bcx^2}{4}+\frac{bx^4\operatorname{atanh}\left(\frac{c}{x^2}\right)}{4}$$

input
```
integrate(x**3*(a+b*atanh(c/x**2)),x)
```

output
```
a*x**4/4 - b*c**2*atanh(c/x**2)/4 + b*c*x**2/4 + b*x**4*atanh(c/x**2)/4
```

3.159.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$$
$$=\frac{1}{4}\,ax^4+\frac{1}{8}\,\left(2\,x^4\operatorname{artanh}\left(\frac{c}{x^2}\right)+\left(2\,x^2-c\log\left(x^2+c\right)+c\log\left(x^2-c\right)\right)c\right)b$$

input
```
integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/8*(2*x^4*arctanh(c/x^2) + (2*x^2 - c*log(x^2 + c) + c*log(x^
2 - c))*c)*b
```

3.159.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. 2(37) = 74.

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 3.77

$$\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{\frac{(x^2+c)bc^3\log\left(\frac{x^2+c}{x^2-c}\right)}{(x^2-c)\left(\frac{(x^2+c)^2}{(x^2-c)^2}-\frac{2(x^2+c)}{x^2-c}+1\right)}+\frac{\frac{2(x^2+c)ac^3}{x^2-c}+\frac{(x^2+c)bc^3}{x^2-c}-bc^3}{\frac{(x^2+c)^2}{(x^2-c)^2}-\frac{2(x^2+c)}{x^2-c}+1}}{2\,c}$$

input
```
integrate(x^3*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

output
```
1/2*((x^2 + c)*b*c^3*log((x^2 + c)/(x^2 - c))/((x^2 - c)*((x^2 + c)^2/(x^2
 - c)^2 - 2*(x^2 + c)/(x^2 - c) + 1)) + (2*(x^2 + c)*a*c^3/(x^2 - c) + (x^
2 + c)*b*c^3/(x^2 - c) - b*c^3)/((x^2 + c)^2/(x^2 - c)^2 - 2*(x^2 + c)/(x^
2 - c) + 1))/c
```

3.159.9 Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33

$$\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{a\,x^4}{4}+\frac{b\,x^4\,\ln\left(x^2+c\right)}{8}+\frac{b\,c\,x^2}{4}-\frac{b\,x^4\,\ln\left(x^2-c\right)}{8}+\frac{b\,c^2\,\mathrm{atan}\left(\frac{x^2\,1\mathrm{i}}{c}\right)\,1\mathrm{i}}{4}$$

input
```
int(x^3*(a + b*atanh(c/x^2)),x)
```

output
```
(a*x^4)/4 + (b*x^4*log(c + x^2))/8 + (b*c^2*atan((x^2*1i)/c)*1i)/4 + (b*c*
x^2)/4 - (b*x^4*log(x^2 - c))/8
```

3.160 $\int x\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$

3.160.1 Optimal result

Integrand size = 12, antiderivative size = 34

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}bc\log\left(c^2-x^4\right)$$

output
```
1/2*x^2*(a+b*arctanh(c/x^2))+1/4*b*c*ln(-x^4+c^2)
```

3.160.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{ax^2}{2}+\frac{1}{2}bx^2\text{arctanh}\left(\frac{c}{x^2}\right)+\frac{1}{4}bc\log\left(-c^2+x^4\right)$$

input
```
Integrate[x*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(a*x^2)/2 + (b*x^2*ArcTanh[c/x^2])/2 + (b*c*Log[-c^2 + x^4])/4
```

3.160.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6452, 795, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$$

$$\Big\downarrow \text{6452}$$

$$bc\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x}dx+\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{795}$$

$$bc\int \frac{x^3}{x^4-c^2}dx+\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{792}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)+\frac{1}{4}bc\log\left(c^2-x^4\right)$$

input
```
Int[x*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(x^2*(a + b*ArcTanh[c/x^2]))/2 + (b*c*Log[c^2 - x^4])/4
```

3.160.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.160.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24

method	result
parallelrisch	$\frac{x^{2} \operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b}{2}+\frac{\ln \left(x^{2}-c\right) b c}{2}+\frac{a\,x^{2}}{2}+\frac{\operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b c}{2}$
derivativedivides	$\frac{a\,x^{2}}{2}-b\left(-\frac{x^{2} \operatorname{arctanh}\left(\frac{c}{x^{2}}\right)}{2}+c\left(-\frac{\ln\left(1+\frac{c}{x^{2}}\right)}{4}+\ln\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{c}{x^{2}}-1\right)}{4}\right)\right)$
default	$\frac{a\,x^{2}}{2}-b\left(-\frac{x^{2} \operatorname{arctanh}\left(\frac{c}{x^{2}}\right)}{2}+c\left(-\frac{\ln\left(1+\frac{c}{x^{2}}\right)}{4}+\ln\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{c}{x^{2}}-1\right)}{4}\right)\right)$
parts	$\frac{a\,x^{2}}{2}+b\left(\frac{x^{2} \operatorname{arctanh}\left(\frac{c}{x^{2}}\right)}{2}-c\left(-\frac{\ln\left(1+\frac{c}{x^{2}}\right)}{4}+\ln\left(\frac{1}{x}\right)-\frac{\ln\left(\frac{c}{x^{2}}-1\right)}{4}\right)\right)$
risch	$\frac{b\,x^{2} \ln \left(x^{2}+c\right)}{4}-\frac{b\,x^{2} \ln \left(-x^{2}+c\right)}{4}-\frac{i \pi b\,x^{2} \operatorname{csgn}\left(\frac{i\left(-x^{2}+c\right)}{x^{2}}\right)^{3}}{8}-\frac{i \pi b\,x^{2} \operatorname{csgn}\left(\frac{i\left(x^{2}+c\right)}{x^{2}}\right)^{3}}{8}+\frac{i \pi b\,x^{2} \operatorname{csgn}\left(\frac{i}{x^{2}}\right) \operatorname{csgn}}{8}$

input
```
int(x*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2*arctanh(c/x^2)*b+1/2*ln(x^2-c)*b*c+1/2*a*x^2+1/2*arctanh(c/x^2)*b*
c
```

3.160.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{4}\,bx^{2}\log\left(\frac{x^{2}+c}{x^{2}-c}\right)+\frac{1}{2}\,ax^{2}+\frac{1}{4}\,bc\log\left(x^{4}-c^{2}\right)$$

input
```
integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

output
```
1/4*b*x^2*log((x^2 + c)/(x^2 - c)) + 1/2*a*x^2 + 1/4*b*c*log(x^4 - c^2)
```

3.160.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(27) = 54$.

Time = 1.37 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.79

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{ax^2}{2}+\frac{bc\log\left(x-\sqrt{-c}\right)}{2}+\frac{bc\log\left(x+\sqrt{-c}\right)}{2}-\frac{bc\,\text{atanh}\left(\frac{c}{x^2}\right)}{2}+\frac{bx^2\,\text{atanh}\left(\frac{c}{x^2}\right)}{2}$$

input

```
integrate(x*(a+b*atanh(c/x**2)),x)
```

output

```
a*x**2/2 + b*c*log(x - sqrt(-c))/2 + b*c*log(x + sqrt(-c))/2 - b*c*atanh(c
/x**2)/2 + b*x**2*atanh(c/x**2)/2
```

3.160.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{2}\,ax^2+\frac{1}{4}\left(2\,x^2\,\text{artanh}\left(\frac{c}{x^2}\right)+c\log\left(x^4-c^2\right)\right)b$$

input

```
integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

output

```
1/2*a*x^2 + 1/4*(2*x^2*arctanh(c/x^2) + c*log(x^4 - c^2))*b
```

3.160.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(30) = 60$.

Time = 0.28 (sec) , antiderivative size = 184, normalized size of antiderivative = 5.41

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx = \frac{1}{2}\,ax^2+\frac{\left(c^2\left(\log\left(\frac{|-x^2-c|}{|-x^2+c|}\right)-\log\left(\left|\frac{x^2+c}{x^2-c}-1\right|\right)\right)+\frac{c^2\log\left(-\frac{\frac{c\left(\frac{x^2+c}{(x^2-c)c}-\frac{1}{c}\right)}{\frac{x^2+c}{x^2-c}+1}+1}{\frac{c\left(\frac{x^2+c}{(x^2-c)c}-\frac{1}{c}\right)}{\frac{x^2+c}{x^2-c}+1}-1}\right)}{\frac{x^2+c}{x^2-c}-1}\right)b}{2\,c}$$

input
```
integrate(x*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

output
```
1/2*a*x^2 + 1/2*(c^2*(log(abs(-x^2 - c)/abs(-x^2 + c)) - log(abs((x^2 + c)
/(x^2 - c) - 1))) + c^2*log(-(c*((x^2 + c)/((x^2 - c)*c) - 1/c)/((x^2 + c)
/(x^2 - c) + 1) + 1)/(c*((x^2 + c)/((x^2 - c)*c) - 1/c)/((x^2 + c)/(x^2 -
c) + 1) - 1))/((x^2 + c)/(x^2 - c) - 1))*b/c
```

3.160.9 Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.38

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx = \frac{a\,x^2}{2}+\frac{b\,x^2\,\ln\left(x^2+c\right)}{4}+\frac{b\,c\,\ln\left(x^4-c^2\right)}{4}-\frac{b\,x^2\,\ln\left(x^2-c\right)}{4}$$

input
```
int(x*(a + b*atanh(c/x^2)),x)
```

output
```
(a*x^2)/2 + (b*x^2*log(c + x^2))/4 + (b*c*log(x^4 - c^2))/4 - (b*x^2*log(x
^2 - c))/4
```

3.161 $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}\,dx$

3.161.1 Optimal result

Integrand size = 14, antiderivative size = 30

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}\,dx = a\log(x)+\frac{1}{4}b\,\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\frac{1}{4}b\,\text{PolyLog}\left(2,\frac{c}{x^2}\right)$$

output
```
a*ln(x)+1/4*b*polylog(2,-c/x^2)-1/4*b*polylog(2,c/x^2)
```

3.161.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}\,dx = a\log(x)+\frac{1}{4}b\left(\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\text{PolyLog}\left(2,\frac{c}{x^2}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])/x,x]
```

output
```
a*Log[x] + (b*(PolyLog[2, -(c/x^2)] - PolyLog[2, c/x^2]))/4
```

3.161.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6450, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x} \, dx$$

$$\Big\downarrow \text{ 6450}$$

$$-\frac{1}{2}\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 6446}$$

$$\frac{1}{2}\left(-a\log\left(\frac{1}{x^2}\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,-\frac{c}{x^2}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{c}{x^2}\right)\right)$$

input
```
Int[(a + b*ArcTanh[c/x^2])/x,x]
```

output
```
(-(a*Log[x^(-2)]) + (b*PolyLog[2, -(c/x^2)])/2 - (b*PolyLog[2, c/x^2])/2)/
2
```

3.161.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.161.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(26) = 52$.

Time = 0.67 (sec) , antiderivative size = 149, normalized size of antiderivative = 4.97

method	result
parts	$a\ln(x)+b\left(-\ln\left(\frac{1}{x}\right)\operatorname{arctanh}\left(\frac{c}{x^2}\right)+2c\left(-\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(1-\frac{\sqrt{c}}{x}\right)+\ln\left(1+\frac{\sqrt{c}}{x}\right)\right)}{4c}-\frac{\operatorname{dilog}\left(1-\frac{\sqrt{c}}{x}\right)+\operatorname{dilog}\left(\right.}{4c}\right.\right.$
derivativedivides	$-a\ln\left(\frac{1}{x}\right)-b\left(\ln\left(\frac{1}{x}\right)\operatorname{arctanh}\left(\frac{c}{x^2}\right)-2c\left(-\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(1-\frac{\sqrt{c}}{x}\right)+\ln\left(1+\frac{\sqrt{c}}{x}\right)\right)}{4c}-\frac{\operatorname{dilog}\left(1-\frac{\sqrt{c}}{x}\right)+\operatorname{dilog}\left(\right.}{4c}\right.\right.$
default	$-a\ln\left(\frac{1}{x}\right)-b\left(\ln\left(\frac{1}{x}\right)\operatorname{arctanh}\left(\frac{c}{x^2}\right)-2c\left(-\frac{\ln\left(\frac{1}{x}\right)\left(\ln\left(1-\frac{\sqrt{c}}{x}\right)+\ln\left(1+\frac{\sqrt{c}}{x}\right)\right)}{4c}-\frac{\operatorname{dilog}\left(1-\frac{\sqrt{c}}{x}\right)+\operatorname{dilog}\left(\right.}{4c}\right.\right.$
risch	$\frac{b\ln(x)\ln(x^2+c)}{2}+\frac{\left(-ib\pi\operatorname{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^3+4a+2ib\pi\operatorname{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^2+ib\pi\operatorname{csgn}\left(\frac{i}{x^2}\right)\operatorname{csgn}\left(i(-x^2+c)\right)\operatorname{csgn}\left(\frac{i(-\right.}{}\right.\right.}{}$

input
```
int((a+b*arctanh(c/x^2))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(-ln(1/x)*arctanh(c/x^2)+2*c*(-1/4*ln(1/x)*(ln(1-1/x*c^(1/2))+ln
(1+1/x*c^(1/2)))/c-1/4*(dilog(1-1/x*c^(1/2))+dilog(1+1/x*c^(1/2)))/c+1/4*l
n(1/x)*(ln(1+(-c)^(1/2)/x)+ln(1-(-c)^(1/2)/x))/c+1/4*(dilog(1+(-c)^(1/2)/x
)+dilog(1-(-c)^(1/2)/x))/c))
```

3.161.5 Fricas [F]

$$\int\frac{a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x}\,dx=\int\frac{b\operatorname{artanh}\left(\frac{c}{x^2}\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))/x,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c/x^2) + a)/x, x)
```

3.161.6 Sympy [F]

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x}\,dx = \int \frac{a + b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{x}\,dx$$

input
```
integrate((a+b*atanh(c/x**2))/x,x)
```

output
```
Integral((a + b*atanh(c/x**2))/x, x)
```

3.161.7 Maxima [F]

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x}\,dx = \int \frac{b\operatorname{artanh}\left(\frac{c}{x^2}\right) + a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))/x,x, algorithm="maxima")
```

output
```
1/2*b*integrate((log(c/x^2 + 1) - log(-c/x^2 + 1))/x, x) + a*log(x)
```

3.161.8 Giac [F]

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x}\,dx = \int \frac{b\operatorname{artanh}\left(\frac{c}{x^2}\right) + a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)/x, x)
```

3.161.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}\,dx=\int \frac{a+b\,\text{atanh}\left(\frac{c}{x^2}\right)}{x}\,dx$$

input
```
int((a + b*atanh(c/x^2))/x,x)
```

output
```
int((a + b*atanh(c/x^2))/x, x)
```

3.162 $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^3} \, dx$

3.162.1 Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^3} \, dx = -\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}$$

output
```
1/2*(-a-b*arctanh(c/x^2))/x^2-1/4*b*ln(1-c^2/x^4)/c
```

3.162.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.14

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^3} \, dx = -\frac{a}{2x^2}-\frac{b\text{arctanh}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*ArcTanh[c/x^2])/(2*x^2) - (b*Log[1 - c^2/x^4])/(4*c)
```

3.162.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6452, 792}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^3} \, dx$$

$$\Big\downarrow \; 6452$$

$$-bc\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^5}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{2x^2}$$

$$\Big\downarrow \; 792$$

$$-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{4c}$$

input
```
Int[(a + b*ArcTanh[c/x^2])/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c/x^2])/x^2 - (b*Log[1 - c^2/x^4])/(4*c)
```

3.162.3.1 Defintions of rubi rules used

rule 792
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.162.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

method	result
parts	$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{2x^2}-\frac{b\ln\left(1-\frac{c^2}{x^4}\right)}{4c}$
derivativedivides	$-\frac{\frac{ca}{x^2}+b\left(\frac{c\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{x^2}+\frac{\ln\left(1-\frac{c^2}{x^4}\right)}{2}\right)}{2c}$
default	$-\frac{\frac{ca}{x^2}+b\left(\frac{c\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{x^2}+\frac{\ln\left(1-\frac{c^2}{x^4}\right)}{2}\right)}{2c}$
parallelrisch	$\frac{2b\ln(x)x^2-\ln(x^2-c)x^2b-x^2\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)b-\mathrm{arctanh}\left(\frac{c}{x^2}\right)bc-ac}{2c\,x^2}$
risch	$-\frac{b\ln(x^2+c)}{4x^2}-\frac{2i\pi bc\,\mathrm{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^2+i\pi bc\,\mathrm{csgn}(i(x^2+c))\,\mathrm{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^2-2i\pi bc+i\pi bc\,\mathrm{csgn}\left(\frac{i}{x^2}\right)\mathrm{csgn}(i(-x^2}{}$

input
```
int((a+b*arctanh(c/x^2))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2-1/2*b/x^2*arctanh(c/x^2)-1/4*b*ln(1-c^2/x^4)/c
```

3.162.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.49

$$\int \frac{a+b\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{x^3}\,dx=-\frac{bx^2\log\left(x^4-c^2\right)-4\,bx^2\log\left(x\right)+bc\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac}{4\,cx^2}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="fricas")
```

output
```
-1/4*(b*x^2*log(x^4 - c^2) - 4*b*x^2*log(x) + b*c*log((x^2 + c)/(x^2 - c))
 + 2*a*c)/(c*x^2)
```

3.162.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(32) = 64$.

Time = 4.21 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.05

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^3}\,dx = \begin{cases} -\frac{a}{2x^2} - \frac{b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2x^2} + \frac{b\log\left(x\right)}{c} - \frac{b\log\left(x-\sqrt{-c}\right)}{2c} - \frac{b\log\left(x+\sqrt{-c}\right)}{2c} + \frac{b\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2c} & \text{for } c \neq 0 \\ -\frac{a}{2x^2} & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*atanh(c/x**2))/x**3,x)
```

output
```
Piecewise((-a/(2*x**2) - b*atanh(c/x**2)/(2*x**2) + b*log(x)/c - b*log(x -
 sqrt(-c))/(2*c) - b*log(x + sqrt(-c))/(2*c) + b*atanh(c/x**2)/(2*c), Ne(c
, 0)), (-a/(2*x**2), True))
```

3.162.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^3}\,dx = -\frac{b\left(\frac{2\,c\operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^2} + \log\left(-\frac{c^2}{x^4} + 1\right)\right)}{4\,c} - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="maxima")
```

output
```
-1/4*b*(2*c*arctanh(c/x^2)/x^2 + log(-c^2/x^4 + 1))/c - 1/2*a/x^2
```

3.162.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.41

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^3}\,dx = -\frac{b\log\left(x^4 - c^2\right)}{4\,c} + \frac{b\log\left(x\right)}{c} - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{4\,x^2} - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^3,x, algorithm="giac")
```

output
```
-1/4*b*log(x^4 - c^2)/c + b*log(x)/c - 1/4*b*log((x^2 + c)/(x^2 - c))/x^2
- 1/2*a/x^2
```

3.162.9 Mupad [B] (verification not implemented)

Time = 3.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.51

$$\int \frac{a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)}{x^3}\,dx = \frac{b\,\ln(x)}{c} - \frac{b\,\ln(x^4-c^2)}{4\,c} - \frac{a}{2\,x^2} - \frac{b\,\ln(x^2+c)}{4\,x^2} + \frac{b\,\ln(x^2-c)}{4\,x^2}$$

input
```
int((a + b*atanh(c/x^2))/x^3,x)
```

output
```
(b*log(x))/c - (b*log(x^4 - c^2))/(4*c) - a/(2*x^2) - (b*log(c + x^2))/(4*
x^2) + (b*log(x^2 - c))/(4*x^2)
```

3.163 $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^5}\,dx$

3.163.1 Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^5}\,dx=-\frac{b}{4cx^2}-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}+\frac{b\text{arctanh}\left(\frac{x^2}{c}\right)}{4c^2}$$

output
```
-1/4*b/c/x^2+1/4*(-a-b*arctanh(c/x^2))/x^4+1/4*b*arctanh(x^2/c)/c^2
```

3.163.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^5}\,dx=-\frac{a}{4x^4}-\frac{b}{4cx^2}-\frac{b\text{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b\log\left(-c+x^2\right)}{8c^2}+\frac{b\log\left(c+x^2\right)}{8c^2}$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])/x^5,x]
```

output
```
-1/4*a/x^4 - b/(4*c*x^2) - (b*ArcTanh[c/x^2])/(4*x^4) - (b*Log[-c + x^2])/
(8*c^2) + (b*Log[c + x^2])/(8*c^2)
```

3.163.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6452, 795, 807, 25, 264, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^5}dx$$

$$\Big\downarrow \text{6452}$$

$$-\frac{1}{2}bc\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^7}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}$$

$$\Big\downarrow \text{795}$$

$$-\frac{1}{2}bc\int \frac{1}{x^3\left(x^4-c^2\right)}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}$$

$$\Big\downarrow \text{807}$$

$$-\frac{1}{4}bc\int -\frac{1}{x^4\left(c^2-x^4\right)}dx^2-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}bc\int \frac{1}{x^4\left(c^2-x^4\right)}dx^2-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}$$

$$\Big\downarrow \text{264}$$

$$-\frac{1}{4}bc\left(\frac{1}{c^2x^2}-\frac{\int \frac{1}{c^2-x^4}dx^2}{c^2}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{1}{4}bc\left(\frac{1}{c^2x^2}-\frac{\text{arctanh}\left(\frac{x^2}{c}\right)}{c^3}\right)$$

input
```
Int[(a + b*ArcTanh[c/x^2])/x^5,x]
```

output
```
-1/4*(a + b*ArcTanh[c/x^2])/x^4 - (b*c*(1/(c^2*x^2) - ArcTanh[x^2/c]/c^3))
/4
```

3.163. $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^5}dx$

3.163.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.163. $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^5}\,dx$

3.163.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

method	result
parallelrisch	$-\frac{-\operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b\,x^{4}+b c\,x^{2}+\operatorname{arctanh}\left(\frac{c}{x^{2}}\right) b\,c^{2}+a\,c^{2}}{4 x^{4} c^{2}}$
derivativedivides	$-\frac{a}{4 x^{4}}-\frac{b \,\operatorname{arctanh}\left(\frac{c}{x^{2}}\right)}{4 x^{4}}-\frac{b}{4 c\,x^{2}}+\frac{b \ln\left(1+\frac{c}{x^{2}}\right)}{8 c^{2}}-\frac{b \ln\left(\frac{c}{x^{2}}-1\right)}{8 c^{2}}$
default	$-\frac{a}{4 x^{4}}-\frac{b \,\operatorname{arctanh}\left(\frac{c}{x^{2}}\right)}{4 x^{4}}-\frac{b}{4 c\,x^{2}}+\frac{b \ln\left(1+\frac{c}{x^{2}}\right)}{8 c^{2}}-\frac{b \ln\left(\frac{c}{x^{2}}-1\right)}{8 c^{2}}$
parts	$-\frac{a}{4 x^{4}}-\frac{b \,\operatorname{arctanh}\left(\frac{c}{x^{2}}\right)}{4 x^{4}}-\frac{b}{4 c\,x^{2}}+\frac{b \ln\left(1+\frac{c}{x^{2}}\right)}{8 c^{2}}-\frac{b \ln\left(\frac{c}{x^{2}}-1\right)}{8 c^{2}}$
risch	$-\frac{b \ln\left(x^{2}+c\right)}{8 x^{4}}-\frac{i \pi b\,c^{2} \operatorname{csgn}\left(i \left(x^{2}+c\right)\right) \operatorname{csgn}\left(\frac{i \left(x^{2}+c\right)}{x^{2}}\right)^{2}-2 i \pi b\,c^{2}+i \pi b\,c^{2} \operatorname{csgn}\left(\frac{i}{x^{2}}\right) \operatorname{csgn}\left(\frac{i \left(x^{2}+c\right)}{x^{2}}\right)^{2}-i \pi b\,c^{2} \operatorname{csgn}\left(\frac{i(-}{}\right.}{}$

input
```
int((a+b*arctanh(c/x^2))/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-arctanh(c/x^2)*b*x^4+b*c*x^2+arctanh(c/x^2)*b*c^2+a*c^2)/x^4/c^2
```

3.163.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^5}\,dx=-\frac{2\,bcx^2+2\,ac^2-(bx^4-bc^2)\log\left(\frac{x^2+c}{x^2-c}\right)}{8\,c^2x^4}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="fricas")
```

output
```
-1/8*(2*b*c*x^2 + 2*a*c^2 - (b*x^4 - b*c^2)*log((x^2 + c)/(x^2 - c)))/(c^2
*x^4)
```

3.163.6 Sympy [A] (verification not implemented)

Time = 5.62 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^5}\,dx=\begin{cases}-\frac{a}{4x^4}-\frac{b\,\text{atanh}\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b}{4cx^2}+\frac{b\,\text{atanh}\left(\frac{c}{x^2}\right)}{4c^2} & \text{for } c\neq 0\\ -\frac{a}{4x^4} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*atanh(c/x**2))/x**5,x)
```

output
```
Piecewise((-a/(4*x**4) - b*atanh(c/x**2)/(4*x**4) - b/(4*c*x**2) + b*atanh
(c/x**2)/(4*c**2), Ne(c, 0)), (-a/(4*x**4), True))
```

3.163.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^5}\,dx=\frac{1}{8}\left(c\left(\frac{\log\left(x^2+c\right)}{c^3}-\frac{\log\left(x^2-c\right)}{c^3}-\frac{2}{c^2x^2}\right)-\frac{2\,\text{artanh}\left(\frac{c}{x^2}\right)}{x^4}\right)b-\frac{a}{4\,x^4}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="maxima")
```

output
```
1/8*(c*(log(x^2 + c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2)) - 2*arctanh(c/x
^2)/x^4)*b - 1/4*a/x^4
```

3.163.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^5}\,dx=\frac{b\log\left(x^2+c\right)}{8\,c^2}-\frac{b\log\left(-x^2+c\right)}{8\,c^2}-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{8\,x^4}-\frac{bx^2+ac}{4\,cx^4}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^5,x, algorithm="giac")
```

output
```
1/8*b*log(x^2 + c)/c^2 - 1/8*b*log(-x^2 + c)/c^2 - 1/8*b*log((x^2 + c)/(x^
2 - c))/x^4 - 1/4*(b*x^2 + a*c)/(c*x^4)
```

3.163.9 Mupad [B] (verification not implemented)

Time = 3.44 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31

$$\int \frac{a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^5}\,dx=\frac{\frac{b\,x^4\,\operatorname{atanh}\left(\frac{x^2}{c}\right)}{4}-\frac{b\,c\,x^2}{4}}{c^2\,x^4}-\frac{\frac{a}{4}-\frac{b\,\ln\left(x^2-c\right)}{8}+\frac{b\,\ln\left(x^2+c\right)}{8}}{x^4}$$

input
```
int((a + b*atanh(c/x^2))/x^5,x)
```

output
```
((b*x^4*atanh(x^2/c))/4 - (b*c*x^2)/4)/(c^2*x^4) - (a/4 - (b*log(x^2 - c))
/8 + (b*log(c + x^2))/8)/x^4
```

3.164 $\int \frac{a+b\mathbf{arctanh}\left(\frac{c}{x^2}\right)}{x^7}\,dx$

3.164.1 Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^7}\,dx = -\frac{b}{12cx^4}-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(c^2-x^4\right)}{12c^3}$$

output
```
-1/12*b/c/x^4+1/6*(-a-b*arctanh(c/x^2))/x^6+1/3*b*ln(x)/c^3-1/12*b*ln(-x^4
+c^2)/c^3
```

3.164.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^7}\,dx = -\frac{a}{6x^6}-\frac{b}{12cx^4}-\frac{b\text{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}+\frac{b\log(x)}{3c^3}-\frac{b\log\left(-c^2+x^4\right)}{12c^3}$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])/x^7,x]
```

output
```
-1/6*a/x^6 - b/(12*c*x^4) - (b*ArcTanh[c/x^2])/(6*x^6) + (b*Log[x])/(3*c^3
) - (b*Log[-c^2 + x^4])/(12*c^3)
```

3.164.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6452, 795, 798, 25, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^7}dx$$

$$\Big\downarrow \text{ 6452}$$

$$-\frac{1}{3}bc\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^9}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}$$

$$\Big\downarrow \text{ 795}$$

$$-\frac{1}{3}bc\int \frac{1}{x^5\left(x^4-c^2\right)}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}$$

$$\Big\downarrow \text{ 798}$$

$$-\frac{1}{12}bc\int -\frac{1}{x^8\left(c^2-x^4\right)}dx^4-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{12}bc\int \frac{1}{x^8\left(c^2-x^4\right)}dx^4-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{1}{12}bc\int \left(\frac{1}{c^4x^4}+\frac{1}{c^2x^8}+\frac{1}{c^4\left(c^2-x^4\right)}\right)dx^4-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{1}{12}bc\left(-\frac{\log\left(x^4\right)}{c^4}+\frac{1}{c^2x^4}+\frac{\log\left(c^2-x^4\right)}{c^4}\right)$$

input
```
Int[(a + b*ArcTanh[c/x^2])/x^7,x]
```

output
```
-1/6*(a + b*ArcTanh[c/x^2])/x^6 - (b*c*(1/(c^2*x^4) - Log[x^4]/c^4 + Log[c
^2 - x^4]/c^4))/12
```

3.164. $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^7}dx$

3.164.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.164.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.79

method	result
derivativedivides	$-\frac{a}{6x^6}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b}{12c\,x^4}-\frac{b\ln\left(\frac{c^2}{x^4}-1\right)}{12c^3}$
default	$-\frac{a}{6x^6}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b}{12c\,x^4}-\frac{b\ln\left(\frac{c^2}{x^4}-1\right)}{12c^3}$
parts	$-\frac{a}{6x^6}-\frac{b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b}{12c\,x^4}-\frac{b\ln\left(\frac{c^2}{x^4}-1\right)}{12c^3}$
parallelrisch	$\frac{4b\ln(x)x^6-2\ln(x^2-c)x^6b-2x^6\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)b-b\,c^2x^2-2\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)b\,c^3-2a\,c^3}{12x^6c^3}$
risch	$-\frac{b\ln(x^2+c)}{12x^6}-\frac{-2i\pi b\,c^3+2i\pi b\,c^3\mathrm{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^2-i\pi b\,c^3\,\mathrm{csgn}\left(\frac{i}{x^2}\right)\mathrm{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^2-i\pi b\,c^3\mathrm{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^3}{}$

input
```
int((a+b*arctanh(c/x^2))/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/6*a/x^6-1/6*b/x^6*arctanh(c/x^2)-1/12*b/c/x^4-1/12*b/c^3*ln(c^2/x^4-1)
```

3.164.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.18

$$\int \frac{a+b\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{x^7}\,dx=-\frac{bx^6\log\left(x^4-c^2\right)-4\,bx^6\log\left(x\right)+bc^2x^2+bc^3\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac^3}{12\,c^3x^6}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="fricas")
```

output
```
-1/12*(b*x^6*log(x^4 - c^2) - 4*b*x^6*log(x) + b*c^2*x^2 + b*c^3*log((x^2
+ c)/(x^2 - c)) + 2*a*c^3)/(c^3*x^6)
```

3.164.6 Sympy [A] (verification not implemented)

Time = 7.76 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.65

$$\int \frac{a+b\mathrm{arctanh}\left(\frac{c}{x^2}\right)}{x^7}\,dx$$
$$=\begin{cases}-\frac{a}{6x^6}-\frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{6x^6}-\frac{b}{12cx^4}+\frac{b\log\left(x\right)}{3c^3}-\frac{b\log\left(x-\sqrt{-c}\right)}{6c^3}-\frac{b\log\left(x+\sqrt{-c}\right)}{6c^3}+\frac{b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)}{6c^3} & \text{for } c\neq 0\\ -\frac{a}{6x^6} & \text{otherwise}\end{cases}$$

input
```
integrate((a+b*atanh(c/x**2))/x**7,x)
```

output
```
Piecewise((-a/(6*x**6) - b*atanh(c/x**2)/(6*x**6) - b/(12*c*x**4) + b*log(
x)/(3*c**3) - b*log(x - sqrt(-c))/(6*c**3) - b*log(x + sqrt(-c))/(6*c**3)
+ b*atanh(c/x**2)/(6*c**3), Ne(c, 0)), (-a/(6*x**6), True))
```

3.164.7 Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^7}\,dx = -\frac{1}{12}\left(c\left(\frac{\log\left(x^4-c^2\right)}{c^4}-\frac{\log\left(x^4\right)}{c^4}+\frac{1}{c^2x^4}\right)+\frac{2\,\text{artanh}\left(\frac{c}{x^2}\right)}{x^6}\right)b-\frac{a}{6\,x^6}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="maxima")
```

output
```
-1/12*(c*(log(x^4 - c^2)/c^4 - log(x^4)/c^4 + 1/(c^2*x^4)) + 2*arctanh(c/x
^2)/x^6)*b - 1/6*a/x^6
```

3.164.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^7}\,dx = -\frac{b\log\left(x^4-c^2\right)}{12\,c^3}+\frac{b\log\left(x\right)}{3\,c^3}-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{12\,x^6}-\frac{bx^2+2\,ac}{12\,cx^6}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^7,x, algorithm="giac")
```

output
```
-1/12*b*log(x^4 - c^2)/c^3 + 1/3*b*log(x)/c^3 - 1/12*b*log((x^2 + c)/(x^2
- c))/x^6 - 1/12*(b*x^2 + 2*a*c)/(c*x^6)
```

3.164.9 Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^7}\,dx = \frac{b\,\ln\left(x\right)}{3\,c^3}-\frac{b\,\ln\left(x^4-c^2\right)}{12\,c^3}-\frac{b}{12\,c\,x^4} -\frac{a}{6\,x^6}-\frac{b\,\ln\left(x^2+c\right)}{12\,x^6}+\frac{b\,\ln\left(x^2-c\right)}{12\,x^6}$$

input
```
int((a + b*atanh(c/x^2))/x^7,x)
```

output
```
(b*log(x))/(3*c^3) - (b*log(x^4 - c^2))/(12*c^3) - b/(12*c*x^4) - a/(6*x^6
) - (b*log(c + x^2))/(12*x^6) + (b*log(x^2 - c))/(12*x^6)
```

3.165 $\int x^4\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$

3.165.1 Optimal result

Integrand size = 14, antiderivative size = 63

$$\int x^4\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{2}{15}bcx^3+\frac{1}{5}bc^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{5}x^5\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)-\frac{1}{5}bc^{5/2}\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)$$

output
```
2/15*b*c*x^3+1/5*b*c^(5/2)*arctan(x/c^(1/2))+1/5*x^5*(a+b*arctanh(c/x^2))-
1/5*b*c^(5/2)*arctanh(x/c^(1/2))
```

3.165.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.40

$$\int x^4\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{2}{15}bcx^3+\frac{ax^5}{5}+\frac{1}{5}bc^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{5}bx^5\operatorname{arctanh}\left(\frac{c}{x^2}\right)+\frac{1}{10}bc^{5/2}\log\left(\sqrt{c}-x\right)-\frac{1}{10}bc^{5/2}\log\left(\sqrt{c}+x\right)$$

input
```
Integrate[x^4*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(2*b*c*x^3)/15 + (a*x^5)/5 + (b*c^(5/2)*ArcTan[x/Sqrt[c]])/5 + (b*x^5*ArcT
anh[c/x^2])/5 + (b*c^(5/2)*Log[Sqrt[c] - x])/10 - (b*c^(5/2)*Log[Sqrt[c] +
 x])/10
```

3.165.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6452, 795, 843, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{2}{5}bc\int \frac{x^2}{1-\frac{c^2}{x^4}}dx+\frac{1}{5}x^5\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{795}$$

$$\frac{2}{5}bc\int \frac{x^6}{x^4-c^2}dx+\frac{1}{5}x^5\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{843}$$

$$\frac{2}{5}bc\left(c^2\int -\frac{x^2}{c^2-x^4}dx+\frac{x^3}{3}\right)+\frac{1}{5}x^5\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{2}{5}bc\left(\frac{x^3}{3}-c^2\int \frac{x^2}{c^2-x^4}dx\right)+\frac{1}{5}x^5\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{827}$$

$$\frac{2}{5}bc\left(\frac{x^3}{3}-c^2\left(\frac{1}{2}\int \frac{1}{c-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+c}dx\right)\right)+\frac{1}{5}x^5\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{216}$$

$$\frac{2}{5}bc\left(\frac{x^3}{3}-c^2\left(\frac{1}{2}\int \frac{1}{c-x^2}dx-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{2\sqrt{c}}\right)\right)+\frac{1}{5}x^5\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{5}x^5\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)+\frac{2}{5}bc\left(\frac{x^3}{3}-c^2\left(\frac{\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{2\sqrt{c}}\right)\right)$$

input
```
Int[x^4*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(x^5*(a + b*ArcTanh[c/x^2]))/5 + (2*b*c*(x^3/3 - c^2*(-1/2*ArcTan[x/Sqrt[c
]]/Sqrt[c] + ArcTanh[x/Sqrt[c]]/(2*Sqrt[c]))))/5
```

3.165.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.165.4 Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

method	result
parts	$\frac{a x^5}{5}+\frac{b x^5 \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{5}-\frac{b c^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5}+\frac{b c^{\frac{5}{2}} \arctan\left(\frac{x}{\sqrt{c}}\right)}{5}+\frac{2bc x^3}{15}$
derivativedivides	$\frac{a x^5}{5}+\frac{b x^5 \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{5}+\frac{2bc x^3}{15}-\frac{b c^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5}-\frac{b c^{\frac{5}{2}} \arctan\left(\frac{\sqrt{c}}{x}\right)}{5}$
default	$\frac{a x^5}{5}+\frac{b x^5 \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{5}+\frac{2bc x^3}{15}-\frac{b c^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5}-\frac{b c^{\frac{5}{2}} \arctan\left(\frac{\sqrt{c}}{x}\right)}{5}$
risch	$\frac{b x^5 \ln(x^2+c)}{10}-\frac{b x^5 \ln(-x^2+c)}{10}-\frac{i\pi b x^5 \operatorname{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^3}{20}+\frac{i\pi b x^5 \operatorname{csgn}\left(\frac{i}{x^2}\right)\operatorname{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^2}{20}-$ $i\pi b x^5 \operatorname{csgn}($

input
```
int(x^4*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/5*a*x^5+1/5*b*x^5*arctanh(c/x^2)-1/5*b*c^(5/2)*arctanh(1/x*c^(1/2))+1/5*
b*c^(5/2)*arctan(x/c^(1/2))+2/15*b*c*x^3
```

3.165.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.70

$$\int x^4\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\left[\frac{1}{10}\,bx^5\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{5}\,ax^5+\frac{2}{15}\,bcx^3+\frac{1}{5}\,bc^{\frac{5}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{10}\,bc^{\frac{5}{2}}\log\left(\frac{x^2-2\sqrt{c}x+c}{x^2-c}\right),\frac{1}{10}\,bx^5\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{5}\,ax^5+\frac{2}{15}\,bcx^3+\frac{1}{5}\,b\sqrt{-c}c^2\arctan\left(\frac{\sqrt{-c}x}{c}\right)+\frac{1}{10}\,b\sqrt{-c}c^2\log\left(\frac{x^2+2\sqrt{-c}x-c}{x^2+c}\right)\right]$$

input
```
integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

output
```
[1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b*c^
(5/2)*arctan(x/sqrt(c)) + 1/10*b*c^(5/2)*log((x^2 - 2*sqrt(c)*x + c)/(x^2
- c)), 1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/
5*b*sqrt(-c)*c^2*arctan(sqrt(-c)*x/c) + 1/10*b*sqrt(-c)*c^2*log((x^2 + 2*s
qrt(-c)*x - c)/(x^2 + c))]
```

3.165.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(58) = 116$.

Time = 3.55 (sec) , antiderivative size = 845, normalized size of antiderivative = 13.41

$$\int x^4\left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right) dx$$

$$= \begin{cases} \frac{ax^5}{5} \\ \frac{x^5(a-\infty b)}{5} \\ \frac{x^5(a+\infty b)}{5} \\ -\frac{6ac^2x^5\sqrt{-c}}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{6ax^9\sqrt{-c}}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} - \frac{6bc^{\frac{9}{2}}\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{3bc^{\frac{9}{2}}\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} + \frac{3bc^{\frac{9}{2}}\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-30c^2\sqrt{-c}+30x^4\sqrt{-c}} - \end{cases}$$

input
```
integrate(x**4*(a+b*atanh(c/x**2)),x)
```

output
```
Piecewise((a*x**5/5, Eq(c, 0)), (x**5*(a - oo*b)/5, Eq(c, -x**2)), (x**5*(
a + oo*b)/5, Eq(c, x**2)), (-6*a*c**2*x**5*sqrt(-c)/(-30*c**2*sqrt(-c) + 3
0*x**4*sqrt(-c)) + 6*a*x**9*sqrt(-c)/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)
) - 6*b*c**(9/2)*sqrt(-c)*log(-sqrt(c) + x)/(-30*c**2*sqrt(-c) + 30*x**4*s
qrt(-c)) + 3*b*c**(9/2)*sqrt(-c)*log(x - sqrt(-c))/(-30*c**2*sqrt(-c) + 30
*x**4*sqrt(-c)) + 3*b*c**(9/2)*sqrt(-c)*log(x + sqrt(-c))/(-30*c**2*sqrt(-
c) + 30*x**4*sqrt(-c)) - 6*b*c**(9/2)*sqrt(-c)*atanh(c/x**2)/(-30*c**2*sqr
t(-c) + 30*x**4*sqrt(-c)) + 6*b*c**(5/2)*x**4*sqrt(-c)*log(-sqrt(c) + x)/(
-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) - 3*b*c**(5/2)*x**4*sqrt(-c)*log(x -
 sqrt(-c))/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) - 3*b*c**(5/2)*x**4*sqrt
(-c)*log(x + sqrt(-c))/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) + 6*b*c**(5/
2)*x**4*sqrt(-c)*atanh(c/x**2)/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) - 3*
b*c**5*log(x - sqrt(-c))/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) + 3*b*c**5
*log(x + sqrt(-c))/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) + 3*b*c**3*x**4*
log(x - sqrt(-c))/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) - 3*b*c**3*x**4*l
og(x + sqrt(-c))/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) - 4*b*c**3*x**3*sq
rt(-c)/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) - 6*b*c**2*x**5*sqrt(-c)*ata
nh(c/x**2)/(-30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) + 4*b*c*x**7*sqrt(-c)/(-
30*c**2*sqrt(-c) + 30*x**4*sqrt(-c)) + 6*b*x**9*sqrt(-c)*atanh(c/x**2)/(-3
0*c**2*sqrt(-c) + 30*x**4*sqrt(-c)), True))
```

3.165.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98

$$\int x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$$
$$=\frac{1}{5}\,ax^5+\frac{1}{30}\left(6\,x^5\,\text{artanh}\left(\frac{c}{x^2}\right)+\left(4\,x^3+6\,c^{\frac{3}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+3\,c^{\frac{3}{2}}\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)\right)c\right)b$$

input
```
integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

output
```
1/5*a*x^5 + 1/30*(6*x^5*arctanh(c/x^2) + (4*x^3 + 6*c^(3/2)*arctan(x/sqrt(
c)) + 3*c^(3/2)*log((x - sqrt(c))/(x + sqrt(c))))*c)*b
```

3.165.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int x^4\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{10}\,bx^5\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{5}\,ax^5+\frac{2}{15}\,bcx^3+\frac{bc^3\arctan\left(\frac{x}{\sqrt{-c}}\right)}{5\,\sqrt{-c}}+\frac{1}{5}\,bc^{\frac{5}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)$$

input
```
integrate(x^4*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

output
```
1/10*b*x^5*log((x^2 + c)/(x^2 - c)) + 1/5*a*x^5 + 2/15*b*c*x^3 + 1/5*b*c^3
*arctan(x/sqrt(-c))/sqrt(-c) + 1/5*b*c^(5/2)*arctan(x/sqrt(c))
```

3.165.9 Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int x^4\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{a\,x^5}{5}+\frac{b\,c^{5/2}\,\mathrm{atan}\left(\frac{x}{\sqrt{c}}\right)}{5}+\frac{b\,x^5\,\ln\left(x^2+c\right)}{10}+\frac{2\,b\,c\,x^3}{15}-\frac{b\,x^5\,\ln\left(x^2-c\right)}{10}+\frac{b\,c^{5/2}\,\mathrm{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{5}$$

input
```
int(x^4*(a + b*atanh(c/x^2)),x)
```

output
```
(a*x^5)/5 + (b*c^(5/2)*atan(x/c^(1/2)))/5 + (b*c^(5/2)*atan((x*1i)/c^(1/2)
)*1i)/5 + (b*x^5*log(c + x^2))/10 + (2*b*c*x^3)/15 - (b*x^5*log(x^2 - c))/
10
```

3.166 $\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx$

3.166.1 Optimal result

Integrand size = 14, antiderivative size = 61

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx=\frac{2bcx}{3}-\frac{1}{3}bc^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}x^3\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)-\frac{1}{3}bc^{3/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)$$

output
```
2/3*b*c*x-1/3*b*c^(3/2)*arctan(x/c^(1/2))+1/3*x^3*(a+b*arctanh(c/x^2))-1/3
*b*c^(3/2)*arctanh(x/c^(1/2))
```

3.166.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.41

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx=\frac{2bcx}{3}+\frac{ax^3}{3}-\frac{1}{3}bc^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{3}bx^3\text{arctanh}\left(\frac{c}{x^2}\right)+\frac{1}{6}bc^{3/2}\log\left(\sqrt{c}-x\right)-\frac{1}{6}bc^{3/2}\log\left(\sqrt{c}+x\right)$$

input
```
Integrate[x^2*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(2*b*c*x)/3 + (a*x^3)/3 - (b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (b*x^3*ArcTanh
[c/x^2])/3 + (b*c^(3/2)*Log[Sqrt[c] - x])/6 - (b*c^(3/2)*Log[Sqrt[c] + x])
/6
```

3.166.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6452, 772, 843, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) \, dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{2}{3} b c \int \frac{1}{1-\frac{c^2}{x^4}}dx+\frac{1}{3} x^3 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{ 772}$$

$$\frac{2}{3} b c \int \frac{x^4}{x^4-c^2}dx+\frac{1}{3} x^3 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{ 843}$$

$$\frac{2}{3} b c \left(c^2 \int \frac{1}{x^4-c^2}dx+x\right)+\frac{1}{3} x^3 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{ 756}$$

$$\frac{2}{3} b c \left(c^2 \left(-\frac{\int \frac{1}{c-x^2}dx}{2 c}-\frac{\int \frac{1}{x^2+c}dx}{2 c}\right)+x\right)+\frac{1}{3} x^3 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{ 216}$$

$$\frac{2}{3} b c \left(c^2 \left(-\frac{\int \frac{1}{c-x^2}dx}{2 c}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{2 c^{3/2}}\right)+x\right)+\frac{1}{3} x^3 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{3} x^3 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)+\frac{2}{3} b c \left(c^2 \left(-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{2 c^{3/2}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)}{2 c^{3/2}}\right)+x\right)$$

input
```
Int[x^2*(a + b*ArcTanh[c/x^2]),x]
```

output
```
(x^3*(a + b*ArcTanh[c/x^2]))/3 + (2*b*c*(x + c^2*(-1/2*ArcTan[x/Sqrt[c]]/c
^(3/2) - ArcTanh[x/Sqrt[c]]/(2*c^(3/2)))))/3
```

3.166.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 772
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(n*p)*(b + a/x^n)^p,
x] /; FreeQ[{a, b}, x] && ILtQ[n, 0] && IntegerQ[p]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.166.4 Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

method	result
parts	$\frac{a x^3}{3}+\frac{b x^3 \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{3}+\frac{2bcx}{3}-\frac{b c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3}-\frac{b c^{\frac{3}{2}} \arctan\left(\frac{x}{\sqrt{c}}\right)}{3}$
derivativedivides	$\frac{a x^3}{3}+\frac{b x^3 \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{3}-\frac{b c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3}+\frac{2bcx}{3}+\frac{b c^{\frac{3}{2}} \arctan\left(\frac{\sqrt{c}}{x}\right)}{3}$
default	$\frac{a x^3}{3}+\frac{b x^3 \operatorname{arctanh}\left(\frac{c}{x^2}\right)}{3}-\frac{b c^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3}+\frac{2bcx}{3}+\frac{b c^{\frac{3}{2}} \arctan\left(\frac{\sqrt{c}}{x}\right)}{3}$
risch	$\frac{b x^3 \ln(x^2+c)}{6}-\frac{b x^3 \ln(-x^2+c)}{6}+\frac{i\pi b x^3 \operatorname{csgn}\left(\frac{i}{x^2}\right) \operatorname{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^2}{12}-\frac{i\pi b x^3 \operatorname{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^3}{12}+\frac{i\pi b x^3 \operatorname{csgn}}{}$

input
```
int(x^2*(a+b*arctanh(c/x^2)),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+1/3*b*x^3*arctanh(c/x^2)+2/3*b*c*x-1/3*b*c^(3/2)*arctanh(1/x*c^(
1/2))-1/3*b*c^(3/2)*arctan(x/c^(1/2))
```

3.166.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.66

$$\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\left[\frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{3}\,ax^3-\frac{1}{3}\,bc^{\frac{3}{2}}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{6}\,bc^{\frac{3}{2}}\log\left(\frac{x^2-2\sqrt{c}x+c}{x^2-c}\right)+\frac{2}{3}\,bcx,\ \frac{1}{6}\,bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{3}\,ax^3+\frac{1}{3}\,b\sqrt{-c}c\arctan\left(\frac{\sqrt{-c}x}{c}\right)+\frac{1}{6}\,b\sqrt{-c}c\log\left(\frac{x^2-2\sqrt{-c}x-c}{x^2+c}\right)+\frac{2}{3}\,bcx\right]$$

input
```
integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

output
```
[1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 - 1/3*b*c^(3/2)*arctan(x/s
qrt(c)) + 1/6*b*c^(3/2)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 - c)) + 2/3*b*c*x
, 1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 + 1/3*b*sqrt(-c)*c*arctan
(sqrt(-c)*x/c) + 1/6*b*sqrt(-c)*c*log((x^2 - 2*sqrt(-c)*x - c)/(x^2 + c))
+ 2/3*b*c*x]
```

3.166.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(56) = 112$.

Time = 2.66 (sec) , antiderivative size = 830, normalized size of antiderivative = 13.61

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$$

$$= \begin{cases} \frac{ax^3}{3} \\ \frac{x^3(a-\infty b)}{3} \\ \frac{x^3(a+\infty b)}{3} \\ -\frac{2ac^2x^3\sqrt{-c}}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}}+\frac{2ax^7\sqrt{-c}}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}}-\frac{2bc^{\frac{7}{2}}\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}}+\frac{bc^{\frac{7}{2}}\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}}+\frac{bc^{\frac{7}{2}}\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-6c^2\sqrt{-c}+6x^4\sqrt{-c}}-\frac{2bc^{\frac{7}{2}}\sqrt{}}{-6c^2} \end{cases}$$

input
```
integrate(x**2*(a+b*atanh(c/x**2)),x)
```

output
```
Piecewise((a*x**3/3, Eq(c, 0)), (x**3*(a - oo*b)/3, Eq(c, -x**2)), (x**3*(
a + oo*b)/3, Eq(c, x**2)), (-2*a*c**2*x**3*sqrt(-c)/(-6*c**2*sqrt(-c) + 6*
x**4*sqrt(-c)) + 2*a*x**7*sqrt(-c)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) -
2*b*c**(7/2)*sqrt(-c)*log(-sqrt(c) + x)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c
)) + b*c**(7/2)*sqrt(-c)*log(x - sqrt(-c))/(-6*c**2*sqrt(-c) + 6*x**4*sqrt
(-c)) + b*c**(7/2)*sqrt(-c)*log(x + sqrt(-c))/(-6*c**2*sqrt(-c) + 6*x**4*s
qrt(-c)) - 2*b*c**(7/2)*sqrt(-c)*atanh(c/x**2)/(-6*c**2*sqrt(-c) + 6*x**4*
sqrt(-c)) + 2*b*c**(3/2)*x**4*sqrt(-c)*log(-sqrt(c) + x)/(-6*c**2*sqrt(-c)
 + 6*x**4*sqrt(-c)) - b*c**(3/2)*x**4*sqrt(-c)*log(x - sqrt(-c))/(-6*c**2*
sqrt(-c) + 6*x**4*sqrt(-c)) - b*c**(3/2)*x**4*sqrt(-c)*log(x + sqrt(-c))/(
-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) + 2*b*c**(3/2)*x**4*sqrt(-c)*atanh(c/x
**2)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) + b*c**4*log(x - sqrt(-c))/(-6*c
**2*sqrt(-c) + 6*x**4*sqrt(-c)) - b*c**4*log(x + sqrt(-c))/(-6*c**2*sqrt(-
c) + 6*x**4*sqrt(-c)) - 4*b*c**3*x*sqrt(-c)/(-6*c**2*sqrt(-c) + 6*x**4*sqr
t(-c)) - b*c**2*x**4*log(x - sqrt(-c))/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)
) + b*c**2*x**4*log(x + sqrt(-c))/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) - 2
*b*c**2*x**3*sqrt(-c)*atanh(c/x**2)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) +
 4*b*c*x**5*sqrt(-c)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)) + 2*b*x**7*sqrt(
-c)*atanh(c/x**2)/(-6*c**2*sqrt(-c) + 6*x**4*sqrt(-c)), True))
```

3.166.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$$
$$=\frac{1}{3}ax^3+\frac{1}{6}\left(2\,x^3\operatorname{artanh}\left(\frac{c}{x^2}\right)-\left(2\,\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)-\sqrt{c}\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)-4\,x\right)c\right)b$$

input
```
integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*x^3*arctanh(c/x^2) - (2*sqrt(c)*arctan(x/sqrt(c)) - sqr
t(c)*log((x - sqrt(c))/(x + sqrt(c))) - 4*x)*c)*b
```

3.166.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13

$$\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{3}bc^3\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}c}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right)+\frac{1}{6}bx^3\log\left(\frac{x^2+c}{x^2-c}\right)+\frac{1}{3}ax^3+\frac{2}{3}bcx$$

input
```
integrate(x^2*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

output
```
1/3*b*c^3*(arctan(x/sqrt(-c))/(sqrt(-c)*c) - arctan(x/sqrt(c))/c^(3/2)) +
1/6*b*x^3*log((x^2 + c)/(x^2 - c)) + 1/3*a*x^3 + 2/3*b*c*x
```

3.166.9 Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.07

$$\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{a\,x^3}{3}-\frac{b\,c^{3/2}\operatorname{atan}\left(\frac{x}{\sqrt{c}}\right)}{3}+\frac{2\,b\,c\,x}{3}+\frac{b\,x^3\,\ln\left(x^2+c\right)}{6}-\frac{b\,x^3\,\ln\left(x^2-c\right)}{6}+\frac{b\,c^{3/2}\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{3}$$

input
```
int(x^2*(a + b*atanh(c/x^2)),x)
```

output
```
(a*x^3)/3 - (b*c^(3/2)*atan(x/c^(1/2)))/3 + (b*c^(3/2)*atan((x*1i)/c^(1/2)
)*1i)/3 + (2*b*c*x)/3 + (b*x^3*log(c + x^2))/6 - (b*x^3*log(x^2 - c))/6
```

3.166. $\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$

3.167 $\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx$

3.167.1 Optimal result

Integrand size = 10, antiderivative size = 44

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=ax+b\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)+bx\text{arctanh}\left(\frac{c}{x^2}\right)-b\sqrt{c}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)$$

output
```
a*x+b*x*arctanh(c/x^2)+b*arctan(x/c^(1/2))*c^(1/2)-b*arctanh(x/c^(1/2))*c^
(1/2)
```

3.167.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=ax+bx\text{arctanh}\left(\frac{c}{x^2}\right)+\frac{1}{2}b\sqrt{c}\left(2\arctan\left(\frac{x}{\sqrt{c}}\right)+\log\left(\sqrt{c}-x\right)-\log\left(\sqrt{c}+x\right)\right)$$

input
```
Integrate[a + b*ArcTanh[c/x^2],x]
```

output
```
a*x + b*x*ArcTanh[c/x^2] + (b*Sqrt[c]*(2*ArcTan[x/Sqrt[c]] + Log[Sqrt[c] -
 x] - Log[Sqrt[c] + x]))/2
```

3.167.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax+b\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)+bx\text{arctanh}\left(\frac{c}{x^2}\right)-b\sqrt{c}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)$$

input
```
Int[a + b*ArcTanh[c/x^2],x]
```

output
```
a*x + b*Sqrt[c]*ArcTan[x/Sqrt[c]] + b*x*ArcTanh[c/x^2] - b*Sqrt[c]*ArcTanh
[x/Sqrt[c]]
```

3.167.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.167.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result
default	$ax-b\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}}{x}\right)+b\arctan\left(\frac{x}{\sqrt{c}}\right)\sqrt{c}+bx\,\text{arctanh}\left(\frac{c}{x^2}\right)$
parts	$ax-b\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}}{x}\right)+b\arctan\left(\frac{x}{\sqrt{c}}\right)\sqrt{c}+bx\,\text{arctanh}\left(\frac{c}{x^2}\right)$
derivativedivides	$ax+bx\,\text{arctanh}\left(\frac{c}{x^2}\right)-b\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}}{x}\right)-b\sqrt{c}\,\arctan\left(\frac{\sqrt{c}}{x}\right)$
risch	$ax+\frac{bx\ln(x^2+c)}{2}-\frac{bx\ln(-x^2+c)}{2}+\frac{ib\pi x\text{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^2}{2}-\frac{ib\pi x}{2}-\frac{ib\pi x\text{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^3}{4}-\frac{ib\pi x\ \text{csgn}(}{}$

input
```
int(a+b*arctanh(c/x^2),x,method=_RETURNVERBOSE)
```

output
```
a*x-b*c^(1/2)*arctanh(1/x*c^(1/2))+b*arctan(x/c^(1/2))*c^(1/2)+b*x*arctanh
(c/x^2)
```

3.167.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.14

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx = \left[\frac{1}{2}\,bx\log\left(\frac{x^2+c}{x^2-c}\right)+b\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{1}{2}\,b\sqrt{c}\log\left(\frac{x^2-2\,\sqrt{c}x+c}{x^2-c}\right)+ax,\frac{1}{2}\,bx\log\left(\frac{x^2+c}{x^2-c}\right)+b\sqrt{-c}\arctan\left(\frac{\sqrt{-c}x}{c}\right)+\frac{1}{2}\,b\sqrt{-c}\log\left(\frac{x^2+2\,\sqrt{-c}x-c}{x^2+c}\right)+ax\right]$$

input
```
integrate(a+b*arctanh(c/x^2),x, algorithm="fricas")
```

output
```
[1/2*b*x*log((x^2 + c)/(x^2 - c)) + b*sqrt(c)*arctan(x/sqrt(c)) + 1/2*b*sq
rt(c)*log((x^2 - 2*sqrt(c)*x + c)/(x^2 - c)) + a*x, 1/2*b*x*log((x^2 + c)/
(x^2 - c)) + b*sqrt(-c)*arctan(sqrt(-c)*x/c) + 1/2*b*sqrt(-c)*log((x^2 + 2
*sqrt(-c)*x - c)/(x^2 + c)) + a*x]
```

3.167.6 Sympy [A] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 632, normalized size of antiderivative = 14.36

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx = ax+b\left(\begin{cases}0\\-\infty x\\\infty x\\-\frac{2c^{\frac{5}{2}}\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}+\frac{c^{\frac{5}{2}}\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}+\frac{c^{\frac{5}{2}}\sqrt{-c}\log\left(x+\sqrt{-c}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}-\frac{2c^{\frac{5}{2}}\sqrt{-c}\,\text{atanh}\left(\frac{c}{x^2}\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}+\frac{2\sqrt{c}x^4\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-2c^2\sqrt{-c}+2x^4\sqrt{-c}}\end{cases}\right.$$

input
```
integrate(a+b*atanh(c/x**2),x)
```

output
```
a*x + b*Piecewise((0, Eq(c, 0)), (-oo*x, Eq(c, -x**2)), (oo*x, Eq(c, x**2)
), (-2*c**(5/2)*sqrt(-c)*log(-sqrt(c) + x)/(-2*c**2*sqrt(-c) + 2*x**4*sqrt
(-c)) + c**(5/2)*sqrt(-c)*log(x - sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**4*sqr
t(-c)) + c**(5/2)*sqrt(-c)*log(x + sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**4*sq
rt(-c)) - 2*c**(5/2)*sqrt(-c)*atanh(c/x**2)/(-2*c**2*sqrt(-c) + 2*x**4*sqr
t(-c)) + 2*sqrt(c)*x**4*sqrt(-c)*log(-sqrt(c) + x)/(-2*c**2*sqrt(-c) + 2*x
**4*sqrt(-c)) - sqrt(c)*x**4*sqrt(-c)*log(x - sqrt(-c))/(-2*c**2*sqrt(-c)
+ 2*x**4*sqrt(-c)) - sqrt(c)*x**4*sqrt(-c)*log(x + sqrt(-c))/(-2*c**2*sqrt
(-c) + 2*x**4*sqrt(-c)) + 2*sqrt(c)*x**4*sqrt(-c)*atanh(c/x**2)/(-2*c**2*s
qrt(-c) + 2*x**4*sqrt(-c)) - c**3*log(x - sqrt(-c))/(-2*c**2*sqrt(-c) + 2*
x**4*sqrt(-c)) + c**3*log(x + sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-c
)) - 2*c**2*x*sqrt(-c)*atanh(c/x**2)/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-c))
+ c*x**4*log(x - sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-c)) - c*x**4*l
og(x + sqrt(-c))/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-c)) + 2*x**5*sqrt(-c)*at
anh(c/x**2)/(-2*c**2*sqrt(-c) + 2*x**4*sqrt(-c)), True))
```

3.167.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\frac{1}{2}\left(c\left(\frac{2\,\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}+\frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{\sqrt{c}}\right)+2\,x\,\text{artanh}\left(\frac{c}{x^2}\right)\right)b+ax$$

input
```
integrate(a+b*arctanh(c/x^2),x, algorithm="maxima")
```

output
```
1/2*(c*(2*arctan(x/sqrt(c))/sqrt(c) + log((x - sqrt(c))/(x + sqrt(c)))/sqr
t(c)) + 2*x*arctanh(c/x^2))*b + a*x
```

3.167.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx$$
$$=\frac{1}{2}\left(2c\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}}+\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}\right)+x\log\left(-\frac{\frac{c}{x^2}+1}{\frac{c}{x^2}-1}\right)\right)b+ax$$

input
```
integrate(a+b*arctanh(c/x^2),x, algorithm="giac")
```

output
```
1/2*(2*c*(arctan(x/sqrt(-c))/sqrt(-c) + arctan(x/sqrt(c))/sqrt(c)) + x*log
(-(c/x^2 + 1)/(c/x^2 - 1)))*b + a*x
```

3.167.9 Mupad [B] (verification not implemented)

Time = 3.23 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx = a\,x+\frac{b\,x\,\ln\left(x^2+c\right)}{2}+b\,\sqrt{c}\,\text{atan}\left(\frac{x}{\sqrt{c}}\right)$$
$$-\frac{b\,x\,\ln\left(x^2-c\right)}{2}+b\,\sqrt{c}\,\text{atan}\left(\frac{x\,1\text{i}}{\sqrt{c}}\right)\,1\text{i}$$

input
```
int(a + b*atanh(c/x^2),x)
```

output
```
a*x + (b*x*log(c + x^2))/2 + b*c^(1/2)*atan(x/c^(1/2)) + b*c^(1/2)*atan((x
*1i)/c^(1/2))*1i - (b*x*log(x^2 - c))/2
```

3.168 $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^2}\,dx$

3.168.1 Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^2}\,dx=\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}+\frac{b\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$$

output
```
(-a-b*arctanh(c/x^2))/x+b*arctan(x/c^(1/2))/c^(1/2)+b*arctanh(x/c^(1/2))/c
^(1/2)
```

3.168.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^2}\,dx=-\frac{a}{x}+\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}-\frac{b\log\left(\sqrt{c}-x\right)}{2\sqrt{c}}+\frac{b\log\left(\sqrt{c}+x\right)}{2\sqrt{c}}$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])/x^2,x]
```

output
```
-(a/x) + (b*ArcTan[x/Sqrt[c]])/Sqrt[c] - (b*ArcTanh[c/x^2])/x - (b*Log[Sqr
t[c] - x])/(2*Sqrt[c]) + (b*Log[Sqrt[c] + x])/(2*Sqrt[c])
```

3.168.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6452, 795, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^2}dx$$

$$\Big\downarrow \text{ 6452}$$

$$-2bc\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^4}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}$$

$$\Big\downarrow \text{ 795}$$

$$-2bc\int \frac{1}{x^4-c^2}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}$$

$$\Big\downarrow \text{ 756}$$

$$-2bc\left(-\frac{\int \frac{1}{c-x^2}dx}{2c}-\frac{\int \frac{1}{x^2+c}dx}{2c}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}$$

$$\Big\downarrow \text{ 216}$$

$$-2bc\left(-\frac{\int \frac{1}{c-x^2}dx}{2c}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{2c^{3/2}}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x}-2bc\left(-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)}{2c^{3/2}}\right)$$

input
```
Int[(a + b*ArcTanh[c/x^2])/x^2,x]
```

output
```
-((a + b*ArcTanh[c/x^2])/x) - 2*b*c*(-1/2*ArcTan[x/Sqrt[c]]/c^(3/2) - ArcT
anh[x/Sqrt[c]]/(2*c^(3/2)))
```

3.168.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.168.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

method	result
parts	$-\frac{a}{x}-\frac{b\,\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{\sqrt{c}}+\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}$
derivativedivides	$-\frac{a}{x}-\frac{b\,\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{\sqrt{c}}-\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{\sqrt{c}}$
default	$-\frac{a}{x}-\frac{b\,\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{\sqrt{c}}-\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{\sqrt{c}}$
risch	$-\frac{b\ln\left(x^2+c\right)}{2x}+\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}}-\frac{-ib\pi\operatorname{csgn}\left(\frac{i\left(x^2+c\right)}{x^2}\right)^3+4a+2ib\pi\operatorname{csgn}\left(\frac{i\left(-x^2+c\right)}{x^2}\right)^2+ib\pi\ \operatorname{csgn}\left(\frac{i}{x^2}\right)\operatorname{csgn}\left(i\left(-x^2+\right.\right.}{}$

input
```
int((a+b*arctanh(c/x^2))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-b/x*arctanh(c/x^2)+b/c^(1/2)*arctanh(1/x*c^(1/2))+b*arctan(x/c^(1/2))
/c^(1/2)
```

3.168.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.

Time = 0.27 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.46

$$\int \frac{a + b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^2} \, dx$$
$$= \left[\frac{2\,b\sqrt{c}x\arctan\left(\frac{x}{\sqrt{c}}\right) + b\sqrt{c}x\log\left(\frac{x^2+2\sqrt{c}x+c}{x^2-c}\right) - bc\log\left(\frac{x^2+c}{x^2-c}\right) - 2\,ac}{2\,cx},\right.$$
$$\left. -\frac{2\,b\sqrt{-c}x\arctan\left(\frac{\sqrt{-c}x}{c}\right) + b\sqrt{-c}x\log\left(\frac{x^2-2\sqrt{-c}x-c}{x^2+c}\right) + bc\log\left(\frac{x^2+c}{x^2-c}\right) + 2\,ac}{2\,cx}\right]$$

input
```
integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="fricas")
```

output
```
[1/2*(2*b*sqrt(c)*x*arctan(x/sqrt(c)) + b*sqrt(c)*x*log((x^2 + 2*sqrt(c)*x
 + c)/(x^2 - c)) - b*c*log((x^2 + c)/(x^2 - c)) - 2*a*c)/(c*x), -1/2*(2*b*
sqrt(-c)*x*arctan(sqrt(-c)*x/c) + b*sqrt(-c)*x*log((x^2 - 2*sqrt(-c)*x - c
)/(x^2 + c)) + b*c*log((x^2 + c)/(x^2 - c)) + 2*a*c)/(c*x)]
```

3.168.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. $2(42) = 84$.

Time = 3.54 (sec) , antiderivative size = 886, normalized size of antiderivative = 19.26

$$\int \frac{a + b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^2} \, dx$$
$$= \begin{cases} -\frac{a}{x} \\ -\frac{a-\infty b}{x} \\ -\frac{a+\infty b}{x} \\ \frac{2ac^{\frac{7}{2}}\sqrt{-c}}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} - \frac{2ac^{\frac{3}{2}}x^4\sqrt{-c}}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} - \frac{bc^{\frac{7}{2}}x\log\left(x-\sqrt{-c}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} + \frac{bc^{\frac{7}{2}}x\log\left(x+\sqrt{-c}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{-c}} + \frac{2bc^{\frac{7}{2}}\sqrt{-c}\,\text{atanh}\left(\frac{c}{x^2}\right)}{-2c^{\frac{7}{2}}x\sqrt{-c}+2c^{\frac{3}{2}}x^5\sqrt{}} \end{cases}$$

input
```
integrate((a+b*atanh(c/x**2))/x**2,x)
```

output
```
Piecewise((-a/x, Eq(c, 0)), (-(a - oo*b)/x, Eq(c, -x**2)), (-(a + oo*b)/x,
 Eq(c, x**2)), (2*a*c**(7/2)*sqrt(-c)/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)
*x**5*sqrt(-c)) - 2*a*c**(3/2)*x**4*sqrt(-c)/(-2*c**(7/2)*x*sqrt(-c) + 2*c
**(3/2)*x**5*sqrt(-c)) - b*c**(7/2)*x*log(x - sqrt(-c))/(-2*c**(7/2)*x*sqr
t(-c) + 2*c**(3/2)*x**5*sqrt(-c)) + b*c**(7/2)*x*log(x + sqrt(-c))/(-2*c**
(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) + 2*b*c**(7/2)*sqrt(-c)*atanh
(c/x**2)/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) + b*c**(3/2)*
x**5*log(x - sqrt(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c))
 - b*c**(3/2)*x**5*log(x + sqrt(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*
x**5*sqrt(-c)) - 2*b*c**(3/2)*x**4*sqrt(-c)*atanh(c/x**2)/(-2*c**(7/2)*x*s
qrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) + 2*b*c**3*x*sqrt(-c)*log(-sqrt(c) + x
)/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) - b*c**3*x*sqrt(-c)*
log(x - sqrt(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) - b*
c**3*x*sqrt(-c)*log(x + sqrt(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**
5*sqrt(-c)) + 2*b*c**3*x*sqrt(-c)*atanh(c/x**2)/(-2*c**(7/2)*x*sqrt(-c) +
2*c**(3/2)*x**5*sqrt(-c)) - 2*b*c*x**5*sqrt(-c)*log(-sqrt(c) + x)/(-2*c**(
7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) + b*c*x**5*sqrt(-c)*log(x - sq
rt(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)) + b*c*x**5*sqr
t(-c)*log(x + sqrt(-c))/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3/2)*x**5*sqrt(-c)
) - 2*b*c*x**5*sqrt(-c)*atanh(c/x**2)/(-2*c**(7/2)*x*sqrt(-c) + 2*c**(3...
```

3.168.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.24

$$\int \frac{a + b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^2}\,dx = \frac{1}{2}\left(c\left(\frac{2\,\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}} - \frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) - \frac{2\,\text{artanh}\left(\frac{c}{x^2}\right)}{x}\right)b - \frac{a}{x}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="maxima")
```

output
```
1/2*(c*(2*arctan(x/sqrt(c))/c^(3/2) - log((x - sqrt(c))/(x + sqrt(c)))/c^(
3/2)) - 2*arctanh(c/x^2)/x)*b - a/x
```

3.168.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.35

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^2}\,dx = -bc\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}c} - \frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{3}{2}}}\right) - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{2\,x} - \frac{a}{x}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^2,x, algorithm="giac")
```

output
```
-b*c*(arctan(x/sqrt(-c))/(sqrt(-c)*c) - arctan(x/sqrt(c))/c^(3/2)) - 1/2*b
*log((x^2 + c)/(x^2 - c))/x - a/x
```

3.168.9 Mupad [B] (verification not implemented)

Time = 3.42 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^2}\,dx = \frac{b\operatorname{atan}\left(\frac{x}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{a}{x} - \frac{b\,\ln\left(x^2+c\right)}{2\,x} + \frac{b\,\ln\left(x^2-c\right)}{2\,x} - \frac{b\operatorname{atan}\left(\frac{x\,1\mathrm{i}}{\sqrt{c}}\right)\,1\mathrm{i}}{\sqrt{c}}$$

input
```
int((a + b*atanh(c/x^2))/x^2,x)
```

output
```
(b*atan(x/c^(1/2)))/c^(1/2) - a/x - (b*atan((x*1i)/c^(1/2))*1i)/c^(1/2) -
(b*log(c + x^2))/(2*x) + (b*log(x^2 - c))/(2*x)
```

3.169 $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^4} dx$

3.169.1 Optimal result

Integrand size = 14, antiderivative size = 65

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^4} dx = -\frac{2b}{3cx} - \frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} - \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{3x^3} + \frac{b\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}}$$

output
```
-2/3*b/c/x-1/3*b*arctan(x/c^(1/2))/c^(3/2)+1/3*(-a-b*arctanh(c/x^2))/x^3+1
/3*b*arctanh(x/c^(1/2))/c^(3/2)
```

3.169.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.38

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^4} dx = -\frac{a}{3x^3} - \frac{2b}{3cx} - \frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{3c^{3/2}} - \frac{b\text{arctanh}\left(\frac{c}{x^2}\right)}{3x^3} - \frac{b\log\left(\sqrt{c}-x\right)}{6c^{3/2}} + \frac{b\log\left(\sqrt{c}+x\right)}{6c^{3/2}}$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])/x^4,x]
```

output
```
-1/3*a/x^3 - (2*b)/(3*c*x) - (b*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) - (b*ArcTan
h[c/x^2])/(3*x^3) - (b*Log[Sqrt[c] - x])/(6*c^(3/2)) + (b*Log[Sqrt[c] + x]
)/(6*c^(3/2))
```

3.169.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {6452, 795, 847, 25, 827, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^4}dx$$

$$\Big\downarrow \text{ 6452}$$

$$-\frac{2}{3}bc\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^6}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}$$

$$\Big\downarrow \text{ 795}$$

$$-\frac{2}{3}bc\int \frac{1}{x^2\left(x^4-c^2\right)}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}$$

$$\Big\downarrow \text{ 847}$$

$$-\frac{2}{3}bc\left(\frac{\int -\frac{x^2}{c^2-x^4}dx}{c^2}+\frac{1}{c^2x}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{2}{3}bc\left(\frac{1}{c^2x}-\frac{\int \frac{x^2}{c^2-x^4}dx}{c^2}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}$$

$$\Big\downarrow \text{ 827}$$

$$-\frac{2}{3}bc\left(\frac{1}{c^2x}-\frac{\frac{1}{2}\int \frac{1}{c-x^2}dx-\frac{1}{2}\int \frac{1}{x^2+c}dx}{c^2}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}$$

$$\Big\downarrow \text{ 216}$$

$$-\frac{2}{3}bc\left(\frac{1}{c^2x}-\frac{\frac{1}{2}\int \frac{1}{c-x^2}dx-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{2\sqrt{c}}}{c^2}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{2}{3}bc\left(\frac{1}{c^2x}-\frac{\frac{\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)}{2\sqrt{c}}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{2\sqrt{c}}}{c^2}\right)$$

input
```
Int[(a + b*ArcTanh[c/x^2])/x^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c/x^2])/x^3 - (2*b*c*(1/(c^2*x) - (-1/2*ArcTan[x/Sqrt[
c]]/Sqrt[c] + ArcTanh[x/Sqrt[c]]/(2*Sqrt[c]))/c^2))/3
```

3.169.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

3.169. $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^4}\,dx$

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.169.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{a}{3x^3}-\frac{b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{2b}{3cx}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3c^{\frac{3}{2}}}-\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{3c^{\frac{3}{2}}}$
derivativedivides	$-\frac{a}{3x^3}-\frac{b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{2b}{3cx}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3c^{\frac{3}{2}}}+\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{3c^{\frac{3}{2}}}$
default	$-\frac{a}{3x^3}-\frac{b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{3x^3}-\frac{2b}{3cx}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{3c^{\frac{3}{2}}}+\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{3c^{\frac{3}{2}}}$
risch	$-\frac{b\ln(x^2+c)}{6x^3}-\frac{-ib\pi\operatorname{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^3+4a+2ib\pi\operatorname{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^2+ib\pi\operatorname{csgn}\left(\frac{i}{x^2}\right)\operatorname{csgn}(i(-x^2+c))\operatorname{csgn}\left(\frac{i(-x^2+}{x^2}\right.}{}$

input
```
int((a+b*arctanh(c/x^2))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3-1/3*b/x^3*arctanh(c/x^2)-2/3*b/c/x+1/3*b/c^(3/2)*arctanh(1/x*c^
(1/2))-1/3*b*arctan(x/c^(1/2))/c^(3/2)
```

3.169.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.91

$$\int\frac{a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^4}\,dx$$
$$=\left[-\frac{2\,b\sqrt{c}x^3\arctan\left(\frac{x}{\sqrt{c}}\right)-b\sqrt{c}x^3\log\left(\frac{x^2+2\,\sqrt{c}x+c}{x^2-c}\right)+4\,bcx^2+bc^2\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac^2}{6\,c^2x^3},\right.$$
$$\left.-\frac{2\,b\sqrt{-c}x^3\arctan\left(\frac{\sqrt{-c}x}{c}\right)+b\sqrt{-c}x^3\log\left(\frac{x^2+2\,\sqrt{-c}x-c}{x^2+c}\right)+4\,bcx^2+bc^2\log\left(\frac{x^2+c}{x^2-c}\right)+2\,ac^2}{6\,c^2x^3}\right]$$

input
```
integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="fricas")
```

output
```
[-1/6*(2*b*sqrt(c)*x^3*arctan(x/sqrt(c)) - b*sqrt(c)*x^3*log((x^2 + 2*sqrt
(c)*x + c)/(x^2 - c)) + 4*b*c*x^2 + b*c^2*log((x^2 + c)/(x^2 - c)) + 2*a*c
^2)/(c^2*x^3), -1/6*(2*b*sqrt(-c)*x^3*arctan(sqrt(-c)*x/c) + b*sqrt(-c)*x^
3*log((x^2 + 2*sqrt(-c)*x - c)/(x^2 + c)) + 4*b*c*x^2 + b*c^2*log((x^2 + c
)/(x^2 - c)) + 2*a*c^2)/(c^2*x^3)]
```

3.169.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(60) = 120$.

Time = 4.82 (sec) , antiderivative size = 1046, normalized size of antiderivative = 16.09

$$\int \frac{a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^4}\,dx$$
$$= \begin{cases} -\frac{a}{3x^3} \\ -\frac{a-\infty b}{3x^3} \\ -\frac{a+\infty b}{3x^3} \\ \frac{2ac^{\frac{17}{2}}\sqrt{-c}}{-6c^{\frac{17}{2}}x^3\sqrt{-c}+6c^{\frac{13}{2}}x^7\sqrt{-c}} - \frac{2ac^{\frac{13}{2}}x^4\sqrt{-c}}{-6c^{\frac{17}{2}}x^3\sqrt{-c}+6c^{\frac{13}{2}}x^7\sqrt{-c}} + \frac{2bc^{\frac{17}{2}}\sqrt{-c}\operatorname{atanh}\left(\frac{c}{x^2}\right)}{-6c^{\frac{17}{2}}x^3\sqrt{-c}+6c^{\frac{13}{2}}x^7\sqrt{-c}} + \frac{bc^{\frac{15}{2}}x^3\log\left(x-\sqrt{-c}\right)}{-6c^{\frac{17}{2}}x^3\sqrt{-c}+6c^{\frac{13}{2}}x^7\sqrt{-c}} - \frac{bc^{\frac{15}{2}}x}{-6c^{\frac{17}{2}}x^3} \end{cases}$$

input
```
integrate((a+b*atanh(c/x**2))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3), Eq(c, 0)), (-(a - oo*b)/(3*x**3), Eq(c, -x**2)), (
-(a + oo*b)/(3*x**3), Eq(c, x**2)), (2*a*c**(17/2)*sqrt(-c)/(-6*c**(17/2)*
x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) - 2*a*c**(13/2)*x**4*sqrt(-c)/(
-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) + 2*b*c**(17/2)*sq
rt(-c)*atanh(c/x**2)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-
c)) + b*c**(15/2)*x**3*log(x - sqrt(-c))/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c
**(13/2)*x**7*sqrt(-c)) - b*c**(15/2)*x**3*log(x + sqrt(-c))/(-6*c**(17/2)
*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) + 4*b*c**(15/2)*x**2*sqrt(-c)/
(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) - 2*b*c**(13/2)*x
**4*sqrt(-c)*atanh(c/x**2)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*
sqrt(-c)) - b*c**(11/2)*x**7*log(x - sqrt(-c))/(-6*c**(17/2)*x**3*sqrt(-c)
 + 6*c**(13/2)*x**7*sqrt(-c)) + b*c**(11/2)*x**7*log(x + sqrt(-c))/(-6*c**
(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) - 4*b*c**(11/2)*x**6*sqr
t(-c)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) + 2*b*c**7*
x**3*sqrt(-c)*log(-sqrt(c) + x)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*
x**7*sqrt(-c)) - b*c**7*x**3*sqrt(-c)*log(x - sqrt(-c))/(-6*c**(17/2)*x**3
*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) - b*c**7*x**3*sqrt(-c)*log(x + sqrt
(-c))/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7*sqrt(-c)) + 2*b*c**7*
x**3*sqrt(-c)*atanh(c/x**2)/(-6*c**(17/2)*x**3*sqrt(-c) + 6*c**(13/2)*x**7
*sqrt(-c)) - 2*b*c**5*x**7*sqrt(-c)*log(-sqrt(c) + x)/(-6*c**(17/2)*x**...
```

3.169.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98

$$\int \frac{a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^4}\,dx = -\frac{1}{6}\left(c\left(\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{5}{2}}}+\frac{4}{c^2x}\right)+\frac{2\operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="maxima")
```

output
```
-1/6*(c*(2*arctan(x/sqrt(c))/c^(5/2) + log((x - sqrt(c))/(x + sqrt(c)))/c^
(5/2) + 4/(c^2*x)) + 2*arctanh(c/x^2)/x^3)*b - 1/3*a/x^3
```

3.169.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int \frac{a + b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^4}\,dx = -\frac{b\arctan\left(\frac{x}{\sqrt{-c}}\right)}{3\sqrt{-c}c} - \frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{3\,c^{\frac{3}{2}}} - \frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{6\,x^3} - \frac{2\,bx^2 + ac}{3\,cx^3}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^4,x, algorithm="giac")
```

output
```
-1/3*b*arctan(x/sqrt(-c))/(sqrt(-c)*c) - 1/3*b*arctan(x/sqrt(c))/c^(3/2) -
 1/6*b*log((x^2 + c)/(x^2 - c))/x^3 - 1/3*(2*b*x^2 + a*c)/(c*x^3)
```

3.169.9 Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{a + b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^4}\,dx = \frac{b\,\ln\left(x^2 - c\right)}{6\,x^3} - \frac{2\,b}{3\,c\,x} - \frac{b\,\text{atan}\left(\frac{x}{\sqrt{c}}\right)}{3\,c^{3/2}} - \frac{b\,\ln\left(x^2 + c\right)}{6\,x^3} - \frac{a}{3\,x^3} - \frac{b\,\text{atan}\left(\frac{x\,1\text{i}}{\sqrt{c}}\right)\,1\text{i}}{3\,c^{3/2}}$$

input
```
int((a + b*atanh(c/x^2))/x^4,x)
```

output
```
(b*log(x^2 - c))/(6*x^3) - (2*b)/(3*c*x) - (b*atan(x/c^(1/2)))/(3*c^(3/2))
 - (b*atan((x*1i)/c^(1/2))*1i)/(3*c^(3/2)) - (b*log(c + x^2))/(6*x^3) - a/
(3*x^3)
```

3.170 $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^6} dx$

3.170.1 Optimal result

Integrand size = 14, antiderivative size = 65

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^6} dx = -\frac{2b}{15cx^3}+\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}+\frac{b\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}$$

output
```
-2/15*b/c/x^3+1/5*b*arctan(x/c^(1/2))/c^(5/2)+1/5*(-a-b*arctanh(c/x^2))/x^
5+1/5*b*arctanh(x/c^(1/2))/c^(5/2)
```

3.170.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.38

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^6} dx = -\frac{a}{5x^5}-\frac{2b}{15cx^3}+\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{5c^{5/2}}-\frac{b\text{arctanh}\left(\frac{c}{x^2}\right)}{5x^5} -\frac{b\log\left(\sqrt{c}-x\right)}{10c^{5/2}}+\frac{b\log\left(\sqrt{c}+x\right)}{10c^{5/2}}$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])/x^6,x]
```

output
```
-1/5*a/x^5 - (2*b)/(15*c*x^3) + (b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (b*Arc
Tanh[c/x^2])/(5*x^5) - (b*Log[Sqrt[c] - x])/(10*c^(5/2)) + (b*Log[Sqrt[c]
+ x])/(10*c^(5/2))
```

3.170.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {6452, 795, 847, 756, 216, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^6}dx$$

$$\Big\downarrow \text{6452}$$

$$-\frac{2}{5}bc\int \frac{1}{\left(1-\frac{c^2}{x^4}\right)x^8}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}$$

$$\Big\downarrow \text{795}$$

$$-\frac{2}{5}bc\int \frac{1}{x^4\left(x^4-c^2\right)}dx-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}$$

$$\Big\downarrow \text{847}$$

$$-\frac{2}{5}bc\left(\frac{\int \frac{1}{x^4-c^2}dx}{c^2}+\frac{1}{3c^2x^3}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}$$

$$\Big\downarrow \text{756}$$

$$-\frac{2}{5}bc\left(\frac{-\frac{\int \frac{1}{c-x^2}dx}{2c}-\frac{\int \frac{1}{x^2+c}dx}{2c}}{c^2}+\frac{1}{3c^2x^3}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}$$

$$\Big\downarrow \text{216}$$

$$-\frac{2}{5}bc\left(\frac{-\frac{\int \frac{1}{c-x^2}dx}{2c}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{2c^{3/2}}}{c^2}+\frac{1}{3c^2x^3}\right)-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}$$

$$\Big\downarrow \text{219}$$

$$-\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{2}{5}bc\left(\frac{-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{2c^{3/2}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)}{2c^{3/2}}}{c^2}+\frac{1}{3c^2x^3}\right)$$

input
```
Int[(a + b*ArcTanh[c/x^2])/x^6,x]
```

3.170. $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^6}dx$

output
```
-1/5*(a + b*ArcTanh[c/x^2])/x^5 - (2*b*c*(1/(3*c^2*x^3) + (-1/2*ArcTan[x/S
qrt[c]]/c^(3/2) - ArcTanh[x/Sqrt[c]]/(2*c^(3/2)))/c^2))/5
```

3.170.3.1 Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.170. $\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^6}\,dx$

3.170.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

method	result
parts	$-\frac{a}{5x^5}-\frac{b\,\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{2b}{15c\,x^3}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5c^{\frac{5}{2}}}+\frac{b\arctan\left(\frac{x}{\sqrt{c}}\right)}{5c^{\frac{5}{2}}}$
derivativedivides	$-\frac{a}{5x^5}-\frac{b\,\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{2b}{15c\,x^3}-\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{5c^{\frac{5}{2}}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5c^{\frac{5}{2}}}$
default	$-\frac{a}{5x^5}-\frac{b\,\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{5x^5}-\frac{2b}{15c\,x^3}-\frac{b\arctan\left(\frac{\sqrt{c}}{x}\right)}{5c^{\frac{5}{2}}}+\frac{b\,\operatorname{arctanh}\left(\frac{\sqrt{c}}{x}\right)}{5c^{\frac{5}{2}}}$
risch	$-\frac{b\ln(x^2+c)}{10x^5}-\frac{-ib\pi\operatorname{csgn}\left(\frac{i(x^2+c)}{x^2}\right)^3+4a+2ib\pi\operatorname{csgn}\left(\frac{i(-x^2+c)}{x^2}\right)^2+ib\pi\ \operatorname{csgn}\left(\frac{i}{x^2}\right)\operatorname{csgn}\left(i(-x^2+c)\right)\operatorname{csgn}\left(\frac{i(-x^2+}{x^2}\right.}{}$

input
```
int((a+b*arctanh(c/x^2))/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/5*a/x^5-1/5*b/x^5*arctanh(c/x^2)-2/15*b/c/x^3+1/5*b/c^(5/2)*arctanh(1/x
*c^(1/2))+1/5*b*arctan(x/c^(1/2))/c^(5/2)
```

3.170.5 Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(49) = 98.

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.02

$$\int\frac{a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^6}\,dx$$
$$=\left[\frac{6\,b\sqrt{c}x^5\arctan\left(\frac{x}{\sqrt{c}}\right)+3\,b\sqrt{c}x^5\log\left(\frac{x^2+2\,\sqrt{c}x+c}{x^2-c}\right)-4\,bc^2x^2-3\,bc^3\log\left(\frac{x^2+c}{x^2-c}\right)-6\,ac^3}{30\,c^3x^5},\right.$$
$$\left.-\frac{6\,b\sqrt{-c}x^5\arctan\left(\frac{\sqrt{-c}x}{c}\right)+3\,b\sqrt{-c}x^5\log\left(\frac{x^2-2\,\sqrt{-c}x-c}{x^2+c}\right)+4\,bc^2x^2+3\,bc^3\log\left(\frac{x^2+c}{x^2-c}\right)+6\,ac^3}{30\,c^3x^5}\right]$$

input
```
integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="fricas")
```

output
```
[1/30*(6*b*sqrt(c)*x^5*arctan(x/sqrt(c)) + 3*b*sqrt(c)*x^5*log((x^2 + 2*sq
rt(c)*x + c)/(x^2 - c)) - 4*b*c^2*x^2 - 3*b*c^3*log((x^2 + c)/(x^2 - c)) -
 6*a*c^3)/(c^3*x^5), -1/30*(6*b*sqrt(-c)*x^5*arctan(sqrt(-c)*x/c) + 3*b*sq
rt(-c)*x^5*log((x^2 - 2*sqrt(-c)*x - c)/(x^2 + c)) + 4*b*c^2*x^2 + 3*b*c^3
*log((x^2 + c)/(x^2 - c)) + 6*a*c^3)/(c^3*x^5)]
```

3.170.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(61) = 122$.

Time = 6.80 (sec) , antiderivative size = 994, normalized size of antiderivative = 15.29

$$\int \frac{a + b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^6}\,dx$$
$$= \begin{cases} -\frac{a}{5x^5} \\ -\frac{a-\infty b}{5x^5} \\ -\frac{a+\infty b}{5x^5} \\ \frac{6ac^{13}\sqrt{-c}}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{6ac^{11}x^4\sqrt{-c}}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} + \frac{6bc^{\frac{21}{2}}x^5\sqrt{-c}\log\left(-\sqrt{c}+x\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{3bc^{\frac{21}{2}}x^5\sqrt{-c}\log\left(x-\sqrt{-c}\right)}{-30c^{13}x^5\sqrt{-c}+30c^{11}x^9\sqrt{-c}} - \frac{3bc^{\frac{2}{}}}{-30c} \end{cases}$$

input
```
integrate((a+b*atanh(c/x**2))/x**6,x)
```

output
```
Piecewise((-a/(5*x**5), Eq(c, 0)), (-(a - oo*b)/(5*x**5), Eq(c, -x**2)), (
-(a + oo*b)/(5*x**5), Eq(c, x**2)), (6*a*c**13*sqrt(-c)/(-30*c**13*x**5*sq
rt(-c) + 30*c**11*x**9*sqrt(-c)) - 6*a*c**11*x**4*sqrt(-c)/(-30*c**13*x**5
*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) + 6*b*c**(21/2)*x**5*sqrt(-c)*log(-sqr
t(c) + x)/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) - 3*b*c**(21/
2)*x**5*sqrt(-c)*log(x - sqrt(-c))/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**
9*sqrt(-c)) - 3*b*c**(21/2)*x**5*sqrt(-c)*log(x + sqrt(-c))/(-30*c**13*x**
5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) + 6*b*c**(21/2)*x**5*sqrt(-c)*atanh(c
/x**2)/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) - 6*b*c**(17/2)*
x**9*sqrt(-c)*log(-sqrt(c) + x)/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*s
qrt(-c)) + 3*b*c**(17/2)*x**9*sqrt(-c)*log(x - sqrt(-c))/(-30*c**13*x**5*s
qrt(-c) + 30*c**11*x**9*sqrt(-c)) + 3*b*c**(17/2)*x**9*sqrt(-c)*log(x + sq
rt(-c))/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) - 6*b*c**(17/2)
*x**9*sqrt(-c)*atanh(c/x**2)/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt
(-c)) + 6*b*c**13*sqrt(-c)*atanh(c/x**2)/(-30*c**13*x**5*sqrt(-c) + 30*c**
11*x**9*sqrt(-c)) + 4*b*c**12*x**2*sqrt(-c)/(-30*c**13*x**5*sqrt(-c) + 30*
c**11*x**9*sqrt(-c)) - 3*b*c**11*x**5*log(x - sqrt(-c))/(-30*c**13*x**5*sq
rt(-c) + 30*c**11*x**9*sqrt(-c)) + 3*b*c**11*x**5*log(x + sqrt(-c))/(-30*c
**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) - 6*b*c**11*x**4*sqrt(-c)*ata
nh(c/x**2)/(-30*c**13*x**5*sqrt(-c) + 30*c**11*x**9*sqrt(-c)) - 4*b*c**...
```

3.170.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^6}\,dx$$
$$=\frac{1}{30}\left(c\left(\frac{6\,\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{7}{2}}}-\frac{3\,\log\left(\frac{x-\sqrt{c}}{x+\sqrt{c}}\right)}{c^{\frac{7}{2}}}-\frac{4}{c^2x^3}\right)-\frac{6\,\text{artanh}\left(\frac{c}{x^2}\right)}{x^5}\right)b-\frac{a}{5\,x^5}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="maxima")
```

output
```
1/30*(c*(6*arctan(x/sqrt(c))/c^(7/2) - 3*log((x - sqrt(c))/(x + sqrt(c)))/
c^(7/2) - 4/(c^2*x^3)) - 6*arctanh(c/x^2)/x^5)*b - 1/5*a/x^5
```

3.170.8 Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^6}\,dx$$
$$= -\frac{1}{5}\,b\left(\frac{\arctan\left(\frac{x}{\sqrt{-c}}\right)}{\sqrt{-c}c^2}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)}{c^{\frac{5}{2}}}\right)-\frac{b\log\left(\frac{x^2+c}{x^2-c}\right)}{10\,x^5}-\frac{2\,bx^2+3\,ac}{15\,cx^5}$$

input
```
integrate((a+b*arctanh(c/x^2))/x^6,x, algorithm="giac")
```

output
```
-1/5*b*(arctan(x/sqrt(-c))/(sqrt(-c)*c^2) - arctan(x/sqrt(c))/c^(5/2)) - 1
/10*b*log((x^2 + c)/(x^2 - c))/x^5 - 1/15*(2*b*x^2 + 3*a*c)/(c*x^5)
```

3.170.9 Mupad [B] (verification not implemented)

Time = 3.54 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^6}\,dx = \frac{b\,\text{atan}\left(\frac{x}{\sqrt{c}}\right)}{5\,c^{5/2}}-\frac{2\,b}{15\,c\,x^3}-\frac{a}{5\,x^5}-\frac{b\,\ln\left(x^2+c\right)}{10\,x^5}$$
$$+\frac{b\,\ln\left(x^2-c\right)}{10\,x^5}-\frac{b\,\text{atan}\left(\frac{x\,1\text{i}}{\sqrt{c}}\right)\,1\text{i}}{5\,c^{5/2}}$$

input
```
int((a + b*atanh(c/x^2))/x^6,x)
```

output
```
(b*atan(x/c^(1/2)))/(5*c^(5/2)) - (2*b)/(15*c*x^3) - a/(5*x^5) - (b*atan((
x*1i)/c^(1/2))*1i)/(5*c^(5/2)) - (b*log(c + x^2))/(10*x^5) + (b*log(x^2 -
c))/(10*x^5)
```

3.171 $\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$

3.171.1 Optimal result

Integrand size = 16, antiderivative size = 94

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx=\frac{1}{2}bcx^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)-\frac{1}{4}c^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{4}x^4\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{4}b^2c^2\log\left(1-\frac{c^2}{x^4}\right)+b^2c^2\log(x)$$

output
```
1/2*b*c*x^2*(a+b*arccoth(x^2/c))-1/4*c^2*(a+b*arccoth(x^2/c))^2+1/4*x^4*(a
+b*arccoth(x^2/c))^2+1/4*b^2*c^2*ln(1-c^2/x^4)+b^2*c^2*ln(x)
```

3.171.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

$$\int x^3\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx=\frac{1}{4}\left(2abcx^2+a^2x^4+2bx^2\left(bc+ax^2\right)\text{arctanh}\left(\frac{c}{x^2}\right)+b^2\left(-c^2+x^4\right)\text{arctanh}\left(\frac{c}{x^2}\right)^2+b(a+b)c^2\log\left(-c+x^2\right)-abc^2\log\left(c+x^2\right)+b^2c^2\log\left(c+x^2\right)\right)$$

input
```
Integrate[x^3*(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
(2*a*b*c*x^2 + a^2*x^4 + 2*b*x^2*(b*c + a*x^2)*ArcTanh[c/x^2] + b^2*(-c^2
+ x^4)*ArcTanh[c/x^2]^2 + b*(a + b)*c^2*Log[-c + x^2] - a*b*c^2*Log[c + x^
2] + b^2*c^2*Log[c + x^2])/4
```

3.171.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {6454, 6452, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

$$\Big\downarrow \text{6454}$$

$$-\frac{1}{2}\int x^6 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 d\frac{1}{x^2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 - bc\int \frac{x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{6544}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 - bc\left(c^2\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}+\int x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)d\frac{1}{x^2}\right)\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 - bc\left(c^2\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}+bc\int \frac{x^2}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}-x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\right)\right)$$

$$\Big\downarrow \text{243}$$

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 - bc\left(c^2\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}+\frac{1}{2}bc\int \frac{x^2}{1-\frac{c^2}{x^4}}d\frac{1}{x^4}-x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\right)\right)$$

$\downarrow$ 47

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2-bc\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}+\frac{1}{2}bc\left(c^2\int\frac{1}{1-\frac{c^2}{x^4}}d\frac{1}{x^4}+\int x^2d\frac{1}{x^4}\right)-x^2\left(a+\right.\right.\right.$$

$\downarrow$ 14

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2-bc\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}+\frac{1}{2}bc\left(c^2\int\frac{1}{1-\frac{c^2}{x^4}}d\frac{1}{x^4}+\log\left(\frac{1}{x^4}\right)\right)-x^2\left(a+\right.\right.\right.$$

$\downarrow$ 16

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2-bc\left(c^2\int\frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}-x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)+\frac{1}{2}bc\left(\log\left(\frac{1}{x^4}\right)-\text{lo}\right.\right.\right.$$

$\downarrow$ 6510

$$\frac{1}{2}\left(\frac{1}{2}x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2-bc\left(-\left(x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\right)+\frac{c\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2b}+\frac{1}{2}bc\left(\log\left(\frac{1}{x^4}\right)-\text{log}\right.\right.\right.$$

input
```
Int[x^3*(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
((x^4*(a + b*ArcTanh[c/x^2])^2)/2 - b*c*(-(x^2*(a + b*ArcTanh[c/x^2])) + (
c*(a + b*ArcTanh[c/x^2])^2)/(2*b) + (b*c*(-Log[1 - c^2/x^4] + Log[x^(-4)])
)/2))/2
```

3.171.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.171.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.50 (sec) , antiderivative size = 805, normalized size of antiderivative = 8.56

Expression too large to display

input
```
int(x^3*(a+b*arctanh(c/x^2))^2,x)
```

output
```
1/4*a^2*x^4-b^2*(-1/4*x^4*arctanh(c/x^2)^2+c*(-1/2*x^2*arctanh(c/x^2)+1/4*
arctanh(c/x^2)*c*ln(1+c/x^2)-1/4*arctanh(c/x^2)*c*ln(c/x^2-1)-1/2*c*(c*(Su
m(-1/4*(ln(1/x-_alpha)*ln(c/x^2-1)-2*c*(1/2*ln(1/x-_alpha)*(ln((RootOf(_Z^
2*c+2*_Z*_alpha*c-2,index=1)-1/x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,ind
ex=1))+ln((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)-1/x+_alpha)/RootOf(_Z^2*
c+2*_Z*_alpha*c-2,index=2)))/c+1/2*(dilog((RootOf(_Z^2*c+2*_Z*_alpha*c-2,i
ndex=1)-1/x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1))+dilog((RootOf(
_Z^2*c+2*_Z*_alpha*c-2,index=2)-1/x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,
index=2)))/c))/c,_alpha=RootOf(_Z^2*c+1))+Sum(1/4*(ln(1/x-_alpha)*ln(c/x^2
-1)-2*c*(1/4/_alpha/c*ln(1/x-_alpha)^2-1/2*_alpha*ln(1/x-_alpha)*ln(1/2*(_
alpha+1/x)/_alpha)-1/2*_alpha*dilog(1/2*(_alpha+1/x)/_alpha)))/c,_alpha=Ro
otOf(_Z^2*c-1)))+1/2*ln(1+c/x^2)+1/2*ln(c/x^2-1)-2*ln(1/x)-c*(Sum(-1/4*(ln
(1/x-_alpha)*ln(1+c/x^2)-2*c*(1/4/_alpha/c*ln(1/x-_alpha)^2+1/2*_alpha*ln(
1/x-_alpha)*ln(1/2*(_alpha+1/x)/_alpha)+1/2*_alpha*dilog(1/2*(_alpha+1/x)/
_alpha)))/c,_alpha=RootOf(_Z^2*c+1))+Sum(1/4*(ln(1/x-_alpha)*ln(1+c/x^2)-2
*c*(1/2*ln(1/x-_alpha)*(ln((RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=1)-1/x+_al
pha)/RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=1))+ln((RootOf(_Z^2*c+2*_Z*_alpha
*c+2,index=2)-1/x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=2)))/c+1/2*(
dilog((RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=1)-1/x+_alpha)/RootOf(_Z^2*c+2*
_Z*_alpha*c+2,index=1))+dilog((RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=2)-1...
```

3.171.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\begin{aligned}\int x^3\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx &= \frac{1}{4}\,a^2x^4+\frac{1}{2}\,abcx^2-\frac{1}{4}\,(ab-b^2)c^2\log\left(x^2+c\right)\\ &\quad+\frac{1}{4}\,(ab+b^2)c^2\log\left(x^2-c\right)\\ &\quad+\frac{1}{16}\,(b^2x^4-b^2c^2)\log\left(\frac{x^2+c}{x^2-c}\right)^2\\ &\quad+\frac{1}{4}\,(abx^4+b^2cx^2)\log\left(\frac{x^2+c}{x^2-c}\right)\end{aligned}$$

input
```
integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

output
```
1/4*a^2*x^4 + 1/2*a*b*c*x^2 - 1/4*(a*b - b^2)*c^2*log(x^2 + c) + 1/4*(a*b
+ b^2)*c^2*log(x^2 - c) + 1/16*(b^2*x^4 - b^2*c^2)*log((x^2 + c)/(x^2 - c)
)^2 + 1/4*(a*b*x^4 + b^2*c*x^2)*log((x^2 + c)/(x^2 - c))
```

3.171.6 Sympy [A] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.61

$$\int x^{3}\left(a+b\text{arctanh}\left(\frac{c}{x^{2}}\right)\right)^{2}dx = \frac{a^{2}x^{4}}{4} - \frac{abc^{2}\operatorname{atanh}\left(\frac{c}{x^{2}}\right)}{2} + \frac{abcx^{2}}{2} + \frac{abx^{4}\operatorname{atanh}\left(\frac{c}{x^{2}}\right)}{2} + \frac{b^{2}c^{2}\log\left(x-\sqrt{-c}\right)}{2} + \frac{b^{2}c^{2}\log\left(x+\sqrt{-c}\right)}{2} - \frac{b^{2}c^{2}\operatorname{atanh}^{2}\left(\frac{c}{x^{2}}\right)}{4} - \frac{b^{2}c^{2}\operatorname{atanh}\left(\frac{c}{x^{2}}\right)}{2} + \frac{b^{2}cx^{2}\operatorname{atanh}\left(\frac{c}{x^{2}}\right)}{2} + \frac{b^{2}x^{4}\operatorname{atanh}^{2}\left(\frac{c}{x^{2}}\right)}{4}$$

input
```
integrate(x**3*(a+b*atanh(c/x**2))**2,x)
```

output
```
a**2*x**4/4 - a*b*c**2*atanh(c/x**2)/2 + a*b*c*x**2/2 + a*b*x**4*atanh(c/x
**2)/2 + b**2*c**2*log(x - sqrt(-c))/2 + b**2*c**2*log(x + sqrt(-c))/2 - b
**2*c**2*atanh(c/x**2)**2/4 - b**2*c**2*atanh(c/x**2)/2 + b**2*c*x**2*atan
h(c/x**2)/2 + b**2*x**4*atanh(c/x**2)**2/4
```

3.171.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.67

$$\int x^{3}\left(a+b\text{arctanh}\left(\frac{c}{x^{2}}\right)\right)^{2}dx = \frac{1}{4}b^{2}x^{4}\operatorname{artanh}\left(\frac{c}{x^{2}}\right)^{2} + \frac{1}{4}a^{2}x^{4} + \frac{1}{4}\left(2x^{4}\operatorname{artanh}\left(\frac{c}{x^{2}}\right) + \left(2x^{2} - c\log\left(x^{2}+c\right) + c\log\left(x^{2}-c\right)\right)c\right)ab + \frac{1}{16}\left(\left(\log\left(x^{2}+c\right)^{2} - 2\left(\log\left(x^{2}+c\right)-2\right)\log\left(x^{2}-c\right) + \log\left(x^{2}-c\right)^{2} + 4\log\left(x^{2}+c\right)\right)c^{2} + 4\left(2x^{2} - c\right.\right.$$

input
```
integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctanh(c/x^2)^2 + 1/4*a^2*x^4 + 1/4*(2*x^4*arctanh(c/x^2) + (
2*x^2 - c*log(x^2 + c) + c*log(x^2 - c))*c)*a*b + 1/16*((log(x^2 + c)^2 -
2*(log(x^2 + c) - 2)*log(x^2 - c) + log(x^2 - c)^2 + 4*log(x^2 + c))*c^2 +
 4*(2*x^2 - c*log(x^2 + c) + c*log(x^2 - c))*c*arctanh(c/x^2))*b^2
```

3.171.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(86) = 172$.

Time = 0.30 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.48

$$\int x^3 \left(a + b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx =$$

$$-\frac{2\,b^2c^3\log\left(\frac{x^2+c}{x^2-c}-1\right) - 2\,b^2c^3\log\left(\frac{x^2+c}{x^2-c}\right) - \frac{(x^2+c)b^2c^3\log\left(\frac{x^2+c}{x^2-c}\right)^2}{(x^2-c)\left(\frac{(x^2+c)^2}{(x^2-c)^2}-\frac{2\,(x^2+c)}{x^2-c}+1\right)} - \frac{2\left(\frac{2\,(x^2+c)abc^3}{x^2-c}+\frac{(x^2+c)b^2c^3}{x^2-c}-b^2c^3\right)\log(}{\frac{(x^2+c)^2}{(x^2-c)^2}-\frac{2\,(x^2+c)}{x^2-c}+1}}{4\,c}$$

input
```
integrate(x^3*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

output
```
-1/4*(2*b^2*c^3*log((x^2 + c)/(x^2 - c) - 1) - 2*b^2*c^3*log((x^2 + c)/(x^
2 - c)) - (x^2 + c)*b^2*c^3*log((x^2 + c)/(x^2 - c))^2/((x^2 - c)*((x^2 +
c)^2/(x^2 - c)^2 - 2*(x^2 + c)/(x^2 - c) + 1)) - 2*(2*(x^2 + c)*a*b*c^3/(x
^2 - c) + (x^2 + c)*b^2*c^3/(x^2 - c) - b^2*c^3)*log((x^2 + c)/(x^2 - c))/
((x^2 + c)^2/(x^2 - c)^2 - 2*(x^2 + c)/(x^2 - c) + 1) - 4*((x^2 + c)*a^2*c
^3/(x^2 - c) + (x^2 + c)*a*b*c^3/(x^2 - c) - a*b*c^3)/((x^2 + c)^2/(x^2 -
c)^2 - 2*(x^2 + c)/(x^2 - c) + 1))/c
```

3.171.9 Mupad [B] (verification not implemented)

Time = 3.81 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.63

$$\int x^3 \left(a + b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \frac{a^2\,x^4}{4} - \frac{a\,b\,c^2\,\ln\left(x^2+c\right)}{4} + \frac{a\,b\,c^2\,\ln\left(x^2-c\right)}{4} + \frac{a\,b\,c\,x^2}{2} + \frac{a\,b\,x^4\,\ln\left(x^2+c\right)}{4} - \frac{a\,b\,x^4\,\ln\left(x^2-c\right)}{4} - \frac{b^2\,c^2\ln\left(x^2+c\right)^2}{16} + \frac{b^2\,c^2\,\ln\left(x^2+c\right)\,\ln\left(x^2-c\right)}{8} + \frac{b^2\,c^2\,\ln\left(x^2+c\right)}{4} - \frac{b^2\,c^2\ln\left(x^2-c\right)^2}{16} + \frac{b^2\,c^2\,\ln\left(x^2-c\right)}{4} + \frac{b^2\,c\,x^2\,\ln\left(x^2+c\right)}{4} - \frac{b^2\,c\,x^2\,\ln\left(x^2-c\right)}{4} + \frac{b^2\,x^4\ln\left(x^2+c\right)^2}{16} - \frac{b^2\,x^4\,\ln\left(x^2+c\right)\,\ln\left(x^2-c\right)}{8} + \frac{b^2\,x^4\ln\left(x^2-c\right)^2}{16}$$

input
```
int(x^3*(a + b*atanh(c/x^2))^2,x)
```

output
```
(a^2*x^4)/4 + (b^2*c^2*log(x^2 - c))/4 - (b^2*c^2*log(c + x^2)^2)/16 + (b^
2*x^4*log(c + x^2)^2)/16 - (b^2*c^2*log(x^2 - c)^2)/16 + (b^2*x^4*log(x^2
- c)^2)/16 + (b^2*c^2*log(c + x^2))/4 + (a*b*x^4*log(c + x^2))/4 + (a*b*c^
2*log(x^2 - c))/4 + (b^2*c^2*log(c + x^2)*log(x^2 - c))/8 + (a*b*c*x^2)/2
- (a*b*x^4*log(x^2 - c))/4 + (b^2*c*x^2*log(c + x^2))/4 - (b^2*x^4*log(c +
 x^2)*log(x^2 - c))/8 - (b^2*c*x^2*log(x^2 - c))/4 - (a*b*c^2*log(c + x^2)
)/4
```

3.171. $\int x^3 \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx$

3.172 $\int x\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$

3.172.1 Optimal result

Integrand size = 14, antiderivative size = 94

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx=-\frac{1}{2}c\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2+\frac{1}{2}x^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2-bc\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\log\left(2-\frac{2}{1+\frac{c}{x^2}}\right)+\frac{1}{2}b^2c\,\text{PolyLog}\left(2,-1+\frac{2}{1+\frac{c}{x^2}}\right)$$

output
```
-1/2*c*(a+b*arccoth(x^2/c))^2+1/2*x^2*(a+b*arccoth(x^2/c))^2-b*c*(a+b*arcc
oth(x^2/c))*ln(2-2/(1+c/x^2))+1/2*b^2*c*polylog(2,-1+2/(1+c/x^2))
```

3.172.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx=\frac{1}{2}\left(b^2(-c+x^2)\,\text{arctanh}\left(\frac{c}{x^2}\right)^2+2b\text{arctanh}\left(\frac{c}{x^2}\right)\left(ax^2-bc\log\left(1-e^{-2\text{arctanh}\left(\frac{c}{x^2}\right)}\right)\right)+a\left(ax^2+bc\log\left(1-\frac{c^2}{x^4}\right)-2bc\log\left(\frac{c}{x^2}\right)\right)+b^2c\,\text{PolyLog}\left(2,e^{-2\text{arctanh}\left(\frac{c}{x^2}\right)}\right)\right)$$

input
```
Integrate[x*(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
(b^2*(-c + x^2)*ArcTanh[c/x^2]^2 + 2*b*ArcTanh[c/x^2]*(a*x^2 - b*c*Log[1 -
 E^(-2*ArcTanh[c/x^2])]) + a*(a*x^2 + b*c*Log[1 - c^2/x^4] - 2*b*c*Log[c/x
^2]) + b^2*c*PolyLog[2, E^(-2*ArcTanh[c/x^2])])/2
```

3.172.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6454, 6452, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2dx$$

$$\Big\downarrow \text{6454}$$

$$-\frac{1}{2}\int x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2-2bc\int \frac{x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}\right)$$

$$\Big\downarrow \text{6550}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2-2bc\left(\int \frac{x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{\frac{c}{x^2}+1}d\frac{1}{x^2}+\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2b}\right)\right)$$

$$\Big\downarrow \text{6494}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2-2bc\left(-bc\int \frac{\log\left(2-\frac{2}{\frac{c}{x^2}+1}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}+\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2b}+\log\left(2-\frac{2}{\frac{c}{x^2}+1}\right)\left(a+\right.\right.\right.$$

$$\Big\downarrow \text{2897}$$

$$\frac{1}{2}\left(x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2-2bc\left(\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2b}+\log\left(2-\frac{2}{\frac{c}{x^2}+1}\right)\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)-\frac{1}{2}b\,\text{PolyLo}\right.\right.$$

input
```
Int[x*(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
(x^2*(a + b*ArcTanh[c/x^2])^2 - 2*b*c*((a + b*ArcTanh[c/x^2])^2/(2*b) + (a
 + b*ArcTanh[c/x^2])*Log[2 - 2/(1 + c/x^2)] - (b*PolyLog[2, -1 + 2/(1 + c/
x^2)])/2))/2
```

3.172.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.172.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 10.81 (sec) , antiderivative size = 889, normalized size of antiderivative = 9.46

method	result	size
derivativedivides	Expression too large to display	889
default	Expression too large to display	889
parts	Expression too large to display	889
risch	Expression too large to display	4795

input
```
int(x*(a+b*arctanh(c/x^2))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2-b^2*(-1/2*x^2*arctanh(c/x^2)^2+2*c*(-1/4*arctanh(c/x^2)*ln(c/x
^2-1)-1/4*arctanh(c/x^2)*ln(1+c/x^2)+ln(1/x)*arctanh(c/x^2)-1/2*c*(Sum(1/4
*(ln(1/x-_alpha)*ln(c/x^2-1)-2*c*(1/4/_alpha/c*ln(1/x-_alpha)^2-1/2*_alpha
*ln(1/x-_alpha)*ln(1/2*(_alpha+1/x)/_alpha)-1/2*_alpha*dilog(1/2*(_alpha+1
/x)/_alpha)))/c,_alpha=RootOf(_Z^2*c-1))+Sum(-1/4*(ln(1/x-_alpha)*ln(c/x^2
-1)-2*c*(1/2*ln(1/x-_alpha)*(ln((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1)-1/
x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1))+ln((RootOf(_Z^2*c+2*_Z*_
alpha*c-2,index=2)-1/x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)))/c+
1/2*(dilog((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=1)-1/x+_alpha)/RootOf(_Z^2
*c+2*_Z*_alpha*c-2,index=1))+dilog((RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)
-1/x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c-2,index=2)))/c))/c,_alpha=RootOf(
_Z^2*c+1))+Sum(1/4*(ln(1/x-_alpha)*ln(1+c/x^2)-2*c*(1/2*ln(1/x-_alpha)*(ln
((RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=1)-1/x+_alpha)/RootOf(_Z^2*c+2*_Z*_a
lpha*c+2,index=1))+ln((RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=2)-1/x+_alpha)/
RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=2)))/c+1/2*(dilog((RootOf(_Z^2*c+2*_Z*
_alpha*c+2,index=1)-1/x+_alpha)/RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=1))+di
log((RootOf(_Z^2*c+2*_Z*_alpha*c+2,index=2)-1/x+_alpha)/RootOf(_Z^2*c+2*_Z
*_alpha*c+2,index=2)))/c))/c,_alpha=RootOf(_Z^2*c-1))+Sum(-1/4*(ln(1/x-_al
pha)*ln(1+c/x^2)-2*c*(1/4/_alpha/c*ln(1/x-_alpha)^2+1/2*_alpha*ln(1/x-_alp
ha)*ln(1/2*(_alpha+1/x)/_alpha)+1/2*_alpha*dilog(1/2*(_alpha+1/x)/_alph...
```

3.172.5 Fricas [F]

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b\operatorname{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 x\,dx$$

input
```
integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x*arctanh(c/x^2)^2 + 2*a*b*x*arctanh(c/x^2) + a^2*x, x)
```

3.172.6 Sympy [F]

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int x\left(a+b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
integrate(x*(a+b*atanh(c/x**2))**2,x)
```

output
```
Integral(x*(a + b*atanh(c/x**2))**2, x)
```

3.172.7 Maxima [F]

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b\operatorname{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 x\,dx$$

input
```
integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*x^2 + 1/2*(2*x^2*arctanh(c/x^2) + c*log(x^4 - c^2))*a*b + 1/8*(x^2
*log(x^2 + c)^2 - 2*(x^2 + c)*log(x^2 + c)*log(x^2 - c) + (x^2 - c)*log(x^
2 - c)^2 + 2*integrate(2*(3*c*x^3 + c^2*x)*log(x^2 + c)/(x^4 - c^2), x))*b
^2
```

3.172.8 Giac [F]

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx=\int \left(b\operatorname{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 x\,dx$$

input
```
integrate(x*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2*x, x)
```

3.172.9 Mupad [F(-1)]

Timed out.

$$\int x\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx=\int x\left(a+b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
int(x*(a + b*atanh(c/x^2))^2,x)
```

output
```
int(x*(a + b*atanh(c/x^2))^2, x)
```

3.173 $\int \frac{\left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx$

3.173.1 Optimal result

Integrand size = 16, antiderivative size = 144

$$\int \frac{\left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx = -\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\text{arctanh}\left(1-\frac{2}{1-\frac{c}{x^2}}\right) + \frac{1}{2}b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right) - \frac{1}{2}b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\text{PolyLog}\left(2,-1+\frac{2}{1-\frac{c}{x^2}}\right) - \frac{1}{4}b^2\,\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x^2}}\right) + \frac{1}{4}b^2\,\text{PolyLog}\left(3,-1+\frac{2}{1-\frac{c}{x^2}}\right)$$

output
```
(a+b*arccoth(x^2/c))^2*arctanh(-1+2/(1-c/x^2))+1/2*b*(a+b*arccoth(x^2/c))*
polylog(2,1-2/(1-c/x^2))-1/2*b*(a+b*arccoth(x^2/c))*polylog(2,-1+2/(1-c/x^
2))-1/4*b^2*polylog(3,1-2/(1-c/x^2))+1/4*b^2*polylog(3,-1+2/(1-c/x^2))
```

3.173.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.27

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx = a^2\log(x)+\frac{1}{2}ab\left(\operatorname{PolyLog}\left(2,-\frac{c}{x^2}\right)-\operatorname{PolyLog}\left(2,\frac{c}{x^2}\right)\right) + \frac{1}{2}b^2\left(-\frac{i\pi^3}{24}+\frac{2}{3}\operatorname{arctanh}\left(\frac{c}{x^2}\right)^3 + \operatorname{arctanh}\left(\frac{c}{x^2}\right)^2\log\left(1+e^{-2\operatorname{arctanh}\left(\frac{c}{x^2}\right)}\right) - \operatorname{arctanh}\left(\frac{c}{x^2}\right)^2\log\left(1-e^{2\operatorname{arctanh}\left(\frac{c}{x^2}\right)}\right) - \operatorname{arctanh}\left(\frac{c}{x^2}\right)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}\left(\frac{c}{x^2}\right)}\right) - \operatorname{arctanh}\left(\frac{c}{x^2}\right)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}\left(\frac{c}{x^2}\right)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}\left(\frac{c}{x^2}\right)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}\left(\frac{c}{x^2}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])^2/x,x]
```

output
```
a^2*Log[x] + (a*b*(PolyLog[2, -(c/x^2)] - PolyLog[2, c/x^2]))/2 + (b^2*((-
1/24*I)*Pi^3 + (2*ArcTanh[c/x^2]^3)/3 + ArcTanh[c/x^2]^2*Log[1 + E^(-2*Arc
Tanh[c/x^2])] - ArcTanh[c/x^2]^2*Log[1 - E^(2*ArcTanh[c/x^2])] - ArcTanh[c
/x^2]*PolyLog[2, -E^(-2*ArcTanh[c/x^2])] - ArcTanh[c/x^2]*PolyLog[2, E^(2*
ArcTanh[c/x^2])] - PolyLog[3, -E^(-2*ArcTanh[c/x^2])]/2 + PolyLog[3, E^(2*
ArcTanh[c/x^2])]/2))/2
```

3.173.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6450, 6448, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{ 6450}$$

$$-\frac{1}{2}\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{ 6448}$$

$$\frac{1}{2}\left(4bc\int \frac{\text{arctanh}\left(1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}-2\text{arctanh}\left(1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\right)$$

$$\Big\downarrow \text{ 6614}$$

$$\frac{1}{2}\left(4bc\left(\frac{1}{2}\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\log\left(2-\frac{2}{1-\frac{c}{x^2}}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}-\frac{1}{2}\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)\log\left(\frac{2}{1-\frac{c}{x^2}}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}\right)-2\text{arctanh}\left(\right.\right.$$

$$\Big\downarrow \text{ 6620}$$

$$\frac{1}{2}\left(4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{2c}-\frac{1}{2}b\int \frac{\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}\right)+\frac{1}{2}\left(\frac{1}{2}b\int \frac{\text{PolyL}}{}\right.\right.\right.$$

$$\Big\downarrow \text{ 7164}$$

$$\frac{1}{2}\left(4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{1-\frac{c}{x^2}}\right)}{4c}\right)+\frac{1}{2}\left(\frac{b\,\text{PolyLog}\left(3,\frac{2}{1-\frac{c}{x^2}}\right.}{4c}\right.\right.\right.$$

input
```
Int[(a + b*ArcTanh[c/x^2])^2/x,x]
```

output
```
(-2*ArcTanh[1 - 2/(1 - c/x^2)]*(a + b*ArcTanh[c/x^2])^2 + 4*b*c*((((a + b*
ArcTanh[c/x^2])*PolyLog[2, 1 - 2/(1 - c/x^2)])/(2*c) - (b*PolyLog[3, 1 - 2
/(1 - c/x^2)])/(4*c))/2 + (-1/2*((a + b*ArcTanh[c/x^2])*PolyLog[2, -1 + 2/
(1 - c/x^2)])/c + (b*PolyLog[3, -1 + 2/(1 - c/x^2)])/(4*c))/2))/2
```

3.173.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.173.4 Maple [F]

$$\int \frac{\left(a+b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x} dx$$

input
```
int((a+b*arctanh(c/x^2))^2/x,x)
```

output
```
int((a+b*arctanh(c/x^2))^2/x,x)
```

3.173.5 Fricas [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x} dx = \int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x} dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x, x)
```

3.173.6 Sympy [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x} dx = \int \frac{\left(a+b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x} dx$$

input
```
integrate((a+b*atanh(c/x**2))**2/x,x)
```

output
```
Integral((a + b*atanh(c/x**2))**2/x, x)
```

3.173.7 Maxima [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="maxima")
```

output
```
a^2*log(x) + integrate(1/4*b^2*(log(c/x^2 + 1) - log(-c/x^2 + 1))^2/x + a*
b*(log(c/x^2 + 1) - log(-c/x^2 + 1))/x, x)
```

3.173.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2/x, x)
```

3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*atanh(c/x^2))^2/x,x)
```

output
```
int((a + b*atanh(c/x^2))^2/x, x)
```

3.174 $\int \frac{\left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx$

3.174.1 Optimal result

Integrand size = 16, antiderivative size = 99

$$\int \frac{\left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx = -\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2c}-\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{2x^2}+\frac{b\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)\log\left(\frac{2}{1-\frac{c}{x^2}}\right)}{c}+\frac{b^2\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)}{2c}$$

output
```
-1/2*(a+b*arccoth(x^2/c))^2/c-1/2*(a+b*arccoth(x^2/c))^2/x^2+b*(a+b*arccot
h(x^2/c))*ln(2/(1-c/x^2))/c+1/2*b^2*polylog(2,1-2/(1-c/x^2))/c
```

3.174.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.15

$$\int \frac{\left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx = -\frac{a^2}{2x^2}-\frac{ab\left(\frac{c\,\text{arctanh}\left(\frac{c}{x^2}\right)}{x^2}-\log\left(\frac{1}{\sqrt{1-\frac{c^2}{x^4}}}\right)\right)}{c}-\frac{b^2\left(\text{arctanh}\left(\frac{c}{x^2}\right)\left(-\text{arctanh}\left(\frac{c}{x^2}\right)+\frac{c\,\text{arctanh}\left(\frac{c}{x^2}\right)}{x^2}-2\log\left(1+e^{-2\text{arctanh}\left(\frac{c}{x^2}\right)}\right)\right)+\text{PolyLog}\left(2,-e^{-2\text{arc}}\right.\right.}{2c}$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])^2/x^3,x]
```

output
```
-1/2*a^2/x^2 - (a*b*((c*ArcTanh[c/x^2])/x^2 - Log[1/Sqrt[1 - c^2/x^4]]))/c
 - (b^2*(ArcTanh[c/x^2]*(-ArcTanh[c/x^2] + (c*ArcTanh[c/x^2])/x^2 - 2*Log[
1 + E^(-2*ArcTanh[c/x^2])]) + PolyLog[2, -E^(-2*ArcTanh[c/x^2])]))/(2*c)
```

3.174.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6454, 6436, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}dx$$

$$\Big\downarrow \text{6454}$$

$$-\frac{1}{2}\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2d\frac{1}{x^2}$$

$$\Big\downarrow \text{6436}$$

$$\frac{1}{2}\left(2bc\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{\left(1-\frac{c^2}{x^4}\right)x^2}d\frac{1}{x^2}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\right)$$

$$\Big\downarrow \text{6546}$$

$$\frac{1}{2}\left(2bc\left(\frac{\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{1-\frac{c}{x^2}}d\frac{1}{x^2}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2bc^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\right)$$

$$\Big\downarrow \text{6470}$$

$$\frac{1}{2}\left(2bc\left(\frac{\frac{\log\left(\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{c}-b\int \frac{\log\left(\frac{2}{1-\frac{c}{x^2}}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2bc^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\right)$$

$$\Big\downarrow \text{2849}$$

$$\frac{1}{2}\left(2bc\left(\frac{\frac{b\int \frac{\log\left(\frac{2}{1-\frac{c}{x^2}}\right)}{1-\frac{2}{1-\frac{c}{x^2}}}d\frac{1}{1-\frac{c}{x^2}}}{c}+\frac{\log\left(\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{c}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2bc^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\right)$$

$$\Big\downarrow \text{2752}$$

$$\frac{1}{2}\left(2bc\left(\frac{\frac{\log\left(\frac{2}{1-\frac{c}{x^2}}\right)\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{c}+\frac{b\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{c}{x^2}}\right)}{2c}}{c}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2bc^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\right)$$

input
```
Int[(a + b*ArcTanh[c/x^2])^2/x^3,x]
```

output
```
(-((a + b*ArcTanh[c/x^2])^2/x^2) + 2*b*c*(-1/2*(a + b*ArcTanh[c/x^2])^2/(b
*c^2) + (((a + b*ArcTanh[c/x^2])*Log[2/(1 - c/x^2)])/c + (b*PolyLog[2, 1 -
 2/(1 - c/x^2)])/(2*c))/c))/2
```

3.174.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

3.174. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx$

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.174.4 Maple [A] (verified)

Time = 4.84 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.35

method	result
derivativedivides	$-\frac{\frac{c a^2}{x^2}+b^2\left(\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2\left(\frac{c}{x^2}-1\right)+2\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2-2\operatorname{arctanh}\left(\frac{c}{x^2}\right)\ln\left(1+\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)-\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)\right)}{2c}$
default	$-\frac{\frac{c a^2}{x^2}+b^2\left(\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2\left(\frac{c}{x^2}-1\right)+2\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2-2\operatorname{arctanh}\left(\frac{c}{x^2}\right)\ln\left(1+\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)-\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)\right)}{2c}$
parts	$-\frac{a^2}{2x^2}-\frac{b^2\left(\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2\left(\frac{c}{x^2}-1\right)+2\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2-2\operatorname{arctanh}\left(\frac{c}{x^2}\right)\ln\left(1+\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)-\operatorname{polylog}\left(2,-\frac{\left(1+\frac{c}{x^2}\right)^2}{1-\frac{c^2}{x^4}}\right)\right)}{2c}$

input
```
int((a+b*arctanh(c/x^2))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c*(c/x^2*a^2+b^2*(arctanh(c/x^2)^2*(c/x^2-1)+2*arctanh(c/x^2)^2-2*arc
tanh(c/x^2)*ln(1+(1+c/x^2)^2/(1-c^2/x^4))-polylog(2,-(1+c/x^2)^2/(1-c^2/x^
4)))+2*a*b*c/x^2*arctanh(c/x^2)+a*b*ln(1-c^2/x^4))
```

3.174.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^3, x)
```

3.174.6 Sympy [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*atanh(c/x**2))**2/x**3,x)
```

output
```
Integral((a + b*atanh(c/x**2))**2/x**3, x)
```

3.174.7 Maxima [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="maxima")
```

output
```
1/8*(8*c^3*integrate(log(x)^2/(c*x^7 - c^3*x^3), x) + c^2*(log(x^2 + c)/c^
3 + log(x^2 - c)/c^3 - 4*log(x)/c^3) - 8*c^2*integrate(x^2*log(x^2 + c)/(c
*x^7 - c^3*x^3), x) + 8*c^2*integrate(x^2*log(x)/(c*x^7 - c^3*x^3), x) + 2
*c*(log(x^2 - c)/c^2 - log(x^2)/c^2 + 1/(c*x^2))*log(-c/x^2 + 1) - c*(log(
x^2 + c)/c^2 - log(x^2 - c)/c^2) - 8*c*integrate(x^4*log(x)^2/(c*x^7 - c^3
*x^3), x) - 4*c*integrate(x^4*log(x^2 + c)/(c*x^7 - c^3*x^3), x) + 16*c*in
tegrate(x^4*log(x)/(c*x^7 - c^3*x^3), x) - log(-c/x^2 + 1)^2/x^2 - (x^2*lo
g(x^2 - c)^2 + 4*x^2*log(x)^2 - 4*x^2*log(x) - 2*(2*x^2*log(x) - x^2)*log(
x^2 - c) + 2*c)/(c*x^2) - (c*log(x^2 + c)^2 - 2*((x^2 + c)*log(x^2 + c) -
2*(x^2 + c)*log(x) - c)*log(x^2 - c))/(c*x^2) - 4*integrate(x^6*log(x^2 +
c)/(c*x^7 - c^3*x^3), x) + 8*integrate(x^6*log(x)/(c*x^7 - c^3*x^3), x))*b
^2 - 1/2*a*b*(2*c*arctanh(c/x^2)/x^2 + log(-c^2/x^4 + 1))/c - 1/2*a^2/x^2
```

3.174.8 Giac [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx=\int \frac{\left(b\operatorname{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2/x^3, x)
```

3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx=\int \frac{\left(a+b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^3}\,dx$$

input
```
int((a + b*atanh(c/x^2))^2/x^3,x)
```

output
```
int((a + b*atanh(c/x^2))^2/x^3, x)
```

3.175 $\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx$

3.175.1 Optimal result

Integrand size = 16, antiderivative size = 97

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx = -\frac{ab}{2cx^2}-\frac{b^2\coth^{-1}\left(\frac{x^2}{c}\right)}{2cx^2}+\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4c^2} -\frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{4x^4}-\frac{b^2\log\left(1-\frac{c^2}{x^4}\right)}{4c^2}$$

output
```
-1/2*a*b/c/x^2-1/2*b^2*arccoth(x^2/c)/c/x^2+1/4*(a+b*arccoth(x^2/c))^2/c^2
-1/4*(a+b*arccoth(x^2/c))^2/x^4-1/4*b^2*ln(1-c^2/x^4)/c^2
```

3.175.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.35

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx = -\frac{a^2c^2+2abcx^2+2bc(ac+bx^2)\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)+b^2(c^2-x^4)\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)^2-4b^2x^4\log(x)+abx^4\log\left(-c+\right.}{4c^2x^4}$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])^2/x^5,x]
```

output
```
-1/4*(a^2*c^2 + 2*a*b*c*x^2 + 2*b*c*(a*c + b*x^2)*ArcTanh[c/x^2] + b^2*(c^
2 - x^4)*ArcTanh[c/x^2]^2 - 4*b^2*x^4*Log[x] + a*b*x^4*Log[-c + x^2] + b^2
*x^4*Log[-c + x^2] - a*b*x^4*Log[c + x^2] + b^2*x^4*Log[c + x^2])/(c^2*x^4
)
```

3.175.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6454, 6452, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}dx$$

$$\Big\downarrow \text{6454}$$

$$-\frac{1}{2}\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}d\frac{1}{x^2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}\left(bc\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{\left(1-\frac{c^2}{x^4}\right)x^4}d\frac{1}{x^2}-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2x^4}\right)$$

$$\Big\downarrow \text{6542}$$

$$\frac{1}{2}\left(bc\left(\frac{\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}}{c^2}-\frac{\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)d\frac{1}{x^2}}{c^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2x^4}\right)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(bc\left(\frac{\int \frac{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}{1-\frac{c^2}{x^4}}d\frac{1}{x^2}}{c^2}-\frac{\frac{a}{x^2}+\frac{b\text{arctanh}\left(\frac{c}{x^2}\right)}{x^2}+\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{2c}}{c^2}\right)-\frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2x^4}\right)$$

$$\Big\downarrow \text{6510}$$

$$\frac{1}{2}\left(bc\left(\frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2bc^3}-\frac{\frac{a}{x^2}+\frac{b\operatorname{arctanh}\left(\frac{c}{x^2}\right)}{x^2}+\frac{b\log\left(1-\frac{c^2}{x^4}\right)}{2c}}{c^2}\right)-\frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{2x^4}\right)$$

input
```
Int[(a + b*ArcTanh[c/x^2])^2/x^5,x]
```

output
```
(-1/2*(a + b*ArcTanh[c/x^2])^2/x^4 + b*c*((a + b*ArcTanh[c/x^2])^2/(2*b*c^
3) - (a/x^2 + (b*ArcTanh[c/x^2])/x^2 + (b*Log[1 - c^2/x^4])/(2*c))/c^2))/2
```

3.175.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.175.4 Maple [A] (verified)

Time = 242.62 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.42

method	result
parallelrisch	$\frac{\operatorname{arctanh}\left(\frac{c}{x^2}\right)^2 b^2 x^4+4b^2\ln(x)x^4-2\ln(x^2-c)x^4b^2+2ab\,x^4\operatorname{arctanh}\left(\frac{c}{x^2}\right)-2x^4\operatorname{arctanh}\left(\frac{c}{x^2}\right)b^2-2\operatorname{arctanh}\left(\frac{c}{x^2}\right)b^2c\,x^2-}{4x^4c^2}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
risch	Expression too large to display

input
```
int((a+b*arctanh(c/x^2))^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/4*(arctanh(c/x^2)^2*b^2*x^4+4*b^2*ln(x)*x^4-2*ln(x^2-c)*x^4*b^2+2*a*b*x^
4*arctanh(c/x^2)-2*x^4*arctanh(c/x^2)*b^2-2*arctanh(c/x^2)*b^2*c*x^2-b^2*c
^2*arctanh(c/x^2)^2-2*a*b*c*x^2-2*a*b*c^2*arctanh(c/x^2)-a^2*c^2)/x^4/c^2
```

3.175.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.47

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx$$
$$=\frac{16\,b^2x^4\log(x)+4\,(ab-b^2)x^4\log(x^2+c)-4\,(ab+b^2)x^4\log(x^2-c)-8\,abcx^2-4\,a^2c^2+(b^2x^4-b^2c^2)\,\mathrm{l}}{16\,c^2x^4}$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="fricas")
```

output
```
1/16*(16*b^2*x^4*log(x) + 4*(a*b - b^2)*x^4*log(x^2 + c) - 4*(a*b + b^2)*x
^4*log(x^2 - c) - 8*a*b*c*x^2 - 4*a^2*c^2 + (b^2*x^4 - b^2*c^2)*log((x^2 +
 c)/(x^2 - c))^2 - 4*(b^2*c*x^2 + a*b*c^2)*log((x^2 + c)/(x^2 - c)))/(c^2*
x^4)
```

3.175.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. 2(78) = 156.

Time = 6.18 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.77

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx$$
$$=\begin{cases} -\frac{a^2}{4x^4}-\frac{ab\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2x^4}-\frac{ab}{2cx^2}+\frac{ab\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2c^2}-\frac{b^2\operatorname{atanh}^2\left(\frac{c}{x^2}\right)}{4x^4}-\frac{b^2\operatorname{atanh}\left(\frac{c}{x^2}\right)}{2cx^2}+\frac{b^2\log(x)}{c^2}-\frac{b^2\log\left(x-\sqrt{-c}\right)}{2c^2}-\frac{b^2\log(x}{2c} \\ -\frac{a^2}{4x^4} \end{cases}$$

input
```
integrate((a+b*atanh(c/x**2))**2/x**5,x)
```

output
```
Piecewise((-a**2/(4*x**4) - a*b*atanh(c/x**2)/(2*x**4) - a*b/(2*c*x**2) +
a*b*atanh(c/x**2)/(2*c**2) - b**2*atanh(c/x**2)**2/(4*x**4) - b**2*atanh(c
/x**2)/(2*c*x**2) + b**2*log(x)/c**2 - b**2*log(x - sqrt(-c))/(2*c**2) - b
**2*log(x + sqrt(-c))/(2*c**2) + b**2*atanh(c/x**2)**2/(4*c**2) + b**2*ata
nh(c/x**2)/(2*c**2), Ne(c, 0)), (-a**2/(4*x**4), True))
```

3.175.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(87) = 174.

Time = 0.19 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.89

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx$$
$$=\frac{1}{4}\left(c\left(\frac{\log\left(x^2+c\right)}{c^3}-\frac{\log\left(x^2-c\right)}{c^3}-\frac{2}{c^2x^2}\right)-\frac{2\operatorname{artanh}\left(\frac{c}{x^2}\right)}{x^4}\right)ab$$
$$-\frac{1}{16}\left(c^2\left(\frac{\log\left(x^2+c\right)^2-2\left(\log\left(x^2+c\right)-2\right)\log\left(x^2-c\right)+\log\left(x^2-c\right)^2+4\log\left(x^2+c\right)}{c^4}-\frac{16\log(x)}{c^4}\right)\right.$$
$$-\frac{b^2\operatorname{artanh}\left(\frac{c}{x^2}\right)^2}{4\,x^4}-\frac{a^2}{4\,x^4}$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="maxima")
```

output
```
1/4*(c*(log(x^2 + c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2)) - 2*arctanh(c/x
^2)/x^4)*a*b - 1/16*(c^2*((log(x^2 + c)^2 - 2*(log(x^2 + c) - 2)*log(x^2 -
 c) + log(x^2 - c)^2 + 4*log(x^2 + c))/c^4 - 16*log(x)/c^4) - 4*c*(log(x^2
 + c)/c^3 - log(x^2 - c)/c^3 - 2/(c^2*x^2))*arctanh(c/x^2))*b^2 - 1/4*b^2*
arctanh(c/x^2)^2/x^4 - 1/4*a^2/x^4
```

3.175.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^5}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^5,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2/x^5, x)
```

3.175.9 Mupad [B] (verification not implemented)

Time = 3.93 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.70

$$\begin{aligned}\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx &= \frac{b^2\ln\left(x^2+c\right)^2}{16\,c^2}-\frac{b^2\,\ln\left(x^2-c\right)}{4\,c^2}-\frac{a^2}{4\,x^4}-\frac{b^2\ln\left(x^2+c\right)^2}{16\,x^4}\\&\quad+\frac{b^2\ln\left(x^2-c\right)^2}{16\,c^2}-\frac{b^2\ln\left(x^2-c\right)^2}{16\,x^4}+\frac{b^2\,\ln\left(x\right)}{c^2}\\&\quad-\frac{b^2\,\ln\left(x^2+c\right)}{4\,c^2}-\frac{a\,b\,\ln\left(x^2+c\right)}{4\,x^4}+\frac{b^2\,\ln\left(x^2-c\right)}{4\,c\,x^2}\\&\quad-\frac{a\,b\,\ln\left(x^2-c\right)}{4\,c^2}-\frac{b^2\,\ln\left(x^2+c\right)\,\ln\left(x^2-c\right)}{8\,c^2}\\&\quad+\frac{a\,b\,\ln\left(x^2-c\right)}{4\,x^4}+\frac{b^2\,\ln\left(x^2+c\right)\,\ln\left(x^2-c\right)}{8\,x^4}\\&\quad-\frac{a\,b}{2\,c\,x^2}-\frac{b^2\,\ln\left(x^2+c\right)}{4\,c\,x^2}+\frac{a\,b\,\ln\left(x^2+c\right)}{4\,c^2}\end{aligned}$$

input
```
int((a + b*atanh(c/x^2))^2/x^5,x)
```

output
```
(b^2*log(c + x^2)^2)/(16*c^2) - (b^2*log(x^2 - c))/(4*c^2) - a^2/(4*x^4) -
 (b^2*log(c + x^2)^2)/(16*x^4) + (b^2*log(x^2 - c)^2)/(16*c^2) - (b^2*log(
x^2 - c)^2)/(16*x^4) + (b^2*log(x))/c^2 - (b^2*log(c + x^2))/(4*c^2) - (a*
b*log(c + x^2))/(4*x^4) + (b^2*log(x^2 - c))/(4*c*x^2) - (a*b*log(x^2 - c)
)/(4*c^2) - (b^2*log(c + x^2)*log(x^2 - c))/(8*c^2) + (a*b*log(x^2 - c))/(
4*x^4) + (b^2*log(c + x^2)*log(x^2 - c))/(8*x^4) - (a*b)/(2*c*x^2) - (b^2*
log(c + x^2))/(4*c*x^2) + (a*b*log(c + x^2))/(4*c^2)
```

3.175. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^5}\,dx$

3.176 $\int x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$

3.176.1 Optimal result

Integrand size = 16, antiderivative size = 1214

$$\int x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx=\text{Too large to display}$$

output
```
8/15*b^2*c^2*x+2/5*a*b*c^(5/2)*arctan(x/c^(1/2))-1/15*b^2*c*x^3*ln(1-c/x^2
)-1/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln(1-c/x^2)+1/15*b*c*x^3*(2*a-b*ln(1-c
/x^2))-1/5*b*c^(5/2)*arctanh(x/c^(1/2))*(2*a-b*ln(1-c/x^2))+2/15*b^2*c*x^3
*ln(1+c/x^2)+1/5*a*b*x^5*ln(1+c/x^2)+1/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln(
1+c/x^2)-1/5*b^2*c^(5/2)*arctanh(x/c^(1/2))*ln(1+c/x^2)-1/10*b^2*x^5*ln(1-
c/x^2)*ln(1+c/x^2)-2/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*x+c^
(1/2)))+1/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1
/2)))-2/5*b^2*c^(5/2)*arctanh(x/c^(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))+1/5*b^2
*c^(5/2)*arctanh(x/c^(1/2))*ln(2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/
2))/(x+c^(1/2)))+1/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln((1-I)*(x+c^(1/2))/(-
I*x+c^(1/2)))+1/5*b^2*c^(5/2)*arctanh(x/c^(1/2))*ln(2*(x+(-c)^(1/2))*c^(1/
2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))+2/5*b^2*c^(5/2)*arctan(x/c^(1/2))*ln(
2-2*c^(1/2)/(-I*x+c^(1/2)))+2/5*b^2*c^(5/2)*arctanh(x/c^(1/2))*ln(2-2*c^(1
/2)/(x+c^(1/2)))+2/15*a*b*c*x^3-1/5*I*b^2*c^(5/2)*arctan(x/c^(1/2))^2-1/5*
I*b^2*c^(5/2)*polylog(2,-I*x/c^(1/2))-1/5*I*b^2*c^(5/2)*polylog(2,-1+2*c^(
1/2)/(-I*x+c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(2,1-(1+I)*(-x+c^(1/2))/(-I
*x+c^(1/2)))-1/10*I*b^2*c^(5/2)*polylog(2,1+(-1+I)*(x+c^(1/2))/(-I*x+c^(1/
2)))-4/15*b^2*c^(5/2)*arctan(x/c^(1/2))-4/15*b^2*c^(5/2)*arctanh(x/c^(1/2)
)+1/5*b^2*c^(5/2)*arctanh(x/c^(1/2))^2+1/20*b^2*x^5*ln(1+c/x^2)^2+1/5*b^2*
c^(5/2)*polylog(2,-x/c^(1/2))-1/5*b^2*c^(5/2)*polylog(2,x/c^(1/2))+1/5*...
```

3.176.2 Mathematica [F]

$$\int x^4\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx=\int x^4\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

input
```
Integrate[x^4*(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
Integrate[x^4*(a + b*ArcTanh[c/x^2])^2, x]
```

3.176.3 Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 1214, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6460, 6457, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$$

$$\Big\downarrow \text{ 6460}$$

$$\int x^4\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2\,dx$$

$$\Big\downarrow \text{ 6457}$$

$$\int\left(\frac{1}{4}x^4\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2-\frac{1}{2}bx^4\log\left(\frac{c}{x^2}+1\right)\left(b\log\left(1-\frac{c}{x^2}\right)-2a\right)+\frac{1}{4}b^2x^4\log^2\left(\frac{c}{x^2}+1\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{20}\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2x^5+\frac{1}{20}b^2\log^2\left(\frac{c}{x^2}+1\right)x^5+\frac{1}{5}ab\log\left(\frac{c}{x^2}+1\right)x^5-\frac{1}{10}b^2\log\left(1-\frac{c}{x^2}\right)\log\left(\frac{c}{x^2}+1\right)x^5+\frac{2}{15}abcx^3-\frac{1}{15}b^2c\log\left(1-\frac{c}{x^2}\right)x^3+\frac{1}{15}bc\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)x^3+\frac{2}{15}b^2c\log\left(\frac{c}{x^2}+1\right)x^3+\frac{8}{15}b^2c^2x-\frac{1}{5}ib^2c^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)^2+\frac{1}{5}b^2c^{5/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)^2-\frac{4}{15}b^2c^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{5}abc^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)-\frac{4}{15}b^2c^{5/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)+\frac{2}{5}b^2c^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)-\frac{1}{5}b^2c^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(1-\frac{c}{x^2}\right)-\frac{1}{5}bc^{5/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)+\frac{1}{5}b^2c^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right)-\frac{1}{5}b^2c^{5/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right)-\frac{2}{5}b^2c^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)+\frac{1}{5}b^2c^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1+i)\left(\sqrt{c}-x\right)}{\sqrt{c}-ix}\right)-\frac{2}{5}b^2c^{5/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}}{x+\sqrt{c}}\right)+\frac{1}{5}b^2c^{5/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}(\sqrt{-c}-x)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(x+\sqrt{c}\right)}\right)+\frac{1}{5}b^2c^{5/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}(x+\sqrt{-c})}{\left(\sqrt{-c}+\sqrt{c}\right)\left(x+\sqrt{c}\right)}\right)+\frac{1}{5}b^2c^{5/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1-i)\left(x+\sqrt{c}\right)}{\sqrt{c}-ix}\right)+\frac{2}{5}b^2c^{5/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{x+\sqrt{c}}\right)+\frac{1}{5}ib^2c^{5/2}\text{PolyLog}\left(2,1-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)-\frac{1}{5}ib^2c^{5/2}\text{PolyLog}\left(2,\frac{2\sqrt{c}}{\sqrt{c}-ix}-1\right)-\frac{1}{10}ib^2c^{5/2}\text{PolyLog}\left(2,1-\frac{(1+i)\left(\sqrt{c}-x\right)}{\sqrt{c}-ix}\right)+\frac{1}{5}b^2c^{5/2}\text{PolyLog}\left(2,-\frac{x}{\sqrt{c}}\right)-\frac{1}{5}ib^2c^{5/2}\text{PolyLog}\left(2,-\frac{ix}{\sqrt{c}}\right)+\frac{1}{5}ib^2c^{5/2}\text{PolyLog}\left(2,\frac{ix}{\sqrt{c}}\right)-\frac{1}{5}b^2c^{5/2}\text{PolyLog}\left(2,\frac{x}{\sqrt{c}}\right)+\frac{1}{5}b^2c^{5/2}\text{PolyLog}\left(2,1-\frac{2\sqrt{c}}{x+\sqrt{c}}\right)-\frac{1}{5}b^2c^{5/2}\text{PolyLog}\left(2,\frac{2\sqrt{c}}{x+\sqrt{c}}-1\right)-\frac{1}{10}b^2c^{5/2}\text{PolyLog}\left(2,1-\frac{2\sqrt{c}(\sqrt{-c}-x)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(x+\sqrt{c}\right)}\right)-\frac{1}{10}b^2c^{5/2}\text{PolyLog}\left(2,1-\frac{2\sqrt{c}(x+\sqrt{-c})}{\left(\sqrt{-c}+\sqrt{c}\right)\left(x+\sqrt{c}\right)}\right)-\frac{1}{10}ib^2c^{5/2}\text{PolyLog}\left(2,1-\frac{(1-i)\left(x+\sqrt{c}\right)}{\sqrt{c}-ix}\right)$$

input
```
Int[x^4*(a + b*ArcTanh[c/x^2])^2,x]
```

3.176. $\int x^4\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$

output
```
(8*b^2*c^2*x)/15 + (2*a*b*c*x^3)/15 + (2*a*b*c^(5/2)*ArcTan[x/Sqrt[c]])/5
- (4*b^2*c^(5/2)*ArcTan[x/Sqrt[c]])/15 - (I/5)*b^2*c^(5/2)*ArcTan[x/Sqrt[c
]]^2 - (4*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]])/15 + (b^2*c^(5/2)*ArcTanh[x/Sqrt
[c]]^2)/5 + (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c]
- I*x)])/5 - (b^2*c*x^3*Log[1 - c/x^2])/15 - (b^2*c^(5/2)*ArcTan[x/Sqrt[c]
]*Log[1 - c/x^2])/5 + (b*c*x^3*(2*a - b*Log[1 - c/x^2]))/15 - (b*c^(5/2)*A
rcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/5 + (x^5*(2*a - b*Log[1 - c/x^
2])^2)/20 + (2*b^2*c*x^3*Log[1 + c/x^2])/15 + (a*b*x^5*Log[1 + c/x^2])/5 +
 (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/5 - (b^2*c^(5/2)*ArcTanh[x
/Sqrt[c]]*Log[1 + c/x^2])/5 - (b^2*x^5*Log[1 - c/x^2]*Log[1 + c/x^2])/10 +
 (b^2*x^5*Log[1 + c/x^2]^2)/20 - (2*b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[(2*S
qrt[c])/(Sqrt[c] - I*x)])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 + I)*
(Sqrt[c] - x))/(Sqrt[c] - I*x)])/5 - (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log
[(2*Sqrt[c])/(Sqrt[c] + x)])/5 + (b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sq
rt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/5 + (b^2*c^(5
/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c]
)*(Sqrt[c] + x))])/5 + (b^2*c^(5/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c
] + x))/(Sqrt[c] - I*x)])/5 + (2*b^2*c^(5/2)*ArcTanh[x/Sqrt[c]]*Log[2 - (2
*Sqrt[c])/(Sqrt[c] + x)])/5 + (I/5)*b^2*c^(5/2)*PolyLog[2, 1 - (2*Sqrt[c])
/(Sqrt[c] - I*x)] - (I/5)*b^2*c^(5/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt...
```

3.176.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6457
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c
)]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6460
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[x^m*(a + b*ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[
p, 1] && ILtQ[n, 0]
```

3.176. $\int x^4 \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$

3.176.4 Maple [F]

$$\int x^4 \left(a + b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
int(x^4*(a+b*arctanh(c/x^2))^2,x)
```

output
```
int(x^4*(a+b*arctanh(c/x^2))^2,x)
```

3.176.5 Fricas [F]

$$\int x^4 \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 x^4 \, dx$$

input
```
integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^4*arctanh(c/x^2)^2 + 2*a*b*x^4*arctanh(c/x^2) + a^2*x^4, x)
```

3.176.6 Sympy [F]

$$\int x^4 \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int x^4 \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
integrate(x**4*(a+b*atanh(c/x**2))**2,x)
```

output
```
Integral(x**4*(a + b*atanh(c/x**2))**2, x)
```

3.176.7 Maxima [F]

$$\int x^4 \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx = \int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 x^4 \, dx$$

input
```
integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

output
```
1/5*a^2*x^5 + 1/15*(6*x^5*arctanh(c/x^2) + (4*x^3 + 6*c^(3/2)*arctan(x/sqr
t(c)) + 3*c^(3/2)*log((x - sqrt(c))/(x + sqrt(c))))*c)*a*b + 1/20*(x^5*log
(x^2 - c)^2 - 5*integrate(-1/5*(5*(x^6 - c*x^4)*log(x^2 + c)^2 - 2*(2*x^6
+ 5*(x^6 - c*x^4)*log(x^2 + c))*log(x^2 - c))/(x^2 - c), x))*b^2
```

3.176.8 Giac [F]

$$\int x^4 \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx = \int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 x^4 \, dx$$

input
```
integrate(x^4*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2*x^4, x)
```

3.176.9 Mupad [F(-1)]

Timed out.

$$\int x^4 \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx = \int x^4 \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx$$

input
```
int(x^4*(a + b*atanh(c/x^2))^2,x)
```

output
```
int(x^4*(a + b*atanh(c/x^2))^2, x)
```

3.177 $\int x^2 \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$

3.177.1 Optimal result

Integrand size = 16, antiderivative size = 1172

$$\int x^2 \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \text{Too large to display}$$

output
```
1/3*b^2*c^(3/2)*arctanh(x/c^(1/2))*ln(2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2
)-c^(1/2))/(x+c^(1/2)))-1/3*b^2*c^(3/2)*arctan(x/c^(1/2))*ln((1-I)*(x+c^(1
/2))/(-I*x+c^(1/2)))+1/3*b^2*c^(3/2)*arctanh(x/c^(1/2))*ln(2*(x+(-c)^(1/2)
)*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))-2/3*b^2*c^(3/2)*arctan(x/c^(1/
2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))+2/3*b^2*c^(3/2)*arctanh(x/c^(1/2))*ln(2
-2*c^(1/2)/(x+c^(1/2)))-2/3*a*b*c^(3/2)*arctan(x/c^(1/2))-2/3*b^2*c*x*ln(1
-c/x^2)+1/3*b^2*c^(3/2)*arctan(x/c^(1/2))*ln(1-c/x^2)-1/3*b*c^(3/2)*arctan
h(x/c^(1/2))*(2*a-b*ln(1-c/x^2))+2/3*b^2*c*x*ln(1+c/x^2)+1/3*a*b*x^3*ln(1+
c/x^2)-1/3*b^2*c^(3/2)*arctan(x/c^(1/2))*ln(1+c/x^2)-1/3*b^2*c^(3/2)*arcta
nh(x/c^(1/2))*ln(1+c/x^2)+4/3*a*b*c*x-1/3*I*b^2*c^(3/2)*polylog(2,I*x/c^(1
/2))-1/3*I*b^2*c^(3/2)*polylog(2,1-2*c^(1/2)/(-I*x+c^(1/2)))-1/6*b^2*x^3*l
n(1-c/x^2)*ln(1+c/x^2)+2/3*b^2*c^(3/2)*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*
x+c^(1/2)))-1/3*b^2*c^(3/2)*arctan(x/c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+
c^(1/2)))-2/3*b^2*c^(3/2)*arctanh(x/c^(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))+1/3
*I*b^2*c^(3/2)*arctan(x/c^(1/2))^2+1/3*I*b^2*c^(3/2)*polylog(2,-I*x/c^(1/2
))+1/3*I*b^2*c^(3/2)*polylog(2,-1+2*c^(1/2)/(-I*x+c^(1/2)))+1/6*I*b^2*c^(3
/2)*polylog(2,1-(1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))+1/6*I*b^2*c^(3/2)*polyl
og(2,1+(-1+I)*(x+c^(1/2))/(-I*x+c^(1/2)))+4/3*b^2*c^(3/2)*arctan(x/c^(1/2)
)-4/3*b^2*c^(3/2)*arctanh(x/c^(1/2))+1/3*b^2*c^(3/2)*arctanh(x/c^(1/2))^2+
1/12*b^2*x^3*ln(1+c/x^2)^2+1/3*b^2*c^(3/2)*polylog(2,-x/c^(1/2))-1/3*b^...
```

3.177.2 Mathematica [F]

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx=\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
Integrate[x^2*(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
Integrate[x^2*(a + b*ArcTanh[c/x^2])^2, x]
```

3.177.3 Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 1172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6460, 6457, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

$$\Big\downarrow \text{ 6460}$$

$$\int x^2\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 dx$$

$$\Big\downarrow \text{ 6457}$$

$$\int \left(\frac{1}{4}x^2\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2-\frac{1}{2}bx^2\log\left(\frac{c}{x^2}+1\right)\left(b\log\left(1-\frac{c}{x^2}\right)-2a\right)+\frac{1}{4}b^2x^2\log^2\left(\frac{c}{x^2}+1\right)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{12}\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2x^3+\frac{1}{12}b^2\log^2\left(\frac{c}{x^2}+1\right)x^3+\frac{1}{3}ab\log\left(\frac{c}{x^2}+1\right)x^3-\frac{1}{6}b^2\log\left(1-\frac{c}{x^2}\right)\log\left(\frac{c}{x^2}+1\right)x^3+\frac{4}{3}abcx-\frac{2}{3}b^2c\log\left(1-\frac{c}{x^2}\right)x+\frac{2}{3}b^2c\log\left(\frac{c}{x^2}+1\right)x+\frac{1}{3}ib^2c^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)^2+\frac{1}{3}b^2c^{3/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)^2+\frac{4}{3}b^2c^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)-\frac{2}{3}abc^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)-\frac{4}{3}b^2c^{3/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)-\frac{2}{3}b^2c^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)+\frac{1}{3}b^2c^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(1-\frac{c}{x^2}\right)-\frac{1}{3}bc^{3/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)-\frac{1}{3}b^2c^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right)-\frac{1}{3}b^2c^{3/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right)+\frac{2}{3}b^2c^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)-\frac{1}{3}b^2c^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right)-\frac{2}{3}b^2c^{3/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}}{x+\sqrt{c}}\right)+\frac{1}{3}b^2c^{3/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}(\sqrt{-c}-x)}{(\sqrt{-c}-\sqrt{c})(x+\sqrt{c})}\right)+\frac{1}{3}b^2c^{3/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}(x+\sqrt{-c})}{(\sqrt{-c}+\sqrt{c})(x+\sqrt{c})}\right)-\frac{1}{3}b^2c^{3/2}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1-i)(x+\sqrt{c})}{\sqrt{c}-ix}\right)+\frac{2}{3}b^2c^{3/2}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{x+\sqrt{c}}\right)-\frac{1}{3}ib^2c^{3/2}\,\text{PolyLog}\left(2,1-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)+\frac{1}{3}ib^2c^{3/2}\,\text{PolyLog}\left(2,\frac{2\sqrt{c}}{\sqrt{c}-ix}-1\right)+\frac{1}{6}ib^2c^{3/2}\,\text{PolyLog}\left(2,1-\frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right)+\frac{1}{3}b^2c^{3/2}\,\text{PolyLog}\left(2,-\frac{x}{\sqrt{c}}\right)+\frac{1}{3}ib^2c^{3/2}\,\text{PolyLog}\left(2,-\frac{ix}{\sqrt{c}}\right)-\frac{1}{3}ib^2c^{3/2}\,\text{PolyLog}\left(2,\frac{ix}{\sqrt{c}}\right)-\frac{1}{3}b^2c^{3/2}\,\text{PolyLog}\left(2,\frac{x}{\sqrt{c}}\right)+\frac{1}{3}b^2c^{3/2}\,\text{PolyLog}\left(2,1-\frac{2\sqrt{c}}{x+\sqrt{c}}\right)-\frac{1}{3}b^2c^{3/2}\,\text{PolyLog}\left(2,\frac{2\sqrt{c}}{x+\sqrt{c}}-1\right)-\frac{1}{6}b^2c^{3/2}\,\text{PolyLog}\left(2,1-\frac{2\sqrt{c}(\sqrt{-c}-x)}{(\sqrt{-c}-\sqrt{c})(x+\sqrt{c})}\right)-\frac{1}{6}b^2c^{3/2}\,\text{PolyLog}\left(2,1-\frac{2\sqrt{c}(x+\sqrt{-c})}{(\sqrt{-c}+\sqrt{c})(x+\sqrt{c})}\right)+\frac{1}{6}ib^2c^{3/2}\,\text{PolyLog}\left(2,1-\frac{(1-i)(x+\sqrt{c})}{\sqrt{c}-ix}\right)$$

input
```
Int[x^2*(a + b*ArcTanh[c/x^2])^2,x]
```

3.177. $\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx$

output
```
(4*a*b*c*x)/3 - (2*a*b*c^(3/2)*ArcTan[x/Sqrt[c]])/3 + (4*b^2*c^(3/2)*ArcTa
n[x/Sqrt[c]])/3 + (I/3)*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]^2 - (4*b^2*c^(3/2)*A
rcTanh[x/Sqrt[c]])/3 + (b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]^2)/3 - (2*b^2*c^(3/
2)*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/3 - (2*b^2*c*x*
Log[1 - c/x^2])/3 + (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/3 - (b*
c^(3/2)*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/3 + (x^3*(2*a - b*Log
[1 - c/x^2])^2)/12 + (2*b^2*c*x*Log[1 + c/x^2])/3 + (a*b*x^3*Log[1 + c/x^2
])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/3 - (b^2*c^(3/2)*Arc
Tanh[x/Sqrt[c]]*Log[1 + c/x^2])/3 - (b^2*x^3*Log[1 - c/x^2]*Log[1 + c/x^2]
)/6 + (b^2*x^3*Log[1 + c/x^2]^2)/12 + (2*b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log
[(2*Sqrt[c])/(Sqrt[c] - I*x)])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1
+ I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/3 - (2*b^2*c^(3/2)*ArcTanh[x/Sqrt[c]
]*Log[(2*Sqrt[c])/(Sqrt[c] + x)])/3 + (b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[
(2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/3 + (b^2
*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sq
rt[c])*(Sqrt[c] + x))])/3 - (b^2*c^(3/2)*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(S
qrt[c] + x))/(Sqrt[c] - I*x)])/3 + (2*b^2*c^(3/2)*ArcTanh[x/Sqrt[c]]*Log[2
 - (2*Sqrt[c])/(Sqrt[c] + x)])/3 - (I/3)*b^2*c^(3/2)*PolyLog[2, 1 - (2*Sqr
t[c])/(Sqrt[c] - I*x)] + (I/3)*b^2*c^(3/2)*PolyLog[2, -1 + (2*Sqrt[c])/(Sq
rt[c] - I*x)] + (I/6)*b^2*c^(3/2)*PolyLog[2, 1 - ((1 + I)*(Sqrt[c] - x)...
```

3.177.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6457
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c
)]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6460
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[x^m*(a + b*ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[
p, 1] && ILtQ[n, 0]
```

3.177. $\int x^2\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$

3.177.4 Maple [F]

$$\int x^2 \left(a + b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
int(x^2*(a+b*arctanh(c/x^2))^2,x)
```

output
```
int(x^2*(a+b*arctanh(c/x^2))^2,x)
```

3.177.5 Fricas [F]

$$\int x^2 \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)^2 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x^2*arctanh(c/x^2)^2 + 2*a*b*x^2*arctanh(c/x^2) + a^2*x^2, x)
```

3.177.6 Sympy [F]

$$\int x^2 \left(a + b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int x^2 \left(a + b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
integrate(x**2*(a+b*atanh(c/x**2))**2,x)
```

output
```
Integral(x**2*(a + b*atanh(c/x**2))**2, x)
```

3.177.7 Maxima [F]

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c/x^2) - (2*sqrt(c)*arctan(x/sqrt(c)) - s
qrt(c)*log((x - sqrt(c))/(x + sqrt(c))) - 4*x)*c)*a*b + 1/12*(x^3*log(x^2
- c)^2 - 3*integrate(-1/3*(3*(x^4 - c*x^2)*log(x^2 + c)^2 - 2*(2*x^4 + 3*(
x^4 - c*x^2)*log(x^2 + c))*log(x^2 - c))/(x^2 - c), x))*b^2
```

3.177.8 Giac [F]

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2*x^2, x)
```

3.177.9 Mupad [F(-1)]

Timed out.

$$\int x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int x^2\left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
int(x^2*(a + b*atanh(c/x^2))^2,x)
```

output
```
int(x^2*(a + b*atanh(c/x^2))^2, x)
```

3.178 $\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx$

3.178.1 Optimal result

Integrand size = 12, antiderivative size = 1549

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx = \text{Too large to display}$$

output
```
-b^2*polylog(2,1-x/(-c)^(1/2))*(-c)^(1/2)+b^2*polylog(2,1+x/(-c)^(1/2))*(-
c)^(1/2)-b^2*polylog(2,1-x/c^(1/2))*c^(1/2)-b^2*polylog(2,x/c^(1/2))*c^(1/
2)+b^2*polylog(2,1+x/c^(1/2))*c^(1/2)+b^2*polylog(2,-x/c^(1/2))*c^(1/2)+b^
2*polylog(2,1-2*c^(1/2)/(x+c^(1/2)))*c^(1/2)-a*b*x*ln(1-c/x^2)-b^2*ln(x/(-
c)^(1/2))*ln(-x+(-c)^(1/2))*(-c)^(1/2)+a*b*x*ln(1+c/x^2)+b^2*ln(-x/(-c)^(1
/2))*ln(x+(-c)^(1/2))*(-c)^(1/2)-b^2*arctan(x/c^(1/2))*ln(1-c/x^2)*c^(1/2)
-b^2*arctanh(x/c^(1/2))*ln(1+c/x^2)*c^(1/2)-b^2*ln(x/c^(1/2))*ln(-x+c^(1/2
))*c^(1/2)+b^2*arctan(x/c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))*c^(
1/2)+b^2*arctanh(x/c^(1/2))*ln(2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/
2))/(x+c^(1/2)))*c^(1/2)+b^2*ln(-x/c^(1/2))*ln(x+c^(1/2))*c^(1/2)+b^2*arct
an(x/c^(1/2))*ln((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))*c^(1/2)+b^2*arctanh(x/c
^(1/2))*ln(2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))*c^(1
/2)+I*b^2*polylog(2,I*x/c^(1/2))*c^(1/2)+I*b^2*polylog(2,1-2*c^(1/2)/(-I*x
+c^(1/2)))*c^(1/2)-1/2*b^2*x*ln(1-c/x^2)*ln(1+c/x^2)-1/2*b^2*ln(1+c/x^2)*l
n(-x+(-c)^(1/2))*(-c)^(1/2)+1/2*b^2*ln(1+c/x^2)*ln(x+(-c)^(1/2))*(-c)^(1/2
)-1/2*b^2*ln(1/2*(-x+(-c)^(1/2))/(-c)^(1/2))*ln(x+(-c)^(1/2))*(-c)^(1/2)+1
/2*b^2*ln(-x+(-c)^(1/2))*ln(1/2*(x+(-c)^(1/2))/(-c)^(1/2))*(-c)^(1/2)+2*a*
b*arctan(x/c^(1/2))*c^(1/2)-2*a*b*arctanh(x/c^(1/2))*c^(1/2)-1/2*b^2*ln(1-
c/x^2)*ln(-x+c^(1/2))*c^(1/2)-2*b^2*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*x+c
^(1/2)))*c^(1/2)-2*b^2*arctanh(x/c^(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))*c^(...
```

3.178.2 Mathematica [A] (verified)

Time = 2.41 (sec) , antiderivative size = 565, normalized size of antiderivative = 0.36

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2\,dx=a^2x-2ab\sqrt{\frac{c}{x^2}}x\left(\arctan\left(\sqrt{\frac{c}{x^2}}\right)+\text{arctanh}\left(\sqrt{\frac{c}{x^2}}\right)\right)+2abx\text{arctanh}\left(\frac{c}{x^2}\right)-\frac{1}{2}b^2\sqrt{\frac{c}{x^2}}x\left(-2i\arctan\left(\sqrt{\frac{c}{x^2}}\right)^2+4\arctan\left(\sqrt{\frac{c}{x^2}}\right)\text{arctanh}\left(\frac{c}{x^2}\right)-\frac{2\text{arctanh}\left(\frac{c}{x^2}\right)^2}{\sqrt{\frac{c}{x^2}}}+2\arctan\left(\sqrt{\frac{c}{x^2}}\right)\log\left(1+e^{4i\arctan\left(\sqrt{\frac{c}{x^2}}\right)}\right)-2\text{arctanh}\left(\frac{c}{x^2}\right)\log\left(1-\sqrt{\frac{c}{x^2}}\right)+\log(2)\log\left(1-\sqrt{\frac{c}{x^2}}\right)-\frac{1}{2}\log^2\left(1-\sqrt{\frac{c}{x^2}}\right)+\log\left(1-\sqrt{\frac{c}{x^2}}\right)\log\left(\left(\frac{1}{2}+\frac{i}{2}\right)\left(-i+\sqrt{\frac{c}{x^2}}\right)\right)+2\text{arctanh}\left(\frac{c}{x^2}\right)\log\left(1+\sqrt{\frac{c}{x^2}}\right)-\log(2)\log\left(1+\sqrt{\frac{c}{x^2}}\right)-\log\left(\frac{1}{2}\left((1+i)-(1-i)\sqrt{\frac{c}{x^2}}\right)\right)\log\left(1+\sqrt{\frac{c}{x^2}}\right)-\log\left(\left(-\frac{1}{2}-\frac{i}{2}\right)\left(i+\sqrt{\frac{c}{x^2}}\right)\right)\log\left(1+\sqrt{\frac{c}{x^2}}\right)+\frac{1}{2}\log^2\left(1+\sqrt{\frac{c}{x^2}}\right)+\log\left(1-\sqrt{\frac{c}{x^2}}\right)\log\left(\frac{1}{2}\left((1+i)+(1-i)\sqrt{\frac{c}{x^2}}\right)\right)-\frac{1}{2}i\,\text{PolyLog}\left(2,-e^{4i\arctan\left(\sqrt{\frac{c}{x^2}}\right)}\right)-\text{PolyLog}\left(2,\frac{1}{2}\left(1-\sqrt{\frac{c}{x^2}}\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}-\frac{i}{2}\right)\left(-1+\sqrt{\frac{c}{x^2}}\right)\right)+\text{PolyLog}\left(2,\left(-\frac{1}{2}+\frac{i}{2}\right)\left(-1+\sqrt{\frac{c}{x^2}}\right)\right)+\text{PolyLog}\left(2,\frac{1}{2}\left(1+\sqrt{\frac{c}{x^2}}\right)\right)-\text{PolyLog}\left(2,\left(\frac{1}{2}-\frac{i}{2}\right)\left(1+\sqrt{\frac{c}{x^2}}\right)\right)-\text{PolyLog}\left(2,\left(\frac{1}{2}+\frac{i}{2}\right)\left(1+\sqrt{\frac{c}{x^2}}\right)\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
a^2*x - 2*a*b*Sqrt[c/x^2]*x*(ArcTan[Sqrt[c/x^2]] + ArcTanh[Sqrt[c/x^2]]) +
 2*a*b*x*ArcTanh[c/x^2] - (b^2*Sqrt[c/x^2]*x*((-2*I)*ArcTan[Sqrt[c/x^2]]^2
 + 4*ArcTan[Sqrt[c/x^2]]*ArcTanh[c/x^2] - (2*ArcTanh[c/x^2]^2)/Sqrt[c/x^2]
 + 2*ArcTan[Sqrt[c/x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c/x^2]])] - 2*ArcTan
h[c/x^2]*Log[1 - Sqrt[c/x^2]] + Log[2]*Log[1 - Sqrt[c/x^2]] - Log[1 - Sqrt
[c/x^2]]^2/2 + Log[1 - Sqrt[c/x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c/x^2])] +
2*ArcTanh[c/x^2]*Log[1 + Sqrt[c/x^2]] - Log[2]*Log[1 + Sqrt[c/x^2]] - Log[
((1 + I) - (1 - I)*Sqrt[c/x^2])/2]*Log[1 + Sqrt[c/x^2]] - Log[(-1/2 - I/2)
*(I + Sqrt[c/x^2])]*Log[1 + Sqrt[c/x^2]] + Log[1 + Sqrt[c/x^2]]^2/2 + Log[
1 - Sqrt[c/x^2]]*Log[((1 + I) + (1 - I)*Sqrt[c/x^2])/2] - (I/2)*PolyLog[2,
 -E^((4*I)*ArcTan[Sqrt[c/x^2]])] - PolyLog[2, (1 - Sqrt[c/x^2])/2] + PolyL
og[2, (-1/2 - I/2)*(-1 + Sqrt[c/x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqr
t[c/x^2])] + PolyLog[2, (1 + Sqrt[c/x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 +
 Sqrt[c/x^2])] - PolyLog[2, (1/2 + I/2)*(1 + Sqrt[c/x^2])]))/2
```

3.178.3 Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 1050, normalized size of antiderivative = 0.68, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6440, 6439, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

$$\Big\downarrow \text{6440}$$

$$\int \left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2 dx$$

$$\Big\downarrow \text{6439}$$

$$\int \left(a^2-ab\log\left(1-\frac{c}{x^2}\right)+ab\log\left(\frac{c}{x^2}+1\right)+\frac{1}{4}b^2\log^2\left(1-\frac{c}{x^2}\right)+\frac{1}{4}b^2\log^2\left(\frac{c}{x^2}+1\right)-\frac{1}{2}b^2\log\left(1-\frac{c}{x^2}\right)\log\left(\frac{c}{x^2}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$xa^2 + 2b\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)a - 2b\sqrt{c}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)a - bx\log\left(1-\frac{c}{x^2}\right)a + bx\log\left(\frac{c}{x^2}+1\right)a - ib^2\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)^2 + b^2\sqrt{c}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)^2 + \frac{1}{4}b^2x\log^2\left(1-\frac{c}{x^2}\right) + \frac{1}{4}b^2x\log^2\left(\frac{c}{x^2}+1\right) + 2b^2\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right) - b^2\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(1-\frac{c}{x^2}\right) + b^2\sqrt{c}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(1-\frac{c}{x^2}\right) + b^2\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right) - b^2\sqrt{c}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right) - \frac{1}{2}b^2x\log\left(1-\frac{c}{x^2}\right)\log\left(\frac{c}{x^2}+1\right) - 2b^2\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}}{\sqrt{c}-ix}\right) + b^2\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1+i)\left(\sqrt{c}-x\right)}{\sqrt{c}-ix}\right) - 2b^2\sqrt{c}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}}{x+\sqrt{c}}\right) + b^2\sqrt{c}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}(\sqrt{-c}-x)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(x+\sqrt{c}\right)}\right) + b^2\sqrt{c}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}(x+\sqrt{-c})}{\left(\sqrt{-c}+\sqrt{c}\right)\left(x+\sqrt{c}\right)}\right) + b^2\sqrt{c}\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1-i)\left(x+\sqrt{c}\right)}{\sqrt{c}-ix}\right) + 2b^2\sqrt{c}\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{x+\sqrt{c}}\right) + ib^2\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right) - ib^2\sqrt{c}\,\text{PolyLog}\left(2,\frac{2\sqrt{c}}{\sqrt{c}-ix}-1\right) - \frac{1}{2}ib^2\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{(1+i)\left(\sqrt{c}-x\right)}{\sqrt{c}-ix}\right) + b^2\sqrt{c}\,\text{PolyLog}\left(2,-\frac{x}{\sqrt{c}}\right) - ib^2\sqrt{c}\,\text{PolyLog}\left(2,-\frac{ix}{\sqrt{c}}\right) + ib^2\sqrt{c}\,\text{PolyLog}\left(2,\frac{ix}{\sqrt{c}}\right) - b^2\sqrt{c}\,\text{PolyLog}\left(2,\frac{x}{\sqrt{c}}\right) + b^2\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{2\sqrt{c}}{x+\sqrt{c}}\right) - b^2\sqrt{c}\,\text{PolyLog}\left(2,\frac{2\sqrt{c}}{x+\sqrt{c}}-1\right) - \frac{1}{2}b^2\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{2\sqrt{c}(\sqrt{-c}-x)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(x+\sqrt{c}\right)}\right) - \frac{1}{2}b^2\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{2\sqrt{c}(x+\sqrt{-c})}{\left(\sqrt{-c}+\sqrt{c}\right)\left(x+\sqrt{c}\right)}\right) - \frac{1}{2}ib^2\sqrt{c}\,\text{PolyLog}\left(2,1-\frac{(1-i)\left(x+\sqrt{c}\right)}{\sqrt{c}-ix}\right)$$

input
```
Int[(a + b*ArcTanh[c/x^2])^2,x]
```

3.178. $\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$

output
```
a^2*x + 2*a*b*Sqrt[c]*ArcTan[x/Sqrt[c]] - I*b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]^
2 - 2*a*b*Sqrt[c]*ArcTanh[x/Sqrt[c]] + b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]^2 +
2*b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)] - a*b
*x*Log[1 - c/x^2] - b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2] + b^2*Sqr
t[c]*ArcTanh[x/Sqrt[c]]*Log[1 - c/x^2] + (b^2*x*Log[1 - c/x^2]^2)/4 + a*b*
x*Log[1 + c/x^2] + b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2] - b^2*Sqrt
[c]*ArcTanh[x/Sqrt[c]]*Log[1 + c/x^2] - (b^2*x*Log[1 - c/x^2]*Log[1 + c/x^
2])/2 + (b^2*x*Log[1 + c/x^2]^2)/4 - 2*b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[(
2*Sqrt[c])/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(
Sqrt[c] - x))/(Sqrt[c] - I*x)] - 2*b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]*Log[(2*S
qrt[c])/(Sqrt[c] + x)] + b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sq
rt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))] + b^2*Sqrt[c]*ArcTanh[x
/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])*(Sqrt[c] +
x))] + b^2*Sqrt[c]*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sqrt[c] + x))/(Sqrt[c]
- I*x)] + 2*b^2*Sqrt[c]*ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] +
x)] + I*b^2*Sqrt[c]*PolyLog[2, 1 - (2*Sqrt[c])/(Sqrt[c] - I*x)] - I*b^2*Sq
rt[c]*PolyLog[2, -1 + (2*Sqrt[c])/(Sqrt[c] - I*x)] - (I/2)*b^2*Sqrt[c]*Pol
yLog[2, 1 - ((1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)] + b^2*Sqrt[c]*PolyLog
[2, -(x/Sqrt[c])] - I*b^2*Sqrt[c]*PolyLog[2, ((-I)*x)/Sqrt[c]] + I*b^2*Sqr
t[c]*PolyLog[2, (I*x)/Sqrt[c]] - b^2*Sqrt[c]*PolyLog[2, x/Sqrt[c]] + b^...
```

3.178.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6439
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[ExpandI
ntegrand[(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c)]/2))^p, x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

rule 6440
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> Int[(a + b*
ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && ILtQ[n, 0
]
```

3.178.4 Maple [F]

$$\int \left(a+b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
int((a+b*arctanh(c/x^2))^2,x)
```

output
```
int((a+b*arctanh(c/x^2))^2,x)
```

3.178.5 Fricas [F]

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2, x)
```

3.178.6 Sympy [F]

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(a+b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
integrate((a+b*atanh(c/x**2))**2,x)
```

output
```
Integral((a + b*atanh(c/x**2))**2, x)
```

3.178.7 Maxima [F]

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

output
```
(c*(2*arctan(x/sqrt(c))/sqrt(c) + log((x - sqrt(c))/(x + sqrt(c)))/sqrt(c)
) + 2*x*arctanh(c/x^2))*a*b + 1/4*(x*log(x^2 - c)^2 - integrate(-((x^2 - c
)*log(x^2 + c)^2 - 2*(2*x^2 + (x^2 - c)*log(x^2 + c))*log(x^2 - c))/(x^2 -
 c), x))*b^2 + a^2*x
```

3.178.8 Giac [F]

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2, x)
```

3.178.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
int((a + b*atanh(c/x^2))^2,x)
```

output
```
int((a + b*atanh(c/x^2))^2, x)
```

3.179 $\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$

3.179.1 Optimal result

Integrand size = 16, antiderivative size = 1117

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx = \text{Too large to display}$$

output
```
2*b^2*arctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))/c^(1/2)-2*b^2*arcta
nh(x/c^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))/c^(1/2)+1/2*b^2*ln(1-c/x^2)*ln(1
+c/x^2)/x-2*a*b*arccot(x/c^(1/2))/c^(1/2)+2*b^2*arccot(x/c^(1/2))*ln(2/(1-
I*c^(1/2)/x))/c^(1/2)+2*b^2*arccoth(x/c^(1/2))*ln(2/(1+1/x*c^(1/2)))/c^(1/
2)-I*b^2*arctan(x/c^(1/2))^2/c^(1/2)-I*b^2*polylog(2,-1+2*c^(1/2)/(-I*x+c^
(1/2)))/c^(1/2)-I*b^2*polylog(2,1-2/(1-I*c^(1/2)/x))/c^(1/2)+1/2*I*b^2*pol
ylog(2,1-(1+I)*(1-1/x*c^(1/2))/(1-I*c^(1/2)/x))/c^(1/2)+1/2*I*b^2*polylog(
2,1+(-1+I)*(1+1/x*c^(1/2))/(1-I*c^(1/2)/x))/c^(1/2)-1/4*b^2*ln(1+c/x^2)^2/
x-2*b^2*arccot(x/c^(1/2))/c^(1/2)-2*b^2*arccoth(x/c^(1/2))/c^(1/2)-2*b^2*a
rctan(x/c^(1/2))/c^(1/2)+2*b^2*arctanh(x/c^(1/2))/c^(1/2)+1/2*b^2*polylog(
2,1+2*(1-(-c)^(1/2)/x)*c^(1/2)/((-c)^(1/2)-c^(1/2))/(1+1/x*c^(1/2)))/c^(1/
2)+1/2*b^2*polylog(2,1-2*(1+(-c)^(1/2)/x)*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+
1/x*c^(1/2)))/c^(1/2)+2*a*b/x-1/4*(2*a-b*ln(1-c/x^2))^2/x+b^2*arccoth(x/c^
(1/2))*ln(1+c/x^2)/c^(1/2)+b^2*arctan(x/c^(1/2))*ln(1+c/x^2)/c^(1/2)-b^2*a
rccot(x/c^(1/2))*ln((1+I)*(1-1/x*c^(1/2))/(1-I*c^(1/2)/x))/c^(1/2)+b^2*pol
ylog(2,-1+2*c^(1/2)/(x+c^(1/2)))/c^(1/2)-b^2*polylog(2,1-2/(1+1/x*c^(1/2))
)/c^(1/2)-b^2*ln(1-c/x^2)/x-b*(2*a-b*ln(1-c/x^2))/x-b^2*arctanh(x/c^(1/2))
^2/c^(1/2)-b^2*arccoth(x/c^(1/2))*ln(-2*(1-(-c)^(1/2)/x)*c^(1/2)/((-c)^(1/
2)-c^(1/2))/(1+1/x*c^(1/2)))/c^(1/2)-b^2*arccoth(x/c^(1/2))*ln(2*(1+(-c)^(
1/2)/x)*c^(1/2)/((-c)^(1/2)+c^(1/2))/(1+1/x*c^(1/2)))/c^(1/2)-b^2*arcco...
```

3.179.2 Mathematica [A] (verified)

Time = 2.13 (sec) , antiderivative size = 568, normalized size of antiderivative = 0.51

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$$

$$= \frac{-2a^2 - \frac{4ab\left(\arctan\left(\sqrt{\frac{c}{x^2}}\right)-\text{arctanh}\left(\sqrt{\frac{c}{x^2}}\right)\right)}{\sqrt{\frac{c}{x^2}}} - 4ab\text{arctanh}\left(\frac{c}{x^2}\right) + \frac{b^2\left(2i\arctan\left(\sqrt{\frac{c}{x^2}}\right)^2-4\arctan\left(\sqrt{\frac{c}{x^2}}\right)\text{arctanh}\left(\frac{c}{x^2}\right)-2\sqrt{\ldots}\right.}{\ldots}}{\ldots}$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])^2/x^2,x]
```

output
```
(-2*a^2 - (4*a*b*(ArcTan[Sqrt[c/x^2]] - ArcTanh[Sqrt[c/x^2]]))/Sqrt[c/x^2]
 - 4*a*b*ArcTanh[c/x^2] + (b^2*((2*I)*ArcTan[Sqrt[c/x^2]]^2 - 4*ArcTan[Sqr
t[c/x^2]]*ArcTanh[c/x^2] - 2*Sqrt[c/x^2]*ArcTanh[c/x^2]^2 - 2*ArcTan[Sqrt[
c/x^2]]*Log[1 + E^((4*I)*ArcTan[Sqrt[c/x^2]])] - 2*ArcTanh[c/x^2]*Log[1 -
Sqrt[c/x^2]] + Log[2]*Log[1 - Sqrt[c/x^2]] - Log[1 - Sqrt[c/x^2]]^2/2 + Lo
g[1 - Sqrt[c/x^2]]*Log[(1/2 + I/2)*(-I + Sqrt[c/x^2])] + 2*ArcTanh[c/x^2]*
Log[1 + Sqrt[c/x^2]] - Log[2]*Log[1 + Sqrt[c/x^2]] - Log[((1 + I) - (1 - I
)*Sqrt[c/x^2])/2]*Log[1 + Sqrt[c/x^2]] - Log[(-1/2 - I/2)*(I + Sqrt[c/x^2]
)]*Log[1 + Sqrt[c/x^2]] + Log[1 + Sqrt[c/x^2]]^2/2 + Log[1 - Sqrt[c/x^2]]*
Log[((1 + I) + (1 - I)*Sqrt[c/x^2])/2] + (I/2)*PolyLog[2, -E^((4*I)*ArcTan
[Sqrt[c/x^2]])] - PolyLog[2, (1 - Sqrt[c/x^2])/2] + PolyLog[2, (-1/2 - I/2
)*(-1 + Sqrt[c/x^2])] + PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[c/x^2])] + Poly
Log[2, (1 + Sqrt[c/x^2])/2] - PolyLog[2, (1/2 - I/2)*(1 + Sqrt[c/x^2])] -
PolyLog[2, (1/2 + I/2)*(1 + Sqrt[c/x^2])]))/Sqrt[c/x^2])/(2*x)
```

3.179.3 Rubi [A] (verified)

Time = 2.44 (sec) , antiderivative size = 1117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6460, 6457, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$$

$$\Big\downarrow \text{ 6460}$$

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{x^2}dx$$
$$\Big\downarrow \text{6457}$$
$$\int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^2}-\frac{b\log\left(\frac{c}{x^2}+1\right)\left(b\log\left(1-\frac{c}{x^2}\right)-2a\right)}{2x^2}+\frac{b^2\log^2\left(\frac{c}{x^2}+1\right)}{4x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{i\arctan\left(\frac{x}{\sqrt{c}}\right)^2b^2}{\sqrt{c}}-\frac{\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)^2b^2}{\sqrt{c}}-\frac{\log^2\left(\frac{c}{x^2}+1\right)b^2}{4x}-\frac{2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)b^2}{\sqrt{c}}-\frac{2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)b^2}{\sqrt{c}}-\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)b^2}{\sqrt{c}}+\frac{2\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)b^2}{\sqrt{c}}+\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)b^2}{\sqrt{c}}+\frac{\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)\log\left(1-\frac{c}{x^2}\right)b^2}{\sqrt{c}}-\frac{\log\left(1-\frac{c}{x^2}\right)b^2}{x}+\frac{\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right)b^2}{\sqrt{c}}+\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right)b^2}{\sqrt{c}}+\frac{\log\left(1-\frac{c}{x^2}\right)\log\left(\frac{c}{x^2}+1\right)b^2}{2x}+\frac{2\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2}{1-\frac{i\sqrt{c}}{x}}\right)b^2}{\sqrt{c}}-\frac{\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1+i)\left(1-\frac{\sqrt{c}}{x}\right)}{1-\frac{i\sqrt{c}}{x}}\right)b^2}{\sqrt{c}}+\frac{2\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2}{\frac{\sqrt{c}}{x}+1}\right)b^2}{\sqrt{c}}-\frac{\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)\log\left(-\frac{2\sqrt{c}\left(1-\frac{\sqrt{-c}}{x}\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(\frac{\sqrt{c}}{x}+1\right)}\right)b^2}{\sqrt{c}}-\frac{\coth^{-1}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}\left(\frac{\sqrt{-c}}{x}+1\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(\frac{\sqrt{c}}{x}+1\right)}\right)b^2}{\sqrt{c}}-\frac{\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1-i)\left(\frac{\sqrt{c}}{x}+1\right)}{1-\frac{i\sqrt{c}}{x}}\right)b^2}{\sqrt{c}}-\frac{2\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{x+\sqrt{c}}\right)b^2}{\sqrt{c}}-\frac{i\,\text{PolyLog}\left(2,1-\frac{2}{1-\frac{i\sqrt{c}}{x}}\right)b^2}{\sqrt{c}}+\frac{i\,\text{PolyLog}\left(2,1-\frac{(1+i)\left(1-\frac{\sqrt{c}}{x}\right)}{1-\frac{i\sqrt{c}}{x}}\right)b^2}{2\sqrt{c}}-\frac{\text{PolyLog}\left(2,1-\frac{2}{\frac{\sqrt{c}}{x}+1}\right)b^2}{\sqrt{c}}+\frac{\text{PolyLog}\left(2,\frac{2\sqrt{c}\left(1-\frac{\sqrt{-c}}{x}\right)}{\left(\sqrt{-c}-\sqrt{c}\right)\left(\frac{\sqrt{c}}{x}+1\right)}+1\right)b^2}{2\sqrt{c}}+\frac{\text{PolyLog}\left(2,1-\frac{2\sqrt{c}\left(\frac{\sqrt{-c}}{x}+1\right)}{\left(\sqrt{-c}+\sqrt{c}\right)\left(\frac{\sqrt{c}}{x}+1\right)}\right)b^2}{2\sqrt{c}}+\frac{i\,\text{PolyLog}\left(2,1-\frac{(1-i)\left(\frac{\sqrt{c}}{x}+1\right)}{1-\frac{i\sqrt{c}}{x}}\right)b^2}{2\sqrt{c}}-\frac{i\,\text{PolyLog}\left(2,\frac{2\sqrt{c}}{\sqrt{c}-ix}-1\right)b^2}{\sqrt{c}}+\frac{\text{PolyLog}\left(2,\frac{2\sqrt{c}}{x+\sqrt{c}}-1\right)b^2}{\sqrt{c}}-\frac{2a\cot^{-1}\left(\frac{x}{\sqrt{c}}\right)b}{\sqrt{c}}+\frac{\text{arctanh}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)b}{\sqrt{c}}-\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)b}{x}-\frac{a\log\left(\frac{c}{x^2}+1\right)b}{x}+\frac{2ab}{x}-\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x}$$

3.179. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}dx$

input
```
Int[(a + b*ArcTanh[c/x^2])^2/x^2,x]
```

output
```
(2*a*b)/x - (2*a*b*ArcCot[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcCot[x/Sqrt[c]])/
Sqrt[c] - (2*b^2*ArcCoth[x/Sqrt[c]])/Sqrt[c] - (2*b^2*ArcTan[x/Sqrt[c]])/S
qrt[c] - (I*b^2*ArcTan[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTanh[x/Sqrt[c]])/
Sqrt[c] - (b^2*ArcTanh[x/Sqrt[c]]^2)/Sqrt[c] + (2*b^2*ArcTan[x/Sqrt[c]]*Lo
g[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/Sqrt[c] - (b^2*Log[1 - c/x^2])/x + (b^
2*ArcCot[x/Sqrt[c]]*Log[1 - c/x^2])/Sqrt[c] - (b*(2*a - b*Log[1 - c/x^2]))
/x + (b*ArcTanh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/Sqrt[c] - (2*a - b*Lo
g[1 - c/x^2])^2/(4*x) - (a*b*Log[1 + c/x^2])/x + (b^2*ArcCoth[x/Sqrt[c]]*L
og[1 + c/x^2])/Sqrt[c] + (b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/Sqrt[c] +
(b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(2*x) - (b^2*Log[1 + c/x^2]^2)/(4*x) +
 (2*b^2*ArcCot[x/Sqrt[c]]*Log[2/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*ArcCo
t[x/Sqrt[c]]*Log[((1 + I)*(1 - Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] +
 (2*b^2*ArcCoth[x/Sqrt[c]]*Log[2/(1 + Sqrt[c]/x)])/Sqrt[c] - (b^2*ArcCoth[
x/Sqrt[c]]*Log[(-2*Sqrt[c]*(1 - Sqrt[-c]/x))/((Sqrt[-c] - Sqrt[c])*(1 + Sq
rt[c]/x))])/Sqrt[c] - (b^2*ArcCoth[x/Sqrt[c]]*Log[(2*Sqrt[c]*(1 + Sqrt[-c]
/x))/((Sqrt[-c] + Sqrt[c])*(1 + Sqrt[c]/x))])/Sqrt[c] - (b^2*ArcCot[x/Sqrt
[c]]*Log[((1 - I)*(1 + Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (2*b^2*
ArcTanh[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/Sqrt[c] - (I*b^2*Po
lyLog[2, 1 - 2/(1 - (I*Sqrt[c])/x)])/Sqrt[c] + ((I/2)*b^2*PolyLog[2, 1 - (
(1 + I)*(1 - Sqrt[c]/x))/(1 - (I*Sqrt[c])/x)])/Sqrt[c] - (b^2*PolyLog[2...
```

3.179.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6457
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c
)]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6460
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[x^m*(a + b*ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[
p, 1] && ILtQ[n, 0]
```

3.179.4 Maple [F]

$$\int \frac{\left(a+b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2} dx$$

input
```
int((a+b*arctanh(c/x^2))^2/x^2,x)
```

output
```
int((a+b*arctanh(c/x^2))^2/x^2,x)
```

3.179.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2} dx = \int \frac{\left(b \operatorname{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^2} dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^2, x)
```

3.179.6 Sympy [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2} dx = \int \frac{\left(a+b \operatorname{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2} dx$$

input
```
integrate((a+b*atanh(c/x**2))**2/x**2,x)
```

output
```
Integral((a + b*atanh(c/x**2))**2/x**2, x)
```

3.179.7 Maxima [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="maxima")
```

output
```
(c*(2*arctan(x/sqrt(c))/c^(3/2) - log((x - sqrt(c))/(x + sqrt(c)))/c^(3/2)
) - 2*arctanh(c/x^2)/x)*a*b - 1/4*b^2*(log(x^2 - c)^2/x + integrate(-((x^2
 - c)*log(x^2 + c)^2 + 2*(2*x^2 - (x^2 - c)*log(x^2 + c))*log(x^2 - c))/(x
^4 - c*x^2), x)) - a^2/x
```

3.179.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2/x^2, x)
```

3.179.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^2}\,dx$$

input
```
int((a + b*atanh(c/x^2))^2/x^2,x)
```

output
```
int((a + b*atanh(c/x^2))^2/x^2, x)
```

3.180 $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$

3.180.1 Optimal result

Integrand size = 16, antiderivative size = 1263

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx = \text{Too large to display}$$

output
```
-2/3*a*b*arctan(x/c^(1/2))/c^(3/2)+1/3*b^2*ln(1-c/x^2)/c/x+1/3*b^2*arctan(
x/c^(1/2))*ln(1-c/x^2)/c^(3/2)-1/3*b*(2*a-b*ln(1-c/x^2))/c/x+1/3*b*arctanh
(x/c^(1/2))*(2*a-b*ln(1-c/x^2))/c^(3/2)-1/3*a*b*ln(1+c/x^2)/x^3-2/3*b^2*ln
(1+c/x^2)/c/x-1/3*b^2*arctan(x/c^(1/2))*ln(1+c/x^2)/c^(3/2)+1/3*b^2*arctan
h(x/c^(1/2))*ln(1+c/x^2)/c^(3/2)+1/6*b^2*ln(1-c/x^2)*ln(1+c/x^2)/x^3+2/3*b
^2*arctan(x/c^(1/2))*ln(2*c^(1/2)/(-I*x+c^(1/2)))/c^(3/2)-1/3*b^2*arctan(x
/c^(1/2))*ln((1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))/c^(3/2)+2/3*b^2*arctanh(x/
c^(1/2))*ln(2*c^(1/2)/(x+c^(1/2)))/c^(3/2)-1/3*b^2*arctanh(x/c^(1/2))*ln(2
*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))/c^(3/2)-1/3*b^2
*arctan(x/c^(1/2))*ln((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))/c^(3/2)-1/3*b^2*ar
ctanh(x/c^(1/2))*ln(2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/
2)))/c^(3/2)-2/3*b^2*arctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))/c^(3
/2)-2/3*b^2*arctanh(x/c^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))/c^(3/2)-1/3*I*b
^2*polylog(2,I*x/c^(1/2))/c^(3/2)-1/3*I*b^2*polylog(2,1-2*c^(1/2)/(-I*x+c^
(1/2)))/c^(3/2)+1/6*b^2*polylog(2,1-2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/(
(-c)^(1/2)+c^(1/2)))/c^(3/2)+2/9*a*b/x^3-2/3*a*b/c/x+1/3*I*b^2*polylog(2,-
I*x/c^(1/2))/c^(3/2)+1/3*I*b^2*polylog(2,-1+2*c^(1/2)/(-I*x+c^(1/2)))/c^(3
/2)+1/6*I*b^2*polylog(2,1-(1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))/c^(3/2)+1/6*I
*b^2*polylog(2,1+(-1+I)*(x+c^(1/2))/(-I*x+c^(1/2)))/c^(3/2)+1/3*I*b^2*arct
an(x/c^(1/2))^2/c^(3/2)+4/3*b^2*arctan(x/c^(1/2))/c^(3/2)+4/3*b^2*arcta...
```

3.180.2 Mathematica [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])^2/x^4,x]
```

output
```
Integrate[(a + b*ArcTanh[c/x^2])^2/x^4, x]
```

3.180.3 Rubi [A] (verified)

Time = 2.49 (sec) , antiderivative size = 1263, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6460, 6457, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{6460}$$

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{6457}$$

$$\int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^4}-\frac{b\log\left(\frac{c}{x^2}+1\right)\left(b\log\left(1-\frac{c}{x^2}\right)-2a\right)}{2x^4}+\frac{b^2\log^2\left(\frac{c}{x^2}+1\right)}{4x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\arctan\left(\frac{x}{\sqrt{c}}\right)^2b^2}{3c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)^2b^2}{3c^{3/2}}-\frac{\log^2\left(\frac{c}{x^2}+1\right)b^2}{12x^3}+\frac{4\arctan\left(\frac{x}{\sqrt{c}}\right)b^2}{3c^{3/2}}+\frac{4\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)b^2}{3c^{3/2}}-\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)b^2}{3c^{3/2}}+\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(1-\frac{c}{x^2}\right)b^2}{3c^{3/2}}+\frac{\log\left(1-\frac{c}{x^2}\right)b^2}{3cx}-\frac{\log\left(1-\frac{c}{x^2}\right)b^2}{9x^3}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right)b^2}{3c^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right)b^2}{3c^{3/2}}+\frac{\log\left(1-\frac{c}{x^2}\right)\log\left(\frac{c}{x^2}+1\right)b^2}{6x^3}-\frac{2\log\left(\frac{c}{x^2}+1\right)b^2}{3cx}+\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)b^2}{3c^{3/2}}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right)b^2}{3c^{3/2}}+\frac{2\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}}{x+\sqrt{c}}\right)b^2}{3c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}(\sqrt{-c}-x)}{(\sqrt{-c}-\sqrt{c})(x+\sqrt{c})}\right)b^2}{3c^{3/2}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}(x+\sqrt{-c})}{(\sqrt{-c}+\sqrt{c})(x+\sqrt{c})}\right)b^2}{3c^{3/2}}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1-i)(x+\sqrt{c})}{\sqrt{c}-ix}\right)b^2}{3c^{3/2}}-\frac{2\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{x+\sqrt{c}}\right)b^2}{3c^{3/2}}-\frac{i\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)b^2}{3c^{3/2}}+\frac{i\operatorname{PolyLog}\left(2,\frac{2\sqrt{c}}{\sqrt{c}-ix}-1\right)b^2}{3c^{3/2}}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right)b^2}{6c^{3/2}}-\frac{\operatorname{PolyLog}\left(2,-\frac{x}{\sqrt{c}}\right)b^2}{3c^{3/2}}+\frac{i\operatorname{PolyLog}\left(2,-\frac{ix}{\sqrt{c}}\right)b^2}{3c^{3/2}}-\frac{i\operatorname{PolyLog}\left(2,\frac{ix}{\sqrt{c}}\right)b^2}{3c^{3/2}}+\frac{\operatorname{PolyLog}\left(2,\frac{x}{\sqrt{c}}\right)b^2}{3c^{3/2}}-\frac{\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{c}}{x+\sqrt{c}}\right)b^2}{3c^{3/2}}+\frac{\operatorname{PolyLog}\left(2,\frac{2\sqrt{c}}{x+\sqrt{c}}-1\right)b^2}{3c^{3/2}}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{c}(\sqrt{-c}-x)}{(\sqrt{-c}-\sqrt{c})(x+\sqrt{c})}\right)b^2}{6c^{3/2}}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{c}(x+\sqrt{-c})}{(\sqrt{-c}+\sqrt{c})(x+\sqrt{c})}\right)b^2}{6c^{3/2}}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{(1-i)(x+\sqrt{c})}{\sqrt{c}-ix}\right)b^2}{6c^{3/2}}-\frac{2a\arctan\left(\frac{x}{\sqrt{c}}\right)b}{3c^{3/2}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)b}{3c^{3/2}}-\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)b}{3cx}-\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)b}{9x^3}-\frac{a\log\left(\frac{c}{x^2}+1\right)b}{3x^3}-\frac{2ab}{3cx}+\frac{2ab}{9x^3}-\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{12x^3}$$

input

```
Int[(a + b*ArcTanh[c/x^2])^2/x^4,x]
```

3.180. $\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$

output
```
(2*a*b)/(9*x^3) - (2*a*b)/(3*c*x) - (2*a*b*ArcTan[x/Sqrt[c]])/(3*c^(3/2))
+ (4*b^2*ArcTan[x/Sqrt[c]])/(3*c^(3/2)) + ((I/3)*b^2*ArcTan[x/Sqrt[c]]^2)/
c^(3/2) + (4*b^2*ArcTanh[x/Sqrt[c]])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]
^2)/(3*c^(3/2)) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] -
I*x)])/(3*c^(3/2)) - (b^2*Log[1 - c/x^2])/(9*x^3) + (b^2*Log[1 - c/x^2])/(
3*c*x) + (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(3*c^(3/2)) - (b*(2*a - b*
Log[1 - c/x^2]))/(9*x^3) - (b*(2*a - b*Log[1 - c/x^2]))/(3*c*x) + (b*ArcTa
nh[x/Sqrt[c]]*(2*a - b*Log[1 - c/x^2]))/(3*c^(3/2)) - (2*a - b*Log[1 - c/x
^2])^2/(12*x^3) - (a*b*Log[1 + c/x^2])/(3*x^3) - (2*b^2*Log[1 + c/x^2])/(3
*c*x) - (b^2*ArcTan[x/Sqrt[c]]*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*ArcTanh[
x/Sqrt[c]]*Log[1 + c/x^2])/(3*c^(3/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2
])/(6*x^3) - (b^2*Log[1 + c/x^2]^2)/(12*x^3) + (2*b^2*ArcTan[x/Sqrt[c]]*Lo
g[(2*Sqrt[c])/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[(
(1 + I)*(Sqrt[c] - x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) + (2*b^2*ArcTanh[x/Sq
rt[c]]*Log[(2*Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c
]]*Log[(2*Sqrt[c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(
3*c^(3/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt
[-c] + Sqrt[c])*(Sqrt[c] + x))])/(3*c^(3/2)) - (b^2*ArcTan[x/Sqrt[c]]*Log[
((1 - I)*(Sqrt[c] + x))/(Sqrt[c] - I*x)])/(3*c^(3/2)) - (2*b^2*ArcTanh[x/S
qrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] + x)])/(3*c^(3/2)) - ((I/3)*b^2*Po...
```

3.180.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6457
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c
)]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6460
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[x^m*(a + b*ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[
p, 1] && ILtQ[n, 0]
```

3.180. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$

3.180.4 Maple [F]

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}dx$$

input
```
int((a+b*arctanh(c/x^2))^2/x^4,x)
```

output
```
int((a+b*arctanh(c/x^2))^2/x^4,x)
```

3.180.5 Fricas [F]

$$\int \frac{\left(a+b\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx = \int \frac{\left(b\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^4, x)
```

3.180.6 Sympy [F]

$$\int \frac{\left(a+b\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx = \int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

input
```
integrate((a+b*atanh(c/x**2))**2/x**4,x)
```

output
```
Integral((a + b*atanh(c/x**2))**2/x**4, x)
```

3.180.7 Maxima [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="maxima")
```

output
```
-1/3*(c*(2*arctan(x/sqrt(c))/c^(5/2) + log((x - sqrt(c))/(x + sqrt(c)))/c^
(5/2) + 4/(c^2*x)) + 2*arctanh(c/x^2)/x^3)*a*b - 1/12*b^2*(log(x^2 - c)^2/
x^3 + 3*integrate(-1/3*(3*(x^2 - c)*log(x^2 + c)^2 + 2*(2*x^2 - 3*(x^2 - c
)*log(x^2 + c))*log(x^2 - c))/(x^6 - c*x^4), x)) - 1/3*a^2/x^3
```

3.180.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx=\int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2/x^4, x)
```

3.180.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^4}\,dx$$

input
```
int((a + b*atanh(c/x^2))^2/x^4,x)
```

output
```
int((a + b*atanh(c/x^2))^2/x^4, x)
```

3.181 $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$

3.181.1 Optimal result

Integrand size = 16, antiderivative size = 1337

$$\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx = \text{Too large to display}$$

output
```
-2/5*b^2*arctanh(x/c^(1/2))*ln(2-2*c^(1/2)/(x+c^(1/2)))/c^(5/2)+2/5*a*b*ar
ctan(x/c^(1/2))/c^(5/2)+1/15*b^2*ln(1-c/x^2)/c/x^3-1/5*b^2*ln(1-c/x^2)/c^2
/x-1/5*b^2*arctan(x/c^(1/2))*ln(1-c/x^2)/c^(5/2)-1/15*b*(2*a-b*ln(1-c/x^2)
)/c/x^3-1/5*b*(2*a-b*ln(1-c/x^2))/c^2/x+1/5*b*arctanh(x/c^(1/2))*(2*a-b*ln
(1-c/x^2))/c^(5/2)-1/5*a*b*ln(1+c/x^2)/x^5-2/15*b^2*ln(1+c/x^2)/c/x^3+1/5*
b^2*arctan(x/c^(1/2))*ln(1+c/x^2)/c^(5/2)+1/5*b^2*arctanh(x/c^(1/2))*ln(1+
c/x^2)/c^(5/2)+1/10*b^2*ln(1-c/x^2)*ln(1+c/x^2)/x^5-2/5*b^2*arctan(x/c^(1/
2))*ln(2*c^(1/2)/(-I*x+c^(1/2)))/c^(5/2)+1/5*b^2*arctan(x/c^(1/2))*ln((1+I
)*(-x+c^(1/2))/(-I*x+c^(1/2)))/c^(5/2)+2/5*b^2*arctanh(x/c^(1/2))*ln(2*c^(
1/2)/(x+c^(1/2)))/c^(5/2)-1/5*b^2*arctanh(x/c^(1/2))*ln(2*(-x+(-c)^(1/2))*
c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))/c^(5/2)+1/5*b^2*arctan(x/c^(1/2)
)*ln((1-I)*(x+c^(1/2))/(-I*x+c^(1/2)))/c^(5/2)-1/5*b^2*arctanh(x/c^(1/2))*
ln(2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/2)+c^(1/2)))/c^(5/2)+2/5*
b^2*arctan(x/c^(1/2))*ln(2-2*c^(1/2)/(-I*x+c^(1/2)))/c^(5/2)-2/15*a*b/c/x^
3+2/5*a*b/c^2/x-1/5*I*b^2*arctan(x/c^(1/2))^2/c^(5/2)-1/5*I*b^2*polylog(2,
-I*x/c^(1/2))/c^(5/2)-1/5*I*b^2*polylog(2,-1+2*c^(1/2)/(-I*x+c^(1/2)))/c^(
5/2)-1/10*I*b^2*polylog(2,1-(1+I)*(-x+c^(1/2))/(-I*x+c^(1/2)))/c^(5/2)-1/1
0*I*b^2*polylog(2,1+(-1+I)*(x+c^(1/2))/(-I*x+c^(1/2)))/c^(5/2)+1/10*b^2*po
lylog(2,1-2*(-x+(-c)^(1/2))*c^(1/2)/((-c)^(1/2)-c^(1/2))/(x+c^(1/2)))/c^(5
/2)+1/10*b^2*polylog(2,1-2*(x+(-c)^(1/2))*c^(1/2)/(x+c^(1/2))/((-c)^(1/...
```

3.181.2 Mathematica [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx=\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

input
```
Integrate[(a + b*ArcTanh[c/x^2])^2/x^6,x]
```

output
```
Integrate[(a + b*ArcTanh[c/x^2])^2/x^6, x]
```

3.181.3 Rubi [A] (verified)

Time = 2.64 (sec) , antiderivative size = 1337, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6460, 6457, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

$$\Big\downarrow \ 6460$$

$$\int \frac{\left(a+b\coth^{-1}\left(\frac{x^2}{c}\right)\right)^2}{x^6}dx$$

$$\Big\downarrow \ 6457$$

$$\int \left(\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{4x^6}-\frac{b\log\left(\frac{c}{x^2}+1\right)\left(b\log\left(1-\frac{c}{x^2}\right)-2a\right)}{2x^6}+\frac{b^2\log^2\left(\frac{c}{x^2}+1\right)}{4x^6}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{i\arctan\left(\frac{x}{\sqrt{c}}\right)^2b^2}{5c^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)^2b^2}{5c^{5/2}}-\frac{\log^2\left(\frac{c}{x^2}+1\right)b^2}{20x^5}-\frac{4\arctan\left(\frac{x}{\sqrt{c}}\right)b^2}{15c^{5/2}}+\frac{4\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)b^2}{15c^{5/2}}+\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)b^2}{5c^{5/2}}-\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(1-\frac{c}{x^2}\right)b^2}{5c^{5/2}}-\frac{\log\left(1-\frac{c}{x^2}\right)b^2}{5c^2x}+\frac{\log\left(1-\frac{c}{x^2}\right)b^2}{15cx^3}-\frac{\log\left(1-\frac{c}{x^2}\right)b^2}{25x^5}+\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right)b^2}{5c^{5/2}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{c}{x^2}+1\right)b^2}{5c^{5/2}}+\frac{\log\left(1-\frac{c}{x^2}\right)\log\left(\frac{c}{x^2}+1\right)b^2}{10x^5}-\frac{2\log\left(\frac{c}{x^2}+1\right)b^2}{15cx^3}-\frac{2\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)b^2}{5c^{5/2}}+\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right)b^2}{5c^{5/2}}+\frac{2\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}}{x+\sqrt{c}}\right)b^2}{5c^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}(\sqrt{-c}-x)}{(\sqrt{-c}-\sqrt{c})(x+\sqrt{c})}\right)b^2}{5c^{5/2}}-\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{2\sqrt{c}(x+\sqrt{-c})}{(\sqrt{-c}+\sqrt{c})(x+\sqrt{c})}\right)b^2}{5c^{5/2}}+\frac{\arctan\left(\frac{x}{\sqrt{c}}\right)\log\left(\frac{(1-i)(x+\sqrt{c})}{\sqrt{c}-ix}\right)b^2}{5c^{5/2}}-\frac{2\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\log\left(2-\frac{2\sqrt{c}}{x+\sqrt{c}}\right)b^2}{5c^{5/2}}+\frac{i\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{c}}{\sqrt{c}-ix}\right)b^2}{5c^{5/2}}-\frac{i\operatorname{PolyLog}\left(2,\frac{2\sqrt{c}}{\sqrt{c}-ix}-1\right)b^2}{5c^{5/2}}-\frac{i\operatorname{PolyLog}\left(2,1-\frac{(1+i)(\sqrt{c}-x)}{\sqrt{c}-ix}\right)b^2}{10c^{5/2}}-\frac{\operatorname{PolyLog}\left(2,-\frac{x}{\sqrt{c}}\right)b^2}{5c^{5/2}}-\frac{i\operatorname{PolyLog}\left(2,-\frac{ix}{\sqrt{c}}\right)b^2}{5c^{5/2}}+\frac{i\operatorname{PolyLog}\left(2,\frac{ix}{\sqrt{c}}\right)b^2}{5c^{5/2}}+\frac{\operatorname{PolyLog}\left(2,\frac{x}{\sqrt{c}}\right)b^2}{5c^{5/2}}-\frac{\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{c}}{x+\sqrt{c}}\right)b^2}{5c^{5/2}}+\frac{\operatorname{PolyLog}\left(2,\frac{2\sqrt{c}}{x+\sqrt{c}}-1\right)b^2}{5c^{5/2}}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{c}(\sqrt{-c}-x)}{(\sqrt{-c}-\sqrt{c})(x+\sqrt{c})}\right)b^2}{10c^{5/2}}+\frac{\operatorname{PolyLog}\left(2,1-\frac{2\sqrt{c}(x+\sqrt{-c})}{(\sqrt{-c}+\sqrt{c})(x+\sqrt{c})}\right)b^2}{10c^{5/2}}-\frac{i\operatorname{PolyLog}\left(2,1-\frac{(1-i)(x+\sqrt{c})}{\sqrt{c}-ix}\right)b^2}{10c^{5/2}}-\frac{8b^2}{15c^2x}+\frac{2a\arctan\left(\frac{x}{\sqrt{c}}\right)b}{5c^{5/2}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{c}}\right)\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)b}{5c^{5/2}}-\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)b}{5c^2x}-\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)b}{15cx^3}-\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)b}{25x^5}-\frac{a\log\left(\frac{c}{x^2}+1\right)b}{5x^5}+\frac{2ab}{5c^2x}-\frac{2ab}{15cx^3}+\frac{2ab}{25x^5}-\frac{\left(2a-b\log\left(1-\frac{c}{x^2}\right)\right)^2}{20x^5}$$

input
```
Int[(a + b*ArcTanh[c/x^2])^2/x^6,x]
```

3.181. $\int \frac{\left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$

output
```
(2*a*b)/(25*x^5) - (2*a*b)/(15*c*x^3) + (2*a*b)/(5*c^2*x) - (8*b^2)/(15*c^
2*x) + (2*a*b*ArcTan[x/Sqrt[c]])/(5*c^(5/2)) - (4*b^2*ArcTan[x/Sqrt[c]])/(
15*c^(5/2)) - ((I/5)*b^2*ArcTan[x/Sqrt[c]]^2)/c^(5/2) + (4*b^2*ArcTanh[x/S
qrt[c]])/(15*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]^2)/(5*c^(5/2)) + (2*b^2*Ar
cTan[x/Sqrt[c]]*Log[2 - (2*Sqrt[c])/(Sqrt[c] - I*x)])/(5*c^(5/2)) - (b^2*L
og[1 - c/x^2])/(25*x^5) + (b^2*Log[1 - c/x^2])/(15*c*x^3) - (b^2*Log[1 - c
/x^2])/(5*c^2*x) - (b^2*ArcTan[x/Sqrt[c]]*Log[1 - c/x^2])/(5*c^(5/2)) - (b
*(2*a - b*Log[1 - c/x^2]))/(25*x^5) - (b*(2*a - b*Log[1 - c/x^2]))/(15*c*x
^3) - (b*(2*a - b*Log[1 - c/x^2]))/(5*c^2*x) + (b*ArcTanh[x/Sqrt[c]]*(2*a
- b*Log[1 - c/x^2]))/(5*c^(5/2)) - (2*a - b*Log[1 - c/x^2])^2/(20*x^5) - (
a*b*Log[1 + c/x^2])/(5*x^5) - (2*b^2*Log[1 + c/x^2])/(15*c*x^3) + (b^2*Arc
Tan[x/Sqrt[c]]*Log[1 + c/x^2])/(5*c^(5/2)) + (b^2*ArcTanh[x/Sqrt[c]]*Log[1
 + c/x^2])/(5*c^(5/2)) + (b^2*Log[1 - c/x^2]*Log[1 + c/x^2])/(10*x^5) - (b
^2*Log[1 + c/x^2]^2)/(20*x^5) - (2*b^2*ArcTan[x/Sqrt[c]]*Log[(2*Sqrt[c])/(
Sqrt[c] - I*x)])/(5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 + I)*(Sqrt[c
] - x))/(Sqrt[c] - I*x)])/(5*c^(5/2)) + (2*b^2*ArcTanh[x/Sqrt[c]]*Log[(2*S
qrt[c])/(Sqrt[c] + x)])/(5*c^(5/2)) - (b^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[
c]*(Sqrt[-c] - x))/((Sqrt[-c] - Sqrt[c])*(Sqrt[c] + x))])/(5*c^(5/2)) - (b
^2*ArcTanh[x/Sqrt[c]]*Log[(2*Sqrt[c]*(Sqrt[-c] + x))/((Sqrt[-c] + Sqrt[c])
*(Sqrt[c] + x))])/(5*c^(5/2)) + (b^2*ArcTan[x/Sqrt[c]]*Log[((1 - I)*(Sq...
```

3.181.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6457
```
Int[((a_.) + ArcCoth[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[ExpandIntegrand[x^m*(a + b*(Log[1 + 1/(x^n*c)]/2) - b*(Log[1 - 1/(x^n*c
)]/2))^p, x], x] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && IGtQ[n, 0] && Inte
gerQ[m]
```

rule 6460
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_)*(x_)^(m_.), x_Symbol] :>
Int[x^m*(a + b*ArcCoth[1/(x^n*c)])^p, x] /; FreeQ[{a, b, c, m}, x] && IGtQ[
p, 1] && ILtQ[n, 0]
```

3.181. $\int \frac{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6} dx$

3.181.4 Maple [F]

$$\int \frac{\left(a+b\,\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}dx$$

input
```
int((a+b*arctanh(c/x^2))^2/x^6,x)
```

output
```
int((a+b*arctanh(c/x^2))^2/x^6,x)
```

3.181.5 Fricas [F]

$$\int \frac{\left(a+b\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx = \int \frac{\left(b\,\mathrm{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}{x^6}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)/x^6, x)
```

3.181.6 Sympy [F]

$$\int \frac{\left(a+b\mathrm{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx = \int \frac{\left(a+b\,\mathrm{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

input
```
integrate((a+b*atanh(c/x**2))**2/x**6,x)
```

output
```
Integral((a + b*atanh(c/x**2))**2/x**6, x)
```

3.181.7 Maxima [F]

$$\int \frac{\left(a + b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx = \int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^6}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="maxima")
```

output
```
1/15*(c*(6*arctan(x/sqrt(c))/c^(7/2) - 3*log((x - sqrt(c))/(x + sqrt(c)))/
c^(7/2) - 4/(c^2*x^3)) - 6*arctanh(c/x^2)/x^5)*a*b - 1/20*b^2*(log(x^2 - c
)^2/x^5 + 5*integrate(-1/5*(5*(x^2 - c)*log(x^2 + c)^2 + 2*(2*x^2 - 5*(x^2
 - c)*log(x^2 + c))*log(x^2 - c))/(x^8 - c*x^6), x)) - 1/5*a^2/x^5
```

3.181.8 Giac [F]

$$\int \frac{\left(a + b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx = \int \frac{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right) + a\right)^2}{x^6}\,dx$$

input
```
integrate((a+b*arctanh(c/x^2))^2/x^6,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2/x^6, x)
```

3.181.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a + b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx = \int \frac{\left(a + b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2}{x^6}\,dx$$

input
```
int((a + b*atanh(c/x^2))^2/x^6,x)
```

output
```
int((a + b*atanh(c/x^2))^2/x^6, x)
```

3.182 $\int (dx)^m \left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx$

3.182.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx = \text{Int}\left((dx)^m \left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

3.182.2 Mathematica [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx = \int (dx)^m \left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^3, x]
```

3.182.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

$$\Big\downarrow \text{6468}$$

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c/x^2])^3,x]
```

output
```
$Aborted
```

3.182.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.182.4 Maple [N/A] (verified)

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

input
```
int((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c/x^2))^3,x)
```

3.182.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^3 (dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c/x^2)^3 + 3*a*b^2*arctanh(c/x^2)^2 + 3*a^2*b*arctan
h(c/x^2) + a^3)*(d*x)^m, x)
```

3.182.6 Sympy [N/A]

Not integrable

Time = 57.81 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx = \int (dx)^m \left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c/x**2))**3,x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c/x**2))**3, x)
```

3.182.7 Maxima [N/A]

Not integrable

Time = 2.64 (sec) , antiderivative size = 398, normalized size of antiderivative = 22.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^3 (dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="maxima")
```

output
```
-1/8*b^3*d^m*x*x^m*log(x^2 - c)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1))
+ integrate(1/8*((b^3*d^m*(m + 1)*x^2 - b^3*c*d^m*(m + 1))*x^m*log(x^2 + c
)^3 + 6*(a*b^2*d^m*(m + 1)*x^2 - a*b^2*c*d^m*(m + 1))*x^m*log(x^2 + c)^2 +
 12*(a^2*b*d^m*(m + 1)*x^2 - a^2*b*c*d^m*(m + 1))*x^m*log(x^2 + c) + 3*((b
^3*d^m*(m + 1)*x^2 - b^3*c*d^m*(m + 1))*x^m*log(x^2 + c) - 2*(a*b^2*c*d^m*
(m + 1) - (a*b^2*d^m*(m + 1) + b^3*d^m)*x^2)*x^m)*log(x^2 - c)^2 - 3*((b^3
*d^m*(m + 1)*x^2 - b^3*c*d^m*(m + 1))*x^m*log(x^2 + c)^2 + 4*(a*b^2*d^m*(m
 + 1)*x^2 - a*b^2*c*d^m*(m + 1))*x^m*log(x^2 + c) + 4*(a^2*b*d^m*(m + 1)*x
^2 - a^2*b*c*d^m*(m + 1))*x^m)*log(x^2 - c))/((m + 1)*x^2 - c*(m + 1)), x)
```

3.182.8 Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^3 (dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c/x^2))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^3*(d*x)^m, x)
```

3.182.9 Mupad [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^3 dx = \int (d\,x)^m \left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^3 dx$$

input
```
int((d*x)^m*(a + b*atanh(c/x^2))^3,x)
```

output
```
int((d*x)^m*(a + b*atanh(c/x^2))^3, x)
```

3.183 $\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$

3.183.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \text{Int}\left((dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c/x^2))^2,x)
```

3.183.2 Mathematica [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2])^2, x]
```

3.183.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx$$

$$\Big\downarrow \text{ 6468}$$

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 \, dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
$Aborted
```

3.183.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.183.4 Maple [N/A] (verified)

Not integrable

Time = 0.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b \,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arctanh(c/x^2))^2,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c/x^2))^2,x)
```

3.183.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2)*(d*x)^m, x)
```

3.183.6 Sympy [N/A]

Not integrable

Time = 36.35 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int (dx)^m \left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c/x**2))**2,x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c/x**2))**2, x)
```

3.183.7 Maxima [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 228, normalized size of antiderivative = 12.67

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*d^m*x*x^m*log(x^2 - c)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1)) -
 integrate(-1/4*((b^2*d^m*(m + 1)*x^2 - b^2*c*d^m*(m + 1))*x^m*log(x^2 + c
)^2 + 4*(a*b*d^m*(m + 1)*x^2 - a*b*c*d^m*(m + 1))*x^m*log(x^2 + c) - 2*((b
^2*d^m*(m + 1)*x^2 - b^2*c*d^m*(m + 1))*x^m*log(x^2 + c) - 2*(a*b*c*d^m*(m
 + 1) - (a*b*d^m*(m + 1) + b^2*d^m)*x^2)*x^m)*log(x^2 - c))/((m + 1)*x^2 -
 c*(m + 1)), x)
```

3.183.8 Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2 (dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)^2*(d*x)^m, x)
```

3.183.9 Mupad [N/A]

Not integrable

Time = 3.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2 dx = \int (d\,x)^m \left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2 dx$$

input
```
int((d*x)^m*(a + b*atanh(c/x^2))^2,x)
```

output
```
int((d*x)^m*(a + b*atanh(c/x^2))^2, x)
```

3.184 $\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx$

3.184.1 Optimal result

Integrand size = 16, antiderivative size = 75

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx = \frac{(dx)^{1+m}\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{d(1+m)} - \frac{2bcd(dx)^{-1+m}\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{c^2}{x^4}\right)}{1-m^2}$$

output
```
(d*x)^(1+m)*(a+b*arctanh(c/x^2))/d/(1+m)-2*b*c*d*(d*x)^(-1+m)*hypergeom([1
, 1/4-1/4*m],[5/4-1/4*m],c^2/x^4)/(-m^2+1)
```

3.184.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.91

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx = \frac{(dx)^m\left((-1+m)x^2\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)+2bc\,\text{Hypergeometric2F1}\left(1,\frac{1}{4}-\frac{m}{4},\frac{5}{4}-\frac{m}{4},\frac{c^2}{x^4}\right)\right)}{(-1+m)(1+m)x}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c/x^2]),x]
```

output
```
((d*x)^m*((-1 + m)*x^2*(a + b*ArcTanh[c/x^2]) + 2*b*c*Hypergeometric2F1[1,
 1/4 - m/4, 5/4 - m/4, c^2/x^4]))/((-1 + m)*(1 + m)*x)
```

3.184.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6464, 862, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) \, dx$$

$$\Big\downarrow \text{ 6464}$$

$$\frac{2bcd^2\int \frac{(dx)^{m-2}}{1-\frac{c^2}{x^4}}dx}{m+1}+\frac{(dx)^{m+1}\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{d(m+1)}$$

$$\Big\downarrow \text{ 862}$$

$$\frac{(dx)^{m+1}\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{d(m+1)}-\frac{2bcd\left(\frac{1}{x}\right)^{m-1}(dx)^{m-1}\int \frac{\left(\frac{1}{x}\right)^{-m}}{1-\frac{c^2}{x^4}}d\frac{1}{x}}{m+1}$$

$$\Big\downarrow \text{ 888}$$

$$\frac{(dx)^{m+1}\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)}{d(m+1)}-\frac{2bcd(dx)^{m-1}\text{Hypergeometric2F1}\left(1,\frac{1-m}{4},\frac{5-m}{4},\frac{c^2}{x^4}\right)}{(1-m)(m+1)}$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c/x^2]),x]
```

output
```
((d*x)^(1 + m)*(a + b*ArcTanh[c/x^2]))/(d*(1 + m)) - (2*b*c*d*(d*x)^(-1 +
m)*Hypergeometric2F1[1, (1 - m)/4, (5 - m)/4, c^2/x^4])/((1 - m)*(1 + m))
```

3.184.3.1 Defintions of rubi rules used

rule 862
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-c^
(-1))*(c*x)^(m + 1)*(1/x)^(m + 1)   Subst[Int[(a + b/x^n)^p/x^(m + 2), x],
x, 1/x], x] /; FreeQ[{a, b, c, m, p}, x] && ILtQ[n, 0] &&  !RationalQ[m]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6464
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))*((d_)*(x_))^(m_), x_Symbol] :
> Simp[(d*x)^(m + 1)*((a + b*ArcTanh[c*x^n])/(d*(m + 1))), x] - Simp[b*c*(n
/(d^n*(m + 1)))   Int[(d*x)^(m + n)/(1 - c^2*x^(2*n)), x], x] /; FreeQ[{a,
b, c, d, m, n}, x] && IntegerQ[n] && NeQ[m, -1]
```

3.184.4 Maple [F]

$$\int (dx)^m \left(a + b \operatorname{arctanh}\left(\frac{c}{x^2}\right)\right) dx$$

input
```
int((d*x)^m*(a+b*arctanh(c/x^2)),x)
```

output
```
int((d*x)^m*(a+b*arctanh(c/x^2)),x)
```

3.184.5 Fricas [F]

$$\int (dx)^m \left(a + b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx = \int \left(b \operatorname{artanh}\left(\frac{c}{x^2}\right) + a\right)(dx)^m dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

output
```
integral((b*arctanh(c/x^2) + a)*(d*x)^m, x)
```

3.184.6 Sympy [F]

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx = \int (dx)^m \left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right) dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c/x**2)),x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c/x**2)), x)
```

3.184.7 Maxima [F]

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

output
```
1/2*(4*c*d^m*integrate(x^2*x^m/((m + 1)*x^4 - c^2*(m + 1)), x) + (d^m*x*x^
m*log(x^2 + c) - d^m*x*x^m*log(x^2 - c))/(m + 1))*b + (d*x)^(m + 1)*a/(d*(
m + 1))
```

3.184.8 Giac [F]

$$\int (dx)^m \left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right) dx = \int \left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)(dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c/x^2) + a)*(d*x)^m, x)
```

3.184.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m \left(a+b\operatorname{arctanh}\left(\frac{c}{x^2}\right)\right)\,dx=\int (d\,x)^m\left(a+b\operatorname{atanh}\left(\frac{c}{x^2}\right)\right)\,dx$$

input
```
int((d*x)^m*(a + b*atanh(c/x^2)),x)
```

output
```
int((d*x)^m*(a + b*atanh(c/x^2)), x)
```

3.185 $\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)} \, dx$

3.185.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)} \, dx=\text{Int}\left(\frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctanh(c/x^2)),x)
```

3.185.2 Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)} \, dx=\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)} \, dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2]), x]
```

3.185.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)} \, dx$$
$$\Big\downarrow \text{ 6468}$$
$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTanh[c/x^2]),x]
```

output
```
$Aborted
```

3.185.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.185.4 Maple [N/A] (verified)

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\,\operatorname{arctanh}\left(\frac{c}{x^2}\right)}dx$$

input
```
int((d*x)^m/(a+b*arctanh(c/x^2)),x)
```

output
```
int((d*x)^m/(a+b*arctanh(c/x^2)),x)
```

3.185.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}\,dx=\int \frac{(dx)^m}{b\,\text{artanh}\left(\frac{c}{x^2}\right)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

3.185.6 Sympy [N/A]

Not integrable

Time = 45.42 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}\,dx=\int \frac{(dx)^m}{a+b\,\text{atanh}\left(\frac{c}{x^2}\right)}\,dx$$

input
```
integrate((d*x)**m/(a+b*atanh(c/x**2)),x)
```

output
```
Integral((d*x)**m/(a + b*atanh(c/x**2)), x)
```

3.185.7 Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}\,dx=\int \frac{(dx)^m}{b\,\text{artanh}\left(\frac{c}{x^2}\right)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

3.185.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}\,dx=\int \frac{(dx)^m}{b\,\text{artanh}\left(\frac{c}{x^2}\right)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c/x^2)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctanh(c/x^2) + a), x)
```

3.185.9 Mupad [N/A]

Not integrable

Time = 3.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(\frac{c}{x^2}\right)}\,dx=\int \frac{(d\,x)^m}{a+b\,\text{atanh}\left(\frac{c}{x^2}\right)}\,dx$$

input
```
int((d*x)^m/(a + b*atanh(c/x^2)),x)
```

output
```
int((d*x)^m/(a + b*atanh(c/x^2)), x)
```

3.186 $\int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2} dx$

3.186.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2} dx = \text{Int}\left(\frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}, x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctanh(c/x^2))^2,x)
```

3.186.2 Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2} dx = \int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2} dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTanh[c/x^2])^2, x]
```

3.186.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}dx$$
$$\Big\downarrow \text{6468}$$
$$\int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTanh[c/x^2])^2,x]
```

output
```
$Aborted
```

3.186.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.186.4 Maple [N/A] (verified)

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{\left(a+b\,\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}dx$$

input
```
int((d*x)^m/(a+b*arctanh(c/x^2))^2,x)
```

output
```
int((d*x)^m/(a+b*arctanh(c/x^2))^2,x)
```

3.186.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arctanh(c/x^2)^2 + 2*a*b*arctanh(c/x^2) + a^2), x)
```

3.186.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atanh(c/x**2))**2,x)
```

output
```
Timed out
```

3.186.7 Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 128, normalized size of antiderivative = 7.11

$$\int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="maxima")
```

output
```
(d^m*x^4 - c^2*d^m)*x^m/(b^2*c*x*log(x^2 + c) - b^2*c*x*log(x^2 - c) + 2*a
*b*c*x) + integrate(-(d^m*(m + 3)*x^4 - c^2*d^m*(m - 1))*x^m/(b^2*c*x^2*lo
g(x^2 + c) - b^2*c*x^2*log(x^2 - c) + 2*a*b*c*x^2), x)
```

3.186.8 Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}\,dx=\int \frac{(dx)^m}{\left(b\,\text{artanh}\left(\frac{c}{x^2}\right)+a\right)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c/x^2))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctanh(c/x^2) + a)^2, x)
```

3.186.9 Mupad [N/A]

Not integrable

Time = 3.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{\left(a+b\text{arctanh}\left(\frac{c}{x^2}\right)\right)^2}\,dx=\int \frac{(d\,x)^m}{\left(a+b\,\text{atanh}\left(\frac{c}{x^2}\right)\right)^2}\,dx$$

input
```
int((d*x)^m/(a + b*atanh(c/x^2))^2,x)
```

output
```
int((d*x)^m/(a + b*atanh(c/x^2))^2, x)
```

3.187 $\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx$

3.187.1 Optimal result

Integrand size = 16, antiderivative size = 88

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}-\frac{b\text{arctanh}\left(c\sqrt{x}\right)}{4c^8}+\frac{1}{4}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)$$

output
```
1/12*b*x^(3/2)/c^5+1/20*b*x^(5/2)/c^3+1/28*b*x^(7/2)/c-1/4*b*arctanh(c*x^(
1/2))/c^8+1/4*x^4*(a+b*arctanh(c*x^(1/2)))+1/4*b*x^(1/2)/c^7
```

3.187.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.30

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\frac{b\sqrt{x}}{4c^7}+\frac{bx^{3/2}}{12c^5}+\frac{bx^{5/2}}{20c^3}+\frac{bx^{7/2}}{28c}+\frac{ax^4}{4}+\frac{1}{4}bx^4\text{arctanh}\left(c\sqrt{x}\right)+\frac{b\log\left(1-c\sqrt{x}\right)}{8c^8}-\frac{b\log\left(1+c\sqrt{x}\right)}{8c^8}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

output
```
(b*Sqrt[x])/(4*c^7) + (b*x^(3/2))/(12*c^5) + (b*x^(5/2))/(20*c^3) + (b*x^(
7/2))/(28*c) + (a*x^4)/4 + (b*x^4*ArcTanh[c*Sqrt[x]])/4 + (b*Log[1 - c*Sqr
t[x]])/(8*c^8) - (b*Log[1 + c*Sqrt[x]])/(8*c^8)
```

3.187.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {6452, 60, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 (a+b\text{arctanh}(c\sqrt{x}))\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{8}bc\int \frac{x^{7/2}}{1-c^2x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{8}bc\left(\frac{\int \frac{x^{5/2}}{1-c^2x}dx}{c^2}-\frac{2x^{7/2}}{7c^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{8}bc\left(\frac{\frac{\int \frac{x^{3/2}}{1-c^2x}dx}{c^2}-\frac{2x^{5/2}}{5c^2}}{c^2}-\frac{2x^{7/2}}{7c^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{8}bc\left(\frac{\frac{\frac{\int \frac{\sqrt{x}}{1-c^2x}dx}{c^2}-\frac{2x^{3/2}}{3c^2}}{c^2}-\frac{2x^{5/2}}{5c^2}}{c^2}-\frac{2x^{7/2}}{7c^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{8}bc\left(\frac{\frac{\frac{\frac{\int \frac{1}{\sqrt{x}(1-c^2x)}dx}{c^2}-\frac{2\sqrt{x}}{c^2}}{c^2}-\frac{2x^{3/2}}{3c^2}}{c^2}-\frac{2x^{5/2}}{5c^2}}{c^2}-\frac{2x^{7/2}}{7c^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}x^4(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{8}bc\left(\frac{\frac{\frac{\frac{2\int\frac{1}{1-c^2x}d\sqrt{x}}{c^2}-\frac{2\sqrt{x}}{c^2}}{c^2}-\frac{2x^{3/2}}{3c^2}}{c^2}-\frac{2x^{5/2}}{5c^2}}{c^2}-\frac{2x^{7/2}}{7c^2}\right)$$

$\downarrow$ 219

$$\frac{1}{4}x^4(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{8}bc\left(\frac{\frac{\frac{\frac{2\text{arctanh}(c\sqrt{x})}{c^3}-\frac{2\sqrt{x}}{c^2}}{c^2}-\frac{2x^{3/2}}{3c^2}}{c^2}-\frac{2x^{5/2}}{5c^2}}{c^2}-\frac{2x^{7/2}}{7c^2}\right)$$

input
```
Int[x^3*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

output
```
(x^4*(a + b*ArcTanh[c*Sqrt[x]]))/4 - (b*c*((-2*x^(7/2))/(7*c^2) + ((-2*x^(
5/2))/(5*c^2) + ((-2*x^(3/2))/(3*c^2) + ((-2*Sqrt[x])/c^2 + (2*ArcTanh[c*S
qrt[x]])/c^3)/c^2)/c^2)/c^2))/8
```

3.187.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.187.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.90

method	result	size
parts	$\frac{a\,x^{4}}{4}+\frac{2b\left(\frac{c^{8}x^{4}\operatorname{arctanh}\left(c\sqrt{x}\right)}{8}+\frac{c^{7}x^{\frac{7}{2}}}{56}+\frac{c^{5}x^{\frac{5}{2}}}{40}+\frac{c^{3}x^{\frac{3}{2}}}{24}+\frac{c\sqrt{x}}{8}+\frac{\ln\left(c\sqrt{x}-1\right)}{16}-\frac{\ln\left(1+c\sqrt{x}\right)}{16}\right)}{c^{8}}$	79
derivativedivides	$\frac{\frac{a\,c^{8}x^{4}}{4}+2b\left(\frac{c^{8}x^{4}\operatorname{arctanh}\left(c\sqrt{x}\right)}{8}+\frac{c^{7}x^{\frac{7}{2}}}{56}+\frac{c^{5}x^{\frac{5}{2}}}{40}+\frac{c^{3}x^{\frac{3}{2}}}{24}+\frac{c\sqrt{x}}{8}+\frac{\ln\left(c\sqrt{x}-1\right)}{16}-\frac{\ln\left(1+c\sqrt{x}\right)}{16}\right)}{c^{8}}$	83
default	$\frac{\frac{a\,c^{8}x^{4}}{4}+2b\left(\frac{c^{8}x^{4}\operatorname{arctanh}\left(c\sqrt{x}\right)}{8}+\frac{c^{7}x^{\frac{7}{2}}}{56}+\frac{c^{5}x^{\frac{5}{2}}}{40}+\frac{c^{3}x^{\frac{3}{2}}}{24}+\frac{c\sqrt{x}}{8}+\frac{\ln\left(c\sqrt{x}-1\right)}{16}-\frac{\ln\left(1+c\sqrt{x}\right)}{16}\right)}{c^{8}}$	83

input
```
int(x^3*(a+b*arctanh(c*x^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+2*b/c^8*(1/8*c^8*x^4*arctanh(c*x^(1/2))+1/56*c^7*x^(7/2)+1/40*c^
5*x^(5/2)+1/24*c^3*x^(3/2)+1/8*c*x^(1/2)+1/16*ln(c*x^(1/2)-1)-1/16*ln(1+c*
x^(1/2)))
```

3.187.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.01

$$\int x^{3}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\,dx$$
$$=\frac{210\,ac^{8}x^{4}+105\left(bc^{8}x^{4}-b\right)\log\left(-\frac{c^{2}x+2\,c\sqrt{x}+1}{c^{2}x-1}\right)+2\left(15\,bc^{7}x^{3}+21\,bc^{5}x^{2}+35\,bc^{3}x+105\,bc\right)\sqrt{x}}{840\,c^{8}}$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

output
```
1/840*(210*a*c^8*x^4 + 105*(b*c^8*x^4 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/
(c^2*x - 1)) + 2*(15*b*c^7*x^3 + 21*b*c^5*x^2 + 35*b*c^3*x + 105*b*c)*sqrt
(x))/c^8
```

3.187.6 Sympy [F]

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\int x^3\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

input
```
integrate(x**3*(a+b*atanh(c*x**(1/2))),x)
```

output
```
Integral(x**3*(a + b*atanh(c*sqrt(x))), x)
```

3.187.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\frac{1}{4}ax^4$$
$$+\frac{1}{840}\left(210\,x^4\,\text{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(15\,c^6x^{\frac{7}{2}}+21\,c^4x^{\frac{5}{2}}+35\,c^2x^{\frac{3}{2}}+105\,\sqrt{x}\right)}{c^8}-\frac{105\,\log\left(c\sqrt{x}+1\right)}{c^9}+\frac{10}{}\right.\right.$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/840*(210*x^4*arctanh(c*sqrt(x)) + c*(2*(15*c^6*x^(7/2) + 21*
c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sqrt(x))/c^8 - 105*log(c*sqrt(x) + 1)/c
^9 + 105*log(c*sqrt(x) - 1)/c^9))*b
```

3.187.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(64) = 128.

Time = 0.29 (sec) , antiderivative size = 359, normalized size of antiderivative = 4.08

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\frac{1}{4}\,ax^4+\frac{2}{105}\,bc\left(\frac{\frac{105\,\left(c\sqrt{x}+1\right)^6}{\left(c\sqrt{x}-1\right)^6}-\frac{315\,\left(c\sqrt{x}+1\right)^5}{\left(c\sqrt{x}-1\right)^5}+\frac{770\,\left(c\sqrt{x}+1\right)^4}{\left(c\sqrt{x}-1\right)^4}-\frac{770\,\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{609\,\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}-\frac{203\,\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+44}{c^9\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^7}+\frac{105\,\big(}{}\right.$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")
```

output
```
1/4*a*x^4 + 2/105*b*c*((105*(c*sqrt(x) + 1)^6/(c*sqrt(x) - 1)^6 - 315*(c*s
qrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 770*(c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^4
 - 770*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 609*(c*sqrt(x) + 1)^2/(c*sqrt(
x) - 1)^2 - 203*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 44)/(c^9*((c*sqrt(x) + 1
)/(c*sqrt(x) - 1) - 1)^7) + 105*((c*sqrt(x) + 1)^7/(c*sqrt(x) - 1)^7 + 7*(
c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 7*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3
 + (c*sqrt(x) + 1)/(c*sqrt(x) - 1))*log(-(c*((c*sqrt(x) + 1)/(c*sqrt(x) -
1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) - c) + 1)/(c*((c*sqrt(x) + 1)/(
c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) - c) - 1))/(c^9*((c
*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^8))
```

3.187.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\frac{\frac{b\,c^3\,x^{3/2}}{12}-\frac{b\,\text{atanh}\left(c\,\sqrt{x}\right)}{4}+\frac{b\,c^5\,x^{5/2}}{20}+\frac{b\,c^7\,x^{7/2}}{28}+\frac{b\,c\,\sqrt{x}}{4}}{c^8}+\frac{b\,\left(105\,x^4\,\ln\left(c\,\sqrt{x}+1\right)-105\,x^4\,\ln\left(1-c\,\sqrt{x}\right)\right)}{840}+\frac{a\,x^4}{4}$$

input
```
int(x^3*(a + b*atanh(c*x^(1/2))),x)
```

output
```
((b*c^3*x^(3/2))/12 - (b*atanh(c*x^(1/2)))/4 + (b*c^5*x^(5/2))/20 + (b*c^7
*x^(7/2))/28 + (b*c*x^(1/2))/4)/c^8 + (b*(105*x^4*log(c*x^(1/2) + 1) - 105
*x^4*log(1 - c*x^(1/2))))/840 + (a*x^4)/4
```

3.187. $\int x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\,dx$

3.188 $\int x^2(a+b\text{arctanh}(c\sqrt{x}))\,dx$

3.188.1 Optimal result

Integrand size = 16, antiderivative size = 75

$$\int x^2(a+b\text{arctanh}(c\sqrt{x}))\,dx=\frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}-\frac{b\text{arctanh}(c\sqrt{x})}{3c^6}+\frac{1}{3}x^3(a+b\text{arctanh}(c\sqrt{x}))$$

output
```
1/9*b*x^(3/2)/c^3+1/15*b*x^(5/2)/c-1/3*b*arctanh(c*x^(1/2))/c^6+1/3*x^3*(a
+b*arctanh(c*x^(1/2)))+1/3*b*x^(1/2)/c^5
```

3.188.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.35

$$\int x^2(a+b\text{arctanh}(c\sqrt{x}))\,dx=\frac{b\sqrt{x}}{3c^5}+\frac{bx^{3/2}}{9c^3}+\frac{bx^{5/2}}{15c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\text{arctanh}(c\sqrt{x})+\frac{b\log(1-c\sqrt{x})}{6c^6}-\frac{b\log(1+c\sqrt{x})}{6c^6}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

output
```
(b*Sqrt[x])/(3*c^5) + (b*x^(3/2))/(9*c^3) + (b*x^(5/2))/(15*c) + (a*x^3)/3
 + (b*x^3*ArcTanh[c*Sqrt[x]])/3 + (b*Log[1 - c*Sqrt[x]])/(6*c^6) - (b*Log[
1 + c*Sqrt[x]])/(6*c^6)
```

3.188.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6452, 60, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right) \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\int \frac{x^{5/2}}{1-c^2x}dx$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\left(\frac{\int \frac{x^{3/2}}{1-c^2x}dx}{c^2}-\frac{2x^{5/2}}{5c^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\left(\frac{\frac{\int \frac{\sqrt{x}}{1-c^2x}dx}{c^2}-\frac{2x^{3/2}}{3c^2}}{c^2}-\frac{2x^{5/2}}{5c^2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\left(\frac{\frac{\frac{\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}dx}{c^2}-\frac{2\sqrt{x}}{c^2}}{c^2}-\frac{2x^{3/2}}{3c^2}}{c^2}-\frac{2x^{5/2}}{5c^2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\left(\frac{\frac{\frac{2\int \frac{1}{1-c^2x}d\sqrt{x}}{c^2}-\frac{2\sqrt{x}}{c^2}}{c^2}-\frac{2x^{3/2}}{3c^2}}{c^2}-\frac{2x^{5/2}}{5c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\left(\frac{\frac{\frac{2\text{arctanh}\left(c\sqrt{x}\right)}{c^3}-\frac{2\sqrt{x}}{c^2}}{c^2}-\frac{2x^{3/2}}{3c^2}}{c^2}-\frac{2x^{5/2}}{5c^2}\right)$$

input
```
Int[x^2*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

output
```
(x^3*(a + b*ArcTanh[c*Sqrt[x]]))/3 - (b*c*((-2*x^(5/2))/(5*c^2) + ((-2*x^(
3/2))/(3*c^2) + ((-2*Sqrt[x])/c^2 + (2*ArcTanh[c*Sqrt[x]])/c^3)/c^2)/c^2))
/6
```

3.188.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.188.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

method	result	size
parts	$\frac{a\,x^{3}}{3}+\frac{2b\left(\frac{c^{6}x^{3}\operatorname{arctanh}(c\sqrt{x})}{6}+\frac{c^{5}x^{\frac{5}{2}}}{30}+\frac{c^{3}x^{\frac{3}{2}}}{18}+\frac{c\sqrt{x}}{6}+\frac{\ln(c\sqrt{x}-1)}{12}-\frac{\ln(1+c\sqrt{x})}{12}\right)}{c^{6}}$	71
derivativedivides	$\frac{\frac{a\,c^{6}x^{3}}{3}+2b\left(\frac{c^{6}x^{3}\operatorname{arctanh}(c\sqrt{x})}{6}+\frac{c^{5}x^{\frac{5}{2}}}{30}+\frac{c^{3}x^{\frac{3}{2}}}{18}+\frac{c\sqrt{x}}{6}+\frac{\ln(c\sqrt{x}-1)}{12}-\frac{\ln(1+c\sqrt{x})}{12}\right)}{c^{6}}$	75
default	$\frac{\frac{a\,c^{6}x^{3}}{3}+2b\left(\frac{c^{6}x^{3}\operatorname{arctanh}(c\sqrt{x})}{6}+\frac{c^{5}x^{\frac{5}{2}}}{30}+\frac{c^{3}x^{\frac{3}{2}}}{18}+\frac{c\sqrt{x}}{6}+\frac{\ln(c\sqrt{x}-1)}{12}-\frac{\ln(1+c\sqrt{x})}{12}\right)}{c^{6}}$	75

input
```
int(x^2*(a+b*arctanh(c*x^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+2*b/c^6*(1/6*c^6*x^3*arctanh(c*x^(1/2))+1/30*c^5*x^(5/2)+1/18*c^
3*x^(3/2)+1/6*c*x^(1/2)+1/12*ln(c*x^(1/2)-1)-1/12*ln(1+c*x^(1/2)))
```

3.188.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int x^{2}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\,dx$$
$$=\frac{30\,ac^{6}x^{3}+15\left(bc^{6}x^{3}-b\right)\log\left(-\frac{c^{2}x+2\,c\sqrt{x}+1}{c^{2}x-1}\right)+2\left(3\,bc^{5}x^{2}+5\,bc^{3}x+15\,bc\right)\sqrt{x}}{90\,c^{6}}$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

output
```
1/90*(30*a*c^6*x^3 + 15*(b*c^6*x^3 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^
2*x - 1)) + 2*(3*b*c^5*x^2 + 5*b*c^3*x + 15*b*c)*sqrt(x))/c^6
```

3.188.6 Sympy [F]

$$\int x^2(a+b\text{arctanh}(c\sqrt{x}))\,dx=\int x^2(a+b\,\text{atanh}\,(c\sqrt{x}))\,dx$$

input
```
integrate(x**2*(a+b*atanh(c*x**(1/2))),x)
```

output
```
Integral(x**2*(a + b*atanh(c*sqrt(x))), x)
```

3.188.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int x^2(a+b\text{arctanh}(c\sqrt{x}))\,dx=\frac{1}{3}\,ax^3+\frac{1}{90}\left(30\,x^3\,\text{artanh}\,(c\sqrt{x})+c\left(\frac{2\left(3\,c^4x^{\frac{5}{2}}+5\,c^2x^{\frac{3}{2}}+15\,\sqrt{x}\right)}{c^6}-\frac{15\,\log\,(c\sqrt{x}+1)}{c^7}+\frac{15\,\log\,(c\sqrt{x}-1)}{c^7}\right)\right)$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/90*(30*x^3*arctanh(c*sqrt(x)) + c*(2*(3*c^4*x^(5/2) + 5*c^2*
x^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sqrt(x) + 1)/c^7 + 15*log(c*sqrt(x) -
 1)/c^7))*b
```

3.188.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(55) = 110.

Time = 0.29 (sec) , antiderivative size = 301, normalized size of antiderivative = 4.01

$$\int x^2(a+b\text{arctanh}(c\sqrt{x}))\,dx=\frac{1}{3}\,ax^3+\frac{2}{45}\,bc\left(\frac{\frac{45\,(c\sqrt{x}+1)^4}{(c\sqrt{x}-1)^4}-\frac{90\,(c\sqrt{x}+1)^3}{(c\sqrt{x}-1)^3}+\frac{140\,(c\sqrt{x}+1)^2}{(c\sqrt{x}-1)^2}-\frac{70\,(c\sqrt{x}+1)}{c\sqrt{x}-1}+23}{c^7\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^5}+\frac{15\left(\frac{3\,(c\sqrt{x}+1)^5}{(c\sqrt{x}-1)^5}+\frac{10\,(c\sqrt{x}+1)^3}{(c\sqrt{x}-1)^3}+\frac{3\,(c\sqrt{x}+1}{c\sqrt{x}-1}\right.}{c^7\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right.}\right.$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")
```

output
```
1/3*a*x^3 + 2/45*b*c*((45*(c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^4 - 90*(c*sqrt
(x) + 1)^3/(c*sqrt(x) - 1)^3 + 140*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 - 7
0*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 23)/(c^7*((c*sqrt(x) + 1)/(c*sqrt(x) -
 1) - 1)^5) + 15*(3*(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 10*(c*sqrt(x) +
1)^3/(c*sqrt(x) - 1)^3 + 3*(c*sqrt(x) + 1)/(c*sqrt(x) - 1))*log(-(c*((c*sq
rt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) - 1) - c) +
1)/(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x)
- 1) - c) - 1))/(c^7*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^6))
```

3.188.9 Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.77

$$\int x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\frac{a\,x^3}{3}+\frac{\frac{b\,c^3\,x^{3/2}}{9}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{3}+\frac{b\,c^5\,x^{5/2}}{15}+\frac{b\,c\,\sqrt{x}}{3}}{c^6}+\frac{b\,x^3\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{3}$$

input
```
int(x^2*(a + b*atanh(c*x^(1/2))),x)
```

output
```
(a*x^3)/3 + ((b*c^3*x^(3/2))/9 - (b*atanh(c*x^(1/2)))/3 + (b*c^5*x^(5/2))/
15 + (b*c*x^(1/2))/3)/c^6 + (b*x^3*atanh(c*x^(1/2)))/3
```

3.189 $\int x(a+b\text{arctanh}(c\sqrt{x}))\,dx$

3.189.1 Optimal result

Integrand size = 14, antiderivative size = 62

$$\int x(a+b\text{arctanh}(c\sqrt{x}))\,dx=\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}-\frac{b\text{arctanh}(c\sqrt{x})}{2c^4}+\frac{1}{2}x^2(a+b\text{arctanh}(c\sqrt{x}))$$

output
```
1/6*b*x^(3/2)/c-1/2*b*arctanh(c*x^(1/2))/c^4+1/2*x^2*(a+b*arctanh(c*x^(1/2
)))+1/2*b*x^(1/2)/c^3
```

3.189.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.42

$$\int x(a+b\text{arctanh}(c\sqrt{x}))\,dx=\frac{b\sqrt{x}}{2c^3}+\frac{bx^{3/2}}{6c}+\frac{ax^2}{2}+\frac{1}{2}bx^2\text{arctanh}(c\sqrt{x})+\frac{b\log(1-c\sqrt{x})}{4c^4}-\frac{b\log(1+c\sqrt{x})}{4c^4}$$

input
```
Integrate[x*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

output
```
(b*Sqrt[x])/(2*c^3) + (b*x^(3/2))/(6*c) + (a*x^2)/2 + (b*x^2*ArcTanh[c*Sqr
t[x]])/2 + (b*Log[1 - c*Sqrt[x]])/(4*c^4) - (b*Log[1 + c*Sqrt[x]])/(4*c^4)
```

3.189.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {6452, 60, 60, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right) \, dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{4}bc\int \frac{x^{3/2}}{1-c^2x}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{4}bc\left(\frac{\int \frac{\sqrt{x}}{1-c^2x}dx}{c^2}-\frac{2x^{3/2}}{3c^2}\right)$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{4}bc\left(\frac{\frac{\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}dx}{c^2}-\frac{2\sqrt{x}}{c^2}}{c^2}-\frac{2x^{3/2}}{3c^2}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{4}bc\left(\frac{\frac{2\int \frac{1}{1-c^2x}d\sqrt{x}}{c^2}-\frac{2\sqrt{x}}{c^2}}{c^2}-\frac{2x^{3/2}}{3c^2}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{4}bc\left(\frac{\frac{2\text{arctanh}\left(c\sqrt{x}\right)}{c^3}-\frac{2\sqrt{x}}{c^2}}{c^2}-\frac{2x^{3/2}}{3c^2}\right)$$

input
```
Int[x*(a + b*ArcTanh[c*Sqrt[x]]),x]
```

output
```
(x^2*(a + b*ArcTanh[c*Sqrt[x]]))/2 - (b*c*((-2*x^(3/2))/(3*c^2) + ((-2*Sqr
t[x])/c^2 + (2*ArcTanh[c*Sqrt[x]])/c^3)/c^2))/4
```

3.189.3.1 Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.189.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{2b\left(\frac{c^{4}x^{2}\operatorname{arctanh}\left(c\sqrt{x}\right)}{4}+\frac{c^{3}x^{\frac{3}{2}}}{12}+\frac{c\sqrt{x}}{4}+\frac{\ln\left(c\sqrt{x}-1\right)}{8}-\frac{\ln\left(1+c\sqrt{x}\right)}{8}\right)}{c^{4}}$	63
derivativedivides	$\frac{\frac{a\,c^{4}x^{2}}{2}+2b\left(\frac{c^{4}x^{2}\operatorname{arctanh}\left(c\sqrt{x}\right)}{4}+\frac{c^{3}x^{\frac{3}{2}}}{12}+\frac{c\sqrt{x}}{4}+\frac{\ln\left(c\sqrt{x}-1\right)}{8}-\frac{\ln\left(1+c\sqrt{x}\right)}{8}\right)}{c^{4}}$	67
default	$\frac{\frac{a\,c^{4}x^{2}}{2}+2b\left(\frac{c^{4}x^{2}\operatorname{arctanh}\left(c\sqrt{x}\right)}{4}+\frac{c^{3}x^{\frac{3}{2}}}{12}+\frac{c\sqrt{x}}{4}+\frac{\ln\left(c\sqrt{x}-1\right)}{8}-\frac{\ln\left(1+c\sqrt{x}\right)}{8}\right)}{c^{4}}$	67

input
```
int(x*(a+b*arctanh(c*x^(1/2))),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+2*b/c^4*(1/4*c^4*x^2*arctanh(c*x^(1/2))+1/12*c^3*x^(3/2)+1/4*c*x
^(1/2)+1/8*ln(c*x^(1/2)-1)-1/8*ln(1+c*x^(1/2)))
```

3.189.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.13

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx = \frac{6\,ac^4x^2+3\,(bc^4x^2-b)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)+2\,(bc^3x+3\,bc)\sqrt{x}}{12\,c^4}$$

input
```
integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="fricas")
```

output
```
1/12*(6*a*c^4*x^2 + 3*(b*c^4*x^2 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*
x - 1)) + 2*(b*c^3*x + 3*b*c)*sqrt(x))/c^4
```

3.189.6 Sympy [F]

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx = \int x\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

input
```
integrate(x*(a+b*atanh(c*x**(1/2))),x)
```

output
```
Integral(x*(a + b*atanh(c*sqrt(x))), x)
```

3.189.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\frac{1}{2}\,ax^2+\frac{1}{12}\left(6\,x^2\operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(c^2x^{\frac{3}{2}}+3\sqrt{x}\right)}{c^4}-\frac{3\,\log\left(c\sqrt{x}+1\right)}{c^5}+\frac{3\,\log\left(c\sqrt{x}-1\right)}{c^5}\right)\right)b$$

input
```
integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/12*(6*x^2*arctanh(c*sqrt(x)) + c*(2*(c^2*x^(3/2) + 3*sqrt(x)
)/c^4 - 3*log(c*sqrt(x) + 1)/c^5 + 3*log(c*sqrt(x) - 1)/c^5))*b
```

3.189.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(46) = 92.

Time = 0.28 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.85

$$\int x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\,dx$$

$$=\frac{1}{2}\,ax^2+\frac{2}{3}\,bc\left(\frac{\frac{3\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2}-\frac{3\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1}+2}{c^5\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^3}+\frac{3\left(\frac{\left(c\sqrt{x}+1\right)^3}{\left(c\sqrt{x}-1\right)^3}+\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)\log\left(-\frac{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}+1}{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{\left(c\sqrt{x}+1\right)c}{c\sqrt{x}-1}-c}-1}\right)}{c^5\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^4}\right)$$

input
```
integrate(x*(a+b*arctanh(c*x^(1/2))),x, algorithm="giac")
```

output
```
1/2*a*x^2 + 2/3*b*c*((3*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 - 3*(c*sqrt(x)
 + 1)/(c*sqrt(x) - 1) + 2)/(c^5*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^3) +
 3*((c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + (c*sqrt(x) + 1)/(c*sqrt(x) - 1))
*log(-(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(
x) - 1) - c) + 1)/(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1
)*c/(c*sqrt(x) - 1) - c) - 1))/(c^5*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)^
4))
```

3.189.9 Mupad [B] (verification not implemented)

Time = 3.59 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.79

$$\int x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\frac{\frac{b\,c^3\,x^{3/2}}{6}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{2}+\frac{b\,c\,\sqrt{x}}{2}}{c^4}+\frac{a\,x^2}{2}+\frac{b\,x^2\,\mathrm{atanh}\left(c\,\sqrt{x}\right)}{2}$$

input
```
int(x*(a + b*atanh(c*x^(1/2))),x)
```

output
```
((b*c^3*x^(3/2))/6 - (b*atanh(c*x^(1/2)))/2 + (b*c*x^(1/2))/2)/c^4 + (a*x^
2)/2 + (b*x^2*atanh(c*x^(1/2)))/2
```

3.190 $\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx$

3.190.1 Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\frac{b\sqrt{x}}{c}+ax-\frac{b\text{arctanh}\left(c\sqrt{x}\right)}{c^2}+bx\text{arctanh}\left(c\sqrt{x}\right)$$

output
```
a*x-b*arctanh(c*x^(1/2))/c^2+b*x*arctanh(c*x^(1/2))+b*x^(1/2)/c
```

3.190.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx=ax+bx\text{arctanh}\left(c\sqrt{x}\right)-bc\left(-\frac{\sqrt{x}}{c^2}+\frac{\text{arctanh}\left(c\sqrt{x}\right)}{c^3}\right)$$

input
```
Integrate[a + b*ArcTanh[c*Sqrt[x]],x]
```

output
```
a*x + b*x*ArcTanh[c*Sqrt[x]] - b*c*(-(Sqrt[x]/c^2) + ArcTanh[c*Sqrt[x]]/c^
3)
```

3.190.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right) \, dx$$

$$\Big\downarrow \text{ 2009}$$

$$ax-\frac{b\text{arctanh}\left(c\sqrt{x}\right)}{c^2}+bx\text{arctanh}\left(c\sqrt{x}\right)+\frac{b\sqrt{x}}{c}$$

input
```
Int[a + b*ArcTanh[c*Sqrt[x]],x]
```

output
```
(b*Sqrt[x])/c + a*x - (b*ArcTanh[c*Sqrt[x]])/c^2 + b*x*ArcTanh[c*Sqrt[x]]
```

3.190.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.190.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.28

method	result	size
default	$ax+\frac{2b\left(\frac{c^{2}x\,\operatorname{arctanh}\left(c\sqrt{x}\right)}{2}+\frac{c\sqrt{x}}{2}+\frac{\ln\left(c\sqrt{x}-1\right)}{4}-\frac{\ln\left(1+c\sqrt{x}\right)}{4}\right)}{c^{2}}$	50
parts	$ax+\frac{2b\left(\frac{c^{2}x\,\operatorname{arctanh}\left(c\sqrt{x}\right)}{2}+\frac{c\sqrt{x}}{2}+\frac{\ln\left(c\sqrt{x}-1\right)}{4}-\frac{\ln\left(1+c\sqrt{x}\right)}{4}\right)}{c^{2}}$	50
derivativedivides	$\frac{a\,c^{2}x+2b\left(\frac{c^{2}x\,\operatorname{arctanh}\left(c\sqrt{x}\right)}{2}+\frac{c\sqrt{x}}{2}+\frac{\ln\left(c\sqrt{x}-1\right)}{4}-\frac{\ln\left(1+c\sqrt{x}\right)}{4}\right)}{c^{2}}$	55

input
```
int(a+b*arctanh(c*x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
a*x+2*b/c^2*(1/2*c^2*x*arctanh(c*x^(1/2))+1/2*c*x^(1/2)+1/4*ln(c*x^(1/2)-1
)-1/4*ln(1+c*x^(1/2)))
```

3.190.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.44

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\frac{2\,ac^2x+2\,bc\sqrt{x}+\left(bc^2x-b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)}{2\,c^2}$$

input
```
integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(2*a*c^2*x + 2*b*c*sqrt(x) + (b*c^2*x - b)*log(-(c^2*x + 2*c*sqrt(x) +
 1)/(c^2*x - 1)))/c^2
```

3.190.6 Sympy [F]

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx=\int \left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)\,dx$$

input
```
integrate(a+b*atanh(c*x**(1/2)),x)
```

output
```
Integral(a + b*atanh(c*sqrt(x)), x)
```

3.190.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.36

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\,dx$$
$$=\frac{1}{2}\left(c\left(\frac{2\sqrt{x}}{c^2}-\frac{\log\left(c\sqrt{x}+1\right)}{c^3}+\frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right)+2\,x\,\text{artanh}\left(c\sqrt{x}\right)\right)b+ax$$

input
```
integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="maxima")
```

output
```
1/2*(c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) +
 2*x*arctanh(c*sqrt(x)))*b + a*x
```

3.190.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(33) = 66$.

Time = 0.32 (sec) , antiderivative size = 174, normalized size of antiderivative = 4.46

$$\int \left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\,dx = 2\,bc\left(\frac{1}{c^3\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)}+\frac{\left(c\sqrt{x}+1\right)\log\left(-\frac{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{(c\sqrt{x}+1)c}{c\sqrt{x}-1}-c}+1}{\frac{c\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}+1\right)}{\frac{(c\sqrt{x}+1)c}{c\sqrt{x}-1}-c}-1}\right)}{\left(c\sqrt{x}-1\right)c^3\left(\frac{c\sqrt{x}+1}{c\sqrt{x}-1}-1\right)^2}\right)+ax$$

input
```
integrate(a+b*arctanh(c*x^(1/2)),x, algorithm="giac")
```

output
```
2*b*c*(1/(c^3*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) - 1)) + (c*sqrt(x) + 1)*log
(-(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/(c*sqrt(x) -
 1) - c) + 1)/(c*((c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1)/((c*sqrt(x) + 1)*c/
(c*sqrt(x) - 1) - c) - 1))/((c*sqrt(x) - 1)*c^3*((c*sqrt(x) + 1)/(c*sqrt(x
) - 1) - 1)^2)) + a*x
```

3.190.9 Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\,dx = a\,x+b\,x\,\operatorname{atanh}\left(c\,\sqrt{x}\right)-\frac{b\,\left(\operatorname{atanh}\left(c\,\sqrt{x}\right)-c\,\sqrt{x}\right)}{c^2}$$

input
```
int(a + b*atanh(c*x^(1/2)),x)
```

output
```
a*x + b*x*atanh(c*x^(1/2)) - (b*(atanh(c*x^(1/2)) - c*x^(1/2)))/c^2
```

3.191 $\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x}\,dx$

3.191.1 Optimal result

Integrand size = 16, antiderivative size = 29

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x}\,dx = a\log(x) - b\,\text{PolyLog}\left(2,-c\sqrt{x}\right) + b\,\text{PolyLog}\left(2,c\sqrt{x}\right)$$

output
```
a*ln(x)-b*polylog(2,-c*x^(1/2))+b*polylog(2,c*x^(1/2))
```

3.191.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x}\,dx = a\log(x) - b\,\text{PolyLog}\left(2,-c\sqrt{x}\right) + b\,\text{PolyLog}\left(2,c\sqrt{x}\right)$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x,x]
```

output
```
a*Log[x] - b*PolyLog[2, -(c*Sqrt[x])] + b*PolyLog[2, c*Sqrt[x]]
```

3.191.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6450, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x}\,dx$$

$$\Big\downarrow \text{ 6450}$$

$$2\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{ 6446}$$

$$2\left(a\log\left(\sqrt{x}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,-c\sqrt{x}\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,c\sqrt{x}\right)\right)$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])/x,x]
```

output
```
2*(a*Log[Sqrt[x]] - (b*PolyLog[2, -(c*Sqrt[x])])/2 + (b*PolyLog[2, c*Sqrt[
x]])/2)
```

3.191.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.191.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.90 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

method	result
parts	$a\ln(x)+b\left(2\ln(c\sqrt{x})\operatorname{arctanh}(c\sqrt{x})-\operatorname{dilog}(1+c\sqrt{x})-\ln(c\sqrt{x})\ln(1+c\sqrt{x})-\text{d}\right.$
derivativedivides	$2a\ln(c\sqrt{x})+2b\left(\ln(c\sqrt{x})\operatorname{arctanh}(c\sqrt{x})-\frac{\operatorname{dilog}(1+c\sqrt{x})}{2}-\frac{\ln(c\sqrt{x})\ln(1+c\sqrt{x})}{2}-\frac{\operatorname{dilog}(c\sqrt{x})}{2}\right.$
default	$2a\ln(c\sqrt{x})+2b\left(\ln(c\sqrt{x})\operatorname{arctanh}(c\sqrt{x})-\frac{\operatorname{dilog}(1+c\sqrt{x})}{2}-\frac{\ln(c\sqrt{x})\ln(1+c\sqrt{x})}{2}-\frac{\operatorname{dilog}(c\sqrt{x})}{2}\right.$

input
```
int((a+b*arctanh(c*x^(1/2)))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(2*ln(c*x^(1/2))*arctanh(c*x^(1/2))-dilog(1+c*x^(1/2))-ln(c*x^(1
/2))*ln(1+c*x^(1/2))-dilog(c*x^(1/2)))
```

3.191.5 Fricas [F]

$$\int \frac{a+b\operatorname{arctanh}(c\sqrt{x})}{x}\,dx=\int \frac{b\operatorname{artanh}(c\sqrt{x})+a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*sqrt(x)) + a)/x, x)
```

3.191.6 Sympy [F]

$$\int \frac{a+b\operatorname{arctanh}(c\sqrt{x})}{x}\,dx=\int \frac{a+b\operatorname{atanh}(c\sqrt{x})}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x**(1/2)))/x,x)
```

output
```
Integral((a + b*atanh(c*sqrt(x)))/x, x)
```

3.191.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(23) = 46$.

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.10

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x}\,dx = -\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\text{Li}_2\left(-c\sqrt{x}+1\right)\right)b + \left(\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}\right)+\text{Li}_2\left(c\sqrt{x}+1\right)\right)b + a\log\left(x\right)$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="maxima")
```

output
```
-(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) + 1))*b + (log(c*s
qrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1))*b + a*log(x)
```

3.191.8 Giac [F]

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x}\,dx = \int \frac{b\,\text{artanh}\left(c\sqrt{x}\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)/x, x)
```

3.191.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x}\,dx = \int \frac{a+b\,\text{atanh}\left(c\,\sqrt{x}\right)}{x}\,dx$$

input
```
int((a + b*atanh(c*x^(1/2)))/x,x)
```

output
```
int((a + b*atanh(c*x^(1/2)))/x, x)
```

3.192 $\int \frac{a+b\operatorname{arctanh}(c\sqrt{x})}{x^2}\,dx$

3.192.1 Optimal result

Integrand size = 16, antiderivative size = 40

$$\int \frac{a+b\operatorname{arctanh}(c\sqrt{x})}{x^2}\,dx = -\frac{bc}{\sqrt{x}} + bc^2\operatorname{arctanh}(c\sqrt{x}) - \frac{a+b\operatorname{arctanh}(c\sqrt{x})}{x}$$

output
```
b*c^2*arctanh(c*x^(1/2))+(-a-b*arctanh(c*x^(1/2)))/x-b*c/x^(1/2)
```

3.192.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.68

$$\int \frac{a+b\operatorname{arctanh}(c\sqrt{x})}{x^2}\,dx = -\frac{a}{x} - \frac{bc}{\sqrt{x}} - \frac{b\operatorname{arctanh}(c\sqrt{x})}{x} - \frac{1}{2}bc^2\log\left(1-c\sqrt{x}\right) + \frac{1}{2}bc^2\log\left(1+c\sqrt{x}\right)$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^2,x]
```

output
```
-(a/x) - (b*c)/Sqrt[x] - (b*ArcTanh[c*Sqrt[x]])/x - (b*c^2*Log[1 - c*Sqrt[
x]])/2 + (b*c^2*Log[1 + c*Sqrt[x]])/2
```

3.192.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6452, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x^2}dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}bc\int \frac{1}{x^{3/2}\left(1-c^2x\right)}dx-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{2}bc\left(c^2\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}dx-\frac{2}{\sqrt{x}}\right)-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}bc\left(2c^2\int \frac{1}{1-c^2x}d\sqrt{x}-\frac{2}{\sqrt{x}}\right)-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}bc\left(2c\text{arctanh}\left(c\sqrt{x}\right)-\frac{2}{\sqrt{x}}\right)-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x}$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])/x^2,x]
```

output
```
-((a + b*ArcTanh[c*Sqrt[x]])/x) + (b*c*(-2/Sqrt[x] + 2*c*ArcTanh[c*Sqrt[x]
]))/2
```

3.192.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.192.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.42

method	result	size
parts	$-\frac{a}{x}+2b\,c^{2}\left(-\frac{\operatorname{arctanh}\left(c\sqrt{x}\right)}{2c^{2}x}-\frac{1}{2c\sqrt{x}}+\frac{\ln\left(1+c\sqrt{x}\right)}{4}-\frac{\ln\left(c\sqrt{x}-1\right)}{4}\right)$	57
derivativedivides	$2c^{2}\left(-\frac{a}{2c^{2}x}+b\left(-\frac{\operatorname{arctanh}\left(c\sqrt{x}\right)}{2c^{2}x}-\frac{1}{2c\sqrt{x}}+\frac{\ln\left(1+c\sqrt{x}\right)}{4}-\frac{\ln\left(c\sqrt{x}-1\right)}{4}\right)\right)$	61
default	$2c^{2}\left(-\frac{a}{2c^{2}x}+b\left(-\frac{\operatorname{arctanh}\left(c\sqrt{x}\right)}{2c^{2}x}-\frac{1}{2c\sqrt{x}}+\frac{\ln\left(1+c\sqrt{x}\right)}{4}-\frac{\ln\left(c\sqrt{x}-1\right)}{4}\right)\right)$	61

input
```
int((a+b*arctanh(c*x^(1/2)))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x+2*b*c^2*(-1/2/c^2/x*arctanh(c*x^(1/2))-1/2/c/x^(1/2)+1/4*ln(1+c*x^(1/
2))-1/4*ln(c*x^(1/2)-1))
```

3.192.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.32

$$\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x^2}\,dx = -\frac{2\,bc\sqrt{x}-\left(bc^2x-b\right)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)+2\,a}{2\,x}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*c*sqrt(x) - (b*c^2*x - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x
- 1)) + 2*a)/x
```

3.192.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(36) = 72.

Time = 2.54 (sec) , antiderivative size = 231, normalized size of antiderivative = 5.78

$$\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x^2}\,dx$$
$$= \begin{cases} -\frac{a}{x}+\frac{b\,\text{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for } c = \\ -\frac{a}{x}-\frac{b\,\text{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} & \text{for } c = \\ -\frac{ac^2x^{\frac{3}{2}}}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{a\sqrt{x}}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{bc^4x^{\frac{5}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{bc^3x^2}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}-\frac{2bc^2x^{\frac{3}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{bcx}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}}+\frac{b\sqrt{x}\,\text{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} & \text{otherw} \end{cases}$$

input
```
integrate((a+b*atanh(c*x**(1/2)))/x**2,x)
```

output
```
Piecewise((-a/x + b*atanh(sqrt(x)*sqrt(1/x))/x, Eq(c, -sqrt(1/x))), (-a/x
- b*atanh(sqrt(x)*sqrt(1/x))/x, Eq(c, sqrt(1/x))), (-a*c**2*x**(3/2)/(c**2
*x**(5/2) - x**(3/2)) + a*sqrt(x)/(c**2*x**(5/2) - x**(3/2)) + b*c**4*x**(
5/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) - b*c**3*x**2/(c**2*x**(5
/2) - x**(3/2)) - 2*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(
3/2)) + b*c*x/(c**2*x**(5/2) - x**(3/2)) + b*sqrt(x)*atanh(c*sqrt(x))/(c**
2*x**(5/2) - x**(3/2)), True))
```

3.192.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.28

$$\int \frac{a + b\text{arctanh}(c\sqrt{x})}{x^2}\, dx$$
$$= \frac{1}{2}\left(\left(c\log\left(c\sqrt{x}+1\right) - c\log\left(c\sqrt{x}-1\right) - \frac{2}{\sqrt{x}}\right)c - \frac{2\ \text{artanh}\left(c\sqrt{x}\right)}{x}\right)b - \frac{a}{x}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="maxima")
```

output
```
1/2*((c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c - 2*arcta
nh(c*sqrt(x))/x)*b - a/x
```

3.192.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(34) = 68.

Time = 0.29 (sec) , antiderivative size = 168, normalized size of antiderivative = 4.20

$$\int \frac{a + b\text{arctanh}(c\sqrt{x})}{x^2}\, dx$$
$$= 2\left(\frac{\left(c\sqrt{x}+1\right)bc\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{\left(c\sqrt{x}-1\right)\left(\frac{\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2} + \frac{2\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1} + 1\right)} + \frac{\frac{2\left(c\sqrt{x}+1\right)ac}{c\sqrt{x}-1} + \frac{\left(c\sqrt{x}+1\right)bc}{c\sqrt{x}-1} + bc}{\frac{\left(c\sqrt{x}+1\right)^2}{\left(c\sqrt{x}-1\right)^2} + \frac{2\left(c\sqrt{x}+1\right)}{c\sqrt{x}-1} + 1}\right)c$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x^2,x, algorithm="giac")
```

output
```
2*((c*sqrt(x) + 1)*b*c*log(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1))/((c*sqrt(x) -
 1)*((c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 2*(c*sqrt(x) + 1)/(c*sqrt(x) -
1) + 1)) + (2*(c*sqrt(x) + 1)*a*c/(c*sqrt(x) - 1) + (c*sqrt(x) + 1)*b*c/(c
*sqrt(x) - 1) + b*c)/((c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 2*(c*sqrt(x) +
 1)/(c*sqrt(x) - 1) + 1))*c
```

3.192.9 Mupad [B] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.30

$$\int \frac{a+b\operatorname{arctanh}\left(c\sqrt{x}\right)}{x^2}\,dx = b\,c\,\mathrm{atan}\left(\frac{c^2\,\sqrt{x}}{\sqrt{-c^2}}\right)\,\sqrt{-c^2}-\frac{a}{x}-\frac{b\,\mathrm{atanh}\left(c\,\sqrt{x}\right)+b\,c\,\sqrt{x}}{x}$$

input
```
int((a + b*atanh(c*x^(1/2)))/x^2,x)
```

output
```
b*c*atan((c^2*x^(1/2))/(-c^2)^(1/2))*(-c^2)^(1/2) - a/x - (b*atanh(c*x^(1/
2)) + b*c*x^(1/2))/x
```

3.193 $\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x^3} dx$

3.193.1 Optimal result

Integrand size = 16, antiderivative size = 60

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x^3} dx = -\frac{bc}{6x^{3/2}} - \frac{bc^3}{2\sqrt{x}} + \frac{1}{2}bc^4\text{arctanh}(c\sqrt{x}) - \frac{a+b\text{arctanh}(c\sqrt{x})}{2x^2}$$

output
```
-1/6*b*c/x^(3/2)+1/2*b*c^4*arctanh(c*x^(1/2))+1/2*(-a-b*arctanh(c*x^(1/2))
)/x^2-1/2*b*c^3/x^(1/2)
```

3.193.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x^3} dx = -\frac{a}{2x^2} - \frac{bc}{6x^{3/2}} - \frac{bc^3}{2\sqrt{x}} - \frac{b\text{arctanh}(c\sqrt{x})}{2x^2} - \frac{1}{4}bc^4\log(1-c\sqrt{x}) + \frac{1}{4}bc^4\log(1+c\sqrt{x})$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*c)/(6*x^(3/2)) - (b*c^3)/(2*Sqrt[x]) - (b*ArcTanh[c*Sqrt[x
]])/(2*x^2) - (b*c^4*Log[1 - c*Sqrt[x]])/4 + (b*c^4*Log[1 + c*Sqrt[x]])/4
```

3.193.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6452, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\operatorname{arctanh}(c\sqrt{x})}{x^3}dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{4}bc\int \frac{1}{x^{5/2}(1-c^2x)}dx-\frac{a+b\operatorname{arctanh}(c\sqrt{x})}{2x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}bc\left(c^2\int \frac{1}{x^{3/2}(1-c^2x)}dx-\frac{2}{3x^{3/2}}\right)-\frac{a+b\operatorname{arctanh}(c\sqrt{x})}{2x^2}$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{4}bc\left(c^2\left(c^2\int \frac{1}{\sqrt{x}(1-c^2x)}dx-\frac{2}{\sqrt{x}}\right)-\frac{2}{3x^{3/2}}\right)-\frac{a+b\operatorname{arctanh}(c\sqrt{x})}{2x^2}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{4}bc\left(c^2\left(2c^2\int \frac{1}{1-c^2x}d\sqrt{x}-\frac{2}{\sqrt{x}}\right)-\frac{2}{3x^{3/2}}\right)-\frac{a+b\operatorname{arctanh}(c\sqrt{x})}{2x^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}bc\left(c^2\left(2c\operatorname{arctanh}(c\sqrt{x})-\frac{2}{\sqrt{x}}\right)-\frac{2}{3x^{3/2}}\right)-\frac{a+b\operatorname{arctanh}(c\sqrt{x})}{2x^2}$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c*Sqrt[x]])/x^2 + (b*c*(-2/(3*x^(3/2)) + c^2*(-2/Sqrt[
x] + 2*c*ArcTanh[c*Sqrt[x]])))/4
```

3.193.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.193.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

method	result	size
parts	$-\frac{a}{2x^2}+2b\,c^4\left(-\frac{\operatorname{arctanh}(c\sqrt{x})}{4c^4x^2}+\frac{\ln(1+c\sqrt{x})}{8}-\frac{1}{12c^3x^{\frac{3}{2}}}-\frac{1}{4c\sqrt{x}}-\frac{\ln(c\sqrt{x}-1)}{8}\right)$	65
derivativedivides	$2c^4\left(-\frac{a}{4c^4x^2}+b\left(-\frac{\operatorname{arctanh}(c\sqrt{x})}{4c^4x^2}+\frac{\ln(1+c\sqrt{x})}{8}-\frac{1}{12c^3x^{\frac{3}{2}}}-\frac{1}{4c\sqrt{x}}-\frac{\ln(c\sqrt{x}-1)}{8}\right)\right)$	69
default	$2c^4\left(-\frac{a}{4c^4x^2}+b\left(-\frac{\operatorname{arctanh}(c\sqrt{x})}{4c^4x^2}+\frac{\ln(1+c\sqrt{x})}{8}-\frac{1}{12c^3x^{\frac{3}{2}}}-\frac{1}{4c\sqrt{x}}-\frac{\ln(c\sqrt{x}-1)}{8}\right)\right)$	69

input
```
int((a+b*arctanh(c*x^(1/2)))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2+2*b*c^4*(-1/4/c^4/x^2*arctanh(c*x^(1/2))+1/8*ln(1+c*x^(1/2))-1/
12/c^3/x^(3/2)-1/4/c/x^(1/2)-1/8*ln(c*x^(1/2)-1))
```

3.193.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

$$\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x^3}\,dx = \frac{3\,(bc^4x^2-b)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)-2\,(3\,bc^3x+bc)\sqrt{x}-6\,a}{12\,x^2}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="fricas")
```

output
```
1/12*(3*(b*c^4*x^2 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) - 2*(3
*b*c^3*x + b*c)*sqrt(x) - 6*a)/x^2
```

3.193.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(56) = 112$.

Time = 8.15 (sec) , antiderivative size = 342, normalized size of antiderivative = 5.70

$$\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x^3}\,dx$$
$$= \begin{cases} -\frac{a}{2x^2}+\frac{b\,\text{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^2} \\ -\frac{a}{2x^2}-\frac{b\,\text{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{2x^2} \\ -\frac{3ac^2x^{\frac{3}{2}}}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3a\sqrt{x}}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{3bc^6x^{\frac{7}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^5x^3}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^4x^{\frac{5}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}+\frac{2bc^3x^2}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}}-\frac{3bc^2x^{\frac{3}{2}}\,\text{atanh}\left(c\sqrt{x}\right.}{6c^2x^{\frac{7}{2}}-6x^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((a+b*atanh(c*x**(1/2)))/x**3,x)
```

output
```
Piecewise((-a/(2*x**2) + b*atanh(sqrt(x)*sqrt(1/x))/(2*x**2), Eq(c, -sqrt(
1/x))), (-a/(2*x**2) - b*atanh(sqrt(x)*sqrt(1/x))/(2*x**2), Eq(c, sqrt(1/x
))), (-3*a*c**2*x**(3/2)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*a*sqrt(x)/(6*c
**2*x**(7/2) - 6*x**(5/2)) + 3*b*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x*
*(7/2) - 6*x**(5/2)) - 3*b*c**5*x**3/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b*
c**4*x**(5/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 2*b*c**3*x
**2/(6*c**2*x**(7/2) - 6*x**(5/2)) - 3*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(6
*c**2*x**(7/2) - 6*x**(5/2)) + b*c*x/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*b*
sqrt(x)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)), True))
```

3.193.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

$$\int \frac{a + b\text{arctanh}\left(c\sqrt{x}\right)}{x^3}\,dx = \frac{1}{12}\left(\left(3\,c^3\log\left(c\sqrt{x}+1\right) - 3\,c^3\log\left(c\sqrt{x}-1\right) - \frac{2\,(3\,c^2x+1)}{x^{\frac{3}{2}}}\right)c - \frac{6\ \text{artanh}\left(c\sqrt{x}\right)}{x^2}\right)b - \frac{a}{2\,x^2}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="maxima")
```

output
```
1/12*((3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^2*x +
1)/x^(3/2))*c - 6*arctanh(c*sqrt(x))/x^2)*b - 1/2*a/x^2
```

3.193.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. 2(44) = 88.

Time = 0.30 (sec) , antiderivative size = 356, normalized size of antiderivative = 5.93

$$\int \frac{a + b\text{arctanh}\left(c\sqrt{x}\right)}{x^3}\,dx = \frac{2}{3}c\left(\frac{3\left(\frac{(c\sqrt{x}+1)^3bc^3}{(c\sqrt{x}-1)^3} + \frac{(c\sqrt{x}+1)bc^3}{c\sqrt{x}-1}\right)\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{\frac{(c\sqrt{x}+1)^4}{(c\sqrt{x}-1)^4} + \frac{4(c\sqrt{x}+1)^3}{(c\sqrt{x}-1)^3} + \frac{6(c\sqrt{x}+1)^2}{(c\sqrt{x}-1)^2} + \frac{4(c\sqrt{x}+1)}{c\sqrt{x}-1} + 1} + \frac{\frac{6(c\sqrt{x}+1)^3ac^3}{(c\sqrt{x}-1)^3} + \frac{6(c\sqrt{x}+1)ac^3}{c\sqrt{x}-1} + \frac{3(c\sqrt{x}+1)^3bc^3}{(c\sqrt{x}-1)^3} + \frac{6(c\sqrt{\cdots}}{\cdots}}{\frac{(c\sqrt{x}+1)^4}{(c\sqrt{x}-1)^4} + \frac{4(c\sqrt{x}+1)^3}{(c\sqrt{x}-1)^3} + \frac{6(c\sqrt{x}+1)^2}{(c\sqrt{x}-1)^2} + \cdots}\right.$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x^3,x, algorithm="giac")
```

output
```
2/3*c*(3*((c*sqrt(x) + 1)^3*b*c^3/(c*sqrt(x) - 1)^3 + (c*sqrt(x) + 1)*b*c^
3/(c*sqrt(x) - 1))*log(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1))/((c*sqrt(x) + 1)^
4/(c*sqrt(x) - 1)^4 + 4*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 6*(c*sqrt(x)
 + 1)^2/(c*sqrt(x) - 1)^2 + 4*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1) + (6*(c
*sqrt(x) + 1)^3*a*c^3/(c*sqrt(x) - 1)^3 + 6*(c*sqrt(x) + 1)*a*c^3/(c*sqrt(
x) - 1) + 3*(c*sqrt(x) + 1)^3*b*c^3/(c*sqrt(x) - 1)^3 + 6*(c*sqrt(x) + 1)^
2*b*c^3/(c*sqrt(x) - 1)^2 + 5*(c*sqrt(x) + 1)*b*c^3/(c*sqrt(x) - 1) + 2*b*
c^3)/((c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^4 + 4*(c*sqrt(x) + 1)^3/(c*sqrt(x)
 - 1)^3 + 6*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 4*(c*sqrt(x) + 1)/(c*sqr
t(x) - 1) + 1))
```

3.193.9 Mupad [B] (verification not implemented)

Time = 3.78 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x^3}\,dx = \frac{b\,c^4\,\text{atanh}\left(c\,\sqrt{x}\right)}{2} - \frac{b\,\left(3\,\ln\left(c\,\sqrt{x}+1\right)-3\,\ln\left(1-c\,\sqrt{x}\right)+2\,c\,\sqrt{x}+6\,c^3\,x^{3/2}\right)}{12\,x^2} - \frac{a}{2\,x^2}$$

input
```
int((a + b*atanh(c*x^(1/2)))/x^3,x)
```

output
```
(b*c^4*atanh(c*x^(1/2)))/2 - (b*(3*log(c*x^(1/2) + 1) - 3*log(1 - c*x^(1/2
)) + 2*c*x^(1/2) + 6*c^3*x^(3/2)))/(12*x^2) - a/(2*x^2)
```

3.194 $\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x^4}\,dx$

3.194.1 Optimal result

Integrand size = 16, antiderivative size = 73

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x^4}\,dx = -\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}+\frac{1}{3}bc^6\text{arctanh}(c\sqrt{x})-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^3}$$

output
```
-1/15*b*c/x^(5/2)-1/9*b*c^3/x^(3/2)+1/3*b*c^6*arctanh(c*x^(1/2))+1/3*(-a-b
*arctanh(c*x^(1/2)))/x^3-1/3*b*c^5/x^(1/2)
```

3.194.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x^4}\,dx = -\frac{a}{3x^3}-\frac{bc}{15x^{5/2}}-\frac{bc^3}{9x^{3/2}}-\frac{bc^5}{3\sqrt{x}}-\frac{b\text{arctanh}(c\sqrt{x})}{3x^3}-\frac{1}{6}bc^6\log\left(1-c\sqrt{x}\right)+\frac{1}{6}bc^6\log\left(1+c\sqrt{x}\right)$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*c)/(15*x^(5/2)) - (b*c^3)/(9*x^(3/2)) - (b*c^5)/(3*Sqrt[x]
) - (b*ArcTanh[c*Sqrt[x]])/(3*x^3) - (b*c^6*Log[1 - c*Sqrt[x]])/6 + (b*c^6
*Log[1 + c*Sqrt[x]])/6
```

3.194.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {6452, 61, 61, 61, 73, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x^4} \, dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{6}bc\int \frac{1}{x^{7/2}(1-c^2x)}dx-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^3}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{6}bc\left(c^2\int \frac{1}{x^{5/2}(1-c^2x)}dx-\frac{2}{5x^{5/2}}\right)-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^3}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{6}bc\left(c^2\left(c^2\int \frac{1}{x^{3/2}(1-c^2x)}dx-\frac{2}{3x^{3/2}}\right)-\frac{2}{5x^{5/2}}\right)-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^3}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{1}{6}bc\left(c^2\left(c^2\left(c^2\int \frac{1}{\sqrt{x}(1-c^2x)}dx-\frac{2}{\sqrt{x}}\right)-\frac{2}{3x^{3/2}}\right)-\frac{2}{5x^{5/2}}\right)-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^3}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{6}bc\left(c^2\left(c^2\left(2c^2\int \frac{1}{1-c^2x}d\sqrt{x}-\frac{2}{\sqrt{x}}\right)-\frac{2}{3x^{3/2}}\right)-\frac{2}{5x^{5/2}}\right)-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^3}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{6}bc\left(c^2\left(c^2\left(2c\text{arctanh}(c\sqrt{x})-\frac{2}{\sqrt{x}}\right)-\frac{2}{3x^{3/2}}\right)-\frac{2}{5x^{5/2}}\right)-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^3}$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])/x^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c*Sqrt[x]])/x^3 + (b*c*(-2/(5*x^(5/2)) + c^2*(-2/(3*x^
(3/2)) + c^2*(-2/Sqrt[x] + 2*c*ArcTanh[c*Sqrt[x]]))))/6
```

3.194.3.1 Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.194.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

method	result	size
parts	$-\frac{a}{3x^{3}}+2 b \,c^{6} \left(-\frac{\operatorname{arctanh}\left(c \sqrt{x}\right)}{6 c^{6} x^{3}}-\frac{1}{30 c^{5} x^{\frac{5}{2}}}-\frac{1}{18 c^{3} x^{\frac{3}{2}}}-\frac{1}{6 c \sqrt{x}}+\frac{\ln\left(1+c \sqrt{x}\right)}{12}-\frac{\ln\left(c \sqrt{x}-1\right)}{12}\right)$	73
derivativedivides	$2 c^{6} \left(-\frac{a}{6 c^{6} x^{3}}+b \left(-\frac{\operatorname{arctanh}\left(c \sqrt{x}\right)}{6 c^{6} x^{3}}-\frac{1}{30 c^{5} x^{\frac{5}{2}}}-\frac{1}{18 c^{3} x^{\frac{3}{2}}}-\frac{1}{6 c \sqrt{x}}+\frac{\ln\left(1+c \sqrt{x}\right)}{12}-\frac{\ln\left(c \sqrt{x}-1\right)}{12}\right)\right)$	77
default	$2 c^{6} \left(-\frac{a}{6 c^{6} x^{3}}+b \left(-\frac{\operatorname{arctanh}\left(c \sqrt{x}\right)}{6 c^{6} x^{3}}-\frac{1}{30 c^{5} x^{\frac{5}{2}}}-\frac{1}{18 c^{3} x^{\frac{3}{2}}}-\frac{1}{6 c \sqrt{x}}+\frac{\ln\left(1+c \sqrt{x}\right)}{12}-\frac{\ln\left(c \sqrt{x}-1\right)}{12}\right)\right)$	77

input
```
int((a+b*arctanh(c*x^(1/2)))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3+2*b*c^6*(-1/6/c^6/x^3*arctanh(c*x^(1/2))-1/30/c^5/x^(5/2)-1/18/
c^3/x^(3/2)-1/6/c/x^(1/2)+1/12*ln(1+c*x^(1/2))-1/12*ln(c*x^(1/2)-1))
```

3.194.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01

$$\int \frac{a + b\text{arctanh}(c\sqrt{x})}{x^4}\,dx = \frac{15\,(bc^6x^3 - b)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right) - 2\,(15\,bc^5x^2 + 5\,bc^3x + 3\,bc)\sqrt{x} - 30\,a}{90\,x^3}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="fricas")
```

output
```
1/90*(15*(b*c^6*x^3 - b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) - 2*(
15*b*c^5*x^2 + 5*b*c^3*x + 3*b*c)*sqrt(x) - 30*a)/x^3
```

3.194.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(68) = 136$.

Time = 19.81 (sec) , antiderivative size = 371, normalized size of antiderivative = 5.08

$$\int \frac{a + b\text{arctanh}(c\sqrt{x})}{x^4}\,dx = \begin{cases} -\frac{a}{3x^3} + \frac{b\,\text{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{3x^3} \\ -\frac{a}{3x^3} - \frac{b\,\text{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{3x^3} \\ -\frac{15ac^2x^{\frac{3}{2}}}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}} + \frac{15a\sqrt{x}}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}} + \frac{15bc^8x^{\frac{9}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}} - \frac{15bc^7x^4}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}} - \frac{15bc^6x^{\frac{7}{2}}\,\text{atanh}\left(c\sqrt{x}\right)}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}} + \frac{10bc^5x^3}{45c^2x^{\frac{9}{2}}-45x^{\frac{7}{2}}} + \frac{2bc}{45c^2x^{\frac{9}{2}}} \end{cases}$$

input
```
integrate((a+b*atanh(c*x**(1/2)))/x**4,x)
```

output
```
Piecewise((-a/(3*x**3) + b*atanh(sqrt(x)*sqrt(1/x))/(3*x**3), Eq(c, -sqrt(
1/x))), (-a/(3*x**3) - b*atanh(sqrt(x)*sqrt(1/x))/(3*x**3), Eq(c, sqrt(1/x
))), (-15*a*c**2*x**(3/2)/(45*c**2*x**(9/2) - 45*x**(7/2)) + 15*a*sqrt(x)/
(45*c**2*x**(9/2) - 45*x**(7/2)) + 15*b*c**8*x**(9/2)*atanh(c*sqrt(x))/(45
*c**2*x**(9/2) - 45*x**(7/2)) - 15*b*c**7*x**4/(45*c**2*x**(9/2) - 45*x**(
7/2)) - 15*b*c**6*x**(7/2)*atanh(c*sqrt(x))/(45*c**2*x**(9/2) - 45*x**(7/2
)) + 10*b*c**5*x**3/(45*c**2*x**(9/2) - 45*x**(7/2)) + 2*b*c**3*x**2/(45*c
**2*x**(9/2) - 45*x**(7/2)) - 15*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(45*c**2
*x**(9/2) - 45*x**(7/2)) + 3*b*c*x/(45*c**2*x**(9/2) - 45*x**(7/2)) + 15*b
*sqrt(x)*atanh(c*sqrt(x))/(45*c**2*x**(9/2) - 45*x**(7/2)), True))
```

3.194.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.99

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x^4}\,dx = \frac{1}{90}\left(\left(15\,c^5\log\left(c\sqrt{x}+1\right)-15\,c^5\log\left(c\sqrt{x}-1\right)-\frac{2\,(15\,c^4x^2+5\,c^2x+3)}{x^{\frac{5}{2}}}\right)c-\frac{30\ \text{artanh}\left(c\sqrt{x}\right)}{x^3}\right)b - \frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="maxima")
```

output
```
1/90*((15*c^5*log(c*sqrt(x) + 1) - 15*c^5*log(c*sqrt(x) - 1) - 2*(15*c^4*x
^2 + 5*c^2*x + 3)/x^(5/2))*c - 30*arctanh(c*sqrt(x))/x^3)*b - 1/3*a/x^3
```

3.194.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(53) = 106.

Time = 0.30 (sec) , antiderivative size = 534, normalized size of antiderivative = 7.32

$$\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x^4}\,dx = \frac{2}{45}\,c\left(\frac{15\left(\frac{3\,(c\sqrt{x}+1)^5bc^5}{(c\sqrt{x}-1)^5}+\frac{10\,(c\sqrt{x}+1)^3bc^5}{(c\sqrt{x}-1)^3}+\frac{3\,(c\sqrt{x}+1)bc^5}{c\sqrt{x}-1}\right)\log\left(-\frac{c\sqrt{x}+1}{c\sqrt{x}-1}\right)}{\frac{(c\sqrt{x}+1)^6}{(c\sqrt{x}-1)^6}+\frac{6\,(c\sqrt{x}+1)^5}{(c\sqrt{x}-1)^5}+\frac{15\,(c\sqrt{x}+1)^4}{(c\sqrt{x}-1)^4}+\frac{20\,(c\sqrt{x}+1)^3}{(c\sqrt{x}-1)^3}+\frac{15\,(c\sqrt{x}+1)^2}{(c\sqrt{x}-1)^2}+\frac{6\,(c\sqrt{x}+1)}{c\sqrt{x}-1}+1}+\frac{\frac{90\,(c\sqrt{x}+1)^5ac^5}{(c\sqrt{x}-1)^5}+\frac{300\,(c}{(c}\right.$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))/x^4,x, algorithm="giac")
```

output
```
2/45*c*(15*(3*(c*sqrt(x) + 1)^5*b*c^5/(c*sqrt(x) - 1)^5 + 10*(c*sqrt(x) +
1)^3*b*c^5/(c*sqrt(x) - 1)^3 + 3*(c*sqrt(x) + 1)*b*c^5/(c*sqrt(x) - 1))*lo
g(-(c*sqrt(x) + 1)/(c*sqrt(x) - 1))/((c*sqrt(x) + 1)^6/(c*sqrt(x) - 1)^6 +
 6*(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 + 15*(c*sqrt(x) + 1)^4/(c*sqrt(x) -
 1)^4 + 20*(c*sqrt(x) + 1)^3/(c*sqrt(x) - 1)^3 + 15*(c*sqrt(x) + 1)^2/(c*s
qrt(x) - 1)^2 + 6*(c*sqrt(x) + 1)/(c*sqrt(x) - 1) + 1) + (90*(c*sqrt(x) +
1)^5*a*c^5/(c*sqrt(x) - 1)^5 + 300*(c*sqrt(x) + 1)^3*a*c^5/(c*sqrt(x) - 1)
^3 + 90*(c*sqrt(x) + 1)*a*c^5/(c*sqrt(x) - 1) + 45*(c*sqrt(x) + 1)^5*b*c^5
/(c*sqrt(x) - 1)^5 + 135*(c*sqrt(x) + 1)^4*b*c^5/(c*sqrt(x) - 1)^4 + 230*(
c*sqrt(x) + 1)^3*b*c^5/(c*sqrt(x) - 1)^3 + 210*(c*sqrt(x) + 1)^2*b*c^5/(c*
sqrt(x) - 1)^2 + 93*(c*sqrt(x) + 1)*b*c^5/(c*sqrt(x) - 1) + 23*b*c^5)/((c*
sqrt(x) + 1)^6/(c*sqrt(x) - 1)^6 + 6*(c*sqrt(x) + 1)^5/(c*sqrt(x) - 1)^5 +
 15*(c*sqrt(x) + 1)^4/(c*sqrt(x) - 1)^4 + 20*(c*sqrt(x) + 1)^3/(c*sqrt(x)
- 1)^3 + 15*(c*sqrt(x) + 1)^2/(c*sqrt(x) - 1)^2 + 6*(c*sqrt(x) + 1)/(c*sqr
t(x) - 1) + 1))
```

3.194.9 Mupad [B] (verification not implemented)

Time = 3.80 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{a + b\text{arctanh}\left(c\sqrt{x}\right)}{x^4}\,dx$$
$$= \frac{b\,c^6\,\text{atanh}\left(c\,\sqrt{x}\right)}{3} - \frac{b\left(15\,\ln\left(c\,\sqrt{x}+1\right) - 15\,\ln\left(1-c\,\sqrt{x}\right) + 6\,c\,\sqrt{x} + 10\,c^3\,x^{3/2} + 30\,c^5\,x^{5/2}\right)}{90\,x^3} - \frac{a}{3\,x^3}$$

input
```
int((a + b*atanh(c*x^(1/2)))/x^4,x)
```

output
```
(b*c^6*atanh(c*x^(1/2)))/3 - (b*(15*log(c*x^(1/2) + 1) - 15*log(1 - c*x^(1
/2)) + 6*c*x^(1/2) + 10*c^3*x^(3/2) + 30*c^5*x^(5/2)))/(90*x^3) - a/(3*x^3
)
```

3.195 $\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$

3.195.1 Optimal result

Integrand size = 18, antiderivative size = 211

$$\begin{aligned}\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx &= \frac{ab\sqrt{x}}{2c^7}+\frac{71b^2x}{420c^6}+\frac{3b^2x^2}{70c^4}+\frac{b^2x^3}{84c^2}\\ &\quad+\frac{b^2\sqrt{x}\text{arctanh}\left(c\sqrt{x}\right)}{2c^7}+\frac{bx^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{6c^5}\\ &\quad+\frac{bx^{5/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{10c^3}\\ &\quad+\frac{bx^{7/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{14c}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{4c^8}\\ &\quad+\frac{1}{4}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2+\frac{44b^2\log\left(1-c^2x\right)}{105c^8}\end{aligned}$$

output
```
71/420*b^2*x/c^6+3/70*b^2*x^2/c^4+1/84*b^2*x^3/c^2+1/6*b*x^(3/2)*(a+b*arct
anh(c*x^(1/2)))/c^5+1/10*b*x^(5/2)*(a+b*arctanh(c*x^(1/2)))/c^3+1/14*b*x^(
7/2)*(a+b*arctanh(c*x^(1/2)))/c-1/4*(a+b*arctanh(c*x^(1/2)))^2/c^8+1/4*x^4
*(a+b*arctanh(c*x^(1/2)))^2+44/105*b^2*ln(-c^2*x+1)/c^8+1/2*a*b*x^(1/2)/c^
7+1/2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c^7
```

3.195.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.06

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$$
$$=\frac{210abc\sqrt{x}+71b^2c^2x+70abc^3x^{3/2}+18b^2c^4x^2+42abc^5x^{5/2}+5b^2c^6x^3+30abc^7x^{7/2}+105a^2c^8x^4+2bc\sqrt{x}}{}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

output
```
(210*a*b*c*Sqrt[x] + 71*b^2*c^2*x + 70*a*b*c^3*x^(3/2) + 18*b^2*c^4*x^2 +
42*a*b*c^5*x^(5/2) + 5*b^2*c^6*x^3 + 30*a*b*c^7*x^(7/2) + 105*a^2*c^8*x^4
+ 2*b*c*Sqrt[x]*(105*a*c^7*x^(7/2) + b*(105 + 35*c^2*x + 21*c^4*x^2 + 15*c
^6*x^3))*ArcTanh[c*Sqrt[x]] + 105*b^2*(-1 + c^8*x^4)*ArcTanh[c*Sqrt[x]]^2
+ b*(105*a + 176*b)*Log[1 - c*Sqrt[x]] - 105*a*b*Log[1 + c*Sqrt[x]] + 176*
b^2*Log[1 + c*Sqrt[x]])/(420*c^8)
```

3.195.3 Rubi [A] (warning: unable to verify)

Time = 2.04 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.45, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.111, Rules used = {6454, 6452, 6542, 6452, 243, 49, 2009, 6542, 6452, 243, 49, 2009, 6542, 6452, 243, 49, 2009, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

$$\Big\downarrow \text{6454}$$

$$2\int x^{7/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\int\frac{x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{6542}$$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\int\frac{x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)d\sqrt{x}}{c^2}\right)\right)$$

$\Big\downarrow$ 6452

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\int\frac{x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{7}x^{7/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{7}bc\int\frac{x^{7/2}}{1-c^2x}d\sqrt{\right.}}{c^2}\right.\right.$$

$\Big\downarrow$ 243

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\int\frac{x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{7}x^{7/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{14}bc\int\frac{x^{3/2}}{1-c^2x}dx}{c^2}\right.\right.$$

$\Big\downarrow$ 49

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\int\frac{x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{7}x^{7/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{14}bc\int\left(-\frac{x}{c^2}-\right.}{c^2}\right.\right.$$

$\Big\downarrow$ 2009

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\int\frac{x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{7}x^{7/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{14}bc\left(-\frac{x}{c^6}-\frac{}{2c}\right.}{c^2}\right.\right.$$

$\Big\downarrow$ 6542

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\frac{\int\frac{x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{7}x^{7/2}\left(a+b\text{arctan}\right.}{}\right.\right.$$

$\Big\downarrow$ 6452

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\frac{\int\frac{x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{5}bc\int\frac{x^{5/2}}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{7}x^{7/2}}{}\right.\right.$$

3.195. $\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$

$\downarrow$ 243

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-\frac{1}{4}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(c\sqrt{x}))}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{10}bc\int\frac{x}{1-c^2x}dx}{c^2}}{c^2}-\frac{\frac{1}{7}x^{7/2}(\ldots}{\ldots}\right.\right.$$

$\downarrow$ 49

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-\frac{1}{4}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(c\sqrt{x}))}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{10}bc\int\left(-\frac{x}{c^2}-\frac{1}{c^4(c^2x-1)}-\ldots\right.}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 2009

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-\frac{1}{4}bc\left(\frac{\frac{\int\frac{x^2(a+b\text{arctanh}(c\sqrt{x}))}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{10}bc\left(-\frac{x}{c^4}-\frac{x}{2c^2}-\frac{\log(1-c^2x\ldots}{c^6}\right.}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 6542

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-\frac{1}{4}bc\left(\frac{\frac{\frac{\int\frac{x(a+b\text{arctanh}(c\sqrt{x}))}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x(a+b\text{arctanh}(c\sqrt{x}))d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{5}x^{5/2}(a+b\text{arctanh}(c\sqrt{x}))-\ldots}{\ldots}}{c^2}\right.\right.$$

$\downarrow$ 6452

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-\frac{1}{4}bc\left(\frac{\frac{\frac{\int\frac{x(a+b\text{arctanh}(c\sqrt{x}))}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{3}bc\int\frac{x^{3/2}}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{5}x^{5/2}(a+b\text{ar}\ldots}{\ldots}}{c^2}\right.\right.$$

$\downarrow$ 243

3.195. $\int x^3\left(a+b\text{arctanh}(c\sqrt{x})\right)^2\,dx$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\frac{\frac{\int\frac{x\left(a+b\text{arctanh}(c\sqrt{x})\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}(c\sqrt{x})\right)-\frac{1}{6}bc\int\frac{x}{1-c^2x}dx}{c^2}}{c^2}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arc}\right.}{}\right.\right.$$

$\Big\downarrow$ 49

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\frac{\frac{\int\frac{x\left(a+b\text{arctanh}(c\sqrt{x})\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}(c\sqrt{x})\right)-\frac{1}{6}bc\int\left(-\frac{1}{c^2}-\frac{1}{c^2\left(c^2x-1\right)}\right)dx}{c^2}}{c^2}}{c^2}-\frac{\frac{1}{5}}{}\right.\right.$$

$\Big\downarrow$ 2009

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\frac{\frac{\int\frac{x\left(a+b\text{arctanh}(c\sqrt{x})\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}(c\sqrt{x})\right)-\frac{1}{6}bc\left(-\frac{x}{c^2}-\frac{\log\left(1-c^2x\right)}{c^4}\right)}{c^2}}{c^2}}{c^2}-\frac{\frac{1}{5}x^{5/}}{}\right.\right.$$

$\Big\downarrow$ 6542

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\frac{\frac{\frac{\int\frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int\left(a+b\text{arctanh}(c\sqrt{x})\right)d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}(c\sqrt{x})\right)-\frac{1}{6}bc\left(-\frac{}{c}\right.}{c^2}}{c^2}}{c^2}\right.\right.$$

$\Big\downarrow$ 2009

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\frac{\frac{\frac{\int\frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}}{c^2}-\frac{a\sqrt{x}+b\sqrt{x}\text{arctanh}(c\sqrt{x})+\frac{b\log\left(1-c^2x\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}(c\sqrt{x}\right.}{}}{c^2}}{c}\right.\right.$$

$\Big\downarrow$ 6510

3.195. $\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{4}bc\left(\frac{\frac{\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{2bc^3}-\frac{a\sqrt{x}+b\sqrt{x}\text{arctanh}(c\sqrt{x})+\frac{b\log\left(1-c^2x\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}(c\sqrt{x})\right)}{c^2}}{c^2}}{c^2}\right.\right.$$

input
```
Int[x^3*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

output
```
2*((x^4*(a + b*ArcTanh[c*Sqrt[x]])^2)/8 - (b*c*(-(((x^(7/2)*(a + b*ArcTanh
[c*Sqrt[x]]))/7 - (b*c*(-(x/c^6) - x/(2*c^4) - x^(3/2)/(3*c^2) - Log[1 - c
^2*x]/c^8))/14)/c^2) + (-(((x^(5/2)*(a + b*ArcTanh[c*Sqrt[x]]))/5 - (b*c*(
-(x/c^4) - x/(2*c^2) - Log[1 - c^2*x]/c^6))/10)/c^2) + (-(((x^(3/2)*(a + b
*ArcTanh[c*Sqrt[x]]))/3 - (b*c*(-(x/c^2) - Log[1 - c^2*x]/c^4))/6)/c^2) +
((a + b*ArcTanh[c*Sqrt[x]])^2/(2*b*c^3) - (a*Sqrt[x] + b*Sqrt[x]*ArcTanh[c
*Sqrt[x]] + (b*Log[1 - c^2*x])/(2*c))/c^2)/c^2)/c^2)/c^2))/4)
```

3.195.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.195.4 Maple [A] (verified)

Time = 7.76 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.46

method	result
parts	$\frac{a^2x^4}{4} + \frac{2b^2\left(\frac{c^8x^4\operatorname{arctanh}(c\sqrt{x})^2}{8} + \frac{\operatorname{arctanh}(c\sqrt{x})c^7x^{\frac{7}{2}}}{28} + \frac{\operatorname{arctanh}(c\sqrt{x})c^5x^{\frac{5}{2}}}{20} + \frac{\operatorname{arctanh}(c\sqrt{x})c^3x^{\frac{3}{2}}}{12} + \frac{\operatorname{arctanh}(c\sqrt{x})c\sqrt{x}}{4} + \operatorname{arcta}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^8x^4}{4} + 2b^2\left(\frac{c^8x^4\operatorname{arctanh}(c\sqrt{x})^2}{8} + \frac{\operatorname{arctanh}(c\sqrt{x})c^7x^{\frac{7}{2}}}{28} + \frac{\operatorname{arctanh}(c\sqrt{x})c^5x^{\frac{5}{2}}}{20} + \frac{\operatorname{arctanh}(c\sqrt{x})c^3x^{\frac{3}{2}}}{12} + \frac{\operatorname{arctanh}(c\sqrt{x})c\sqrt{x}}{4} + \operatorname{arctan}\right.}{}$
default	$\frac{\frac{a^2c^8x^4}{4} + 2b^2\left(\frac{c^8x^4\operatorname{arctanh}(c\sqrt{x})^2}{8} + \frac{\operatorname{arctanh}(c\sqrt{x})c^7x^{\frac{7}{2}}}{28} + \frac{\operatorname{arctanh}(c\sqrt{x})c^5x^{\frac{5}{2}}}{20} + \frac{\operatorname{arctanh}(c\sqrt{x})c^3x^{\frac{3}{2}}}{12} + \frac{\operatorname{arctanh}(c\sqrt{x})c\sqrt{x}}{4} + \operatorname{arctan}\right.}{}$

input
```
int(x^3*(a+b*arctanh(c*x^(1/2)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*a^2*x^4+2*b^2/c^8*(1/8*c^8*x^4*arctanh(c*x^(1/2))^2+1/28*arctanh(c*x^(
1/2))*c^7*x^(7/2)+1/20*arctanh(c*x^(1/2))*c^5*x^(5/2)+1/12*arctanh(c*x^(1/
2))*c^3*x^(3/2)+1/4*arctanh(c*x^(1/2))*c*x^(1/2)+1/8*arctanh(c*x^(1/2))*ln
(c*x^(1/2)-1)-1/8*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))+1/32*ln(c*x^(1/2)-1)^
2-1/16*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)+1/32*ln(1+c*x^(1/2))^2-1/16*(
ln(1+c*x^(1/2))-ln(1/2*c*x^(1/2)+1/2))*ln(-1/2*c*x^(1/2)+1/2)+1/168*c^6*x^
3+3/140*c^4*x^2+71/840*c^2*x+22/105*ln(c*x^(1/2)-1)+22/105*ln(1+c*x^(1/2))
)+4*a*b/c^8*(1/8*c^8*x^4*arctanh(c*x^(1/2))+1/56*c^7*x^(7/2)+1/40*c^5*x^(5
/2)+1/24*c^3*x^(3/2)+1/8*c*x^(1/2)+1/16*ln(c*x^(1/2)-1)-1/16*ln(1+c*x^(1/2
)))
```

3.195.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.29

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$$
$$=\frac{420\,a^2c^8x^4+20\,b^2c^6x^3+72\,b^2c^4x^2+284\,b^2c^2x+105\,(b^2c^8x^4-b^2)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\,(105\,abc^8-10}{}$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
1/1680*(420*a^2*c^8*x^4 + 20*b^2*c^6*x^3 + 72*b^2*c^4*x^2 + 284*b^2*c^2*x
+ 105*(b^2*c^8*x^4 - b^2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 +
4*(105*a*b*c^8 - 105*a*b + 176*b^2)*log(c*sqrt(x) + 1) - 4*(105*a*b*c^8 -
105*a*b - 176*b^2)*log(c*sqrt(x) - 1) + 4*(105*a*b*c^8*x^4 - 105*a*b*c^8 +
 (15*b^2*c^7*x^3 + 21*b^2*c^5*x^2 + 35*b^2*c^3*x + 105*b^2*c)*sqrt(x))*log
(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) + 8*(15*a*b*c^7*x^3 + 21*a*b*c^5*
x^2 + 35*a*b*c^3*x + 105*a*b*c)*sqrt(x))/c^8
```

3.195.6 Sympy [F]

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx=\int x^3\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

input
```
integrate(x**3*(a+b*atanh(c*x**(1/2)))**2,x)
```

output
```
Integral(x**3*(a + b*atanh(c*sqrt(x)))**2, x)
```

3.195.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.26

$$\int x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx=\frac{1}{4}\,b^2x^4\operatorname{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{4}\,a^2x^4$$
$$+\frac{1}{420}\left(210\,x^4\operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(15\,c^6x^{\frac{7}{2}}+21\,c^4x^{\frac{5}{2}}+35\,c^2x^{\frac{3}{2}}+105\,\sqrt{x}\right)}{c^8}-\frac{105\,\log\left(c\sqrt{x}+1\right)}{c^9}+\frac{10}{}\right.\right.$$
$$+\frac{1}{1680}\left(4\,c\left(\frac{2\left(15\,c^6x^{\frac{7}{2}}+21\,c^4x^{\frac{5}{2}}+35\,c^2x^{\frac{3}{2}}+105\,\sqrt{x}\right)}{c^8}-\frac{105\,\log\left(c\sqrt{x}+1\right)}{c^9}+\frac{105\,\log\left(c\sqrt{x}-1\right)}{c^9}\right)a\right.$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x^4*arctanh(c*sqrt(x))^2 + 1/4*a^2*x^4 + 1/420*(210*x^4*arctanh(c*
sqrt(x)) + c*(2*(15*c^6*x^(7/2) + 21*c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*sq
rt(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9 + 105*log(c*sqrt(x) - 1)/c^9))*a*b
 + 1/1680*(4*c*(2*(15*c^6*x^(7/2) + 21*c^4*x^(5/2) + 35*c^2*x^(3/2) + 105*
sqrt(x))/c^8 - 105*log(c*sqrt(x) + 1)/c^9 + 105*log(c*sqrt(x) - 1)/c^9)*ar
ctanh(c*sqrt(x)) + (20*c^6*x^3 + 72*c^4*x^2 + 284*c^2*x - 2*(105*log(c*sqr
t(x) - 1) - 352)*log(c*sqrt(x) + 1) + 105*log(c*sqrt(x) + 1)^2 + 105*log(c
*sqrt(x) - 1)^2 + 704*log(c*sqrt(x) - 1))/c^8)*b^2
```

3.195.8 Giac [F]

$$\int x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx=\int\left(b\operatorname{artanh}\left(c\sqrt{x}\right)+a\right)^2x^3\,dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^2*x^3, x)
```

3.195.9 Mupad [B] (verification not implemented)

Time = 6.55 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.15

$$\int x^3 \left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx = \frac{a^2\,x^4}{4}+\frac{44\,b^2\,\ln\left(c\,\sqrt{x}-1\right)}{105\,c^8}+\frac{44\,b^2\,\ln\left(c\,\sqrt{x}+1\right)}{105\,c^8}+\frac{71\,b^2\,x}{420\,c^6}-\frac{b^2\ln\left(c\,\sqrt{x}+1\right)^2}{16\,c^8}-\frac{b^2\ln\left(1-c\,\sqrt{x}\right)^2}{16\,c^8}+\frac{b^2\,x^3}{84\,c^2}+\frac{3\,b^2\,x^2}{70\,c^4}+\frac{b^2\,x^4\ln\left(c\,\sqrt{x}+1\right)^2}{16}+\frac{b^2\,x^4\ln\left(1-c\,\sqrt{x}\right)^2}{16}+\frac{b^2\,x^{7/2}\,\ln\left(c\,\sqrt{x}+1\right)}{28\,c}+\frac{b^2\,x^{5/2}\,\ln\left(c\,\sqrt{x}+1\right)}{20\,c^3}+\frac{b^2\,x^{3/2}\,\ln\left(c\,\sqrt{x}+1\right)}{12\,c^5}+\frac{b^2\,\sqrt{x}\,\ln\left(c\,\sqrt{x}+1\right)}{4\,c^7}-\frac{b^2\,x^{7/2}\,\ln\left(1-c\,\sqrt{x}\right)}{28\,c}-\frac{b^2\,x^{5/2}\,\ln\left(1-c\,\sqrt{x}\right)}{20\,c^3}-\frac{b^2\,x^{3/2}\,\ln\left(1-c\,\sqrt{x}\right)}{12\,c^5}-\frac{b^2\,\sqrt{x}\,\ln\left(1-c\,\sqrt{x}\right)}{4\,c^7}+\frac{a\,b\,\ln\left(c\,\sqrt{x}-1\right)}{4\,c^8}-\frac{a\,b\,\ln\left(c\,\sqrt{x}+1\right)}{4\,c^8}+\frac{a\,b\,x^4\,\ln\left(c\,\sqrt{x}+1\right)}{4}-\frac{a\,b\,x^4\,\ln\left(1-c\,\sqrt{x}\right)}{4}+\frac{b^2\,\ln\left(c\,\sqrt{x}+1\right)\,\ln\left(1-c\,\sqrt{x}\right)}{8\,c^8}+\frac{a\,b\,x^{7/2}}{14\,c}+\frac{a\,b\,x^{5/2}}{10\,c^3}+\frac{a\,b\,x^{3/2}}{6\,c^5}+\frac{a\,b\,\sqrt{x}}{2\,c^7}-\frac{b^2\,x^4\,\ln\left(c\,\sqrt{x}+1\right)\,\ln\left(1-c\,\sqrt{x}\right)}{8}$$

input
```
int(x^3*(a + b*atanh(c*x^(1/2)))^2,x)
```

output
```
(a^2*x^4)/4 + (44*b^2*log(c*x^(1/2) - 1))/(105*c^8) + (44*b^2*log(c*x^(1/2
) + 1))/(105*c^8) + (71*b^2*x)/(420*c^6) - (b^2*log(c*x^(1/2) + 1)^2)/(16*
c^8) - (b^2*log(1 - c*x^(1/2))^2)/(16*c^8) + (b^2*x^3)/(84*c^2) + (3*b^2*x
^2)/(70*c^4) + (b^2*x^4*log(c*x^(1/2) + 1)^2)/16 + (b^2*x^4*log(1 - c*x^(1
/2))^2)/16 + (b^2*x^(7/2)*log(c*x^(1/2) + 1))/(28*c) + (b^2*x^(5/2)*log(c*
x^(1/2) + 1))/(20*c^3) + (b^2*x^(3/2)*log(c*x^(1/2) + 1))/(12*c^5) + (b^2*
x^(1/2)*log(c*x^(1/2) + 1))/(4*c^7) - (b^2*x^(7/2)*log(1 - c*x^(1/2)))/(28
*c) - (b^2*x^(5/2)*log(1 - c*x^(1/2)))/(20*c^3) - (b^2*x^(3/2)*log(1 - c*x
^(1/2)))/(12*c^5) - (b^2*x^(1/2)*log(1 - c*x^(1/2)))/(4*c^7) + (a*b*log(c*
x^(1/2) - 1))/(4*c^8) - (a*b*log(c*x^(1/2) + 1))/(4*c^8) + (a*b*x^4*log(c*
x^(1/2) + 1))/4 - (a*b*x^4*log(1 - c*x^(1/2)))/4 + (b^2*log(c*x^(1/2) + 1)
*log(1 - c*x^(1/2)))/(8*c^8) + (a*b*x^(7/2))/(14*c) + (a*b*x^(5/2))/(10*c^
3) + (a*b*x^(3/2))/(6*c^5) + (a*b*x^(1/2))/(2*c^7) - (b^2*x^4*log(c*x^(1/2
) + 1)*log(1 - c*x^(1/2)))/8
```

3.195. $\int x^3 \left(a + b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2 dx$

3.196 $\int x^2 \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 \, dx$

3.196.1 Optimal result

Integrand size = 18, antiderivative size = 173

$$\int x^2 \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 \, dx = \frac{2ab\sqrt{x}}{3c^5}+\frac{8b^2x}{45c^4}+\frac{b^2x^2}{30c^2}+\frac{2b^2\sqrt{x}\text{arctanh}\left(c\sqrt{x}\right)}{3c^5}+\frac{2bx^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{9c^3}+\frac{2bx^{5/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{15c}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{3c^6}+\frac{1}{3}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2+\frac{23b^2\log\left(1-c^2x\right)}{45c^6}$$

output
```
8/45*b^2*x/c^4+1/30*b^2*x^2/c^2+2/9*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))/c^3
+2/15*b*x^(5/2)*(a+b*arctanh(c*x^(1/2)))/c-1/3*(a+b*arctanh(c*x^(1/2)))^2/
c^6+1/3*x^3*(a+b*arctanh(c*x^(1/2)))^2+23/45*b^2*ln(-c^2*x+1)/c^6+2/3*a*b*
x^(1/2)/c^5+2/3*b^2*arctanh(c*x^(1/2))*x^(1/2)/c^5
```

3.196.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.12

$$\int x^2 \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 \, dx$$
$$= \frac{60abc\sqrt{x}+16b^2c^2x+20abc^3x^{3/2}+3b^2c^4x^2+12abc^5x^{5/2}+30a^2c^6x^3+4bc\sqrt{x}\left(15ac^5x^{5/2}+b(15+5c^2x+\right.}{}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

output
```
(60*a*b*c*Sqrt[x] + 16*b^2*c^2*x + 20*a*b*c^3*x^(3/2) + 3*b^2*c^4*x^2 + 12
*a*b*c^5*x^(5/2) + 30*a^2*c^6*x^3 + 4*b*c*Sqrt[x]*(15*a*c^5*x^(5/2) + b*(1
5 + 5*c^2*x + 3*c^4*x^2))*ArcTanh[c*Sqrt[x]] + 30*b^2*(-1 + c^6*x^3)*ArcTa
nh[c*Sqrt[x]]^2 + 2*b*(15*a + 23*b)*Log[1 - c*Sqrt[x]] - 30*a*b*Log[1 + c*
Sqrt[x]] + 46*b^2*Log[1 + c*Sqrt[x]])/(90*c^6)
```

3.196.3 Rubi [A] (warning: unable to verify)

Time = 1.43 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.31, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {6454, 6452, 6542, 6452, 243, 49, 2009, 6542, 6452, 243, 49, 2009, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2dx$$

$$\Big\downarrow \text{6454}$$

$$2\int x^{5/2}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{6}x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\int\frac{x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{6542}$$

$$2\left(\frac{1}{6}x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\int\frac{x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)d\sqrt{x}}{c^2}\right)\right)$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{6}x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\int\frac{x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{5}bc\int\frac{x^{5/2}}{1-c^2x}d\sqrt{x}}{c^2}\right.\right.$$

$$\Big\downarrow \text{243}$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\int \frac{x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{10}bc\int \frac{x}{1-c^2x}dx}{c^2}\right.\right.$$

$\downarrow$ 49

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\int \frac{x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{10}bc\int \left(-\frac{x}{c^2}-\right.}{c^2}\right.\right.$$

$\downarrow$ 2009

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\int \frac{x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{10}bc\left(-\frac{x}{c^4}-\frac{x}{2c}\right.}{c^2}\right.\right.$$

$\downarrow$ 6542

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\frac{\int \frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctanh}\right.}{}\right.\right.$$

$\downarrow$ 6452

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\frac{\int \frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{3}bc\int \frac{x^{3/2}}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(\right.}{}\right.\right.$$

$\downarrow$ 243

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\frac{\int \frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\int \frac{x}{1-c^2x}dx}{c^2}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a\right.}{}\right.\right.$$

$\downarrow$ 49

3.196. $\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\int\left(-\frac{1}{c^2}-\frac{1}{c^2\left(c^2x-1\right)}\right)dx}{c^2}}{c^2}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\left(-\frac{x}{c^2}-\frac{\log\left(1-c^2x\right)}{c^4}\right)}{c^2}}{c^2}-\right.\right.$$

$$\Big\downarrow \text{6542}$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\frac{\frac{\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\left(\right.}{c^2}}{c^2}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\frac{\frac{\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{a\sqrt{x}+b\sqrt{x}\text{arctanh}\left(c\sqrt{x}\right)+\frac{b\log\left(1-c^2x\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\right.\right.}{}}{c^2}\right.\right.$$

$$\Big\downarrow \text{6510}$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{3}bc\left(\frac{\frac{\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2bc^3}-\frac{a\sqrt{x}+b\sqrt{x}\text{arctanh}\left(c\sqrt{x}\right)+\frac{b\log\left(1-c^2x\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right.\right.}{}}{c^2}\right.\right.$$

input
```
Int[x^2*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

3.196. $\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$

output
```
2*((x^3*(a + b*ArcTanh[c*Sqrt[x]])^2)/6 - (b*c*(-(((x^(5/2)*(a + b*ArcTanh
[c*Sqrt[x]]))/5 - (b*c*(-(x/c^4) - x/(2*c^2) - Log[1 - c^2*x]/c^6))/10)/c^
2) + (-(((x^(3/2)*(a + b*ArcTanh[c*Sqrt[x]]))/3 - (b*c*(-(x/c^2) - Log[1 -
 c^2*x]/c^4))/6)/c^2) + ((a + b*ArcTanh[c*Sqrt[x]])^2/(2*b*c^3) - (a*Sqrt[
x] + b*Sqrt[x]*ArcTanh[c*Sqrt[x]] + (b*Log[1 - c^2*x])/(2*c))/c^2)/c^2)/c^
2))/3)
```

3.196.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.196.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(137) = 274$.

Time = 7.55 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.61

method	result
parts	$\frac{a^2x^3}{3}+\frac{2b^2\left(\frac{c^6x^3\operatorname{arctanh}(c\sqrt{x})^2}{6}+\frac{\operatorname{arctanh}(c\sqrt{x})c^5x^{\frac{5}{2}}}{15}+\frac{\operatorname{arctanh}(c\sqrt{x})c^3x^{\frac{3}{2}}}{9}+\frac{\operatorname{arctanh}(c\sqrt{x})c\sqrt{x}}{3}+\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{6}-\right.}{}$
derivativedivides	$\frac{\frac{a^2c^6x^3}{3}+2b^2\left(\frac{c^6x^3\operatorname{arctanh}(c\sqrt{x})^2}{6}+\frac{\operatorname{arctanh}(c\sqrt{x})c^5x^{\frac{5}{2}}}{15}+\frac{\operatorname{arctanh}(c\sqrt{x})c^3x^{\frac{3}{2}}}{9}+\frac{\operatorname{arctanh}(c\sqrt{x})c\sqrt{x}}{3}+\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{6}-\right.}{}$
default	$\frac{\frac{a^2c^6x^3}{3}+2b^2\left(\frac{c^6x^3\operatorname{arctanh}(c\sqrt{x})^2}{6}+\frac{\operatorname{arctanh}(c\sqrt{x})c^5x^{\frac{5}{2}}}{15}+\frac{\operatorname{arctanh}(c\sqrt{x})c^3x^{\frac{3}{2}}}{9}+\frac{\operatorname{arctanh}(c\sqrt{x})c\sqrt{x}}{3}+\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{6}-\right.}{}$

input
```
int(x^2*(a+b*arctanh(c*x^(1/2)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+2*b^2/c^6*(1/6*c^6*x^3*arctanh(c*x^(1/2))^2+1/15*arctanh(c*x^(
1/2))*c^5*x^(5/2)+1/9*arctanh(c*x^(1/2))*c^3*x^(3/2)+1/3*arctanh(c*x^(1/2)
)*c*x^(1/2)+1/6*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/6*arctanh(c*x^(1/2))*
ln(1+c*x^(1/2))-1/12*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)+1/24*ln(c*x^(1/
2)-1)^2+1/24*ln(1+c*x^(1/2))^2-1/12*(ln(1+c*x^(1/2))-ln(1/2*c*x^(1/2)+1/2)
)*ln(-1/2*c*x^(1/2)+1/2)+1/60*c^4*x^2+4/45*c^2*x+23/90*ln(c*x^(1/2)-1)+23/
90*ln(1+c*x^(1/2)))+4*a*b/c^6*(1/6*c^6*x^3*arctanh(c*x^(1/2))+1/30*c^5*x^(
5/2)+1/18*c^3*x^(3/2)+1/6*c*x^(1/2)+1/12*ln(c*x^(1/2)-1)-1/12*ln(1+c*x^(1/
2)))
```

3.196.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.39

$$\int x^2 \left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2 dx$$
$$= \frac{60\,a^2c^6x^3+6\,b^2c^4x^2+32\,b^2c^2x+15\,(b^2c^6x^3-b^2)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\,(15\,abc^6-15\,ab+23\,b^2)\log\left(c\right.}{}$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
1/180*(60*a^2*c^6*x^3 + 6*b^2*c^4*x^2 + 32*b^2*c^2*x + 15*(b^2*c^6*x^3 - b
^2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*(15*a*b*c^6 - 15*a*b
 + 23*b^2)*log(c*sqrt(x) + 1) - 4*(15*a*b*c^6 - 15*a*b - 23*b^2)*log(c*sqr
t(x) - 1) + 4*(15*a*b*c^6*x^3 - 15*a*b*c^6 + (3*b^2*c^5*x^2 + 5*b^2*c^3*x
+ 15*b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)) + 8*(3*a*
b*c^5*x^2 + 5*a*b*c^3*x + 15*a*b*c)*sqrt(x))/c^6
```

3.196.6 Sympy [F]

$$\int x^2 \left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2 dx = \int x^2 \left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^2 dx$$

input
```
integrate(x**2*(a+b*atanh(c*x**(1/2)))**2,x)
```

output
```
Integral(x**2*(a + b*atanh(c*sqrt(x)))**2, x)
```

3.196.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.39

$$\int x^2 \left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2 dx = \frac{1}{3}\,b^2x^3\operatorname{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{3}\,a^2x^3$$
$$+\frac{1}{45}\left(30\,x^3\operatorname{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(3\,c^4x^{\frac{5}{2}}+5\,c^2x^{\frac{3}{2}}+15\,\sqrt{x}\right)}{c^6}-\frac{15\,\log\left(c\sqrt{x}+1\right)}{c^7}+\frac{15\,\log\left(c\sqrt{x}-1\right)}{c^7}\right)\right.$$
$$+\frac{1}{180}\left(4\,c\left(\frac{2\left(3\,c^4x^{\frac{5}{2}}+5\,c^2x^{\frac{3}{2}}+15\,\sqrt{x}\right)}{c^6}-\frac{15\,\log\left(c\sqrt{x}+1\right)}{c^7}+\frac{15\,\log\left(c\sqrt{x}-1\right)}{c^7}\right)\operatorname{artanh}\left(c\sqrt{x}\right)+\frac{6}{}\right.$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3*arctanh(c*sqrt(x))^2 + 1/3*a^2*x^3 + 1/45*(30*x^3*arctanh(c*sq
rt(x)) + c*(2*(3*c^4*x^(5/2) + 5*c^2*x^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*
sqrt(x) + 1)/c^7 + 15*log(c*sqrt(x) - 1)/c^7))*a*b + 1/180*(4*c*(2*(3*c^4*
x^(5/2) + 5*c^2*x^(3/2) + 15*sqrt(x))/c^6 - 15*log(c*sqrt(x) + 1)/c^7 + 15
*log(c*sqrt(x) - 1)/c^7)*arctanh(c*sqrt(x)) + (6*c^4*x^2 + 32*c^2*x - 2*(1
5*log(c*sqrt(x) - 1) - 46)*log(c*sqrt(x) + 1) + 15*log(c*sqrt(x) + 1)^2 +
15*log(c*sqrt(x) - 1)^2 + 92*log(c*sqrt(x) - 1))/c^6)*b^2
```

3.196.8 Giac [F]

$$\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx=\int\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^2*x^2, x)
```

3.196.9 Mupad [B] (verification not implemented)

Time = 3.91 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.07

$$\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$$
$$=\frac{46\,b^2\,\ln\left(c^2\,x-1\right)-30\,b^2\,\text{atanh}\left(c\,\sqrt{x}\right)^2-60\,a\,b\,\text{atanh}\left(c\,\sqrt{x}\right)+16\,b^2\,c^2\,x+30\,a^2\,c^6\,x^3+3\,b^2\,c^4\,x^2+30\,b^2}{}$$

input
```
int(x^2*(a + b*atanh(c*x^(1/2)))^2,x)
```

output
```
(46*b^2*log(c^2*x - 1) - 30*b^2*atanh(c*x^(1/2))^2 - 60*a*b*atanh(c*x^(1/2
)) + 16*b^2*c^2*x + 30*a^2*c^6*x^3 + 3*b^2*c^4*x^2 + 30*b^2*c^6*x^3*atanh(
c*x^(1/2))^2 + 60*b^2*c*x^(1/2)*atanh(c*x^(1/2)) + 60*a*b*c*x^(1/2) + 20*b
^2*c^3*x^(3/2)*atanh(c*x^(1/2)) + 12*b^2*c^5*x^(5/2)*atanh(c*x^(1/2)) + 20
*a*b*c^3*x^(3/2) + 12*a*b*c^5*x^(5/2) + 60*a*b*c^6*x^3*atanh(c*x^(1/2)))/(
90*c^6)
```

3.197 $\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$

3.197.1 Optimal result

Integrand size = 16, antiderivative size = 129

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx=\frac{ab\sqrt{x}}{c^3}+\frac{b^2x}{6c^2}+\frac{b^2\sqrt{x}\text{arctanh}\left(c\sqrt{x}\right)}{c^3}+\frac{bx^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{3c}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2c^4}+\frac{1}{2}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2+\frac{2b^2\log\left(1-c^2x\right)}{3c^4}$$

output
```
1/6*b^2*x/c^2+1/3*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))/c-1/2*(a+b*arctanh(c*
x^(1/2)))^2/c^4+1/2*x^2*(a+b*arctanh(c*x^(1/2)))^2+2/3*b^2*ln(-c^2*x+1)/c^
4+a*b*x^(1/2)/c^3+b^2*arctanh(c*x^(1/2))*x^(1/2)/c^3
```

3.197.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.24

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx=\frac{6abc\sqrt{x}+b^2c^2x+2abc^3x^{3/2}+3a^2c^4x^2+2bc\sqrt{x}\left(3ac^3x^{3/2}+b(3+c^2x)\right)\text{arctanh}\left(c\sqrt{x}\right)+3b^2(-1+c^4x^2)}{6c^4}$$

input
```
Integrate[x*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

output
```
(6*a*b*c*Sqrt[x] + b^2*c^2*x + 2*a*b*c^3*x^(3/2) + 3*a^2*c^4*x^2 + 2*b*c*S
qrt[x]*(3*a*c^3*x^(3/2) + b*(3 + c^2*x))*ArcTanh[c*Sqrt[x]] + 3*b^2*(-1 +
c^4*x^2)*ArcTanh[c*Sqrt[x]]^2 + b*(3*a + 4*b)*Log[1 - c*Sqrt[x]] - 3*a*b*L
og[1 + c*Sqrt[x]] + 4*b^2*Log[1 + c*Sqrt[x]])/(6*c^4)
```

3.197.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {6454, 6452, 6542, 6452, 243, 49, 2009, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2dx$$

$$\Big\downarrow \text{6454}$$

$$2\int x^{3/2}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{4}x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{2}bc\int\frac{x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{6542}$$

$$2\left(\frac{1}{4}x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{2}bc\left(\frac{\int\frac{x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)d\sqrt{x}}{c^2}\right)\right)$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{4}x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{2}bc\left(\frac{\int\frac{x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{3}bc\int\frac{x^{3/2}}{1-c^2x}d\sqrt{x}}{c^2}\right)\right)$$

$$\Big\downarrow \text{243}$$

$$2\left(\frac{1}{4}x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{2}bc\left(\frac{\int\frac{x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\int\frac{x}{1-c^2x}dx}{c^2}\right)\right)$$

$\Big\downarrow$ 49

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{2}bc\left(\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\int\left(-\frac{1}{c^2}-\frac{}{c}\right.}{c^2}\right.\right.$$

$\Big\downarrow$ 2009

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{2}bc\left(\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)-\frac{1}{6}bc\left(-\frac{x}{c^2}-\frac{\log(}{}\right.}{c^2}\right.\right.$$

$\Big\downarrow$ 6542

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{\int\frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int\left(a+b\text{arctanh}(c\sqrt{x})\right)d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{\right.\right.}{}\right.\right.$$

$\Big\downarrow$ 2009

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{\int\frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}}{c^2}-\frac{a\sqrt{x}+b\sqrt{x}\text{arctanh}(c\sqrt{x})+\frac{b\log\left(1-c^2x\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+\text{barc}\right.}{}\right.\right.$$

$\Big\downarrow$ 6510

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{1}{2}bc\left(\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{2bc^3}-\frac{a\sqrt{x}+b\sqrt{x}\text{arctanh}(c\sqrt{x})+\frac{b\log\left(1-c^2x\right)}{2c}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+\text{barct}\right.}{}\right.\right.$$

input
```
Int[x*(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

output
```
2*((x^2*(a + b*ArcTanh[c*Sqrt[x]])^2)/4 - (b*c*(-(((x^(3/2)*(a + b*ArcTanh
[c*Sqrt[x]]))/3 - (b*c*(-(x/c^2) - Log[1 - c^2*x]/c^4))/6)/c^2) + ((a + b*
ArcTanh[c*Sqrt[x]])^2/(2*b*c^3) - (a*Sqrt[x] + b*Sqrt[x]*ArcTanh[c*Sqrt[x]
] + (b*Log[1 - c^2*x])/(2*c))/c^2)/c^2))/2)
```

3.197. $\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$

3.197.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.197.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 248 vs. 2(105) = 210.

Time = 7.60 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.93

method	result
parts	$\frac{a^2x^2}{2}+\frac{2b^2\left(\frac{c^4x^2\operatorname{arctanh}(c\sqrt{x})^2}{4}+\frac{\operatorname{arctanh}(c\sqrt{x})c^3x^{\frac{3}{2}}}{6}+\frac{\operatorname{arctanh}(c\sqrt{x})c\sqrt{x}}{2}+\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{4}-\frac{\operatorname{arctanh}(c\sqrt{x})\ln(1+c}{4}\right.}{}$
derivativedivides	$\frac{\frac{a^2c^4x^2}{2}+2b^2\left(\frac{c^4x^2\operatorname{arctanh}(c\sqrt{x})^2}{4}+\frac{\operatorname{arctanh}(c\sqrt{x})c^3x^{\frac{3}{2}}}{6}+\frac{\operatorname{arctanh}(c\sqrt{x})c\sqrt{x}}{2}+\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{4}-\frac{\operatorname{arctanh}(c\sqrt{x})\ln(1+c}{4}\right.}{}$
default	$\frac{\frac{a^2c^4x^2}{2}+2b^2\left(\frac{c^4x^2\operatorname{arctanh}(c\sqrt{x})^2}{4}+\frac{\operatorname{arctanh}(c\sqrt{x})c^3x^{\frac{3}{2}}}{6}+\frac{\operatorname{arctanh}(c\sqrt{x})c\sqrt{x}}{2}+\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{4}-\frac{\operatorname{arctanh}(c\sqrt{x})\ln(1+c}{4}\right.}{}$

input
```
int(x*(a+b*arctanh(c*x^(1/2)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*x^2+2*b^2/c^4*(1/4*c^4*x^2*arctanh(c*x^(1/2))^2+1/6*arctanh(c*x^(1
/2))*c^3*x^(3/2)+1/2*arctanh(c*x^(1/2))*c*x^(1/2)+1/4*arctanh(c*x^(1/2))*l
n(c*x^(1/2)-1)-1/4*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))-1/8*ln(c*x^(1/2)-1)*
ln(1/2*c*x^(1/2)+1/2)+1/16*ln(c*x^(1/2)-1)^2+1/16*ln(1+c*x^(1/2))^2-1/8*(l
n(1+c*x^(1/2))-ln(1/2*c*x^(1/2)+1/2))*ln(-1/2*c*x^(1/2)+1/2)+1/12*c^2*x+1/
3*ln(c*x^(1/2)-1)+1/3*ln(1+c*x^(1/2)))+4*a*b/c^4*(1/4*c^4*x^2*arctanh(c*x^
(1/2))+1/12*c^3*x^(3/2)+1/4*c*x^(1/2)+1/8*ln(c*x^(1/2)-1)-1/8*ln(1+c*x^(1/
2)))
```

3.197.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.60

$$\int x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx$$
$$=\frac{12\,a^2c^4x^2+4\,b^2c^2x+3\,(b^2c^4x^2-b^2)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\,(3\,abc^4-3\,ab+4\,b^2)\log\left(c\sqrt{x}+1\right)-4\,(3\,a}{}$$

input
```
integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
1/24*(12*a^2*c^4*x^2 + 4*b^2*c^2*x + 3*(b^2*c^4*x^2 - b^2)*log(-(c^2*x + 2
*c*sqrt(x) + 1)/(c^2*x - 1))^2 + 4*(3*a*b*c^4 - 3*a*b + 4*b^2)*log(c*sqrt(
x) + 1) - 4*(3*a*b*c^4 - 3*a*b - 4*b^2)*log(c*sqrt(x) - 1) + 4*(3*a*b*c^4*
x^2 - 3*a*b*c^4 + (b^2*c^3*x + 3*b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x)
 + 1)/(c^2*x - 1)) + 8*(a*b*c^3*x + 3*a*b*c)*sqrt(x))/c^4
```

3.197.6 Sympy [F]

$$\int x\left(a+b\text{arctanh}(c\sqrt{x})\right)^2\,dx=\int x\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^2\,dx$$

input
```
integrate(x*(a+b*atanh(c*x**(1/2)))**2,x)
```

output
```
Integral(x*(a + b*atanh(c*sqrt(x)))**2, x)
```

3.197.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(105)=210$.

Time = 0.20 (sec), antiderivative size = 215, normalized size of antiderivative = 1.67

$$\int x\left(a+b\text{arctanh}(c\sqrt{x})\right)^2\,dx=\frac{1}{2}\,b^2x^2\,\text{artanh}\left(c\sqrt{x}\right)^2+\frac{1}{2}\,a^2x^2$$
$$+\frac{1}{6}\left(6\,x^2\,\text{artanh}\left(c\sqrt{x}\right)+c\left(\frac{2\left(c^2x^{\frac{3}{2}}+3\sqrt{x}\right)}{c^4}-\frac{3\,\log\left(c\sqrt{x}+1\right)}{c^5}+\frac{3\,\log\left(c\sqrt{x}-1\right)}{c^5}\right)\right)ab$$
$$+\frac{1}{24}\left(4\,c\left(\frac{2\left(c^2x^{\frac{3}{2}}+3\sqrt{x}\right)}{c^4}-\frac{3\,\log\left(c\sqrt{x}+1\right)}{c^5}+\frac{3\,\log\left(c\sqrt{x}-1\right)}{c^5}\right)\text{artanh}\left(c\sqrt{x}\right)+\frac{4\,c^2x-2\,(3\,\log\left(\right.}{}\right.$$

input
```
integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
1/2*b^2*x^2*arctanh(c*sqrt(x))^2 + 1/2*a^2*x^2 + 1/6*(6*x^2*arctanh(c*sqrt
(x)) + c*(2*(c^2*x^(3/2) + 3*sqrt(x))/c^4 - 3*log(c*sqrt(x) + 1)/c^5 + 3*l
og(c*sqrt(x) - 1)/c^5))*a*b + 1/24*(4*c*(2*(c^2*x^(3/2) + 3*sqrt(x))/c^4 -
 3*log(c*sqrt(x) + 1)/c^5 + 3*log(c*sqrt(x) - 1)/c^5)*arctanh(c*sqrt(x)) +
 (4*c^2*x - 2*(3*log(c*sqrt(x) - 1) - 8)*log(c*sqrt(x) + 1) + 3*log(c*sqrt
(x) + 1)^2 + 3*log(c*sqrt(x) - 1)^2 + 16*log(c*sqrt(x) - 1))/c^4)*b^2
```

3.197.8 Giac [F]

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx=\int\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^2x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^2*x, x)
```

3.197.9 Mupad [B] (verification not implemented)

Time = 3.65 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.11

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\,dx=\frac{a^2\,x^2}{2}-\frac{b^2\,\text{atanh}\left(c\,\sqrt{x}\right)^2}{2\,c^4}+\frac{2\,b^2\,\ln\left(c^2\,x-1\right)}{3\,c^4}+\frac{b^2\,x^2\,\text{atanh}\left(c\,\sqrt{x}\right)^2}{2}+\frac{b^2\,x}{6\,c^2}+\frac{b^2\,x^{3/2}\,\text{atanh}\left(c\,\sqrt{x}\right)}{3\,c}+\frac{b^2\,\sqrt{x}\,\text{atanh}\left(c\,\sqrt{x}\right)}{c^3}+\frac{a\,b\,x^{3/2}}{3\,c}+\frac{a\,b\,\sqrt{x}}{c^3}-\frac{a\,b\,\text{atanh}\left(c\,\sqrt{x}\right)}{c^4}+a\,b\,x^2\,\text{atanh}\left(c\,\sqrt{x}\right)$$

input
```
int(x*(a + b*atanh(c*x^(1/2)))^2,x)
```

output
```
(a^2*x^2)/2 - (b^2*atanh(c*x^(1/2))^2)/(2*c^4) + (2*b^2*log(c^2*x - 1))/(3
*c^4) + (b^2*x^2*atanh(c*x^(1/2))^2)/2 + (b^2*x)/(6*c^2) + (b^2*x^(3/2)*at
anh(c*x^(1/2)))/(3*c) + (b^2*x^(1/2)*atanh(c*x^(1/2)))/c^3 + (a*b*x^(3/2))
/(3*c) + (a*b*x^(1/2))/c^3 - (a*b*atanh(c*x^(1/2)))/c^4 + a*b*x^2*atanh(c*
x^(1/2))
```

3.198 $\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 dx$

3.198.1 Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 dx=\frac{2ab\sqrt{x}}{c}+\frac{2b^2\sqrt{x}\text{arctanh}\left(c\sqrt{x}\right)}{c}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{c^2}+x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2+\frac{b^2\log\left(1-c^2x\right)}{c^2}$$

output
```
-(a+b*arctanh(c*x^(1/2)))^2/c^2+x*(a+b*arctanh(c*x^(1/2)))^2+b^2*ln(-c^2*x
+1)/c^2+2*a*b*x^(1/2)/c+2*b^2*arctanh(c*x^(1/2))*x^(1/2)/c
```

3.198.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.35

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 dx$$
$$=\frac{2abc\sqrt{x}+a^2c^2x+2bc\left(b+ac\sqrt{x}\right)\sqrt{x}\text{arctanh}\left(c\sqrt{x}\right)+b^2\left(-1+c^2x\right)\text{arctanh}\left(c\sqrt{x}\right)^2+b(a+b)\log\left(1-c\right.}{c^2}$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

output
```
(2*a*b*c*Sqrt[x] + a^2*c^2*x + 2*b*c*(b + a*c*Sqrt[x])*Sqrt[x]*ArcTanh[c*S
qrt[x]] + b^2*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^2 + b*(a + b)*Log[1 - c*Sqrt
[x]] - a*b*Log[1 + c*Sqrt[x]] + b^2*Log[1 + c*Sqrt[x]])/c^2
```

3.198.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {6442, 6452, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 dx$$

$$\Big\downarrow \text{6442}$$

$$2\int \sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 d\sqrt{x}$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-bc\int \frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{6542}$$

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-bc\left(\frac{\int \frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)d\sqrt{x}}{c^2}\right)\right)$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-bc\left(\frac{\int \frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}}{c^2}-\frac{a\sqrt{x}+b\sqrt{x}\text{arctanh}\left(c\sqrt{x}\right)+\frac{b\log(1-c^2x)}{2c}}{c^2}\right)\right)$$

$$\Big\downarrow \text{6510}$$

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-bc\left(\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2bc^3}-\frac{a\sqrt{x}+b\sqrt{x}\text{arctanh}\left(c\sqrt{x}\right)+\frac{b\log(1-c^2x)}{2c}}{c^2}\right)\right)$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])^2,x]
```

output
```
2*((x*(a + b*ArcTanh[c*Sqrt[x]])^2)/2 - b*c*((a + b*ArcTanh[c*Sqrt[x]])^2/
(2*b*c^3) - (a*Sqrt[x] + b*Sqrt[x]*ArcTanh[c*Sqrt[x]] + (b*Log[1 - c^2*x])
/(2*c))/c^2))
```

3.198.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6442
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> With[{k = D
enominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*ArcTanh[c*x^(k*n)])^p,
x], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && FractionQ[n]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.198.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(75) = 150$.

Time = 7.61 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.51

method	result
parts	$a^2x+\frac{2b^2\left(\frac{c^2x\,\mathrm{arctanh}(c\sqrt{x})^2}{2}+\mathrm{arctanh}(c\sqrt{x})c\sqrt{x}+\frac{\mathrm{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{2}-\frac{\mathrm{arctanh}(c\sqrt{x})\ln(1+c\sqrt{x})}{2}+\frac{\ln(c\sqrt{x}-1)^2}{8}-\cdots\right.}{c^2}$
derivativedivides	$\frac{a^2c^2x+2b^2\left(\frac{c^2x\,\mathrm{arctanh}(c\sqrt{x})^2}{2}+\mathrm{arctanh}(c\sqrt{x})c\sqrt{x}+\frac{\mathrm{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{2}-\frac{\mathrm{arctanh}(c\sqrt{x})\ln(1+c\sqrt{x})}{2}+\frac{\ln(c\sqrt{x}-1)^2}{8}-\cdots\right.}{}$
default	$\frac{a^2c^2x+2b^2\left(\frac{c^2x\,\mathrm{arctanh}(c\sqrt{x})^2}{2}+\mathrm{arctanh}(c\sqrt{x})c\sqrt{x}+\frac{\mathrm{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{2}-\frac{\mathrm{arctanh}(c\sqrt{x})\ln(1+c\sqrt{x})}{2}+\frac{\ln(c\sqrt{x}-1)^2}{8}-\cdots\right.}{}$

input
```
int((a+b*arctanh(c*x^(1/2)))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2*x+2*b^2/c^2*(1/2*c^2*x*arctanh(c*x^(1/2))^2+arctanh(c*x^(1/2))*c*x^(1/
2)+1/2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/2*arctanh(c*x^(1/2))*ln(1+c*x^
(1/2))+1/8*ln(c*x^(1/2)-1)^2-1/4*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)+1/2
*ln(c*x^(1/2)-1)+1/2*ln(1+c*x^(1/2))-1/4*(ln(1+c*x^(1/2))-ln(1/2*c*x^(1/2)
+1/2))*ln(-1/2*c*x^(1/2)+1/2)+1/8*ln(1+c*x^(1/2))^2)+4*a*b/c^2*(1/2*c^2*x*
arctanh(c*x^(1/2))+1/2*c*x^(1/2)+1/4*ln(c*x^(1/2)-1)-1/4*ln(1+c*x^(1/2)))
```

3.198.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(75) = 150$.

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.94

$$\int \left(a+b\mathrm{arctanh}(c\sqrt{x})\right)^2\,dx$$
$$=\frac{4\,a^2c^2x+8\,abc\sqrt{x}+(b^2c^2x-b^2)\log\left(-\frac{c^2x+2\,c\sqrt{x}+1}{c^2x-1}\right)^2+4\,(abc^2-ab+b^2)\log\left(c\sqrt{x}+1\right)-4\,(abc^2-ab-\cdots}{4\,c^2}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="fricas")
```

output
```
1/4*(4*a^2*c^2*x + 8*a*b*c*sqrt(x) + (b^2*c^2*x - b^2)*log(-(c^2*x + 2*c*s
qrt(x) + 1)/(c^2*x - 1))^2 + 4*(a*b*c^2 - a*b + b^2)*log(c*sqrt(x) + 1) -
4*(a*b*c^2 - a*b - b^2)*log(c*sqrt(x) - 1) + 4*(a*b*c^2*x - a*b*c^2 + b^2*
c*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)))/c^2
```

3.198.6 Sympy [F]

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 \, dx = \int \left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^2 \, dx$$

input
```
integrate((a+b*atanh(c*x**(1/2)))**2,x)
```

output
```
Integral((a + b*atanh(c*sqrt(x)))**2, x)
```

3.198.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(75) = 150$.

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.06

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 \, dx$$
$$= \left(c\left(\frac{2\sqrt{x}}{c^2} - \frac{\log\left(c\sqrt{x}+1\right)}{c^3} + \frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right) + 2\,x\,\text{artanh}\left(c\sqrt{x}\right)\right)ab$$
$$+ \frac{1}{4}\left(4\,c\left(\frac{2\sqrt{x}}{c^2} - \frac{\log\left(c\sqrt{x}+1\right)}{c^3} + \frac{\log\left(c\sqrt{x}-1\right)}{c^3}\right)\text{artanh}\left(c\sqrt{x}\right) + 4\,x\,\text{artanh}\left(c\sqrt{x}\right)^2 - \frac{2\left(\log\left(c\sqrt{x}-\right.\right.}{}\right.$$
$$+ a^2x$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="maxima")
```

output
```
(c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) + 2*x
*arctanh(c*sqrt(x)))*a*b + 1/4*(4*c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^
3 + log(c*sqrt(x) - 1)/c^3)*arctanh(c*sqrt(x)) + 4*x*arctanh(c*sqrt(x))^2
- (2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log(c*sqrt(x) + 1)^2 -
log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1))/c^2)*b^2 + a^2*x
```

3.198.8 Giac [F]

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 \,dx=\int \left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^2 \,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^2, x)
```

3.198.9 Mupad [B] (verification not implemented)

Time = 3.51 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 \,dx=a^2\,x+\frac{c\,\left(2\,b^2\,\sqrt{x}\,\text{atanh}\left(c\,\sqrt{x}\right)+2\,a\,b\,\sqrt{x}\right)-b^2\,\text{atanh}\left(c\,\sqrt{x}\right)^2+b^2\,\ln\left(c^2\,x-1\right)-2\,a\,b\,\text{atanh}\left(c\,\sqrt{x}\right)}{c^2}+b^2\,x\,\text{atanh}\left(c\,\sqrt{x}\right)^2+2\,a\,b\,x\,\text{atanh}\left(c\,\sqrt{x}\right)$$

input
```
int((a + b*atanh(c*x^(1/2)))^2,x)
```

output
```
a^2*x + (c*(2*b^2*x^(1/2)*atanh(c*x^(1/2)) + 2*a*b*x^(1/2)) - b^2*atanh(c*
x^(1/2))^2 + b^2*log(c^2*x - 1) - 2*a*b*atanh(c*x^(1/2)))/c^2 + b^2*x*atan
h(c*x^(1/2))^2 + 2*a*b*x*atanh(c*x^(1/2))
```

3.199 $\int \frac{(a+b\text{arctanh}(c\sqrt{x}))^2}{x}\,dx$

3.199.1 Optimal result

Integrand size = 18, antiderivative size = 145

$$\begin{aligned}\int \frac{(a+b\text{arctanh}(c\sqrt{x}))^2}{x}\,dx &= 4\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)(a+b\text{arctanh}(c\sqrt{x}))^2 \\ &\quad - 2b(a+b\text{arctanh}(c\sqrt{x}))\operatorname{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right) \\ &\quad + 2b(a+b\text{arctanh}(c\sqrt{x}))\operatorname{PolyLog}\left(2,-1+\frac{2}{1-c\sqrt{x}}\right) \\ &\quad + b^2\operatorname{PolyLog}\left(3,1-\frac{2}{1-c\sqrt{x}}\right) \\ &\quad - b^2\operatorname{PolyLog}\left(3,-1+\frac{2}{1-c\sqrt{x}}\right)\end{aligned}$$

output
```
-4*(a+b*arctanh(c*x^(1/2)))^2*arctanh(-1+2/(1-c*x^(1/2)))-2*b*(a+b*arctanh
(c*x^(1/2)))*polylog(2,1-2/(1-c*x^(1/2)))+2*b*(a+b*arctanh(c*x^(1/2)))*pol
ylog(2,-1+2/(1-c*x^(1/2)))+b^2*polylog(3,1-2/(1-c*x^(1/2)))-b^2*polylog(3,
-1+2/(1-c*x^(1/2)))
```

3.199.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.40

$$\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{x}\,dx = a^2\log(x)+2ab\left(-\operatorname{PolyLog}\left(2,-c\sqrt{x}\right)+\operatorname{PolyLog}\left(2,c\sqrt{x}\right)\right) + 2b^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\operatorname{arctanh}\left(c\sqrt{x}\right)^3 - \operatorname{arctanh}\left(c\sqrt{x}\right)^2\log\left(1+e^{-2\operatorname{arctanh}\left(c\sqrt{x}\right)}\right) + \operatorname{arctanh}\left(c\sqrt{x}\right)^2\log\left(1-e^{2\operatorname{arctanh}\left(c\sqrt{x}\right)}\right) + \operatorname{arctanh}\left(c\sqrt{x}\right)\operatorname{PolyLog}\left(2,-e^{-2\operatorname{arctanh}\left(c\sqrt{x}\right)}\right) + \operatorname{arctanh}\left(c\sqrt{x}\right)\operatorname{PolyLog}\left(2,e^{2\operatorname{arctanh}\left(c\sqrt{x}\right)}\right) + \frac{1}{2}\operatorname{PolyLog}\left(3,-e^{-2\operatorname{arctanh}\left(c\sqrt{x}\right)}\right) - \frac{1}{2}\operatorname{PolyLog}\left(3,e^{2\operatorname{arctanh}\left(c\sqrt{x}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x,x]
```

output
```
a^2*Log[x] + 2*a*b*(-PolyLog[2, -(c*Sqrt[x])] + PolyLog[2, c*Sqrt[x]]) + 2
*b^2*((I/24)*Pi^3 - (2*ArcTanh[c*Sqrt[x]]^3)/3 - ArcTanh[c*Sqrt[x]]^2*Log[
1 + E^(-2*ArcTanh[c*Sqrt[x]])] + ArcTanh[c*Sqrt[x]]^2*Log[1 - E^(2*ArcTanh
[c*Sqrt[x]])] + ArcTanh[c*Sqrt[x]]*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])]
+ ArcTanh[c*Sqrt[x]]*PolyLog[2, E^(2*ArcTanh[c*Sqrt[x]])] + PolyLog[3, -E^
(-2*ArcTanh[c*Sqrt[x]])]/2 - PolyLog[3, E^(2*ArcTanh[c*Sqrt[x]])]/2)
```

3.199.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {6450, 6448, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x}dx$$

$\Big\downarrow$ 6450

$$2\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{\sqrt{x}}d\sqrt{x}$$

$\Big\downarrow$ 6448

$$2\left(2\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-4bc\int \frac{\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}\right)$$

$\Big\downarrow$ 6614

$$2\left(2\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-4bc\left(\frac{1}{2}\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\log\left(2-\frac{2}{1-c\sqrt{x}}\right)}{1-c^2x}d\sqrt{x}-\frac{1}{2}\right.\right.$$

$\Big\downarrow$ 6620

$$2\left(2\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{2c}-\right.\right.\right.$$

$\Big\downarrow$ 7164

$$2\left(2\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{2c}-\right.\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x,x]
```

output
```
2*(2*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^2 - 4*b*c*(
(((a + b*ArcTanh[c*Sqrt[x]])*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/(2*c) - (b
*PolyLog[3, 1 - 2/(1 - c*Sqrt[x])])/(4*c))/2 + (-1/2*((a + b*ArcTanh[c*Sqr
t[x]])*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])])/c + (b*PolyLog[3, -1 + 2/(1 - c
*Sqrt[x])])/(4*c))/2))
```

3.199. $\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x}dx$

3.199.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.199. $\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x}\,dx$

3.199.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 562.75 (sec) , antiderivative size = 662, normalized size of antiderivative = 4.57

method	result
parts	$a^2 \ln(x) + b^2 \left(2\ln(c\sqrt{x}) \operatorname{arctanh}(c\sqrt{x})^2 - 2 \operatorname{arctanh}(c\sqrt{x}) \operatorname{polylog}\left(2, -\frac{(1+c\sqrt{x})^2}{-c^2x+1}\right) -\right.$
derivativedivides	$2a^2 \ln(c\sqrt{x}) + 2b^2 \left(\ln(c\sqrt{x}) \operatorname{arctanh}(c\sqrt{x})^2 - \operatorname{arctanh}(c\sqrt{x}) \operatorname{polylog}\left(2, -\frac{(1+c\sqrt{x})^2}{-c^2x+1}\right)\right.$
default	$2a^2 \ln(c\sqrt{x}) + 2b^2 \left(\ln(c\sqrt{x}) \operatorname{arctanh}(c\sqrt{x})^2 - \operatorname{arctanh}(c\sqrt{x}) \operatorname{polylog}\left(2, -\frac{(1+c\sqrt{x})^2}{-c^2x+1}\right)\right.$

input
```
int((a+b*arctanh(c*x^(1/2)))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2*(2*ln(c*x^(1/2))*arctanh(c*x^(1/2))^2-2*arctanh(c*x^(1/2))*p
olylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))+polylog(3,-(1+c*x^(1/2))^2/(-c^2*x+1
))-2*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))^2/(-c^2*x+1)-1)+2*arctanh(c*x^(
1/2))^2*ln(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+4*arctanh(c*x^(1/2))*polylog(
2,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1)^(
1/2))+2*arctanh(c*x^(1/2))^2*ln(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+4*arctan
h(c*x^(1/2))*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-4*polylog(3,(1+c*x^
(1/2))/(-c^2*x+1)^(1/2))+I*Pi*csgn(I*(-(1+c*x^(1/2))^2/(c^2*x-1)-1)/(1-(1+
c*x^(1/2))^2/(c^2*x-1)))*(csgn(I*(-(1+c*x^(1/2))^2/(c^2*x-1)-1))*csgn(I/(1
-(1+c*x^(1/2))^2/(c^2*x-1)))-csgn(I*(-(1+c*x^(1/2))^2/(c^2*x-1)-1))*csgn(I
*(-(1+c*x^(1/2))^2/(c^2*x-1)-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))-csgn(I/(1-(
1+c*x^(1/2))^2/(c^2*x-1)))*csgn(I*(-(1+c*x^(1/2))^2/(c^2*x-1)-1)/(1-(1+c*x
^(1/2))^2/(c^2*x-1)))+csgn(I*(-(1+c*x^(1/2))^2/(c^2*x-1)-1)/(1-(1+c*x^(1/2
))^2/(c^2*x-1)))^2)*arctanh(c*x^(1/2))^2)+2*a*b*(2*ln(c*x^(1/2))*arctanh(c
*x^(1/2))-dilog(1+c*x^(1/2))-ln(c*x^(1/2))*ln(1+c*x^(1/2))-dilog(c*x^(1/2)
))
```

3.199.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*sqrt(x))^2 + 2*a*b*arctanh(c*sqrt(x)) + a^2)/x, x)
```

3.199.6 Sympy [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^2}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x**(1/2)))**2/x,x)
```

output
```
Integral((a + b*atanh(c*sqrt(x)))**2/x, x)
```

3.199.7 Maxima [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="maxima")
```

output
```
1/4*b^2*integrate(log(c*sqrt(x) + 1)^2/x, x) - 1/2*b^2*integrate(log(c*sqr
t(x) + 1)*log(-c*sqrt(x) + 1)/x, x) + 1/4*b^2*integrate(log(-c*sqrt(x) + 1
)^2/x, x) + a*b*integrate(log(c*sqrt(x) + 1)/x, x) - a*b*integrate(log(-c*
sqrt(x) + 1)/x, x) + a^2*log(x)
```

3.199.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^2/x, x)
```

3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(c\,\sqrt{x}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*atanh(c*x^(1/2)))^2/x,x)
```

output
```
int((a + b*atanh(c*x^(1/2)))^2/x, x)
```

3.200 $\int \frac{\left(a+b\,\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$

3.200.1 Optimal result

Integrand size = 18, antiderivative size = 85

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx = -\frac{2bc\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{\sqrt{x}} + c^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2 - \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x} + b^2c^2\log(x) - b^2c^2\log\left(1-c^2x\right)$$

output
```
c^2*(a+b*arctanh(c*x^(1/2)))^2-(a+b*arctanh(c*x^(1/2)))^2/x+b^2*c^2*ln(x)-
b^2*c^2*ln(-c^2*x+1)-2*b*c*(a+b*arctanh(c*x^(1/2)))/x^(1/2)
```

3.200.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.52

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx = -\frac{a^2+2abc\sqrt{x}+2b\left(a+bc\sqrt{x}\right)\text{arctanh}\left(c\sqrt{x}\right)-b^2\left(-1+c^2x\right)\text{arctanh}\left(c\sqrt{x}\right)^2+b(a+b)c^2x\log\left(1-c\sqrt{\right.}}{x}$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x^2,x]
```

output
```
-((a^2 + 2*a*b*c*Sqrt[x] + 2*b*(a + b*c*Sqrt[x])*ArcTanh[c*Sqrt[x]] - b^2*
(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^2 + b*(a + b)*c^2*x*Log[1 - c*Sqrt[x]] - a
*b*c^2*x*Log[1 + c*Sqrt[x]] + b^2*c^2*x*Log[1 + c*Sqrt[x]] - b^2*c^2*x*Log
[x])/x)
```

3.200.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6454, 6452, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}dx$$

$$\Big\downarrow \text{ 6454}$$

$$2\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 6452}$$

$$2\left(bc\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x\left(1-c^2x\right)}d\sqrt{x}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2x}\right)$$

$$\Big\downarrow \text{ 6544}$$

$$2\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{1-c^2x}d\sqrt{x}+\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x}d\sqrt{x}\right)-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2x}\right)$$

$$\Big\downarrow \text{ 6452}$$

$$2\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{1-c^2x}d\sqrt{x}+bc\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}d\sqrt{x}-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}}\right)-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2x}\right)$$

$$\Big\downarrow \text{ 243}$$

$$2\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{1-c^2x}d\sqrt{x}+\frac{1}{2}bc\int \frac{1}{\sqrt{x}\left(1-c^2x\right)}dx-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}}\right)-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2x}\right)$$

$$\Big\downarrow \text{ 47}$$

$$2\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{1-c^2x}d\sqrt{x}+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x}dx+\int \frac{1}{\sqrt{x}}dx\right)-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}}\right)-\frac{(a+b\text{arcta}}{2x}\right.$$

$\downarrow$ 14

$$2\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{1-c^2x}d\sqrt{x}+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x}dx+\log(x)\right)-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}}\right)-\frac{(a+b\text{arctanh}}{2x}\right.$$

$\downarrow$ 16

$$2\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{1-c^2x}d\sqrt{x}-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}}+\frac{1}{2}bc\left(\log(x)-\log\left(1-c^2x\right)\right)\right)-\frac{(a+b\text{arctanh}(c\sqrt{\cdot}}{2x}\right.$$

$\downarrow$ 6510

$$2\left(bc\left(\frac{c\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2b}-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}}+\frac{1}{2}bc\left(\log(x)-\log\left(1-c^2x\right)\right)\right)-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2x}\right)$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x^2,x]
```

output
```
2*(-1/2*(a + b*ArcTanh[c*Sqrt[x]])^2/x + b*c*(-((a + b*ArcTanh[c*Sqrt[x]])
/Sqrt[x]) + (c*(a + b*ArcTanh[c*Sqrt[x]])^2)/(2*b) + (b*c*(Log[x] - Log[1
- c^2*x]))/2))
```

3.200.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.200.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(77) = 154$.

Time = 1.00 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.72

method	result
parts	$-\frac{a^2}{x}+2b^2c^2\left(-\frac{\operatorname{arctanh}(c\sqrt{x})^2}{2c^2x}-\frac{\operatorname{arctanh}(c\sqrt{x})}{c\sqrt{x}}-\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{2}+\frac{\operatorname{arctanh}(c\sqrt{x})\ln(1+c\sqrt{x})}{2}\right.$
derivativedivides	$2c^2\left(-\frac{a^2}{2c^2x}+b^2\left(-\frac{\operatorname{arctanh}(c\sqrt{x})^2}{2c^2x}-\frac{\operatorname{arctanh}(c\sqrt{x})}{c\sqrt{x}}-\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{2}+\frac{\operatorname{arctanh}(c\sqrt{x})\ln(1+c}{2}\right.\right.$
default	$2c^2\left(-\frac{a^2}{2c^2x}+b^2\left(-\frac{\operatorname{arctanh}(c\sqrt{x})^2}{2c^2x}-\frac{\operatorname{arctanh}(c\sqrt{x})}{c\sqrt{x}}-\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{2}+\frac{\operatorname{arctanh}(c\sqrt{x})\ln(1+c}{2}\right.\right.$

input
```
int((a+b*arctanh(c*x^(1/2)))^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a^2/x+2*b^2*c^2*(-1/2/c^2/x*arctanh(c*x^(1/2))^2-arctanh(c*x^(1/2))/c/x^(
1/2)-1/2*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)+1/2*arctanh(c*x^(1/2))*ln(1+c*
x^(1/2))-1/8*ln(c*x^(1/2)-1)^2+1/4*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)-1
/2*ln(c*x^(1/2)-1)-1/2*ln(1+c*x^(1/2))+ln(c*x^(1/2))+1/4*(ln(1+c*x^(1/2))-
ln(1/2*c*x^(1/2)+1/2))*ln(-1/2*c*x^(1/2)+1/2)-1/8*ln(1+c*x^(1/2))^2)+4*a*b
*c^2*(-1/2/c^2/x*arctanh(c*x^(1/2))-1/2/c/x^(1/2)-1/4*ln(c*x^(1/2)-1)+1/4*
ln(1+c*x^(1/2)))
```

3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(77) = 154$.

Time = 0.27 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.85

$$\int \frac{\left(a + b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}\, dx$$
$$= \frac{8\,b^2c^2x\log\left(\sqrt{x}\right) + 4\,(ab - b^2)c^2x\log\left(c\sqrt{x} + 1\right) - 4\,(ab + b^2)c^2x\log\left(c\sqrt{x} - 1\right) - 8\,abc\sqrt{x} + (b^2c^2x - b^2)}{4\,x}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="fricas")
```

output
```
1/4*(8*b^2*c^2*x*log(sqrt(x)) + 4*(a*b - b^2)*c^2*x*log(c*sqrt(x) + 1) - 4
*(a*b + b^2)*c^2*x*log(c*sqrt(x) - 1) - 8*a*b*c*sqrt(x) + (b^2*c^2*x - b^2
)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 - 4*a^2 - 4*(b^2*c*sqrt(x)
 + a*b)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1)))/x
```

3.200.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(78) = 156$.

Time = 2.62 (sec) , antiderivative size = 680, normalized size of antiderivative = 8.00

$$\int \frac{\left(a + b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$$
$$= \begin{cases} -\frac{a^2}{x} \\ -\frac{a^2}{x} + \frac{2ab\operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} - \frac{b^2\operatorname{atanh}^2\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} \\ -\frac{a^2}{x} - \frac{2ab\operatorname{atanh}\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} - \frac{b^2\operatorname{atanh}^2\left(\sqrt{x}\sqrt{\frac{1}{x}}\right)}{x} \\ -\frac{a^2c^2x^{\frac{3}{2}}}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{a^2\sqrt{x}}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{2abc^4x^{\frac{5}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} - \frac{2abc^3x^2}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} - \frac{4abc^2x^{\frac{3}{2}}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{2abcx}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} + \frac{2ab\sqrt{x}\operatorname{atanh}\left(c\sqrt{x}\right)}{c^2x^{\frac{5}{2}}-x^{\frac{3}{2}}} + b^2 \end{cases}$$

input
```
integrate((a+b*atanh(c*x**(1/2)))**2/x**2,x)
```

output
```
Piecewise((-a**2/x, Eq(c, 0)), (-a**2/x + 2*a*b*atanh(sqrt(x)*sqrt(1/x))/x
 - b**2*atanh(sqrt(x)*sqrt(1/x))**2/x, Eq(c, -sqrt(1/x))), (-a**2/x - 2*a*
b*atanh(sqrt(x)*sqrt(1/x))/x - b**2*atanh(sqrt(x)*sqrt(1/x))**2/x, Eq(c, s
qrt(1/x))), (-a**2*c**2*x**(3/2)/(c**2*x**(5/2) - x**(3/2)) + a**2*sqrt(x)
/(c**2*x**(5/2) - x**(3/2)) + 2*a*b*c**4*x**(5/2)*atanh(c*sqrt(x))/(c**2*x
**(5/2) - x**(3/2)) - 2*a*b*c**3*x**2/(c**2*x**(5/2) - x**(3/2)) - 4*a*b*c
**2*x**(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + 2*a*b*c*x/(c**2
*x**(5/2) - x**(3/2)) + 2*a*b*sqrt(x)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x*
*(3/2)) + b**2*c**4*x**(5/2)*log(x)/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c*
*4*x**(5/2)*log(sqrt(x) - 1/c)/(c**2*x**(5/2) - x**(3/2)) + b**2*c**4*x**(
5/2)*atanh(c*sqrt(x))**2/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**4*x**(5/2)
*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) - 2*b**2*c**3*x**2*atanh(c*sq
rt(x))/(c**2*x**(5/2) - x**(3/2)) - b**2*c**2*x**(3/2)*log(x)/(c**2*x**(5/
2) - x**(3/2)) + 2*b**2*c**2*x**(3/2)*log(sqrt(x) - 1/c)/(c**2*x**(5/2) -
x**(3/2)) - 2*b**2*c**2*x**(3/2)*atanh(c*sqrt(x))**2/(c**2*x**(5/2) - x**(
3/2)) + 2*b**2*c**2*x**(3/2)*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) +
 2*b**2*c*x*atanh(c*sqrt(x))/(c**2*x**(5/2) - x**(3/2)) + b**2*sqrt(x)*ata
nh(c*sqrt(x))**2/(c**2*x**(5/2) - x**(3/2)), True))
```

3.200. $\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$

3.200.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(77) = 154$.

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.05

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx$$
$$= \left(\left(c\log\left(c\sqrt{x}+1\right)-c\log\left(c\sqrt{x}-1\right)-\frac{2}{\sqrt{x}}\right)c-\frac{2\,\text{artanh}\left(c\sqrt{x}\right)}{x}\right)ab$$
$$+\frac{1}{4}\left(\left(2\left(\log\left(c\sqrt{x}-1\right)-2\right)\log\left(c\sqrt{x}+1\right)-\log\left(c\sqrt{x}+1\right)^2-\log\left(c\sqrt{x}-1\right)^2-4\log\left(c\sqrt{x}-1\right)+4\,\text{lo}\right.\right.$$
$$-\frac{b^2\,\text{artanh}\left(c\sqrt{x}\right)^2}{x}-\frac{a^2}{x}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="maxima")
```

output
```
((c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c - 2*arctanh(c
*sqrt(x))/x)*a*b + 1/4*((2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - l
og(c*sqrt(x) + 1)^2 - log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1) + 4*log(
x))*c^2 + 4*(c*log(c*sqrt(x) + 1) - c*log(c*sqrt(x) - 1) - 2/sqrt(x))*c*ar
ctanh(c*sqrt(x)))*b^2 - b^2*arctanh(c*sqrt(x))^2/x - a^2/x
```

3.200.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx = \int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^2/x^2, x)
```

3.200.9 Mupad [B] (verification not implemented)

Time = 4.23 (sec) , antiderivative size = 278, normalized size of antiderivative = 3.27

$$\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}\,dx = 2\,b^2\,c^2\,\ln\left(\sqrt{x}\right)-\frac{a^2}{x}-b^2\,c^2\,\ln\left(c\,\sqrt{x}-1\right)-b^2\,c^2\,\ln\left(c\,\sqrt{x}+1\right)+\frac{b^2\,c^2\ln\left(c\,\sqrt{x}+1\right)^2}{4}+\frac{b^2\,c^2\ln\left(1-c\,\sqrt{x}\right)^2}{4}-\frac{b^2\ln\left(c\,\sqrt{x}+1\right)^2}{4\,x}-\frac{b^2\ln\left(1-c\,\sqrt{x}\right)^2}{4\,x}-a\,b\,c^2\,\ln\left(c\,\sqrt{x}-1\right)+a\,b\,c^2\,\ln\left(c\,\sqrt{x}+1\right)-\frac{2\,a\,b\,c}{\sqrt{x}}-\frac{a\,b\,\ln\left(c\,\sqrt{x}+1\right)}{x}+\frac{a\,b\,\ln\left(1-c\,\sqrt{x}\right)}{x}-\frac{b^2\,c^2\,\ln\left(c\,\sqrt{x}+1\right)\,\ln\left(1-c\,\sqrt{x}\right)}{2}-\frac{b^2\,c\,\ln\left(c\,\sqrt{x}+1\right)}{\sqrt{x}}+\frac{b^2\,c\,\ln\left(1-c\,\sqrt{x}\right)}{\sqrt{x}}+\frac{b^2\,\ln\left(c\,\sqrt{x}+1\right)\,\ln\left(1-c\,\sqrt{x}\right)}{2\,x}$$

input
```
int((a + b*atanh(c*x^(1/2)))^2/x^2,x)
```

output
```
2*b^2*c^2*log(x^(1/2)) - a^2/x - b^2*c^2*log(c*x^(1/2) - 1) - b^2*c^2*log(
c*x^(1/2) + 1) + (b^2*c^2*log(c*x^(1/2) + 1)^2)/4 + (b^2*c^2*log(1 - c*x^(
1/2))^2)/4 - (b^2*log(c*x^(1/2) + 1)^2)/(4*x) - (b^2*log(1 - c*x^(1/2))^2)
/(4*x) - a*b*c^2*log(c*x^(1/2) - 1) + a*b*c^2*log(c*x^(1/2) + 1) - (2*a*b*
c)/x^(1/2) - (a*b*log(c*x^(1/2) + 1))/x + (a*b*log(1 - c*x^(1/2)))/x - (b^
2*c^2*log(c*x^(1/2) + 1)*log(1 - c*x^(1/2)))/2 - (b^2*c*log(c*x^(1/2) + 1)
)/x^(1/2) + (b^2*c*log(1 - c*x^(1/2)))/x^(1/2) + (b^2*log(c*x^(1/2) + 1)*l
og(1 - c*x^(1/2)))/(2*x)
```

3.201 $\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$

3.201.1 Optimal result

Integrand size = 18, antiderivative size = 133

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx = -\frac{b^2c^2}{6x}-\frac{bc\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{3x^{3/2}}-\frac{bc^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{\sqrt{x}} +\frac{1}{2}c^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2x^2} +\frac{2}{3}b^2c^4\log(x)-\frac{2}{3}b^2c^4\log\left(1-c^2x\right)$$

output
```
-1/6*b^2*c^2/x-1/3*b*c*(a+b*arctanh(c*x^(1/2)))/x^(3/2)+1/2*c^4*(a+b*arcta
nh(c*x^(1/2)))^2-1/2*(a+b*arctanh(c*x^(1/2)))^2/x^2+2/3*b^2*c^4*ln(x)-2/3*
b^2*c^4*ln(-c^2*x+1)-b*c^3*(a+b*arctanh(c*x^(1/2)))/x^(1/2)
```

3.201.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.34

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx = -\frac{3a^2+2abc\sqrt{x}+b^2c^2x+6abc^3x^{3/2}+2b\left(3a+bc\sqrt{x}\left(1+3c^2x\right)\right)\text{arctanh}\left(c\sqrt{x}\right)-3b^2\left(-1+c^4x^2\right)\text{arctan}}{}$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])^2/x^3,x]
```

output
```
-1/6*(3*a^2 + 2*a*b*c*Sqrt[x] + b^2*c^2*x + 6*a*b*c^3*x^(3/2) + 2*b*(3*a +
 b*c*Sqrt[x]*(1 + 3*c^2*x))*ArcTanh[c*Sqrt[x]] - 3*b^2*(-1 + c^4*x^2)*ArcT
anh[c*Sqrt[x]]^2 + b*(3*a + 4*b)*c^4*x^2*Log[1 - c*Sqrt[x]] - 3*a*b*c^4*x^
2*Log[1 + c*Sqrt[x]] + 4*b^2*c^4*x^2*Log[1 + c*Sqrt[x]] - 4*b^2*c^4*x^2*Lo
g[x])/x^2
```

3.201.3 Rubi [A] (warning: unable to verify)

Time = 1.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {6454, 6452, 6544, 6452, 243, 54, 2009, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^3}dx$$

$$\Big\downarrow \textbf{6454}$$

$$2\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \textbf{6452}$$

$$2\left(\frac{1}{2}bc\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x^2\left(1-c^2x\right)}d\sqrt{x}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{4x^2}\right)$$

$$\Big\downarrow \textbf{6544}$$

$$2\left(\frac{1}{2}bc\left(c^2\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x\left(1-c^2x\right)}d\sqrt{x}+\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x^2}d\sqrt{x}\right)-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{4x^2}\right)$$

$$\Big\downarrow \textbf{6452}$$

$$2\left(\frac{1}{2}bc\left(c^2\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x\left(1-c^2x\right)}d\sqrt{x}+\frac{1}{3}bc\int \frac{1}{x^{3/2}\left(1-c^2x\right)}d\sqrt{x}-\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{3x^{3/2}}\right)-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{4x^2}\right)$$

$$\Big\downarrow \textbf{243}$$

$$2\left(\frac{1}{2}bc\left(c^2\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x(1-c^2x)}d\sqrt{x}+\frac{1}{6}bc\int \frac{1}{x(1-c^2x)}dx-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^{3/2}}\right)-\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{4x^2}\right.$$

$$\Big\downarrow \text{54}$$

$$2\left(\frac{1}{2}bc\left(c^2\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x(1-c^2x)}d\sqrt{x}+\frac{1}{6}bc\int \left(-\frac{c^4}{c^2x-1}+\frac{c^2}{\sqrt{x}}+\frac{1}{x}\right)dx-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^{3/2}}\right)-\frac{(a+b\text{arct}}{4}\right.$$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{2}bc\left(c^2\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x(1-c^2x)}d\sqrt{x}-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^{3/2}}+\frac{1}{6}bc\left(c^2\log(x)-c^2\log(1-c^2x)-\frac{1}{\sqrt{x}}\right)\right)-\frac{(a+}{}\right.$$

$$\Big\downarrow \text{6544}$$

$$2\left(\frac{1}{2}bc\left(c^2\left(c^2\int \frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}+\int \frac{a+b\text{arctanh}(c\sqrt{x})}{x}d\sqrt{x}\right)-\frac{a+b\text{arctanh}(c\sqrt{x})}{3x^{3/2}}+\frac{1}{6}bc\left(c^2\log(x)\right.\right.\right.$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{2}bc\left(c^2\left(c^2\int \frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}+bc\int \frac{1}{\sqrt{x}(1-c^2x)}d\sqrt{x}-\frac{a+b\text{arctanh}(c\sqrt{x})}{\sqrt{x}}\right)-\frac{a+b\text{arctanh}(c\sqrt{x}}{3x^{3/2}}\right.\right.$$

$$\Big\downarrow \text{243}$$

$$2\left(\frac{1}{2}bc\left(c^2\left(c^2\int \frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}+\frac{1}{2}bc\int \frac{1}{\sqrt{x}(1-c^2x)}dx-\frac{a+b\text{arctanh}(c\sqrt{x})}{\sqrt{x}}\right)-\frac{a+b\text{arctanh}(c\sqrt{x}}{3x^{3/2}}\right.\right.$$

$$\Big\downarrow \text{47}$$

$$2\left(\frac{1}{2}bc\left(c^2\left(c^2\int \frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x}dx+\int \frac{1}{\sqrt{x}}dx\right)-\frac{a+b\text{arctanh}(c\sqrt{x})}{\sqrt{x}}\right)-\frac{a+ba}{}\right.\right.$$

$$\Big\downarrow \text{14}$$

$$2\left(\frac{1}{2}bc\left(c^2\left(c^2\int \frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x}dx+\log(x)\right)-\frac{a+b\text{arctanh}(c\sqrt{x})}{\sqrt{x}}\right)-\frac{a+b\text{arct}}{3x}\right.\right.$$

$$\Big\downarrow \text{16}$$

3.201. $\int \frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{x^3}dx$

$$2\left(\frac{1}{2}bc\left(c^2\left(c^2\int\frac{a+b\text{arctanh}(c\sqrt{x})}{1-c^2x}d\sqrt{x}-\frac{a+b\text{arctanh}(c\sqrt{x})}{\sqrt{x}}+\frac{1}{2}bc(\log(x)-\log(1-c^2x))\right)-\frac{a+b\text{arctanh}}{3x^{3/2}}\right.\right.$$

$$\Big\downarrow \text{6510}$$

$$2\left(\frac{1}{2}bc\left(c^2\left(\frac{c(a+b\text{arctanh}(c\sqrt{x}))^2}{2b}-\frac{a+b\text{arctanh}(c\sqrt{x})}{\sqrt{x}}+\frac{1}{2}bc(\log(x)-\log(1-c^2x))\right)-\frac{a+b\text{arctanh}(c\sqrt{x}}{3x^{3/2}}\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])^2/x^3,x]
```

output
```
2*(-1/4*(a + b*ArcTanh[c*Sqrt[x]])^2/x^2 + (b*c*(-1/3*(a + b*ArcTanh[c*Sqr
t[x]])/x^(3/2) + c^2*(-((a + b*ArcTanh[c*Sqrt[x]])/Sqrt[x]) + (c*(a + b*Ar
cTanh[c*Sqrt[x]])^2)/(2*b) + (b*c*(Log[x] - Log[1 - c^2*x]))/2) + (b*c*(-(
1/Sqrt[x]) + c^2*Log[x] - c^2*Log[1 - c^2*x]))/6))/2)
```

3.201.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

3.201. $\int\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{x^3}\,dx$

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.201.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(109) = 218$.

Time = 0.99 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.98

method	result
parts	$-\frac{a^2}{2x^2}+2b^2c^4\left(-\frac{\operatorname{arctanh}(c\sqrt{x})^2}{4c^4x^2}-\frac{\operatorname{arctanh}(c\sqrt{x})}{6c^3x^{\frac{3}{2}}}-\frac{\operatorname{arctanh}(c\sqrt{x})}{2c\sqrt{x}}-\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{4}+\right.$
derivativedivides	$2c^4\left(-\frac{a^2}{4c^4x^2}+b^2\left(-\frac{\operatorname{arctanh}(c\sqrt{x})^2}{4c^4x^2}-\frac{\operatorname{arctanh}(c\sqrt{x})}{6c^3x^{\frac{3}{2}}}-\frac{\operatorname{arctanh}(c\sqrt{x})}{2c\sqrt{x}}-\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{4}+\right.\right.$
default	$2c^4\left(-\frac{a^2}{4c^4x^2}+b^2\left(-\frac{\operatorname{arctanh}(c\sqrt{x})^2}{4c^4x^2}-\frac{\operatorname{arctanh}(c\sqrt{x})}{6c^3x^{\frac{3}{2}}}-\frac{\operatorname{arctanh}(c\sqrt{x})}{2c\sqrt{x}}-\frac{\operatorname{arctanh}(c\sqrt{x})\ln(c\sqrt{x}-1)}{4}+\right.\right.$

3.201. $\int \frac{\left(a+b\operatorname{arctanh}(c\sqrt{x})\right)^2}{x^3}\,dx$

input
```
int((a+b*arctanh(c*x^(1/2)))^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2/x^2+2*b^2*c^4*(-1/4/c^4/x^2*arctanh(c*x^(1/2))^2-1/6*arctanh(c*x^
(1/2))/c^3/x^(3/2)-1/2*arctanh(c*x^(1/2))/c/x^(1/2)-1/4*arctanh(c*x^(1/2))
*ln(c*x^(1/2)-1)+1/4*arctanh(c*x^(1/2))*ln(1+c*x^(1/2))+1/8*ln(c*x^(1/2)-1
)*ln(1/2*c*x^(1/2)+1/2)-1/16*ln(c*x^(1/2)-1)^2+1/8*(ln(1+c*x^(1/2))-ln(1/2
*c*x^(1/2)+1/2))*ln(-1/2*c*x^(1/2)+1/2)-1/16*ln(1+c*x^(1/2))^2-1/12/c^2/x+
2/3*ln(c*x^(1/2))-1/3*ln(c*x^(1/2)-1)-1/3*ln(1+c*x^(1/2)))+4*a*b*c^4*(-1/4
/c^4/x^2*arctanh(c*x^(1/2))+1/8*ln(1+c*x^(1/2))-1/12/c^3/x^(3/2)-1/4/c/x^(
1/2)-1/8*ln(c*x^(1/2)-1))
```

3.201.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.51

$$\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$$
$$= \frac{32\,b^2c^4x^2\log\left(\sqrt{x}\right)+4\,(3\,ab-4\,b^2)c^4x^2\log\left(c\sqrt{x}+1\right)-4\,(3\,ab+4\,b^2)c^4x^2\log\left(c\sqrt{x}-1\right)-4\,b^2c^2x+3\,(b}{}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="fricas")
```

output
```
1/24*(32*b^2*c^4*x^2*log(sqrt(x)) + 4*(3*a*b - 4*b^2)*c^4*x^2*log(c*sqrt(x
) + 1) - 4*(3*a*b + 4*b^2)*c^4*x^2*log(c*sqrt(x) - 1) - 4*b^2*c^2*x + 3*(b
^2*c^4*x^2 - b^2)*log(-(c^2*x + 2*c*sqrt(x) + 1)/(c^2*x - 1))^2 - 12*a^2 -
 4*(3*a*b + (3*b^2*c^3*x + b^2*c)*sqrt(x))*log(-(c^2*x + 2*c*sqrt(x) + 1)/
(c^2*x - 1)) - 8*(3*a*b*c^3*x + a*b*c)*sqrt(x))/x^2
```

3.201.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(122) = 244$.

Time = 7.30 (sec) , antiderivative size = 972, normalized size of antiderivative = 7.31

$$\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*atanh(c*x**(1/2)))**2/x**3,x)
```

output
```
Piecewise((-a**2/(2*x**2), Eq(c, 0)), (-a**2/(2*x**2) + a*b*atanh(sqrt(x)*
sqrt(1/x))/x**2 - b**2*atanh(sqrt(x)*sqrt(1/x))**2/(2*x**2), Eq(c, -sqrt(1
/x))), (-a**2/(2*x**2) - a*b*atanh(sqrt(x)*sqrt(1/x))/x**2 - b**2*atanh(sq
rt(x)*sqrt(1/x))**2/(2*x**2), Eq(c, sqrt(1/x))), (-3*a**2*c**2*x**(3/2)/(6
*c**2*x**(7/2) - 6*x**(5/2)) + 3*a**2*sqrt(x)/(6*c**2*x**(7/2) - 6*x**(5/2
)) + 6*a*b*c**6*x**(7/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) -
 6*a*b*c**5*x**3/(6*c**2*x**(7/2) - 6*x**(5/2)) - 6*a*b*c**4*x**(5/2)*atan
h(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*a*b*c**3*x**2/(6*c**2*x**(
7/2) - 6*x**(5/2)) - 6*a*b*c**2*x**(3/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2)
 - 6*x**(5/2)) + 2*a*b*c*x/(6*c**2*x**(7/2) - 6*x**(5/2)) + 6*a*b*sqrt(x)*
atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*b**2*c**6*x**(7/2)*log
(x)/(6*c**2*x**(7/2) - 6*x**(5/2)) - 8*b**2*c**6*x**(7/2)*log(sqrt(x) - 1/
c)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 3*b**2*c**6*x**(7/2)*atanh(c*sqrt(x))*
*2/(6*c**2*x**(7/2) - 6*x**(5/2)) - 8*b**2*c**6*x**(7/2)*atanh(c*sqrt(x))/
(6*c**2*x**(7/2) - 6*x**(5/2)) - 6*b**2*c**5*x**3*atanh(c*sqrt(x))/(6*c**2
*x**(7/2) - 6*x**(5/2)) - 4*b**2*c**4*x**(5/2)*log(x)/(6*c**2*x**(7/2) - 6
*x**(5/2)) + 8*b**2*c**4*x**(5/2)*log(sqrt(x) - 1/c)/(6*c**2*x**(7/2) - 6*
x**(5/2)) - 3*b**2*c**4*x**(5/2)*atanh(c*sqrt(x))**2/(6*c**2*x**(7/2) - 6*
x**(5/2)) + 8*b**2*c**4*x**(5/2)*atanh(c*sqrt(x))/(6*c**2*x**(7/2) - 6*x**
(5/2)) - b**2*c**4*x**(5/2)/(6*c**2*x**(7/2) - 6*x**(5/2)) + 4*b**2*c**...
```

3.201.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(109) = 218$.

Time = 0.22 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.76

$$\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx$$
$$= \frac{1}{6}\left(\left(3\,c^3\log\left(c\sqrt{x}+1\right)-3\,c^3\log\left(c\sqrt{x}-1\right)-\frac{2\,(3\,c^2x+1)}{x^{\frac{3}{2}}}\right)c-\frac{6\,\operatorname{artanh}\left(c\sqrt{x}\right)}{x^2}\right)ab$$
$$+\frac{1}{24}\left(\left(16\,c^2\log\left(x\right)-\frac{3\,c^2x\log\left(c\sqrt{x}+1\right)^2+3\,c^2x\log\left(c\sqrt{x}-1\right)^2+16\,c^2x\log\left(c\sqrt{x}-1\right)-2\left(3\,c^2x\log\right.}{x}\right.\right.$$
$$-\frac{b^2\,\operatorname{artanh}\left(c\sqrt{x}\right)^2}{2\,x^2}-\frac{a^2}{2\,x^2}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="maxima")
```

output
```
1/6*((3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*(3*c^2*x + 1
)/x^(3/2))*c - 6*arctanh(c*sqrt(x))/x^2)*a*b + 1/24*((16*c^2*log(x) - (3*c
^2*x*log(c*sqrt(x) + 1)^2 + 3*c^2*x*log(c*sqrt(x) - 1)^2 + 16*c^2*x*log(c*
sqrt(x) - 1) - 2*(3*c^2*x*log(c*sqrt(x) - 1) - 8*c^2*x)*log(c*sqrt(x) + 1)
 + 4)/x)*c^2 + 4*(3*c^3*log(c*sqrt(x) + 1) - 3*c^3*log(c*sqrt(x) - 1) - 2*
(3*c^2*x + 1)/x^(3/2))*c*arctanh(c*sqrt(x)))*b^2 - 1/2*b^2*arctanh(c*sqrt(
x))^2/x^2 - 1/2*a^2/x^2
```

3.201.8 Giac [F]

$$\int \frac{\left(a + b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx = \int \frac{\left(b\operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^2}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^2/x^3, x)
```

3.201.9 Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.56

$$\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^3}\,dx = \frac{4\,b^2\,c^4\,\ln\left(\sqrt{x}\right)}{3}-\frac{a^2}{2\,x^2}-\frac{2\,b^2\,c^4\,\ln\left(c\,\sqrt{x}-1\right)}{3}-\frac{2\,b^2\,c^4\,\ln\left(c\,\sqrt{x}+1\right)}{3}+\frac{b^2\,c^4\ln\left(c\,\sqrt{x}+1\right)^2}{8}+\frac{b^2\,c^4\ln\left(1-c\,\sqrt{x}\right)^2}{8}-\frac{b^2\,c^2}{6\,x}-\frac{b^2\ln\left(c\,\sqrt{x}+1\right)^2}{8\,x^2}-\frac{b^2\ln\left(1-c\,\sqrt{x}\right)^2}{8\,x^2}-\frac{b^2\,c^3\,\ln\left(c\,\sqrt{x}+1\right)}{2\,\sqrt{x}}+\frac{b^2\,c^3\,\ln\left(1-c\,\sqrt{x}\right)}{2\,\sqrt{x}}-\frac{a\,b\,c^4\,\ln\left(c\,\sqrt{x}-1\right)}{2}+\frac{a\,b\,c^4\,\ln\left(c\,\sqrt{x}+1\right)}{2}-\frac{a\,b\,c}{3\,x^{3/2}}-\frac{a\,b\,\ln\left(c\,\sqrt{x}+1\right)}{2\,x^2}+\frac{a\,b\,\ln\left(1-c\,\sqrt{x}\right)}{2\,x^2}-\frac{b^2\,c^4\,\ln\left(c\,\sqrt{x}+1\right)\,\ln\left(1-c\,\sqrt{x}\right)}{4}-\frac{a\,b\,c^3}{\sqrt{x}}-\frac{b^2\,c\,\ln\left(c\,\sqrt{x}+1\right)}{6\,x^{3/2}}+\frac{b^2\,c\,\ln\left(1-c\,\sqrt{x}\right)}{6\,x^{3/2}}+\frac{b^2\,\ln\left(c\,\sqrt{x}+1\right)\,\ln\left(1-c\,\sqrt{x}\right)}{4\,x^2}$$

input
```
int((a + b*atanh(c*x^(1/2)))^2/x^3,x)
```

output
```
(4*b^2*c^4*log(x^(1/2)))/3 - a^2/(2*x^2) - (2*b^2*c^4*log(c*x^(1/2) - 1))/
3 - (2*b^2*c^4*log(c*x^(1/2) + 1))/3 + (b^2*c^4*log(c*x^(1/2) + 1)^2)/8 +
(b^2*c^4*log(1 - c*x^(1/2))^2)/8 - (b^2*c^2)/(6*x) - (b^2*log(c*x^(1/2) +
1)^2)/(8*x^2) - (b^2*log(1 - c*x^(1/2))^2)/(8*x^2) - (b^2*c^3*log(c*x^(1/2
) + 1))/(2*x^(1/2)) + (b^2*c^3*log(1 - c*x^(1/2)))/(2*x^(1/2)) - (a*b*c^4*
log(c*x^(1/2) - 1))/2 + (a*b*c^4*log(c*x^(1/2) + 1))/2 - (a*b*c)/(3*x^(3/2
)) - (a*b*log(c*x^(1/2) + 1))/(2*x^2) + (a*b*log(1 - c*x^(1/2)))/(2*x^2) -
 (b^2*c^4*log(c*x^(1/2) + 1)*log(1 - c*x^(1/2)))/4 - (a*b*c^3)/x^(1/2) - (
b^2*c*log(c*x^(1/2) + 1))/(6*x^(3/2)) + (b^2*c*log(1 - c*x^(1/2)))/(6*x^(3
/2)) + (b^2*log(c*x^(1/2) + 1)*log(1 - c*x^(1/2)))/(4*x^2)
```

3.202 $\int x^3 (a + b\,\text{arctanh}(c\sqrt{x}))^3 \, dx$

3.202.1 Optimal result

Integrand size = 18, antiderivative size = 374

$$\int x^3 (a+b\,\text{arctanh}(c\sqrt{x}))^3 \, dx = \frac{47b^3\sqrt{x}}{70c^7} + \frac{23b^3x^{3/2}}{420c^5} + \frac{b^3x^{5/2}}{140c^3} - \frac{47b^3\text{arctanh}(c\sqrt{x})}{70c^8} + \frac{71b^2x(a+b\,\text{arctanh}(c\sqrt{x}))}{140c^6} + \frac{9b^2x^2(a+b\,\text{arctanh}(c\sqrt{x}))}{70c^4} + \frac{b^2x^3(a+b\,\text{arctanh}(c\sqrt{x}))}{28c^2} + \frac{44b(a+b\,\text{arctanh}(c\sqrt{x}))^2}{35c^8} + \frac{3b\sqrt{x}(a+b\,\text{arctanh}(c\sqrt{x}))^2}{4c^7} + \frac{bx^{3/2}(a+b\,\text{arctanh}(c\sqrt{x}))^2}{4c^5} + \frac{3bx^{5/2}(a+b\,\text{arctanh}(c\sqrt{x}))^2}{20c^3} + \frac{3bx^{7/2}(a+b\,\text{arctanh}(c\sqrt{x}))^2}{28c} - \frac{(a+b\,\text{arctanh}(c\sqrt{x}))^3}{4c^8} + \frac{1}{4}x^4(a+b\,\text{arctanh}(c\sqrt{x}))^3 - \frac{88b^2(a+b\,\text{arctanh}(c\sqrt{x}))\log\left(\frac{2}{1-c\sqrt{x}}\right)}{35c^8} - \frac{44b^3\,\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{35c^8}$$

output
```
23/420*b^3*x^(3/2)/c^5+1/140*b^3*x^(5/2)/c^3-47/70*b^3*arctanh(c*x^(1/2))/
c^8+71/140*b^2*x*(a+b*arctanh(c*x^(1/2)))/c^6+9/70*b^2*x^2*(a+b*arctanh(c*
x^(1/2)))/c^4+1/28*b^2*x^3*(a+b*arctanh(c*x^(1/2)))/c^2+44/35*b*(a+b*arcta
nh(c*x^(1/2)))^2/c^8+1/4*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))^2/c^5+3/20*b*x
^(5/2)*(a+b*arctanh(c*x^(1/2)))^2/c^3+3/28*b*x^(7/2)*(a+b*arctanh(c*x^(1/2
)))^2/c-1/4*(a+b*arctanh(c*x^(1/2)))^3/c^8+1/4*x^4*(a+b*arctanh(c*x^(1/2))
)^3-88/35*b^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^8-44/35*b^3*p
olylog(2,1-2/(1-c*x^(1/2)))/c^8+47/70*b^3*x^(1/2)/c^7+3/4*b*(a+b*arctanh(c
*x^(1/2)))^2*x^(1/2)/c^7
```

3.202.2 Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.12

$$\int x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$$
$$=\frac{-564ab^2+630a^2bc\sqrt{x}+564b^3c\sqrt{x}+426ab^2c^2x+210a^2bc^3x^{3/2}+46b^3c^3x^{3/2}+108ab^2c^4x^2+126a^2bc^5x^5}{}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

output
```
(-564*a*b^2 + 630*a^2*b*c*Sqrt[x] + 564*b^3*c*Sqrt[x] + 426*a*b^2*c^2*x +
210*a^2*b*c^3*x^(3/2) + 46*b^3*c^3*x^(3/2) + 108*a*b^2*c^4*x^2 + 126*a^2*b
*c^5*x^(5/2) + 6*b^3*c^5*x^(5/2) + 30*a*b^2*c^6*x^3 + 90*a^2*b*c^7*x^(7/2)
 + 210*a^3*c^8*x^4 + 6*b^2*(b*(-176 + 105*c*Sqrt[x] + 35*c^3*x^(3/2) + 21*
c^5*x^(5/2) + 15*c^7*x^(7/2)) + 105*a*(-1 + c^8*x^4))*ArcTanh[c*Sqrt[x]]^2
 + 210*b^3*(-1 + c^8*x^4)*ArcTanh[c*Sqrt[x]]^3 + 6*b*ArcTanh[c*Sqrt[x]]*(1
05*a^2*c^8*x^4 + b^2*(-94 + 71*c^2*x + 18*c^4*x^2 + 5*c^6*x^3) + 2*a*b*c*S
qrt[x]*(105 + 35*c^2*x + 21*c^4*x^2 + 15*c^6*x^3) - 352*b^2*Log[1 + E^(-2*
ArcTanh[c*Sqrt[x]])]) + 315*a^2*b*Log[1 - c*Sqrt[x]] - 315*a^2*b*Log[1 + c
*Sqrt[x]] + 1056*a*b^2*Log[1 - c^2*x] + 1056*b^3*PolyLog[2, -E^(-2*ArcTanh
[c*Sqrt[x]])])/(840*c^8)
```

3.202.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 949 vs. 2(374) = 748.

Time = 5.46 (sec) , antiderivative size = 949, normalized size of antiderivative = 2.54, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.222, Rules used = {6454, 6452, 6542, 6452, 6542, 6452, 254, 2009, 6542, 6452, 254, 2009, 6542, 6436, 6452, 262, 219, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$$
$$\Big\downarrow \text{6454}$$
$$2\int x^{7/2}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,d\sqrt{x}$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\int \frac{x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{6542}$$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\int \frac{x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}}{c^2}\right)\right)$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\int \frac{x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{7}x^{7/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{2}{7}bc\int \frac{x^{7/2}\left(a-\right.}{}}{c^2}\right.\right.$$

$$\Big\downarrow \text{6542}$$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\int \frac{x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{7}x^{7/2}\left(a+b\text{arcta}\right.}{}\right.\right.$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\int \frac{x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{2}{5}bc\int \frac{x^{5/2}\left(a+b\text{arctanh}\left(\right.\right.}{1-c^2x}}{c^2}}{c^2}\right.\right.$$

$$\Big\downarrow \text{254}$$

3.202. $\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\int\frac{x^2\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-\frac{2}{5}bc\int\frac{x^{5/2}\left(a+b\text{arctanh}(\right.}{1-c^2x}}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 2009

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\int\frac{x^2\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-\frac{2}{5}bc\int\frac{x^{5/2}\left(a+b\text{arctanh}(\right.}{1-c^2x}}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 6542

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\int\frac{x\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x\left(a+b\text{arctanh}(c\sqrt{x})\right)^2d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctanh}(c\sqrt{x})\right.}{}}{c^2}\right.\right.$$

$\downarrow$ 6452

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\int\frac{x\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-\frac{2}{3}bc\int\frac{x^{3/2}\left(a+b\text{arctanh}(c\sqrt{x})\right)}{1-c^2x}}{c^2}}{c^2}}{}\right.\right.$$

$\downarrow$ 254

3.202. $\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{2}{3}bc\int\frac{x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 2009

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{2}{3}bc\int\frac{x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 6542

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{2}{3}}{c^2}}{}\right.\right.$$

$\downarrow$ 6436

3.202. $\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}-\cdots}{\cdots}\right.\right.$$

$\big\downarrow$ 6452

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}-\cdots}{\cdots}\right.\right.$$

$\big\downarrow$ 262

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}-\cdots}{\cdots}\right.\right.$$

$\big\downarrow$ 219

3.202. $\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$$2\left(\frac{1}{8}x^4\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\int\frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}-\cdots}{\cdots}\right.\right.$$

$\downarrow$ 6510

$$2\left(\frac{1}{8}x^4\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\cdots}{\cdots}}{\cdots}\right.\right.$$

$\downarrow$ 6546

$$2\left(\frac{1}{8}x^4\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\left(\frac{\int\frac{a+b\operatorname{arctanh}\left(c\sqrt{x}\right)}{1-c\sqrt{x}}d\sqrt{x}}{c}-\frac{\left(a+b\operatorname{arct}\cdots\right.}{\cdots}\right.}{c^2}}{c^2}\cdots}{\cdots}\right.\right.$$

$\downarrow$ 6470

3.202. $\int x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}(c\sqrt{x})\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-2bc\left(\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)\log\left(\frac{2}{1-c\sqrt{x}}\right)}{c}-\cdots}{c}\right.}{c^2}}{c^2}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 2849$$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}(c\sqrt{x})\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-2bc\left(\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)\log\left(\frac{2}{1-c\sqrt{x}}\right)}{c}+\cdots}{c}\right.}{c^2}}{c^2}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 2752$$

$$2\left(\frac{1}{8}x^4\left(a+b\text{arctanh}(c\sqrt{x})\right)^3-\frac{3}{8}bc\left(\frac{\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-2bc\left(\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)\log\left(\frac{2}{1-c\sqrt{x}}\right)}{c}+\cdots}{c}\right.}{c^2}}{c^2}}{\cdots}\right.\right.$$

3.202. $\int x^3\left(a+b\text{arctanh}(c\sqrt{x})\right)^3\,dx$

input
```
Int[x^3*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

output
```
2*((x^4*(a + b*ArcTanh[c*Sqrt[x]])^3)/8 - (3*b*c*(-(((x^(7/2)*(a + b*ArcTa
nh[c*Sqrt[x]])^2)/7 - (2*b*c*(-(((x^3*(a + b*ArcTanh[c*Sqrt[x]]))/6 - (b*c
*(-(Sqrt[x]/c^6) - x^(3/2)/(3*c^4) - x^(5/2)/(5*c^2) + ArcTanh[c*Sqrt[x]]/
c^7))/6)/c^2) + (-(((x^2*(a + b*ArcTanh[c*Sqrt[x]]))/4 - (b*c*(-(Sqrt[x]/c
^4) - x^(3/2)/(3*c^2) + ArcTanh[c*Sqrt[x]]/c^5))/4)/c^2) + (-(((x*(a + b*A
rcTanh[c*Sqrt[x]]))/2 - (b*c*(-(Sqrt[x]/c^2) + ArcTanh[c*Sqrt[x]]/c^3))/2)
/c^2) + (-1/2*(a + b*ArcTanh[c*Sqrt[x]])^2/(b*c^2) + (((a + b*ArcTanh[c*Sq
rt[x]])*Log[2/(1 - c*Sqrt[x])])/c + (b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/
(2*c))/c)/c^2)/c^2)/c^2))/7)/c^2) + (-(((x^(5/2)*(a + b*ArcTanh[c*Sqrt[x]]
)^2)/5 - (2*b*c*(-(((x^2*(a + b*ArcTanh[c*Sqrt[x]]))/4 - (b*c*(-(Sqrt[x]/c
^4) - x^(3/2)/(3*c^2) + ArcTanh[c*Sqrt[x]]/c^5))/4)/c^2) + (-(((x*(a + b*A
rcTanh[c*Sqrt[x]]))/2 - (b*c*(-(Sqrt[x]/c^2) + ArcTanh[c*Sqrt[x]]/c^3))/2)
/c^2) + (-1/2*(a + b*ArcTanh[c*Sqrt[x]])^2/(b*c^2) + (((a + b*ArcTanh[c*Sq
rt[x]])*Log[2/(1 - c*Sqrt[x])])/c + (b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/
(2*c))/c)/c^2)/c^2))/5)/c^2) + (-(((x^(3/2)*(a + b*ArcTanh[c*Sqrt[x]])^2)/
3 - (2*b*c*(-(((x*(a + b*ArcTanh[c*Sqrt[x]]))/2 - (b*c*(-(Sqrt[x]/c^2) + A
rcTanh[c*Sqrt[x]]/c^3))/2)/c^2) + (-1/2*(a + b*ArcTanh[c*Sqrt[x]])^2/(b*c^
2) + (((a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/c + (b*PolyLog[2
, 1 - 2/(1 - c*Sqrt[x])])/(2*c))/c)/c^2))/3)/c^2) + ((a + b*ArcTanh[c*Sqrt
[x]])^3/(3*b*c^3) - (Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^2 - 2*b*c*(-1/2...
```

3.202.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

3.202. $\int x^3\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.202.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 3.41 (sec) , antiderivative size = 1341, normalized size of antiderivative = 3.59

Expression too large to display

input
```
int(x^3*(a+b*arctanh(c*x^(1/2)))^3,x)
```

output
```
2/c^8*(-11/30*b^3-3/16*I*b^3*Pi*arctanh(c*x^(1/2))^2+9/140*b^3*arctanh(c*x
^(1/2))*c^4*x^2+1/56*b^3*arctanh(c*x^(1/2))*c^6*x^3+3/56*b^3*arctanh(c*x^(
1/2))^2*c^7*x^(7/2)+3/40*b^3*arctanh(c*x^(1/2))^2*c^5*x^(5/2)+1/8*b^3*arct
anh(c*x^(1/2))^2*c^3*x^(3/2)+3/8*b^3*arctanh(c*x^(1/2))^2*c*x^(1/2)+1/8*b^
3*c^8*x^4*arctanh(c*x^(1/2))^3+71/280*b^3*arctanh(c*x^(1/2))*c^2*x+3/32*I*
b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^3*arctanh(c*x^(1/2))^2+3/32*I*b^3
*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^3*arct
anh(c*x^(1/2))^2+3/16*I*b^3*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^2*arc
tanh(c*x^(1/2))^2-3/16*I*b^3*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^3*ar
ctanh(c*x^(1/2))^2-3/32*I*b^3*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))*csg
n(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^
(1/2))^2/(c^2*x-1)))*arctanh(c*x^(1/2))^2+3/32*I*b^3*Pi*csgn(I/(1-(1+c*x^(
1/2))^2/(c^2*x-1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^(1/2))^2/(c
^2*x-1)))^2*arctanh(c*x^(1/2))^2-3/32*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2
*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^2*a
rctanh(c*x^(1/2))^2+3/16*I*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))*c
sgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2*arctanh(c*x^(1/2))^2+3/32*I*b^3*Pi*csgn
(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*arc
tanh(c*x^(1/2))^2-47/140*b^3*arctanh(c*x^(1/2))-1/8*b^3*arctanh(c*x^(1/2))
^3+22/35*b^3*arctanh(c*x^(1/2))^2-44/35*b^3*dilog(1-I*(1+c*x^(1/2))/(-c...
```

3.202.5 Fricas [F]

$$\int x^3 \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3 \, dx = \int \left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*x^3*arctanh(c*sqrt(x))^2 +
 3*a^2*b*x^3*arctanh(c*sqrt(x)) + a^3*x^3, x)
```

3.202.6 Sympy [F]

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\int x^3\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

input
```
integrate(x**3*(a+b*atanh(c*x**(1/2)))**3,x)
```

output
```
Integral(x**3*(a + b*atanh(c*sqrt(x)))**3, x)
```

3.202.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1972 vs. 2(297) = 594.

Time = 0.85 (sec) , antiderivative size = 1972, normalized size of antiderivative = 5.27

$$\int x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

output
```
1/4*a^3*x^4 - 1/26880*a*b^2*c*((315*c^7*x^4 + 500*c^5*x^3 + 1002*c^3*x^2 +
 3684*c*x - 12*(105*c^7*x^4 + 120*c^6*x^(7/2) + 140*c^5*x^3 + 168*c^4*x^(5
/2) + 210*c^3*x^2 + 280*c^2*x^(3/2) + 420*c*x + 840*sqrt(x))*log(c*sqrt(x)
 + 1))/c^8 - 6396*log(c*sqrt(x) + 1)/c^9 - 6396*log(c*sqrt(x) - 1)/c^9) -
1/2240*(840*x^4*log(c*sqrt(x) + 1) - c*((105*c^7*x^4 - 120*c^6*x^(7/2) + 1
40*c^5*x^3 - 168*c^4*x^(5/2) + 210*c^3*x^2 - 280*c^2*x^(3/2) + 420*c*x - 8
40*sqrt(x))/c^8 + 840*log(c*sqrt(x) + 1)/c^9))*a*b^2*log(-c*sqrt(x) + 1) +
 1/2240*(840*x^4*log(c*sqrt(x) + 1) - c*((105*c^7*x^4 - 120*c^6*x^(7/2) +
140*c^5*x^3 - 168*c^4*x^(5/2) + 210*c^3*x^2 - 280*c^2*x^(3/2) + 420*c*x -
840*sqrt(x))/c^8 + 840*log(c*sqrt(x) + 1)/c^9))*a^2*b - 1/2240*(840*x^4*lo
g(-c*sqrt(x) + 1) - c*((105*c^7*x^4 + 120*c^6*x^(7/2) + 140*c^5*x^3 + 168*
c^4*x^(5/2) + 210*c^3*x^2 + 280*c^2*x^(3/2) + 420*c*x + 840*sqrt(x))/c^8 +
 840*log(c*sqrt(x) - 1)/c^9))*a^2*b + 1/1881600*(11025*(32*log(-c*sqrt(x)
+ 1)^2 - 8*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^8 + 57600*(49*log(-c*s
qrt(x) + 1)^2 - 14*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1)^7 + 548800*(18
*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1)^6 + 79
0272*(25*log(-c*sqrt(x) + 1)^2 - 10*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x) -
1)^5 + 3087000*(8*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sq
rt(x) - 1)^4 + 2195200*(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) +
2)*(c*sqrt(x) - 1)^3 + 4939200*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt...
```

3.202.8 Giac [F]

$$\int x^3 \left(a + b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3 \, dx = \int \left(b \operatorname{artanh}\left(c\sqrt{x}\right) + a\right)^3 x^3 \, dx$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^3*x^3, x)
```

3.202.9 Mupad [F(-1)]

Timed out.

$$\int x^3 \left(a + b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3 \, dx = \int x^3 \left(a + b \operatorname{atanh}\left(c\sqrt{x}\right)\right)^3 \, dx$$

input
```
int(x^3*(a + b*atanh(c*x^(1/2)))^3,x)
```

output
```
int(x^3*(a + b*atanh(c*x^(1/2)))^3, x)
```

3.203 $\int x^2 \left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3 \, dx$

3.203.1 Optimal result

Integrand size = 18, antiderivative size = 304

$$\int x^2 \left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3 \, dx = \frac{19b^3\sqrt{x}}{30c^5}+\frac{b^3x^{3/2}}{30c^3}-\frac{19b^3 \operatorname{arctanh}\left(c\sqrt{x}\right)}{30c^6}+\frac{8b^2x\left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{15c^4}+\frac{b^2x^2\left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{10c^2}+\frac{23b\left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{15c^6}+\frac{b\sqrt{x}\left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{c^5}+\frac{bx^{3/2}\left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{3c^3}+\frac{bx^{5/2}\left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{5c}-\frac{\left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{3c^6}+\frac{1}{3}x^3\left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{46b^2\left(a+b \operatorname{arctanh}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1-c\sqrt{x}}\right)}{15c^6}-\frac{23b^3 \operatorname{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{15c^6}$$

output
```
1/30*b^3*x^(3/2)/c^3-19/30*b^3*arctanh(c*x^(1/2))/c^6+8/15*b^2*x*(a+b*arct
anh(c*x^(1/2)))/c^4+1/10*b^2*x^2*(a+b*arctanh(c*x^(1/2)))/c^2+23/15*b*(a+b
*arctanh(c*x^(1/2)))^2/c^6+1/3*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))^2/c^3+1/
5*b*x^(5/2)*(a+b*arctanh(c*x^(1/2)))^2/c-1/3*(a+b*arctanh(c*x^(1/2)))^3/c^
6+1/3*x^3*(a+b*arctanh(c*x^(1/2)))^3-46/15*b^2*(a+b*arctanh(c*x^(1/2)))*ln
(2/(1-c*x^(1/2)))/c^6-23/15*b^3*polylog(2,1-2/(1-c*x^(1/2)))/c^6+19/30*b^3
*x^(1/2)/c^5+b*(a+b*arctanh(c*x^(1/2)))^2*x^(1/2)/c^5
```

3.203.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.15

$$\int x^2 \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3 dx$$
$$= \frac{-19ab^2+30a^2bc\sqrt{x}+19b^3c\sqrt{x}+16ab^2c^2x+10a^2bc^3x^{3/2}+b^3c^3x^{3/2}+3ab^2c^4x^2+6a^2bc^5x^{5/2}+10a^3c^6x^3}{}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

output
```
(-19*a*b^2 + 30*a^2*b*c*Sqrt[x] + 19*b^3*c*Sqrt[x] + 16*a*b^2*c^2*x + 10*a
^2*b*c^3*x^(3/2) + b^3*c^3*x^(3/2) + 3*a*b^2*c^4*x^2 + 6*a^2*b*c^5*x^(5/2)
 + 10*a^3*c^6*x^3 + 2*b^2*(b*(-23 + 15*c*Sqrt[x] + 5*c^3*x^(3/2) + 3*c^5*x
^(5/2)) + 15*a*(-1 + c^6*x^3))*ArcTanh[c*Sqrt[x]]^2 + 10*b^3*(-1 + c^6*x^3
)*ArcTanh[c*Sqrt[x]]^3 + b*ArcTanh[c*Sqrt[x]]*(30*a^2*c^6*x^3 + 4*a*b*c*Sq
rt[x]*(15 + 5*c^2*x + 3*c^4*x^2) + b^2*(-19 + 16*c^2*x + 3*c^4*x^2) - 92*b
^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + 15*a^2*b*Log[1 - c*Sqrt[x]] - 15*
a^2*b*Log[1 + c*Sqrt[x]] + 46*a*b^2*Log[1 - c^2*x] + 46*b^3*PolyLog[2, -E^
(-2*ArcTanh[c*Sqrt[x]])])/(30*c^6)
```

3.203.3 Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 611 vs. 2(304) = 608.

Time = 3.53 (sec) , antiderivative size = 611, normalized size of antiderivative = 2.01, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {6454, 6452, 6542, 6452, 6542, 6452, 254, 2009, 6542, 6436, 6452, 262, 219, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3 dx$$
$$\Big\downarrow \text{ 6454}$$
$$2\int x^{5/2} \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3 d\sqrt{x}$$
$$\Big\downarrow \text{ 6452}$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\int\frac{x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}\right)$$

$\Big\downarrow$ 6542

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\int\frac{x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}}{c^2}\right)\right)$$

$\Big\downarrow$ 6452

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\int\frac{x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{2}{5}bc\int\frac{x^{5/2}\left(a-\right.}{}}{c^2}\right.\right.$$

$\Big\downarrow$ 6542

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{5}x^{5/2}\left(a+b\text{arctan}\right.}{}\right.\right.$$

$\Big\downarrow$ 6452

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{2}{3}bc\int\frac{x^{3/2}\left(a+b\text{arctanh}\left(c\right.\right.}{1-c^2x}}{c^2}}{c^2}\right.\right.$$

$\Big\downarrow$ 254

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{2}{3}bc\int\frac{x^{3/2}\left(a+b\text{arctanh}\left(c\right.\right.}{1-c^2x}}{c^2}}{c^2}\right.\right.$$

3.203. $\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$$\Big\downarrow \text{2009}$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\int \frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{2}{3}bc\int \frac{x^{3/2}\left(a+b\text{arctanh}(c\right.}{1-c^2x}}{c^2}}{c^2}\right.\right.$$

$$\Big\downarrow \text{6542}$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\frac{\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{}}{c^2}\right.\right.$$

$$\Big\downarrow \text{6436}$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\frac{\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int \frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}}{}\right.\right.$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\frac{\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int \frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}}{}\right.\right.$$

3.203. $\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$\downarrow$ 262

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\int\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 219

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\int\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 6510

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{3}x^3\ldots}{\ldots}\right.\right.$$

$\downarrow$ 6546

3.203. $\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}(c\sqrt{x})\right)^3-\frac{1}{2}bc\left(\frac{\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-2bc\left(\frac{\int\frac{a+b\text{arctanh}(c\sqrt{x})}{1-c\sqrt{x}}d\sqrt{x}}{c}-\frac{\left(a+b\text{arct}\cdots\right.}{\cdots}\right.}{c^2}}{c^2}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 6470$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}(c\sqrt{x})\right)^3-\frac{1}{2}bc\left(\frac{\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}(c\sqrt{x})\right)}{c}-b\cdots}{c}\right.}{c^2}}{c^2}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 2849$$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}(c\sqrt{x})\right)^3-\frac{1}{2}bc\left(\frac{\frac{\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}(c\sqrt{x})\right)^2-2bc\left(\frac{b\int\frac{\log\left(\frac{2}{1-c\sqrt{x}}\right)}{1-\frac{2}{1-c\sqrt{x}}}d\frac{1}{1-c\sqrt{x}}}{c}+\frac{\log\left(\frac{2}{1-c\sqrt{x}}\right)\cdots}{c}\right.}{c^2}}{c^2}}{\cdots}\right.\right.$$

$$\Big\downarrow \; 2752$$

3.203. $\int x^2\left(a+b\text{arctanh}(c\sqrt{x})\right)^3\,dx$

$$2\left(\frac{1}{6}x^3\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{1}{2}bc\left(\frac{\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{c}+\cdots}{c}\cdots\right.}{c^2}\cdots}{c^2}\cdots\right.\right.$$

input
```
Int[x^2*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

output
```
2*((x^3*(a + b*ArcTanh[c*Sqrt[x]])^3)/6 - (b*c*(-(((x^(5/2)*(a + b*ArcTanh
[c*Sqrt[x]])^2)/5 - (2*b*c*(-(((x^2*(a + b*ArcTanh[c*Sqrt[x]]))/4 - (b*c*(
-(Sqrt[x]/c^4) - x^(3/2)/(3*c^2) + ArcTanh[c*Sqrt[x]]/c^5))/4)/c^2) + (-((
(x*(a + b*ArcTanh[c*Sqrt[x]]))/2 - (b*c*(-(Sqrt[x]/c^2) + ArcTanh[c*Sqrt[x
]]/c^3))/2)/c^2) + (-1/2*(a + b*ArcTanh[c*Sqrt[x]])^2/(b*c^2) + (((a + b*A
rcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/c + (b*PolyLog[2, 1 - 2/(1 - c*
Sqrt[x])])/(2*c))/c)/c^2)/c^2))/5)/c^2) + (-(((x^(3/2)*(a + b*ArcTanh[c*Sq
rt[x]])^2)/3 - (2*b*c*(-(((x*(a + b*ArcTanh[c*Sqrt[x]]))/2 - (b*c*(-(Sqrt[
x]/c^2) + ArcTanh[c*Sqrt[x]]/c^3))/2)/c^2) + (-1/2*(a + b*ArcTanh[c*Sqrt[x
]])^2/(b*c^2) + (((a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/c + (
b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/(2*c))/c)/c^2))/3)/c^2) + ((a + b*Arc
Tanh[c*Sqrt[x]])^3/(3*b*c^3) - (Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^2 - 2*b
*c*(-1/2*(a + b*ArcTanh[c*Sqrt[x]])^2/(b*c^2) + (((a + b*ArcTanh[c*Sqrt[x]
])*Log[2/(1 - c*Sqrt[x])])/c + (b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/(2*c)
)/c))/c^2)/c^2)/c^2))/2)
```

3.203.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 254
```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Int[PolynomialDivide[x^m,
 a + b*x^2, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 3]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

3.203. $\int x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.203.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 1264, normalized size of antiderivative = 4.16

Expression too large to display

input
```
int(x^2*(a+b*arctanh(c*x^(1/2)))^3,x)
```

output
```
2/c^6*(1/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x
^(1/2))^2/(c^2*x-1))*arctanh(c*x^(1/2))^2+1/4*I*b^3*Pi*csgn(I*(1+c*x^(1/2)
)/(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2*arctanh(c*x^(1/2))
^2+1/8*I*b^3*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))*csgn(I*(1+c*x^(1/2))
^2/(c^2*x-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^2*arctanh(c*x^(1/2))^2-1/8*I*b
^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(
1-(1+c*x^(1/2))^2/(c^2*x-1)))^2*arctanh(c*x^(1/2))^2-1/3*b^3-1/8*I*b^3*Pi*
csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*cs
gn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))*arctanh(c*x^
(1/2))^2+1/6*c^6*x^3*a^3+3*a*b^2*(1/6*c^6*x^3*arctanh(c*x^(1/2))^2+1/15*ar
ctanh(c*x^(1/2))*c^5*x^(5/2)+1/9*arctanh(c*x^(1/2))*c^3*x^(3/2)+1/3*arctan
h(c*x^(1/2))*c*x^(1/2)+1/6*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)-1/6*arctanh(
c*x^(1/2))*ln(1+c*x^(1/2))-1/12*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)+1/24
*ln(c*x^(1/2)-1)^2+1/24*ln(1+c*x^(1/2))^2-1/12*(ln(1+c*x^(1/2))-ln(1/2*c*x
^(1/2)+1/2))*ln(-1/2*c*x^(1/2)+1/2)+1/60*c^4*x^2+4/45*c^2*x+23/90*ln(c*x^(
1/2)-1)+23/90*ln(1+c*x^(1/2)))+3*a^2*b*(1/6*c^6*x^3*arctanh(c*x^(1/2))+1/3
0*c^5*x^(5/2)+1/18*c^3*x^(3/2)+1/6*c*x^(1/2)+1/12*ln(c*x^(1/2)-1)-1/12*ln(
1+c*x^(1/2)))+1/4*I*b^3*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^2*arctanh
(c*x^(1/2))^2-1/4*I*b^3*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^3*arctanh
(c*x^(1/2))^2+1/8*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^3*arctanh(...
```

3.203.5 Fricas [F]

$$\int x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\int \left(b\operatorname{artanh}\left(c\sqrt{x}\right)+a\right)^3x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x^2*arctanh(c*sqrt(x))^3 + 3*a*b^2*x^2*arctanh(c*sqrt(x))^2 +
 3*a^2*b*x^2*arctanh(c*sqrt(x)) + a^3*x^2, x)
```

3.203.6 Sympy [F]

$$\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\int x^2\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

input
```
integrate(x**2*(a+b*atanh(c*x**(1/2)))**3,x)
```

output
```
Integral(x**2*(a + b*atanh(c*sqrt(x)))**3, x)
```

3.203.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1579 vs. $2(243) = 486$.

Time = 0.79 (sec) , antiderivative size = 1579, normalized size of antiderivative = 5.19

$$\int x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

output
```
1/3*a^3*x^3 - 1/720*a*b^2*c*((20*c^5*x^3 + 39*c^3*x^2 + 138*c*x - 6*(10*c^
5*x^3 + 12*c^4*x^(5/2) + 15*c^3*x^2 + 20*c^2*x^(3/2) + 30*c*x + 60*sqrt(x)
)*log(c*sqrt(x) + 1))/c^6 - 222*log(c*sqrt(x) + 1)/c^7 - 222*log(c*sqrt(x)
 - 1)/c^7) - 1/120*(60*x^3*log(c*sqrt(x) + 1) - c*((10*c^5*x^3 - 12*c^4*x^
(5/2) + 15*c^3*x^2 - 20*c^2*x^(3/2) + 30*c*x - 60*sqrt(x))/c^6 + 60*log(c*
sqrt(x) + 1)/c^7))*a*b^2*log(-c*sqrt(x) + 1) + 1/120*(60*x^3*log(c*sqrt(x)
 + 1) - c*((10*c^5*x^3 - 12*c^4*x^(5/2) + 15*c^3*x^2 - 20*c^2*x^(3/2) + 30
*c*x - 60*sqrt(x))/c^6 + 60*log(c*sqrt(x) + 1)/c^7))*a^2*b - 1/120*(60*x^3
*log(-c*sqrt(x) + 1) - c*((10*c^5*x^3 + 12*c^4*x^(5/2) + 15*c^3*x^2 + 20*c
^2*x^(3/2) + 30*c*x + 60*sqrt(x))/c^6 + 60*log(c*sqrt(x) - 1)/c^7))*a^2*b
+ 1/7200*(100*(18*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 1)*(c*sq
rt(x) - 1)^6 + 432*(25*log(-c*sqrt(x) + 1)^2 - 10*log(-c*sqrt(x) + 1) + 2)
*(c*sqrt(x) - 1)^5 + 3375*(8*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1)
 + 1)*(c*sqrt(x) - 1)^4 + 4000*(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x)
 + 1) + 2)*(c*sqrt(x) - 1)^3 + 13500*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*s
qrt(x) + 1) + 1)*(c*sqrt(x) - 1)^2 + 10800*(log(-c*sqrt(x) + 1)^2 - 2*log(
-c*sqrt(x) + 1) + 2)*(c*sqrt(x) - 1))*a*b^2/c^6 - 1/864000*(1000*(36*log(-
c*sqrt(x) + 1)^3 - 18*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 1)*(
c*sqrt(x) - 1)^6 + 1728*(125*log(-c*sqrt(x) + 1)^3 - 75*log(-c*sqrt(x) + 1
)^2 + 30*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(x) - 1)^5 + 16875*(32*log(-c*...
```

3.203.8 Giac [F]

$$\int x^2 \left(a + b\text{arctanh}\left(c\sqrt{x}\right)\right)^3 \, dx = \int \left(b\,\text{artanh}\left(c\sqrt{x}\right) + a\right)^3 x^2 \, dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^3*x^2, x)
```

3.203.9 Mupad [F(-1)]

Timed out.

$$\int x^2 \left(a + b\text{arctanh}\left(c\sqrt{x}\right)\right)^3 \, dx = \int x^2 \left(a + b\,\text{atanh}\left(c\,\sqrt{x}\right)\right)^3 \, dx$$

input
```
int(x^2*(a + b*atanh(c*x^(1/2)))^3,x)
```

output
```
int(x^2*(a + b*atanh(c*x^(1/2)))^3, x)
```

3.204 $\int x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

3.204.1 Optimal result

Integrand size = 16, antiderivative size = 234

$$\int x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\frac{b^3\sqrt{x}}{2c^3}-\frac{b^3\operatorname{arctanh}\left(c\sqrt{x}\right)}{2c^4}+\frac{b^2x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)}{2c^2}+\frac{2b\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{c^4}+\frac{3b\sqrt{x}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{2c^3}+\frac{bx^{3/2}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{2c}-\frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{2c^4}+\frac{1}{2}x^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{4b^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1-c\sqrt{x}}\right)}{c^4}-\frac{2b^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{c^4}$$

output
```
-1/2*b^3*arctanh(c*x^(1/2))/c^4+1/2*b^2*x*(a+b*arctanh(c*x^(1/2)))/c^2+2*b
*(a+b*arctanh(c*x^(1/2)))^2/c^4+1/2*b*x^(3/2)*(a+b*arctanh(c*x^(1/2)))^2/c
-1/2*(a+b*arctanh(c*x^(1/2)))^3/c^4+1/2*x^2*(a+b*arctanh(c*x^(1/2)))^3-4*b
^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^4-2*b^3*polylog(2,1-2/(1
-c*x^(1/2)))/c^4+1/2*b^3*x^(1/2)/c^3+3/2*b*(a+b*arctanh(c*x^(1/2)))^2*x^(1
/2)/c^3
```

3.204.2 Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.22

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$$
$$=\frac{-2ab^2+6a^2bc\sqrt{x}+2b^3c\sqrt{x}+2ab^2c^2x+2a^2bc^3x^{3/2}+2a^3c^4x^2+2b^2\left(b\left(-4+3c\sqrt{x}+c^3x^{3/2}\right)+3a(-1+\right.}{}$$

input
```
Integrate[x*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

output
```
(-2*a*b^2 + 6*a^2*b*c*Sqrt[x] + 2*b^3*c*Sqrt[x] + 2*a*b^2*c^2*x + 2*a^2*b*
c^3*x^(3/2) + 2*a^3*c^4*x^2 + 2*b^2*(b*(-4 + 3*c*Sqrt[x] + c^3*x^(3/2)) +
3*a*(-1 + c^4*x^2))*ArcTanh[c*Sqrt[x]]^2 + 2*b^3*(-1 + c^4*x^2)*ArcTanh[c*
Sqrt[x]]^3 + 2*b*ArcTanh[c*Sqrt[x]]*(3*a^2*c^4*x^2 + b^2*(-1 + c^2*x) + 2*
a*b*c*Sqrt[x]*(3 + c^2*x) - 8*b^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + 3*
a^2*b*Log[1 - c*Sqrt[x]] - 3*a^2*b*Log[1 + c*Sqrt[x]] + 8*a*b^2*Log[1 - c^
2*x] + 8*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/(4*c^4)
```

3.204.3 Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.52, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {6454, 6452, 6542, 6452, 6542, 6436, 6452, 262, 219, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

$$\Big\downarrow \ 6454$$

$$2\int x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3d\sqrt{x}$$

$$\Big\downarrow \ 6452$$

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\int\frac{x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}\right)$$

$$\Big\downarrow \ 6542$$

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}}{c^2}\right)\right)$$

$\Big\downarrow$ 6452

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\int\frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{2}{3}bc\int\frac{x^{3/2}\left(a+\right.}{}}{c^2}\right.\right.$$

$\Big\downarrow$ 6542

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\frac{\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}}{c^2}}{c^2}-\frac{\frac{1}{3}x^{3/2}\left(a+b\text{arctanh}\left(\right.\right.}{}\right.\right.$$

$\Big\downarrow$ 6436

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\frac{\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d}{c^2}}{c^2}\right.\right.$$

$\Big\downarrow$ 6452

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\frac{\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d}{c^2}}{c^2}\right.\right.$$

$\Big\downarrow$ 262

3.204. $\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\frac{\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 219

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\frac{\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 6510

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)}{1-c^2x}d\sqrt{x}}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 6546

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\left(\frac{\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{1-c\sqrt{x}}d\sqrt{x}}{c}-\frac{(a+}{}\right.}{c^2}}{c^2}\right.\right.$$

$\downarrow$ 6470

3.204. $\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{\right.\right.}{c}}{c}}{c^2}}{c^2}\right.\right.$$

$$\Big\downarrow \text{2849}$$

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\left(\frac{\frac{b\int\frac{\log\left(\frac{2}{1-c\sqrt{x}}\right)}{1-\frac{2}{1-c\sqrt{x}}}d\frac{1}{1-c\sqrt{x}}}{c}+\frac{\log\left(\frac{1}{\right.}}{c}\right.}{c^2}}{c^2}\right.\right.$$

$$\Big\downarrow \text{2752}$$

$$2\left(\frac{1}{4}x^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{4}bc\left(\frac{\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{\right.\right.}{c}}{c}}{c^2}}{c^2}\right.\right.$$

input
```
Int[x*(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

3.204. $\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

output
```
2*((x^2*(a + b*ArcTanh[c*Sqrt[x]])^3)/4 - (3*b*c*(-(((x^(3/2)*(a + b*ArcTa
nh[c*Sqrt[x]])^2)/3 - (2*b*c*(-(((x*(a + b*ArcTanh[c*Sqrt[x]]))/2 - (b*c*(
-(Sqrt[x]/c^2) + ArcTanh[c*Sqrt[x]]/c^3))/2)/c^2) + (-1/2*(a + b*ArcTanh[c
*Sqrt[x]])^2/(b*c^2) + (((a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*Sqrt[x])]
)/c + (b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/(2*c))/c)/c^2))/3)/c^2) + ((a
+ b*ArcTanh[c*Sqrt[x]])^3/(3*b*c^3) - (Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^
2 - 2*b*c*(-1/2*(a + b*ArcTanh[c*Sqrt[x]])^2/(b*c^2) + (((a + b*ArcTanh[c*
Sqrt[x]])*Log[2/(1 - c*Sqrt[x])])/c + (b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])]
)/(2*c))/c))/c^2)/c^2))/4)
```

3.204.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.204. $\int x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

3.204.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 30.44 (sec) , antiderivative size = 1145, normalized size of antiderivative = 4.89

method	result	size
derivativedivides	Expression too large to display	1145
default	Expression too large to display	1145
parts	Expression too large to display	1147

input
```
int(x*(a+b*arctanh(c*x^(1/2)))^3,x,method=_RETURNVERBOSE)
```

output
```
2/c^4*(1/4*a^3*c^4*x^2+b^3*(-1/4+arctanh(c*x^(1/2))^2+1/4*c*x^(1/2)+1/4*ar
ctanh(c*x^(1/2))^2*c^3*x^(3/2)+3/4*arctanh(c*x^(1/2))^2*c*x^(1/2)-1/4*arct
anh(c*x^(1/2))^3+3/8*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)-3/8*arctanh(c*x^
(1/2))^2*ln(1+c*x^(1/2))-2*arctanh(c*x^(1/2))*ln(1+I*(1+c*x^(1/2))/(-c^2*x
+1)^(1/2))-2*arctanh(c*x^(1/2))*ln(1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/4
*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/16*I*Pi*csgn(I/
(1-(1+c*x^(1/2))^2/(c^2*x-1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^
(1/2))^2/(c^2*x-1)))^2*arctanh(c*x^(1/2))^2+3/8*I*Pi*csgn(I*(1+c*x^(1/2))/
(-c^2*x+1)^(1/2))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^2*arctanh(c*x^(1/2))^2
-3/16*I*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x
-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^2*arctanh(c*x^(1/2))^2+3/16*I*Pi*csgn(I
*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))^2*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))*arcta
nh(c*x^(1/2))^2+3/16*I*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^(1/2)
)^2/(c^2*x-1)))^3*arctanh(c*x^(1/2))^2+3/16*I*Pi*csgn(I*(1+c*x^(1/2))^2/(c
^2*x-1))^3*arctanh(c*x^(1/2))^2+3/8*I*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-
1)))^2*arctanh(c*x^(1/2))^2-3/8*I*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))
^3*arctanh(c*x^(1/2))^2-1/4*arctanh(c*x^(1/2))*(1+c*x^(1/2))^2+1/2*(1+c*x^
(1/2))*arctanh(c*x^(1/2))-3/8*I*Pi*arctanh(c*x^(1/2))^2+1/4*c^4*x^2*arctan
h(c*x^(1/2))^3+1/2*(c*x^(1/2)-1)*(1+c*x^(1/2))*arctanh(c*x^(1/2))-2*dilog(
1-I*(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-2*dilog(1+I*(1+c*x^(1/2))/(-c^2*x+1...
```

3.204.5 Fricas [F]

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\int \left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

output
```
integral(b^3*x*arctanh(c*sqrt(x))^3 + 3*a*b^2*x*arctanh(c*sqrt(x))^2 + 3*a
^2*b*x*arctanh(c*sqrt(x)) + a^3*x, x)
```

3.204.6 Sympy [F]

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\int x\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

input
```
integrate(x*(a+b*atanh(c*x**(1/2)))**3,x)
```

output
```
Integral(x*(a + b*atanh(c*sqrt(x)))**3, x)
```

3.204.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1184 vs. $2(191) = 382$.

Time = 0.71 (sec) , antiderivative size = 1184, normalized size of antiderivative = 5.06

$$\int x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*x^2 - 1/32*a*b^2*c*((3*c^3*x^2 + 10*c*x - 2*(3*c^3*x^2 + 4*c^2*x^(
3/2) + 6*c*x + 12*sqrt(x))*log(c*sqrt(x) + 1))/c^4 - 14*log(c*sqrt(x) + 1)
/c^5 - 14*log(c*sqrt(x) - 1)/c^5) - 1/16*(12*x^2*log(c*sqrt(x) + 1) - c*((
3*c^3*x^2 - 4*c^2*x^(3/2) + 6*c*x - 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) + 1
)/c^5))*a*b^2*log(-c*sqrt(x) + 1) + 1/16*(12*x^2*log(c*sqrt(x) + 1) - c*((
3*c^3*x^2 - 4*c^2*x^(3/2) + 6*c*x - 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) + 1
)/c^5))*a^2*b - 1/16*(12*x^2*log(-c*sqrt(x) + 1) - c*((3*c^3*x^2 + 4*c^2*x
^(3/2) + 6*c*x + 12*sqrt(x))/c^4 + 12*log(c*sqrt(x) - 1)/c^5))*a^2*b + 1/1
92*(9*(8*log(-c*sqrt(x) + 1)^2 - 4*log(-c*sqrt(x) + 1) + 1)*(c*sqrt(x) - 1
)^4 + 32*(9*log(-c*sqrt(x) + 1)^2 - 6*log(-c*sqrt(x) + 1) + 2)*(c*sqrt(x)
- 1)^3 + 216*(2*log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 1)*(c*sqrt
(x) - 1)^2 + 288*(log(-c*sqrt(x) + 1)^2 - 2*log(-c*sqrt(x) + 1) + 2)*(c*sq
rt(x) - 1))*a*b^2/c^4 - 1/4608*(9*(32*log(-c*sqrt(x) + 1)^3 - 24*log(-c*sq
rt(x) + 1)^2 + 12*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^4 + 128*(9*log(
-c*sqrt(x) + 1)^3 - 9*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 2)*(
c*sqrt(x) - 1)^3 + 432*(4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x) + 1)^2
+ 6*log(-c*sqrt(x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 1152*(log(-c*sqrt(x) + 1)
^3 - 3*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(x) - 1))
*b^3/c^4 + 2*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*s
qrt(x) + 1/2))*b^3/c^4 - 319/384*b^3*log(c*sqrt(x) - 1)/c^4 + 1/16*(25*...
```

3.204.8 Giac [F]

$$\int x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\int \left(b\operatorname{artanh}\left(c\sqrt{x}\right)+a\right)^3x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^3*x, x)
```

3.204.9 Mupad [F(-1)]

Timed out.

$$\int x\big(a+b\text{arctanh}\big(c\sqrt{x}\big)\big)^3\,dx=\int x\,\big(a+b\,\text{atanh}\big(c\,\sqrt{x}\big)\big)^3\,dx$$

input
```
int(x*(a + b*atanh(c*x^(1/2)))^3,x)
```

output
```
int(x*(a + b*atanh(c*x^(1/2)))^3, x)
```

3.205 $\int \left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

3.205.1 Optimal result

Integrand size = 14, antiderivative size = 142

$$\int \left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\frac{3b\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{c^2}+\frac{3b\sqrt{x}\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{c}-\frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{c^2}+x\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{6b^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\log\left(\frac{2}{1-c\sqrt{x}}\right)}{c^2}-\frac{3b^3\operatorname{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)}{c^2}$$

output
```
3*b*(a+b*arctanh(c*x^(1/2)))^2/c^2-(a+b*arctanh(c*x^(1/2)))^3/c^2+x*(a+b*a
rctanh(c*x^(1/2)))^3-6*b^2*(a+b*arctanh(c*x^(1/2)))*ln(2/(1-c*x^(1/2)))/c^
2-3*b^3*polylog(2,1-2/(1-c*x^(1/2)))/c^2+3*b*(a+b*arctanh(c*x^(1/2)))^2*x^
(1/2)/c
```

3.205.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.42

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$$
$$= \frac{6b^2\left(-1+c\sqrt{x}\right)\left(a+b+ac\sqrt{x}\right)\text{arctanh}\left(c\sqrt{x}\right)^2+2b^3(-1+c^2x)\,\text{arctanh}\left(c\sqrt{x}\right)^3+6b\text{arctanh}\left(c\sqrt{x}\right)\left(2ab\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

output
```
(6*b^2*(-1 + c*Sqrt[x])*(a + b + a*c*Sqrt[x])*ArcTanh[c*Sqrt[x]]^2 + 2*b^3
*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^3 + 6*b*ArcTanh[c*Sqrt[x]]*(2*a*b*c*Sqrt[
x] + a^2*c^2*x - 2*b^2*Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])]) + a*(6*a*b*c*Sq
rt[x] + 2*a^2*c^2*x + 3*a*b*Log[1 - c*Sqrt[x]] - 3*a*b*Log[1 + c*Sqrt[x]]
+ 6*b^2*Log[1 - c^2*x]) + 6*b^3*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])])/(2
*c^2)
```

3.205.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.643, Rules used = {6442, 6452, 6542, 6436, 6510, 6546, 6470, 2849, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

$$\Big\downarrow \text{6442}$$

$$2\int \sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3 d\sqrt{x}$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{2}bc\int \frac{x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{6542}$$

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{2}bc\left(\frac{\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2d\sqrt{x}}{c^2}\right)\right)$$

$\big\downarrow$ 6436

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{2}bc\left(\frac{\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}}{c^2}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int \frac{\sqrt{x}\left(a+b\text{arcta}\right.}{1-c^2}}{c^2}\right.\right.$$

$\big\downarrow$ 6510

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\int \frac{\sqrt{x}\left(a+b\text{arctanh}\right.}{1-c^2x}}{c^2}\right.\right.$$

$\big\downarrow$ 6546

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\left(\frac{\int \frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{1-c\sqrt{x}}}{c}\right.}{c^2}\right.\right.$$

$\big\downarrow$ 6470

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\left(\frac{\frac{\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{ar}\right.}{c}}{}\right.}{}\right.\right.$$

$\big\downarrow$ 2849

3.205. $\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\left(\frac{\frac{b\int \frac{\log\left(\frac{2}{1-c\sqrt{x}}\right)}{1-\frac{2}{1-c\sqrt{x}}}d\frac{}{1-\cdots}}{c}}{}\right.}{}\right.\right.$$

$\downarrow$ 2752

$$2\left(\frac{1}{2}x\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{3}{2}bc\left(\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3bc^3}-\frac{\sqrt{x}\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2-2bc\left(\frac{\log\left(\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{ar}\cdots\right.}{c}\right.}{}\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])^3,x]
```

output
```
2*((x*(a + b*ArcTanh[c*Sqrt[x]])^3)/2 - (3*b*c*((a + b*ArcTanh[c*Sqrt[x]])
^3/(3*b*c^3) - (Sqrt[x]*(a + b*ArcTanh[c*Sqrt[x]])^2 - 2*b*c*(-1/2*(a + b*
ArcTanh[c*Sqrt[x]])^2/(b*c^2) + (((a + b*ArcTanh[c*Sqrt[x]])*Log[2/(1 - c*
Sqrt[x])])/c + (b*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/(2*c))/c))/c^2))/2)
```

3.205.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2849
```
Int[Log[(c_.)/((d_) + (e_.)*(x_))]/((f_) + (g_.)*(x_)^2), x_Symbol] :> Simp
[-e/g   Subst[Int[Log[2*d*x]/(1 - 2*d*x), x], x, 1/(d + e*x)], x] /; FreeQ[
{c, d, e, f, g}, x] && EqQ[c, 2*d] && EqQ[e^2*f + d^2*g, 0]
```

3.205. $\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

rule 6442
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] :> With[{k = D
enominator[n]}, Simp[k   Subst[Int[x^(k - 1)*(a + b*ArcTanh[c*x^(k*n)])^p,
x], x, x^(1/k)], x]] /; FreeQ[{a, b, c}, x] && IGtQ[p, 1] && FractionQ[n]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6470
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symbol
] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(Log[2/(1 + e*(x/d))]/e), x] + Simp[b*c
*(p/e)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2/(1 + e*(x/d))]/(1 - c^2*x^
2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d^2 - e^2
, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

rule 6546
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*e*(p + 1)), x] + Simp[1/
(c*d)   Int[(a + b*ArcTanh[c*x])^p/(1 - c*x), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

3.205. $\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx$

3.205.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.19 (sec) , antiderivative size = 5673, normalized size of antiderivative = 39.95

method	result	size
derivativedivides	Expression too large to display	5673
default	Expression too large to display	5673
parts	Expression too large to display	5674

input
```
int((a+b*arctanh(c*x^(1/2)))^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.205.5 Fricas [F]

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\int \left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="fricas")
```

output
```
integral(b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*b
*arctanh(c*sqrt(x)) + a^3, x)
```

3.205.6 Sympy [F]

$$\int \left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3\,dx=\int \left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^3\,dx$$

input
```
integrate((a+b*atanh(c*x**(1/2)))**3,x)
```

output
```
Integral((a + b*atanh(c*sqrt(x)))**3, x)
```

3.205.7 Maxima [F]

$$\int \left(a+b\text{arctanh}(c\sqrt{x})\right)^3\,dx=\int \left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="maxima")
```

output
```
3/2*(c*(2*sqrt(x)/c^2 - log(c*sqrt(x) + 1)/c^3 + log(c*sqrt(x) - 1)/c^3) +
 2*x*arctanh(c*sqrt(x)))*a^2*b + 3/4*(4*c*(2*sqrt(x)/c^2 - log(c*sqrt(x) +
 1)/c^3 + log(c*sqrt(x) - 1)/c^3)*arctanh(c*sqrt(x)) + 4*x*arctanh(c*sqrt(
x))^2 - (2*(log(c*sqrt(x) - 1) - 2)*log(c*sqrt(x) + 1) - log(c*sqrt(x) + 1
)^2 - log(c*sqrt(x) - 1)^2 - 4*log(c*sqrt(x) - 1))/c^2)*a*b^2 + a^3*x - 1/
32*b^3*(((4*log(-c*sqrt(x) + 1)^3 - 6*log(-c*sqrt(x) + 1)^2 + 6*log(-c*sqr
t(x) + 1) - 3)*(c*sqrt(x) - 1)^2 + 8*(log(-c*sqrt(x) + 1)^3 - 3*log(-c*sqr
t(x) + 1)^2 + 6*log(-c*sqrt(x) + 1) - 6)*(c*sqrt(x) - 1))/c^2 - 4*integrat
e(log(c*sqrt(x) + 1)^3 - 3*log(c*sqrt(x) + 1)^2*log(-c*sqrt(x) + 1) + 3*lo
g(c*sqrt(x) + 1)*log(-c*sqrt(x) + 1)^2, x))
```

3.205.8 Giac [F]

$$\int \left(a+b\text{arctanh}(c\sqrt{x})\right)^3\,dx=\int \left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^3, x)
```

3.205.9 Mupad [F(-1)]

Timed out.

$$\int \left(a+b\text{arctanh}(c\sqrt{x})\right)^3\,dx=\int \left(a+b\,\text{atanh}\left(c\,\sqrt{x}\right)\right)^3\,dx$$

input
```
int((a + b*atanh(c*x^(1/2)))^3,x)
```

output
```
int((a + b*atanh(c*x^(1/2)))^3, x)
```

3.206 $\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx$

3.206.1 Optimal result

Integrand size = 18, antiderivative size = 224

$$\begin{aligned}\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx =\ & 4\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3 \\ & -3b\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right) \\ & +3b\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2\text{PolyLog}\left(2,-1+\frac{2}{1-c\sqrt{x}}\right) \\ & +3b^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\text{PolyLog}\left(3,1-\frac{2}{1-c\sqrt{x}}\right) \\ & -3b^2\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)\text{PolyLog}\left(3,-1+\frac{2}{1-c\sqrt{x}}\right) \\ & -\frac{3}{2}b^3\text{PolyLog}\left(4,1-\frac{2}{1-c\sqrt{x}}\right) \\ & +\frac{3}{2}b^3\text{PolyLog}\left(4,-1+\frac{2}{1-c\sqrt{x}}\right)\end{aligned}$$

output
```
-4*(a+b*arctanh(c*x^(1/2)))^3*arctanh(-1+2/(1-c*x^(1/2)))-3*b*(a+b*arctanh
(c*x^(1/2)))^2*polylog(2,1-2/(1-c*x^(1/2)))+3*b*(a+b*arctanh(c*x^(1/2)))^2
*polylog(2,-1+2/(1-c*x^(1/2)))+3*b^2*(a+b*arctanh(c*x^(1/2)))*polylog(3,1-
2/(1-c*x^(1/2)))-3*b^2*(a+b*arctanh(c*x^(1/2)))*polylog(3,-1+2/(1-c*x^(1/2
)))-3/2*b^3*polylog(4,1-2/(1-c*x^(1/2)))+3/2*b^3*polylog(4,-1+2/(1-c*x^(1/
2)))
```

3.206.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.89

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx = a^3\log(x)+3a^2b\left(-\text{PolyLog}\left(2,-c\sqrt{x}\right)+\text{PolyLog}\left(2,c\sqrt{x}\right)\right)$$
$$+6ab^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\text{arctanh}\left(c\sqrt{x}\right)^3-\text{arctanh}\left(c\sqrt{x}\right)^2\log\left(1+e^{-2\text{arctanh}\left(c\sqrt{x}\right)}\right)+\text{arctanh}\left(c\sqrt{x}\right)^2\log\left(1-e^{2\text{arctanh}\left(c\sqrt{x}\right)}\right)+\text{arctanh}\left(c\sqrt{x}\right)\text{PolyLog}\left(2,-e^{-2\text{arctanh}\left(c\sqrt{x}\right)}\right)+\text{arctanh}\left(c\sqrt{x}\right)\text{PolyLog}\left(2,e^{2\text{arctanh}\left(c\sqrt{x}\right)}\right)+\frac{1}{2}\text{PolyLog}\left(3,-e^{-2\text{arctanh}\left(c\sqrt{x}\right)}\right)-\frac{1}{2}\text{PolyLog}\left(3,e^{2\text{arctanh}\left(c\sqrt{x}\right)}\right)\right)$$
$$+\frac{1}{32}b^3\left(\pi^4-32\text{arctanh}\left(c\sqrt{x}\right)^4-64\text{arctanh}\left(c\sqrt{x}\right)^3\log\left(1+e^{-2\text{arctanh}\left(c\sqrt{x}\right)}\right)+64\text{arctanh}\left(c\sqrt{x}\right)^3\log\left(1-e^{2\text{arctanh}\left(c\sqrt{x}\right)}\right)+96\text{arctanh}\left(c\sqrt{x}\right)^2\text{PolyLog}\left(2,-e^{-2\text{arctanh}\left(c\sqrt{x}\right)}\right)+96\text{arctanh}\left(c\sqrt{x}\right)^2\text{PolyLog}\left(2,e^{2\text{arctanh}\left(c\sqrt{x}\right)}\right)+96\text{arctanh}\left(c\sqrt{x}\right)\text{PolyLog}\left(3,-e^{-2\text{arctanh}\left(c\sqrt{x}\right)}\right)-96\text{arctanh}\left(c\sqrt{x}\right)\text{PolyLog}\left(3,e^{2\text{arctanh}\left(c\sqrt{x}\right)}\right)+48\,\text{PolyLog}\left(4,-e^{-2\text{arctanh}\left(c\sqrt{x}\right)}\right)+48\,\text{PolyLog}\left(4,e^{2\text{arctanh}\left(c\sqrt{x}\right)}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x,x]
```

output
```
a^3*Log[x] + 3*a^2*b*(-PolyLog[2, -(c*Sqrt[x])] + PolyLog[2, c*Sqrt[x]]) +
 6*a*b^2*((I/24)*Pi^3 - (2*ArcTanh[c*Sqrt[x]]^3)/3 - ArcTanh[c*Sqrt[x]]^2*
Log[1 + E^(-2*ArcTanh[c*Sqrt[x]])] + ArcTanh[c*Sqrt[x]]^2*Log[1 - E^(2*Arc
Tanh[c*Sqrt[x]])] + ArcTanh[c*Sqrt[x]]*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]
])] + ArcTanh[c*Sqrt[x]]*PolyLog[2, E^(2*ArcTanh[c*Sqrt[x]])] + PolyLog[3,
 -E^(-2*ArcTanh[c*Sqrt[x]])]/2 - PolyLog[3, E^(2*ArcTanh[c*Sqrt[x]])]/2) +
 (b^3*(Pi^4 - 32*ArcTanh[c*Sqrt[x]]^4 - 64*ArcTanh[c*Sqrt[x]]^3*Log[1 + E^
(-2*ArcTanh[c*Sqrt[x]])] + 64*ArcTanh[c*Sqrt[x]]^3*Log[1 - E^(2*ArcTanh[c*
Sqrt[x]])] + 96*ArcTanh[c*Sqrt[x]]^2*PolyLog[2, -E^(-2*ArcTanh[c*Sqrt[x]])
] + 96*ArcTanh[c*Sqrt[x]]^2*PolyLog[2, E^(2*ArcTanh[c*Sqrt[x]])] + 96*ArcT
anh[c*Sqrt[x]]*PolyLog[3, -E^(-2*ArcTanh[c*Sqrt[x]])] - 96*ArcTanh[c*Sqrt[
x]]*PolyLog[3, E^(2*ArcTanh[c*Sqrt[x]])] + 48*PolyLog[4, -E^(-2*ArcTanh[c*
Sqrt[x]])] + 48*PolyLog[4, E^(2*ArcTanh[c*Sqrt[x]])]))/32
```

3.206.3 Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6450, 6448, 6614, 6620, 6624, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(c\sqrt{x}))^3}{x}dx$$

$$\Big\downarrow \text{6450}$$

$$2\int \frac{(a+b\text{arctanh}(c\sqrt{x}))^3}{\sqrt{x}}d\sqrt{x}$$

$$\Big\downarrow \text{6448}$$

$$2\left(2\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)(a+b\text{arctanh}(c\sqrt{x}))^3-6bc\int \frac{\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)(a+b\text{arctanh}(c\sqrt{x}))^2}{1-c^2x}d\sqrt{x}\right)$$

$$\Big\downarrow \text{6614}$$

$$2\left(2\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)(a+b\text{arctanh}(c\sqrt{x}))^3-6bc\left(\frac{1}{2}\int \frac{(a+b\text{arctanh}(c\sqrt{x}))^2\log\left(2-\frac{2}{1-c\sqrt{x}}\right)}{1-c^2x}d\sqrt{x}-\frac{1}{2}\right.\right.$$

$\downarrow$ 6620

$$2\left(2\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}(c\sqrt{x})\right)^3-6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{2c}-\right.\right.\right.$$

$\downarrow$ 6624

$$2\left(2\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}(c\sqrt{x})\right)^3-6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{2c}-\right.\right.\right.$$

$\downarrow$ 7164

$$2\left(2\text{arctanh}\left(1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}(c\sqrt{x})\right)^3-6bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-c\sqrt{x}}\right)\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{2c}-\right.\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x,x]
```

output
```
2*(2*ArcTanh[1 - 2/(1 - c*Sqrt[x])]*(a + b*ArcTanh[c*Sqrt[x]])^3 - 6*b*c*(
(((a + b*ArcTanh[c*Sqrt[x]])^2*PolyLog[2, 1 - 2/(1 - c*Sqrt[x])])/(2*c) -
b*(((a + b*ArcTanh[c*Sqrt[x]])*PolyLog[3, 1 - 2/(1 - c*Sqrt[x])])/(2*c) -
(b*PolyLog[4, 1 - 2/(1 - c*Sqrt[x])])/(4*c)))/2 + (-1/2*((a + b*ArcTanh[c*
Sqrt[x]])^2*PolyLog[2, -1 + 2/(1 - c*Sqrt[x])])/c + b*(((a + b*ArcTanh[c*S
qrt[x]])*PolyLog[3, -1 + 2/(1 - c*Sqrt[x])])/(2*c) - (b*PolyLog[4, -1 + 2/
(1 - c*Sqrt[x])])/(4*c)))/2))
```

3.206.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.206. $\int \frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{x}\,dx$

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6624
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*PolyLog[k_, u_])/((d_) + (e_
.)*(x_)^2), x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(PolyLog[k + 1, u]/(2*
c*d)), x] - Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[k + 1,
 u]/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, k}, x] && IGtQ[p, 0] && E
qQ[c^2*d + e, 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.206.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 13.89 (sec) , antiderivative size = 1363, normalized size of antiderivative = 6.08

method	result	size
parts	Expression too large to display	1363
derivativedivides	Expression too large to display	1364
default	Expression too large to display	1364

input
```
int((a+b*arctanh(c*x^(1/2)))^3/x,x,method=_RETURNVERBOSE)
```

output
```
a^3*ln(x)+b^3*(2*ln(c*x^(1/2))*arctanh(c*x^(1/2))^3-2*arctanh(c*x^(1/2))^3
*ln((1+c*x^(1/2))^2/(-c^2*x+1)-1)+2*arctanh(c*x^(1/2))^3*ln(1+(1+c*x^(1/2)
)/(-c^2*x+1)^(1/2))+6*arctanh(c*x^(1/2))^2*polylog(2,-(1+c*x^(1/2))/(-c^2*
x+1)^(1/2))-12*arctanh(c*x^(1/2))*polylog(3,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2
))+12*polylog(4,-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+2*arctanh(c*x^(1/2))^3*ln
(1-(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+6*arctanh(c*x^(1/2))^2*polylog(2,(1+c*x
^(1/2))/(-c^2*x+1)^(1/2))-12*arctanh(c*x^(1/2))*polylog(3,(1+c*x^(1/2))/(-
c^2*x+1)^(1/2))+12*polylog(4,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+I*Pi*csgn(I*(
-(1+c*x^(1/2))^2/(c^2*x-1)-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))*(csgn(I*(-(1+
c*x^(1/2))^2/(c^2*x-1)-1))*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))-csgn(I*(-
(1+c*x^(1/2))^2/(c^2*x-1)-1))*csgn(I*(-(1+c*x^(1/2))^2/(c^2*x-1)-1)/(1-(1+
c*x^(1/2))^2/(c^2*x-1)))-csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))*csgn(I*(-(1
+c*x^(1/2))^2/(c^2*x-1)-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))+csgn(I*(-(1+c*x^
(1/2))^2/(c^2*x-1)-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^2)*arctanh(c*x^(1/2))
^3-3*arctanh(c*x^(1/2))^2*polylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))+3*arctanh
(c*x^(1/2))*polylog(3,-(1+c*x^(1/2))^2/(-c^2*x+1))-3/2*polylog(4,-(1+c*x^(
1/2))^2/(-c^2*x+1)))+3*a*b^2*(2*ln(c*x^(1/2))*arctanh(c*x^(1/2))^2-2*arcta
nh(c*x^(1/2))*polylog(2,-(1+c*x^(1/2))^2/(-c^2*x+1))+polylog(3,-(1+c*x^(1/
2))^2/(-c^2*x+1))-2*arctanh(c*x^(1/2))^2*ln((1+c*x^(1/2))^2/(-c^2*x+1)-1)+
2*arctanh(c*x^(1/2))^2*ln(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+4*arctanh(c...
```

3.206.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*
b*arctanh(c*sqrt(x)) + a^3)/x, x)
```

3.206.6 Sympy [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x**(1/2)))**3/x,x)
```

output
```
Integral((a + b*atanh(c*sqrt(x)))**3/x, x)
```

3.206.7 Maxima [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="maxima")
```

output
```
1/8*b^3*integrate(log(c*sqrt(x) + 1)^3/x, x) - 3/8*b^3*integrate(log(c*sqr
t(x) + 1)^2*log(-c*sqrt(x) + 1)/x, x) + 3/8*b^3*integrate(log(c*sqrt(x) +
1)*log(-c*sqrt(x) + 1)^2/x, x) - 1/8*b^3*integrate(log(-c*sqrt(x) + 1)^3/x
, x) + 3/4*a*b^2*integrate(log(c*sqrt(x) + 1)^2/x, x) - 3/2*a*b^2*integrat
e(log(c*sqrt(x) + 1)*log(-c*sqrt(x) + 1)/x, x) + 3/4*a*b^2*integrate(log(-
c*sqrt(x) + 1)^2/x, x) + 3/2*a^2*b*integrate(log(c*sqrt(x) + 1)/x, x) - 3/
2*a^2*b*integrate(log(-c*sqrt(x) + 1)/x, x) + a^3*log(x)
```

3.206.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^3/x, x)
```

3.206.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(c\,\sqrt{x}\right)\right)^3}{x}\,dx$$

input
```
int((a + b*atanh(c*x^(1/2)))^3/x,x)
```

output
```
int((a + b*atanh(c*x^(1/2)))^3/x, x)
```

3.207 $\int \frac{\left(a+b\mathbf{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$

3.207.1 Optimal result

Integrand size = 18, antiderivative size = 142

$$\begin{aligned}\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx &= 3bc^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2-\frac{3bc\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^2}{\sqrt{x}}\\ &\quad +c^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{x}\\ &\quad +6b^2c^2\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)\log\left(2-\frac{2}{1+c\sqrt{x}}\right)\\ &\quad -3b^3c^2\operatorname{PolyLog}\left(2,-1+\frac{2}{1+c\sqrt{x}}\right)\end{aligned}$$

output
```
3*b*c^2*(a+b*arctanh(c*x^(1/2)))^2+c^2*(a+b*arctanh(c*x^(1/2)))^3-(a+b*arc
tanh(c*x^(1/2)))^3/x+6*b^2*c^2*(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2
)))-3*b^3*c^2*polylog(2,-1+2/(1+c*x^(1/2)))-3*b*c*(a+b*arctanh(c*x^(1/2)))
^2/x^(1/2)
```

3.207.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.62

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$
$$= \frac{6b^2\left(-1+c\sqrt{x}\right)\left(a+ac\sqrt{x}+bc\sqrt{x}\right)\text{arctanh}\left(c\sqrt{x}\right)^2+2b^3(-1+c^2x)\,\text{arctanh}\left(c\sqrt{x}\right)^3-6b\text{arctanh}\left(c\sqrt{x}\right)}{}$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x^2,x]
```

output
```
(6*b^2*(-1 + c*Sqrt[x])*(a + a*c*Sqrt[x] + b*c*Sqrt[x])*ArcTanh[c*Sqrt[x]]
^2 + 2*b^3*(-1 + c^2*x)*ArcTanh[c*Sqrt[x]]^3 - 6*b*ArcTanh[c*Sqrt[x]]*(a^2
 + 2*a*b*c*Sqrt[x] - 2*b^2*c^2*x*Log[1 - E^(-2*ArcTanh[c*Sqrt[x]])]) + a*(
-2*a^2 - 6*a*b*c*Sqrt[x] - 3*a*b*c^2*x*Log[1 - c*Sqrt[x]] + 3*a*b*c^2*x*Lo
g[1 + c*Sqrt[x]] + 12*b^2*c^2*x*Log[(c*Sqrt[x])/Sqrt[1 - c^2*x]]) - 6*b^3*
c^2*x*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])/(2*x)
```

3.207.3 Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {6454, 6452, 6544, 6452, 6510, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$

$$\Big\downarrow \text{ 6454}$$

$$2\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^{3/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 6452}$$

$$2\left(\frac{3}{2}bc\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x\left(1-c^2x\right)}d\sqrt{x}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{2x}\right)$$

$$\Big\downarrow \text{ 6544}$$

$$2\left(\frac{3}{2}bc\left(c^2\int \frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{1-c^2x}d\sqrt{x}+\int \frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{x}d\sqrt{x}\right)-\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{2x}\right)$$

$\downarrow$ 6452

$$2\left(\frac{3}{2}bc\left(c^2\int \frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{1-c^2x}d\sqrt{x}+2bc\int \frac{a+b\text{arctanh}(c\sqrt{x})}{\sqrt{x}\left(1-c^2x\right)}d\sqrt{x}-\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{\sqrt{x}}\right)-\frac{(a+ba$$

$\downarrow$ 6510

$$2\left(\frac{3}{2}bc\left(2bc\int \frac{a+b\text{arctanh}(c\sqrt{x})}{\sqrt{x}\left(1-c^2x\right)}d\sqrt{x}+\frac{c\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{3b}-\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{\sqrt{x}}\right)-\frac{(a+b\text{arctanh}(c}{2x}$$

$\downarrow$ 6550

$$2\left(\frac{3}{2}bc\left(2bc\left(\int \frac{a+b\text{arctanh}(c\sqrt{x})}{\left(\sqrt{x}c+1\right)\sqrt{x}}d\sqrt{x}+\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{2b}\right)+\frac{c\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{3b}-\frac{(a+b\text{arctanh}(}{\sqrt{x}}$$

$\downarrow$ 6494

$$2\left(\frac{3}{2}bc\left(2bc\left(-bc\int \frac{\log\left(2-\frac{2}{\sqrt{x}c+1}\right)}{1-c^2x}d\sqrt{x}+\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{2b}+\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\text{arctanh}(c\sqrt{x})\right)$$

$\downarrow$ 2897

$$2\left(\frac{3}{2}bc\left(2bc\left(\frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^2}{2b}+\log\left(2-\frac{2}{c\sqrt{x}+1}\right)\left(a+b\text{arctanh}(c\sqrt{x})\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{\sqrt{x}c+1}-1\right.$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x^2,x]
```

output
```
2*(-1/2*(a + b*ArcTanh[c*Sqrt[x]])^3/x + (3*b*c*(-((a + b*ArcTanh[c*Sqrt[x
]])^2/Sqrt[x]) + (c*(a + b*ArcTanh[c*Sqrt[x]])^3)/(3*b) + 2*b*c*((a + b*Ar
cTanh[c*Sqrt[x]])^2/(2*b) + (a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1 + c*Sq
rt[x])] - (b*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])])/2)))/2)
```

3.207. $\int \frac{\left(a+b\text{arctanh}(c\sqrt{x})\right)^3}{x^2}\,dx$

3.207.3.1 Defintions of rubi rules used

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.207. $\int \frac{\left(a+b\operatorname{arctanh}(c\sqrt{x})\right)^3}{x^2}\,dx$

3.207.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 5.79 (sec) , antiderivative size = 4700, normalized size of antiderivative = 33.10

method	result	size
derivativedivides	Expression too large to display	4700
default	Expression too large to display	4700
parts	Expression too large to display	4702

input
```
int((a+b*arctanh(c*x^(1/2)))^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
2*c^2*(-1/2*a^3/c^2/x+b^3*(-3/2*arctanh(c*x^(1/2))^2+3/2*polylog(2,-(1+c*x
^(1/2))/(-c^2*x+1)^(1/2))+3/2*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/
8*I*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))*csgn(I*(1+c*x^(1/2))^2/(c^2*x
-1))*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))*arcta
nh(c*x^(1/2))^2+1/2*arctanh(c*x^(1/2))^3-3/4*arctanh(c*x^(1/2))^2*ln(c*x^(
1/2)-1)+3/4*arctanh(c*x^(1/2))^2*ln(1+c*x^(1/2))-3/2*arctanh(c*x^(1/2))^2*
ln((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/2*dilog(1+(1+c*x^(1/2))/(-c^2*x+1)^(1
/2))-3/2*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/4*I*Pi*csgn(I/(1-(1+c*x^(
1/2))^2/(c^2*x-1)))^2*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/4*I*Pi*csgn(
I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^3*polylog(2,-(1+c*x^(1/2))/(-c^2*x+1)^(1/
2))-3/4*I*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^3*polylog(2,(1+c*x^(1/2
))/(-c^2*x+1)^(1/2))+3/4*I*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^3*dilo
g(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/4*I*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^
2*x-1)))^3*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/8*I*Pi*csgn(I*(1+c*x^(1
/2))^2/(c^2*x-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^3*dilog(1+(1+c*x^(1/2))/(-
c^2*x+1)^(1/2))+3/8*I*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^(1/2))
^2/(c^2*x-1)))^3*dilog((1+c*x^(1/2))/(-c^2*x+1)^(1/2))+3/8*I*Pi*csgn(I*(1+
c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^3*polylog(2,-(1+c*x^
(1/2))/(-c^2*x+1)^(1/2))+3/8*I*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c
*x^(1/2))^2/(c^2*x-1)))^3*polylog(2,(1+c*x^(1/2))/(-c^2*x+1)^(1/2))-3/8...
```

3.207.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*
b*arctanh(c*sqrt(x)) + a^3)/x^2, x)
```

3.207.6 Sympy [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*atanh(c*x**(1/2)))**3/x**2,x)
```

output
```
Integral((a + b*atanh(c*sqrt(x)))**3/x**2, x)
```

3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(125)=250$.

Time = 1.16 (sec) , antiderivative size = 528, normalized size of antiderivative = 3.72

$$\begin{aligned}
&\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx \\
&=-3\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}\,c\sqrt{x}+\frac{1}{2}\right)+\text{Li}_2\left(\frac{1}{2}\,c\sqrt{x}+\frac{1}{2}\right)\right)b^3c^2 \\
&\quad-3\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\text{Li}_2\left(-c\sqrt{x}+1\right)\right)b^3c^2 \\
&\quad+3\left(\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}\right)+\text{Li}_2\left(c\sqrt{x}+1\right)\right)b^3c^2-3\,ab^2c^2\log\left(c\sqrt{x}-1\right) \\
&\quad-\frac{3}{4}\left(\left(2\,c\log\left(c\sqrt{x}-1\right)-c\log\left(x\right)+\frac{2}{\sqrt{x}}\right)c-\frac{2\,\log\left(-c\sqrt{x}+1\right)}{x}\right)a^2b \\
&\quad-\frac{a^3}{x}+\frac{3}{2}\left(a^2bc^2-2\,ab^2c^2\right)\log\left(c\sqrt{x}+1\right)-\frac{3}{4}\left(a^2bc^2-4\,ab^2c^2\right)\log\left(x\right) \\
&\quad-\frac{12\,a^2bc\sqrt{x}-\left(b^3c^2x-b^3\right)\log\left(c\sqrt{x}+1\right)^3+\left(b^3c^2x-b^3\right)\log\left(-c\sqrt{x}+1\right)^3+6\left(b^3c\sqrt{x}+ab^2-\left(ab^2c^2-\right.\right.}{}
\end{aligned}$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="maxima")
```

output
```
-3*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1
/2))*b^3*c^2 - 3*(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) +
1))*b^3*c^2 + 3*(log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1)
)*b^3*c^2 - 3*a*b^2*c^2*log(c*sqrt(x) - 1) - 3/4*((2*c*log(c*sqrt(x) - 1)
- c*log(x) + 2/sqrt(x))*c - 2*log(-c*sqrt(x) + 1)/x)*a^2*b - a^3/x + 3/2*(
a^2*b*c^2 - 2*a*b^2*c^2)*log(c*sqrt(x) + 1) - 3/4*(a^2*b*c^2 - 4*a*b^2*c^2
)*log(x) - 1/8*(12*a^2*b*c*sqrt(x) - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1)^
3 + (b^3*c^2*x - b^3)*log(-c*sqrt(x) + 1)^3 + 6*(b^3*c*sqrt(x) + a*b^2 - (
a*b^2*c^2 - b^3*c^2)*x)*log(c*sqrt(x) + 1)^2 + 3*(2*b^3*c*sqrt(x) + 2*a*b^
2 - 2*(a*b^2*c^2 + b^3*c^2)*x - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1))*log(
-c*sqrt(x) + 1)^2 + 12*(2*a*b^2*c*sqrt(x) + a^2*b)*log(c*sqrt(x) + 1) - 3*
(8*a*b^2*c*sqrt(x) - (b^3*c^2*x - b^3)*log(c*sqrt(x) + 1)^2 + 4*(b^3*c*sqr
t(x) + a*b^2 - (a*b^2*c^2 - b^3*c^2)*x)*log(c*sqrt(x) + 1))*log(-c*sqrt(x)
 + 1))/x
```

3.207.8 Giac [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(b\operatorname{artanh}\left(c\sqrt{x}\right)+a\right)^3}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^3/x^2, x)
```

3.207.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^2}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(c\,\sqrt{x}\right)\right)^3}{x^2}\,dx$$

input
```
int((a + b*atanh(c*x^(1/2)))^3/x^2,x)
```

output
```
int((a + b*atanh(c*x^(1/2)))^3/x^2, x)
```

3.208 $\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$

3.208.1 Optimal result

Integrand size = 18, antiderivative size = 234

$$\begin{aligned}\int \frac{\left(a+b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx = & -\frac{b^3c^3}{2\sqrt{x}}+\frac{1}{2}b^3c^4\,\mathrm{arctanh}\left(c\sqrt{x}\right)\\ & -\frac{b^2c^2\left(a+b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\right)}{2x}+2bc^4\left(a+b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\right)^2\\ & -\frac{bc\left(a+b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\right)^2}{2x^{3/2}}-\frac{3bc^3\left(a+b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\right)^2}{2\sqrt{x}}\\ & +\frac{1}{2}c^4\left(a+b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\right)^3-\frac{\left(a+b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\right)^3}{2x^2}\\ & +4b^2c^4\left(a+b\,\mathrm{arctanh}\left(c\sqrt{x}\right)\right)\log\left(2-\frac{2}{1+c\sqrt{x}}\right)\\ & -2b^3c^4\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1+c\sqrt{x}}\right)\end{aligned}$$

output
```
1/2*b^3*c^4*arctanh(c*x^(1/2))-1/2*b^2*c^2*(a+b*arctanh(c*x^(1/2)))/x+2*b*
c^4*(a+b*arctanh(c*x^(1/2)))^2-1/2*b*c*(a+b*arctanh(c*x^(1/2)))^2/x^(3/2)+
1/2*c^4*(a+b*arctanh(c*x^(1/2)))^3-1/2*(a+b*arctanh(c*x^(1/2)))^3/x^2+4*b^
2*c^4*(a+b*arctanh(c*x^(1/2)))*ln(2-2/(1+c*x^(1/2)))-2*b^3*c^4*polylog(2,-
1+2/(1+c*x^(1/2)))-1/2*b^3*c^3/x^(1/2)-3/2*b*c^3*(a+b*arctanh(c*x^(1/2)))^
2/x^(1/2)
```

3.208.2 Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.42

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx =$$
$$-\frac{2a^3+2a^2bc\sqrt{x}+2ab^2c^2x+6a^2bc^3x^{3/2}+2b^3c^3x^{3/2}-2ab^2c^4x^2-2b^2\left(bc\sqrt{x}\left(-1-3c^2x+4c^3x^{3/2}\right)+3c\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*Sqrt[x]])^3/x^3,x]
```

output
```
-1/4*(2*a^3 + 2*a^2*b*c*Sqrt[x] + 2*a*b^2*c^2*x + 6*a^2*b*c^3*x^(3/2) + 2*
b^3*c^3*x^(3/2) - 2*a*b^2*c^4*x^2 - 2*b^2*(b*c*Sqrt[x]*(-1 - 3*c^2*x + 4*c
^3*x^(3/2)) + 3*a*(-1 + c^4*x^2))*ArcTanh[c*Sqrt[x]]^2 - 2*b^3*(-1 + c^4*x
^2)*ArcTanh[c*Sqrt[x]]^3 + 2*b*ArcTanh[c*Sqrt[x]]*(3*a^2 + b^2*c^2*x*(1 -
c^2*x) + 2*a*b*c*Sqrt[x]*(1 + 3*c^2*x) - 8*b^2*c^4*x^2*Log[1 - E^(-2*ArcTa
nh[c*Sqrt[x]])]) + 3*a^2*b*c^4*x^2*Log[1 - c*Sqrt[x]] - 3*a^2*b*c^4*x^2*Lo
g[1 + c*Sqrt[x]] - 16*a*b^2*c^4*x^2*Log[(c*Sqrt[x])/Sqrt[1 - c^2*x]] + 8*b
^3*c^4*x^2*PolyLog[2, E^(-2*ArcTanh[c*Sqrt[x]])])/x^2
```

3.208.3 Rubi [A] (verified)

Time = 1.91 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {6454, 6452, 6544, 6452, 6544, 6452, 264, 219, 6510, 6550, 6494, 2897}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}dx$$

$$\Big\downarrow \text{ 6454}$$

$$2\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^{5/2}}d\sqrt{x}$$

$$\Big\downarrow \text{ 6452}$$

$$2\left(\frac{3}{4}bc\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2\left(1-c^2x\right)}d\sqrt{x}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{4x^2}\right)$$

$$\Big\downarrow \text{6544}$$

$$2\left(\frac{3}{4}bc\left(c^2\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x\left(1-c^2x\right)}d\sqrt{x}+\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x^2}d\sqrt{x}\right)-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{4x^2}\right)$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{3}{4}bc\left(\frac{2}{3}bc\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x^{3/2}\left(1-c^2x\right)}d\sqrt{x}+c^2\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{x\left(1-c^2x\right)}d\sqrt{x}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{3x^{3/2}}\right)-\frac{(a+ba}{}\right.$$

$$\Big\downarrow \text{6544}$$

$$2\left(\frac{3}{4}bc\left(\frac{2}{3}bc\left(c^2\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}\left(1-c^2x\right)}d\sqrt{x}+\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{x^{3/2}}d\sqrt{x}\right)+c^2\left(c^2\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{}\right.\right.\right.$$

$$\Big\downarrow \text{6452}$$

$$2\left(\frac{3}{4}bc\left(c^2\left(c^2\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}+2bc\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}\left(1-c^2x\right)}d\sqrt{x}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{\sqrt{x}}\right)+\frac{2}{3}bc\right.\right.$$

$$\Big\downarrow \text{264}$$

$$2\left(\frac{3}{4}bc\left(c^2\left(c^2\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}+2bc\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}\left(1-c^2x\right)}d\sqrt{x}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{\sqrt{x}}\right)+\frac{2}{3}bc\right.\right.$$

$$\Big\downarrow \text{219}$$

$$2\left(\frac{3}{4}bc\left(c^2\left(c^2\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{1-c^2x}d\sqrt{x}+2bc\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}\left(1-c^2x\right)}d\sqrt{x}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{\sqrt{x}}\right)+\frac{2}{3}bc\right.\right.$$

$$\Big\downarrow \text{6510}$$

$$2\left(\frac{3}{4}bc\left(c^2\left(2bc\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\sqrt{x}\left(1-c^2x\right)}d\sqrt{x}+\frac{c\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3b}-\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{\sqrt{x}}\right)+\frac{2}{3}bc\left(c^2\int\frac{a}{}\right.\right.\right.$$

$$\Big\downarrow \text{6550}$$

$$2\left(\frac{3}{4}bc\left(c^2\left(2bc\left(\int\frac{a+b\text{arctanh}\left(c\sqrt{x}\right)}{\left(\sqrt{x}c+1\right)\sqrt{x}}d\sqrt{x}+\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^2}{2b}\right)+\frac{c\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{3b}-\frac{(a+\text{barcta}}{\sqrt{}}\right.\right.\right.$$

$$\Big\downarrow \text{6494}$$

3.208. $\displaystyle\int\frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$

$$2\left(\frac{3}{4}bc\left(c^2\left(2bc\left(-bc\int\frac{\log\left(2-\frac{2}{\sqrt{x}c+1}\right)}{1-c^2x}d\sqrt{x}+\frac{(a+b\text{arctanh}(c\sqrt{x}))^2}{2b}+\log\left(2-\frac{2}{c\sqrt{x}+1}\right)(a+b\text{arctanh}(c\sqrt{\right.\right.\right.\right.\right.$$

$$\Big\downarrow \text{2897}$$

$$2\left(\frac{3}{4}bc\left(c^2\left(2bc\left(\frac{(a+b\text{arctanh}(c\sqrt{x}))^2}{2b}+\log\left(2-\frac{2}{c\sqrt{x}+1}\right)(a+b\text{arctanh}(c\sqrt{x}))-\frac{1}{2}b\,\text{PolyLog}\left(2,\frac{2}{\sqrt{x}c+1}\right.\right.\right.\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*Sqrt[x]])^3/x^3,x]
```

output
```
2*(-1/4*(a + b*ArcTanh[c*Sqrt[x]])^3/x^2 + (3*b*c*(-1/3*(a + b*ArcTanh[c*S
qrt[x]])^2/x^(3/2) + c^2*(-((a + b*ArcTanh[c*Sqrt[x]])^2/Sqrt[x]) + (c*(a
+ b*ArcTanh[c*Sqrt[x]])^3)/(3*b) + 2*b*c*((a + b*ArcTanh[c*Sqrt[x]])^2/(2*
b) + (a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1 + c*Sqrt[x])] - (b*PolyLog[2,
 -1 + 2/(1 + c*Sqrt[x])])/2)) + (2*b*c*(-1/2*(a + b*ArcTanh[c*Sqrt[x]])/x
+ (b*c*(-(1/Sqrt[x]) + c*ArcTanh[c*Sqrt[x]]))/2 + c^2*((a + b*ArcTanh[c*Sq
rt[x]])^2/(2*b) + (a + b*ArcTanh[c*Sqrt[x]])*Log[2 - 2/(1 + c*Sqrt[x])] -
(b*PolyLog[2, -1 + 2/(1 + c*Sqrt[x])])/2)))/3))/4)
```

3.208.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2897
```
Int[Log[u_]*(Pq_)^(m_.), x_Symbol] :> With[{C = FullSimplify[Pq^m*((1 - u)/
D[u, x])]}, Simp[C*PolyLog[2, 1 - u], x] /; FreeQ[C, x]] /; IntegerQ[m] &&
PolyQ[Pq, x] && RationalFunctionQ[u, x] && LeQ[RationalFunctionExponents[u,
 x][[2]], Expon[Pq, x]]
```

3.208. $\int \frac{(a+b\text{arctanh}(c\sqrt{x}))^3}{x^3}\,dx$

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6494
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_))), x
_Symbol] :> Simp[(a + b*ArcTanh[c*x])^p*(Log[2 - 2/(1 + e*(x/d))]/d), x] -
Simp[b*c*(p/d)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(Log[2 - 2/(1 + e*(x/d))]
/(1 - c^2*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c
^2*d^2 - e^2, 0]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

rule 6550
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*d*(p + 1)), x] + Simp[1/
d   Int[(a + b*ArcTanh[c*x])^p/(x*(1 + c*x)), x], x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[c^2*d + e, 0] && GtQ[p, 0]
```

3.208. $\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$

3.208.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 4.70 (sec) , antiderivative size = 1219, normalized size of antiderivative = 5.21

method	result	size
derivativedivides	Expression too large to display	1219
default	Expression too large to display	1219
parts	Expression too large to display	1275

input
```
int((a+b*arctanh(c*x^(1/2)))^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
2*c^4*(-3/4*b^3/c/x^(1/2)*arctanh(c*x^(1/2))^2-2*b^3*dilog((1+c*x^(1/2))/(
-c^2*x+1)^(1/2))+1/4*b^3*arctanh(c*x^(1/2))+1/4*b^3*arctanh(c*x^(1/2))^3-b
^3*arctanh(c*x^(1/2))^2-3/8*b^3*arctanh(c*x^(1/2))^2*ln(c*x^(1/2)-1)+3/8*b
^3*arctanh(c*x^(1/2))^2*ln(1+c*x^(1/2))-3/4*b^3*arctanh(c*x^(1/2))^2*ln((1
+c*x^(1/2))/(-c^2*x+1)^(1/2))+2*b^3*dilog(1+(1+c*x^(1/2))/(-c^2*x+1)^(1/2)
)+3/8*I*b^3*Pi*arctanh(c*x^(1/2))^2-1/4*b^3/c^2/x*arctanh(c*x^(1/2))-1/4*b
^3/c^3/x^(3/2)*arctanh(c*x^(1/2))^2-1/4*b^3/c^4/x^2*arctanh(c*x^(1/2))^3-1
/4*a^3/c^4/x^2+3*a^2*b*(-1/4/c^4/x^2*arctanh(c*x^(1/2))+1/8*ln(1+c*x^(1/2)
)-1/12/c^3/x^(3/2)-1/4/c/x^(1/2)-1/8*ln(c*x^(1/2)-1))+3*a*b^2*(-1/4/c^4/x^
2*arctanh(c*x^(1/2))^2-1/6*arctanh(c*x^(1/2))/c^3/x^(3/2)-1/2*arctanh(c*x^
(1/2))/c/x^(1/2)-1/4*arctanh(c*x^(1/2))*ln(c*x^(1/2)-1)+1/4*arctanh(c*x^(1
/2))*ln(1+c*x^(1/2))+1/8*ln(c*x^(1/2)-1)*ln(1/2*c*x^(1/2)+1/2)-1/16*ln(c*x
^(1/2)-1)^2+1/8*(ln(1+c*x^(1/2))-ln(1/2*c*x^(1/2)+1/2))*ln(-1/2*c*x^(1/2)+
1/2)-1/16*ln(1+c*x^(1/2))^2-1/12/c^2/x+2/3*ln(c*x^(1/2))-1/3*ln(c*x^(1/2)-
1)-1/3*ln(1+c*x^(1/2)))-1/4*b^3/(c*x^(1/2)-(-c^2*x+1)^(1/2)+1)*(-c^2*x+1)^
(1/2)+1/4*b^3/(c*x^(1/2)+(-c^2*x+1)^(1/2)+1)*(-c^2*x+1)^(1/2)-3/16*I*b^3*P
i*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1)/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^3*arctan
h(c*x^(1/2))^2+3/8*I*b^3*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^3*arctan
h(c*x^(1/2))^2-3/16*I*b^3*Pi*csgn(I*(1+c*x^(1/2))^2/(c^2*x-1))^3*arctanh(c
*x^(1/2))^2-3/8*I*b^3*Pi*csgn(I/(1-(1+c*x^(1/2))^2/(c^2*x-1)))^2*arctan...
```

3.208.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\,\text{artanh}\left(c\sqrt{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="fricas")
```

output
```
integral((b^3*arctanh(c*sqrt(x))^3 + 3*a*b^2*arctanh(c*sqrt(x))^2 + 3*a^2*
b*arctanh(c*sqrt(x)) + a^3)/x^3, x)
```

3.208.6 Sympy [F]

$$\int \frac{\left(a+b\text{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\,\text{atanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*atanh(c*x**(1/2)))**3/x**3,x)
```

output
```
Integral((a + b*atanh(c*sqrt(x)))**3/x**3, x)
```

3.208.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(193) = 386$.

Time = 1.33 (sec) , antiderivative size = 703, normalized size of antiderivative = 3.00

$$
\begin{aligned}
&\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx \\
&= -2\left(\log\left(c\sqrt{x}+1\right)\log\left(-\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)+\operatorname{Li}_2\left(\frac{1}{2}c\sqrt{x}+\frac{1}{2}\right)\right)b^3c^4 \\
&\quad - 2\left(\log\left(c\sqrt{x}\right)\log\left(-c\sqrt{x}+1\right)+\operatorname{Li}_2\left(-c\sqrt{x}+1\right)\right)b^3c^4 \\
&\quad + 2\left(\log\left(c\sqrt{x}+1\right)\log\left(-c\sqrt{x}\right)+\operatorname{Li}_2\left(c\sqrt{x}+1\right)\right)b^3c^4 \\
&\quad - \frac{1}{8}\left(\left(6\,c^3\log\left(c\sqrt{x}-1\right)-3\,c^3\log\left(x\right)+\frac{6\,c^2x+3\,c\sqrt{x}+2}{x^{\frac{3}{2}}}\right)c-\frac{6\,\log\left(-c\sqrt{x}+1\right)}{x^2}\right)a^2b \\
&\quad + \frac{1}{4}\left(3\,a^2bc^4-8\,ab^2c^4+b^3c^4\right)\log\left(c\sqrt{x}+1\right) \\
&\quad - \frac{1}{4}\left(8\,ab^2c^4+b^3c^4\right)\log\left(c\sqrt{x}-1\right)-\frac{1}{8}\left(3\,a^2bc^4-16\,ab^2c^4\right)\log\left(x\right)-\frac{a^3}{2\,x^2} \\
&\quad - \frac{4\,a^2bc\sqrt{x}-\left(b^3c^4x^2-b^3\right)\log\left(c\sqrt{x}+1\right)^3+\left(b^3c^4x^2-b^3\right)\log\left(-c\sqrt{x}+1\right)^3+2\left(3\,b^3c^3x^{\frac{3}{2}}+b^3c\sqrt{x}+3\,a\right.}{}
\end{aligned}
$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="maxima")
```

output
```
-2*(log(c*sqrt(x) + 1)*log(-1/2*c*sqrt(x) + 1/2) + dilog(1/2*c*sqrt(x) + 1
/2))*b^3*c^4 - 2*(log(c*sqrt(x))*log(-c*sqrt(x) + 1) + dilog(-c*sqrt(x) +
1))*b^3*c^4 + 2*(log(c*sqrt(x) + 1)*log(-c*sqrt(x)) + dilog(c*sqrt(x) + 1)
)*b^3*c^4 - 1/8*((6*c^3*log(c*sqrt(x) - 1) - 3*c^3*log(x) + (6*c^2*x + 3*c
*sqrt(x) + 2)/x^(3/2))*c - 6*log(-c*sqrt(x) + 1)/x^2)*a^2*b + 1/4*(3*a^2*b
*c^4 - 8*a*b^2*c^4 + b^3*c^4)*log(c*sqrt(x) + 1) - 1/4*(8*a*b^2*c^4 + b^3*
c^4)*log(c*sqrt(x) - 1) - 1/8*(3*a^2*b*c^4 - 16*a*b^2*c^4)*log(x) - 1/2*a^
3/x^2 - 1/16*(4*a^2*b*c*sqrt(x) - (b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^3
 + (b^3*c^4*x^2 - b^3)*log(-c*sqrt(x) + 1)^3 + 2*(3*b^3*c^3*x^(3/2) + b^3*
c*sqrt(x) + 3*a*b^2 - (3*a*b^2*c^4 - 4*b^3*c^4)*x^2)*log(c*sqrt(x) + 1)^2
+ (6*b^3*c^3*x^(3/2) + 2*b^3*c*sqrt(x) + 6*a*b^2 - 2*(3*a*b^2*c^4 + 4*b^3*
c^4)*x^2 - 3*(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1))*log(-c*sqrt(x) + 1)^2
 + 4*(3*a^2*b*c^3 + 2*b^3*c^3)*x^(3/2) - 2*(3*a^2*b*c^2 - 4*a*b^2*c^2)*x +
 4*(6*a*b^2*c^3*x^(3/2) + b^3*c^2*x + 2*a*b^2*c*sqrt(x) + 3*a^2*b)*log(c*s
qrt(x) + 1) - (24*a*b^2*c^3*x^(3/2) + 4*b^3*c^2*x + 8*a*b^2*c*sqrt(x) - 3*
(b^3*c^4*x^2 - b^3)*log(c*sqrt(x) + 1)^2 + 4*(3*b^3*c^3*x^(3/2) + b^3*c*sq
rt(x) + 3*a*b^2 - (3*a*b^2*c^4 - 4*b^3*c^4)*x^2)*log(c*sqrt(x) + 1))*log(-
c*sqrt(x) + 1))/x^2
```

3.208.8 Giac [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(b\operatorname{artanh}\left(c\sqrt{x}\right)+a\right)^3}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(1/2)))^3/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*sqrt(x)) + a)^3/x^3, x)
```

3.208.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{arctanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx=\int \frac{\left(a+b\operatorname{atanh}\left(c\sqrt{x}\right)\right)^3}{x^3}\,dx$$

input
```
int((a + b*atanh(c*x^(1/2)))^3/x^3,x)
```

output
```
int((a + b*atanh(c*x^(1/2)))^3/x^3, x)
```

3.209 $\int x^{3/2}\text{arctanh}\left(\sqrt{x}\right)\,dx$

3.209.1 Optimal result

Integrand size = 12, antiderivative size = 38

$$\int x^{3/2}\text{arctanh}\left(\sqrt{x}\right)\,dx=\frac{x}{5}+\frac{x^2}{10}+\frac{2}{5}x^{5/2}\text{arctanh}\left(\sqrt{x}\right)+\frac{1}{5}\log(1-x)$$

output
```
1/5*x+1/10*x^2+2/5*x^(5/2)*arctanh(x^(1/2))+1/5*ln(1-x)
```

3.209.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.82

$$\int x^{3/2}\text{arctanh}\left(\sqrt{x}\right)\,dx=\frac{1}{10}\left(x(2+x)+4x^{5/2}\text{arctanh}\left(\sqrt{x}\right)+2\log(1-x)\right)$$

input
```
Integrate[x^(3/2)*ArcTanh[Sqrt[x]],x]
```

output
```
(x*(2 + x) + 4*x^(5/2)*ArcTanh[Sqrt[x]] + 2*Log[1 - x])/10
```

3.209.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6452, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}\text{arctanh}\left(\sqrt{x}\right)\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}\left(\sqrt{x}\right)-\frac{1}{5}\int \frac{x^2}{1-x}dx$$

$$\Big\downarrow \text{ 49}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}\left(\sqrt{x}\right)-\frac{1}{5}\int \left(-x+\frac{1}{1-x}-1\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}x^{5/2}\text{arctanh}\left(\sqrt{x}\right)+\frac{1}{5}\left(\frac{x^2}{2}+x+\log(1-x)\right)$$

input
```
Int[x^(3/2)*ArcTanh[Sqrt[x]],x]
```

output
```
(2*x^(5/2)*ArcTanh[Sqrt[x]])/5 + (x + x^2/2 + Log[1 - x])/5
```

3.209.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.209.4 Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{2x^{\frac{5}{2}} \operatorname{arctanh}\left(\sqrt{x}\right)}{5}+\frac{x^{2}}{10}+\frac{x}{5}+\frac{\ln\left(-1+\sqrt{x}\right)}{5}+\frac{\ln\left(1+\sqrt{x}\right)}{5}$	35
default	$\frac{2x^{\frac{5}{2}} \operatorname{arctanh}\left(\sqrt{x}\right)}{5}+\frac{x^{2}}{10}+\frac{x}{5}+\frac{\ln\left(-1+\sqrt{x}\right)}{5}+\frac{\ln\left(1+\sqrt{x}\right)}{5}$	35
meijerg	$\frac{x(3x+6)}{30}-\frac{x^{\frac{5}{2}}\left(\ln\left(1-\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)\right)}{5}+\frac{\ln(1-x)}{5}$	40

input
```
int(x^(3/2)*arctanh(x^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
2/5*x^(5/2)*arctanh(x^(1/2))+1/10*x^2+1/5*x+1/5*ln(-1+x^(1/2))+1/5*ln(1+x^
(1/2))
```

3.209.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int x^{3/2}\operatorname{arctanh}\left(\sqrt{x}\right)\,dx=\frac{1}{5}\,x^{\frac{5}{2}}\log\left(-\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{10}\,x^{2}+\frac{1}{5}\,x+\frac{1}{5}\,\log\left(x-1\right)$$

input
```
integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="fricas")
```

output
```
1/5*x^(5/2)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + 1/10*x^2 + 1/5*x + 1/5*log
(x - 1)
```

3.209.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(29) = 58$.

Time = 1.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.18

$$\int x^{3/2}\text{arctanh}\left(\sqrt{x}\right)\,dx = \frac{4x^{\frac{7}{2}}\,\text{atanh}\left(\sqrt{x}\right)}{10x-10} - \frac{4x^{\frac{5}{2}}\,\text{atanh}\left(\sqrt{x}\right)}{10x-10} + \frac{x^3}{10x-10} + \frac{x^2}{10x-10} + \frac{4x\log\left(\sqrt{x}+1\right)}{10x-10} - \frac{4x\,\text{atanh}\left(\sqrt{x}\right)}{10x-10} - \frac{4\log\left(\sqrt{x}+1\right)}{10x-10} + \frac{4\,\text{atanh}\left(\sqrt{x}\right)}{10x-10} - \frac{2}{10x-10}$$

input
```
integrate(x**(3/2)*atanh(x**(1/2)),x)
```

output
```
4*x**(7/2)*atanh(sqrt(x))/(10*x - 10) - 4*x**(5/2)*atanh(sqrt(x))/(10*x -
10) + x**3/(10*x - 10) + x**2/(10*x - 10) + 4*x*log(sqrt(x) + 1)/(10*x - 1
0) - 4*x*atanh(sqrt(x))/(10*x - 10) - 4*log(sqrt(x) + 1)/(10*x - 10) + 4*a
tanh(sqrt(x))/(10*x - 10) - 2/(10*x - 10)
```

3.209.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int x^{3/2}\text{arctanh}\left(\sqrt{x}\right)\,dx = \frac{2}{5}\,x^{\frac{5}{2}}\,\text{artanh}\left(\sqrt{x}\right) + \frac{1}{10}\,x^2 + \frac{1}{5}\,x + \frac{1}{5}\,\log\left(x-1\right)$$

input
```
integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="maxima")
```

output
```
2/5*x^(5/2)*arctanh(sqrt(x)) + 1/10*x^2 + 1/5*x + 1/5*log(x - 1)
```

3.209.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(26) = 52$.

Time = 0.28 (sec) , antiderivative size = 170, normalized size of antiderivative = 4.47

$$\int x^{3/2}\text{arctanh}\left(\sqrt{x}\right)\,dx = \frac{8\left(\frac{\left(\sqrt{x}+1\right)^{3}}{\left(\sqrt{x}-1\right)^{3}} - \frac{\left(\sqrt{x}+1\right)^{2}}{\left(\sqrt{x}-1\right)^{2}} + \frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{5\left(\frac{\sqrt{x}+1}{\sqrt{x}-1} - 1\right)^{4}} + \frac{2\left(\frac{5\left(\sqrt{x}+1\right)^{4}}{\left(\sqrt{x}-1\right)^{4}} + \frac{10\left(\sqrt{x}+1\right)^{2}}{\left(\sqrt{x}-1\right)^{2}} + 1\right)\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{5\left(\frac{\sqrt{x}+1}{\sqrt{x}-1} - 1\right)^{5}} + \frac{2}{5}\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right) - \frac{2}{5}\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1} + 1\right|\right)$$

input
```
integrate(x^(3/2)*arctanh(x^(1/2)),x, algorithm="giac")
```

output
```
8/5*((sqrt(x) + 1)^3/(sqrt(x) - 1)^3 - (sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + (
sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^4 + 2/5*(5*(
sqrt(x) + 1)^4/(sqrt(x) - 1)^4 + 10*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 1)*l
og(-(sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^5 + 2/5
*log((sqrt(x) + 1)/abs(sqrt(x) - 1)) - 2/5*log(abs(-(sqrt(x) + 1)/(sqrt(x)
 - 1) + 1))
```

3.209.9 Mupad [B] (verification not implemented)

Time = 3.49 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.63

$$\int x^{3/2}\text{arctanh}\left(\sqrt{x}\right)\,dx = \frac{x}{5} + \frac{\ln\left(x-1\right)}{5} + \frac{2\,x^{5/2}\,\text{atanh}\left(\sqrt{x}\right)}{5} + \frac{x^2}{10}$$

input
```
int(x^(3/2)*atanh(x^(1/2)),x)
```

output
```
x/5 + log(x - 1)/5 + (2*x^(5/2)*atanh(x^(1/2)))/5 + x^2/10
```

3.210 $\int \sqrt{x}\,\text{arctanh}\left(\sqrt{x}\right)\,dx$

3.210.1 Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \sqrt{x}\,\text{arctanh}\left(\sqrt{x}\right)\,dx=\frac{x}{3}+\frac{2}{3}x^{3/2}\text{arctanh}\left(\sqrt{x}\right)+\frac{1}{3}\log(1-x)$$

output
```
1/3*x+2/3*x^(3/2)*arctanh(x^(1/2))+1/3*ln(1-x)
```

3.210.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \sqrt{x}\,\text{arctanh}\left(\sqrt{x}\right)\,dx=\frac{1}{3}\left(x+2x^{3/2}\text{arctanh}\left(\sqrt{x}\right)+\log(1-x)\right)$$

input
```
Integrate[Sqrt[x]*ArcTanh[Sqrt[x]],x]
```

output
```
(x + 2*x^(3/2)*ArcTanh[Sqrt[x]] + Log[1 - x])/3
```

3.210.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {6452, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}\text{arctanh}\left(\sqrt{x}\right) \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}\left(\sqrt{x}\right)-\frac{1}{3}\int \frac{x}{1-x}dx$$

$$\Big\downarrow \text{49}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}\left(\sqrt{x}\right)-\frac{1}{3}\int \left(\frac{1}{1-x}-1\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}x^{3/2}\text{arctanh}\left(\sqrt{x}\right)+\frac{1}{3}(x+\log(1-x))$$

input
```
Int[Sqrt[x]*ArcTanh[Sqrt[x]],x]
```

output
```
(2*x^(3/2)*ArcTanh[Sqrt[x]])/3 + (x + Log[1 - x])/3
```

3.210.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.210.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{\ln\left(-1+\sqrt{x}\right)}{3}+\frac{\ln\left(1+\sqrt{x}\right)}{3}$	30
default	$\frac{2x^{\frac{3}{2}}\operatorname{arctanh}\left(\sqrt{x}\right)}{3}+\frac{x}{3}+\frac{\ln\left(-1+\sqrt{x}\right)}{3}+\frac{\ln\left(1+\sqrt{x}\right)}{3}$	30
meijerg	$\frac{x}{3}-\frac{x^{\frac{3}{2}}\left(\ln\left(1-\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)\right)}{3}+\frac{\ln\left(1-x\right)}{3}$	35

input
```
int(arctanh(x^(1/2))*x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x^(3/2)*arctanh(x^(1/2))+1/3*x+1/3*ln(-1+x^(1/2))+1/3*ln(1+x^(1/2))
```

3.210.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \sqrt{x}\operatorname{arctanh}\left(\sqrt{x}\right)\,dx=\frac{1}{3}\,x^{\frac{3}{2}}\log\left(-\frac{x+2\,\sqrt{x}+1}{x-1}\right)+\frac{1}{3}\,x+\frac{1}{3}\,\log\left(x-1\right)$$

input
```
integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="fricas")
```

output
```
1/3*x^(3/2)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + 1/3*x + 1/3*log(x - 1)
```

3.210.6 Sympy [F]

$$\int \sqrt{x}\text{arctanh}(\sqrt{x})\ dx = \int \sqrt{x}\,\text{atanh}\left(\sqrt{x}\right)dx$$

input
```
integrate(atanh(x**(1/2))*x**(1/2),x)
```

output
```
Integral(sqrt(x)*atanh(sqrt(x)), x)
```

3.210.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.61

$$\int \sqrt{x}\text{arctanh}(\sqrt{x})\ dx = \frac{2}{3}\,x^{\frac{3}{2}}\,\text{artanh}\left(\sqrt{x}\right) + \frac{1}{3}\,x + \frac{1}{3}\,\log\left(x-1\right)$$

input
```
integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="maxima")
```

output
```
2/3*x^(3/2)*arctanh(sqrt(x)) + 1/3*x + 1/3*log(x - 1)
```

3.210.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(21) = 42.

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.90

$$\int \sqrt{x}\text{arctanh}(\sqrt{x})\ dx = \frac{2\left(\frac{3\left(\sqrt{x}+1\right)^2}{\left(\sqrt{x}-1\right)^2}+1\right)\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{3\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^3} + \frac{4\left(\sqrt{x}+1\right)}{3\left(\sqrt{x}-1\right)\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right)^2} + \frac{2}{3}\log\left(\frac{\sqrt{x}+1}{\left|\sqrt{x}-1\right|}\right) - \frac{2}{3}\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right|\right)$$

input
```
integrate(arctanh(x^(1/2))*x^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(x) + 1)^2/(sqrt(x) - 1)^2 + 1)*log(-(sqrt(x) + 1)/(sqrt(x) -
1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^3 + 4/3*(sqrt(x) + 1)/((sqrt(x) - 1)
*((sqrt(x) + 1)/(sqrt(x) - 1) - 1)^2) + 2/3*log((sqrt(x) + 1)/abs(sqrt(x)
- 1)) - 2/3*log(abs(-(sqrt(x) + 1)/(sqrt(x) - 1) + 1))
```

3.210.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}\text{arctanh}\left(\sqrt{x}\right)\,dx=\int \sqrt{x}\,\mathrm{atanh}\left(\sqrt{x}\right)\,dx$$

input
```
int(x^(1/2)*atanh(x^(1/2)),x)
```

output
```
int(x^(1/2)*atanh(x^(1/2)), x)
```

3.211 $\int \frac{\operatorname{arctanh}(\sqrt{x})}{\sqrt{x}} dx$

3.211.1 Optimal result

Integrand size = 12, antiderivative size = 20

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{\sqrt{x}} dx = 2\sqrt{x}\operatorname{arctanh}(\sqrt{x}) + \log(1-x)$$

output
```
ln(1-x)+2*arctanh(x^(1/2))*x^(1/2)
```

3.211.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{\sqrt{x}} dx = 2\sqrt{x}\operatorname{arctanh}(\sqrt{x}) + \log(1-x)$$

input
```
Integrate[ArcTanh[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTanh[Sqrt[x]] + Log[1 - x]
```

3.211.3 Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6452, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\sqrt{x})}{\sqrt{x}} \, dx$$

$$\Big\downarrow \text{6452}$$

$$2\sqrt{x}\text{arctanh}(\sqrt{x}) - \int \frac{1}{1-x}dx$$

$$\Big\downarrow \text{16}$$

$$2\sqrt{x}\text{arctanh}(\sqrt{x}) + \log(1-x)$$

input
```
Int[ArcTanh[Sqrt[x]]/Sqrt[x],x]
```

output
```
2*Sqrt[x]*ArcTanh[Sqrt[x]] + Log[1 - x]
```

3.211.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.211.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\ln(1-x)+2\,\operatorname{arctanh}\left(\sqrt{x}\right)\sqrt{x}$	17
default	$\ln(1-x)+2\,\operatorname{arctanh}\left(\sqrt{x}\right)\sqrt{x}$	17
meijerg	$-\sqrt{x}\left(\ln\left(1-\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)\right)+\ln(1-x)$	30

input
```
int(arctanh(x^(1/2))/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(1-x)+2*arctanh(x^(1/2))*x^(1/2)
```

3.211.5 Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{\sqrt{x}}\,dx = \sqrt{x}\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)+\log(x-1)$$

input
```
integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x)*log(-(x + 2*sqrt(x) + 1)/(x - 1)) + log(x - 1)
```

3.211.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(17) = 34.

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.35

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{\sqrt{x}}\,dx = \frac{2x^{\frac{3}{2}}\operatorname{atanh}\left(\sqrt{x}\right)}{x-1}-\frac{2\sqrt{x}\operatorname{atanh}\left(\sqrt{x}\right)}{x-1}+\frac{2x\log\left(\sqrt{x}+1\right)}{x-1}-\frac{2x\operatorname{atanh}\left(\sqrt{x}\right)}{x-1}-\frac{2\log\left(\sqrt{x}+1\right)}{x-1}+\frac{2\operatorname{atanh}\left(\sqrt{x}\right)}{x-1}$$

input
```
integrate(atanh(x**(1/2))/x**(1/2),x)
```

output
```
2*x**(3/2)*atanh(sqrt(x))/(x - 1) - 2*sqrt(x)*atanh(sqrt(x))/(x - 1) + 2*x
*log(sqrt(x) + 1)/(x - 1) - 2*x*atanh(sqrt(x))/(x - 1) - 2*log(sqrt(x) + 1
)/(x - 1) + 2*atanh(sqrt(x))/(x - 1)
```

3.211.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{\sqrt{x}}\,dx = 2\sqrt{x}\operatorname{artanh}\left(\sqrt{x}\right) + \log\left(-x+1\right)$$

input
```
integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(x)*arctanh(sqrt(x)) + log(-x + 1)
```

3.211.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(16) = 32.

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.60

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{\sqrt{x}}\,dx = \frac{2\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1}-1} + 2\log\left(\frac{\sqrt{x}+1}{|\sqrt{x}-1|}\right) - 2\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}+1\right|\right)$$

input
```
integrate(arctanh(x^(1/2))/x^(1/2),x, algorithm="giac")
```

output
```
2*log(-(sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) - 1) + 2*
log((sqrt(x) + 1)/abs(sqrt(x) - 1)) - 2*log(abs(-(sqrt(x) + 1)/(sqrt(x) -
1) + 1))
```

3.211.9 Mupad [B] (verification not implemented)

Time = 3.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{\operatorname{arctanh}\left(\sqrt{x}\right)}{\sqrt{x}}\,dx = \ln\left(x-1\right)+2\,\sqrt{x}\,\operatorname{atanh}\left(\sqrt{x}\right)$$

input
```
int(atanh(x^(1/2))/x^(1/2),x)
```

output
```
log(x - 1) + 2*x^(1/2)*atanh(x^(1/2))
```

3.212 $\int \frac{\operatorname{arctanh}(\sqrt{x})}{x^{3/2}}\,dx$

3.212.1 Optimal result

Integrand size = 12, antiderivative size = 24

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{x^{3/2}}\,dx = -\frac{2\operatorname{arctanh}(\sqrt{x})}{\sqrt{x}} - \log(1-x) + \log(x)$$

output
```
-ln(1-x)+ln(x)-2*arctanh(x^(1/2))/x^(1/2)
```

3.212.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{x^{3/2}}\,dx = -\frac{2\operatorname{arctanh}(\sqrt{x})}{\sqrt{x}} - \log(1-x) + \log(x)$$

input
```
Integrate[ArcTanh[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcTanh[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

3.212.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6452, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(\sqrt{x})}{x^{3/2}}\,dx$$

$$\Big\downarrow \; 6452$$

$$\int \frac{1}{(1-x)x}dx - \frac{2\text{arctanh}(\sqrt{x})}{\sqrt{x}}$$

$$\Big\downarrow \; 47$$

$$\int \frac{1}{1-x}dx + \int \frac{1}{x}dx - \frac{2\text{arctanh}(\sqrt{x})}{\sqrt{x}}$$

$$\Big\downarrow \; 14$$

$$\int \frac{1}{1-x}dx - \frac{2\text{arctanh}(\sqrt{x})}{\sqrt{x}} + \log(x)$$

$$\Big\downarrow \; 16$$

$$-\frac{2\text{arctanh}(\sqrt{x})}{\sqrt{x}} - \log(1-x) + \log(x)$$

input
```
Int[ArcTanh[Sqrt[x]]/x^(3/2),x]
```

output
```
(-2*ArcTanh[Sqrt[x]])/Sqrt[x] - Log[1 - x] + Log[x]
```

3.212.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.212.4 Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

method	result	size
derivativedivides	$-\frac{2\,\mathrm{arctanh}\left(\sqrt{x}\right)}{\sqrt{x}}-\ln\left(1+\sqrt{x}\right)+\ln\left(x\right)-\ln\left(-1+\sqrt{x}\right)$	29
default	$-\frac{2\,\mathrm{arctanh}\left(\sqrt{x}\right)}{\sqrt{x}}-\ln\left(1+\sqrt{x}\right)+\ln\left(x\right)-\ln\left(-1+\sqrt{x}\right)$	29
meijerg	$\ln\left(x\right)+i\pi+\frac{\ln\left(1-\sqrt{x}\right)-\ln\left(1+\sqrt{x}\right)}{\sqrt{x}}-\ln\left(1-x\right)$	37

input
```
int(arctanh(x^(1/2))/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh(x^(1/2))/x^(1/2)-ln(1+x^(1/2))+ln(x)-ln(-1+x^(1/2))
```

3.212.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

$$\int \frac{\mathrm{arctanh}\left(\sqrt{x}\right)}{x^{3/2}}\,dx=-\frac{x\log\left(x-1\right)-x\log\left(x\right)+\sqrt{x}\log\left(-\frac{x+2\sqrt{x}+1}{x-1}\right)}{x}$$

input
```
integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="fricas")
```

output
```
-(x*log(x - 1) - x*log(x) + sqrt(x)*log(-(x + 2*sqrt(x) + 1)/(x - 1)))/x
```

3.212.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(20) = 40$.

Time = 0.41 (sec) , antiderivative size = 126, normalized size of antiderivative = 5.25

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{x^{3/2}}\,dx = -\frac{2x^{\frac{3}{2}}\operatorname{atanh}(\sqrt{x})}{x^2-x} + \frac{2\sqrt{x}\operatorname{atanh}(\sqrt{x})}{x^2-x} + \frac{x^2\log(x)}{x^2-x} - \frac{2x^2\log(\sqrt{x}+1)}{x^2-x} + \frac{2x^2\operatorname{atanh}(\sqrt{x})}{x^2-x} - \frac{x\log(x)}{x^2-x} + \frac{2x\log(\sqrt{x}+1)}{x^2-x} - \frac{2x\operatorname{atanh}(\sqrt{x})}{x^2-x}$$

input
```
integrate(atanh(x**(1/2))/x**(3/2),x)
```

output
```
-2*x**(3/2)*atanh(sqrt(x))/(x**2 - x) + 2*sqrt(x)*atanh(sqrt(x))/(x**2 - x
) + x**2*log(x)/(x**2 - x) - 2*x**2*log(sqrt(x) + 1)/(x**2 - x) + 2*x**2*a
tanh(sqrt(x))/(x**2 - x) - x*log(x)/(x**2 - x) + 2*x*log(sqrt(x) + 1)/(x**
2 - x) - 2*x*atanh(sqrt(x))/(x**2 - x)
```

3.212.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{x^{3/2}}\,dx = -\frac{2\operatorname{artanh}(\sqrt{x})}{\sqrt{x}} - \log(x-1) + \log(x)$$

input
```
integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="maxima")
```

output
```
-2*arctanh(sqrt(x))/sqrt(x) - log(x - 1) + log(x)
```

3.212.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(20) = 40$.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 3.00

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{x^{3/2}}\,dx = \frac{2\log\left(-\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)}{\frac{\sqrt{x}+1}{\sqrt{x}-1}+1} - 2\log\left(\frac{\sqrt{x}+1}{|\sqrt{x}-1|}\right) + 2\log\left(\left|-\frac{\sqrt{x}+1}{\sqrt{x}-1}-1\right|\right)$$

input
```
integrate(arctanh(x^(1/2))/x^(3/2),x, algorithm="giac")
```

output
```
2*log(-(sqrt(x) + 1)/(sqrt(x) - 1))/((sqrt(x) + 1)/(sqrt(x) - 1) + 1) - 2*
log((sqrt(x) + 1)/abs(sqrt(x) - 1)) + 2*log(abs(-(sqrt(x) + 1)/(sqrt(x) -
1) - 1))
```

3.212.9 Mupad [B] (verification not implemented)

Time = 3.82 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{arctanh}(\sqrt{x})}{x^{3/2}}\,dx = 2\ln\left(\sqrt{x}\right) - \ln(x-1) - \frac{2\,\operatorname{atanh}(\sqrt{x})}{\sqrt{x}}$$

input
```
int(atanh(x^(1/2))/x^(3/2),x)
```

output
```
2*log(x^(1/2)) - log(x - 1) - (2*atanh(x^(1/2)))/x^(1/2)
```

3.213 $\int x^3\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx$

3.213.1 Optimal result

Integrand size = 16, antiderivative size = 190

$$\int x^3\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{3bx^{5/2}}{20c}-\frac{\sqrt{3}b\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}+\frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}}-\frac{b\operatorname{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)}{4c^{8/3}}+\frac{1}{4}x^4\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{16c^{8/3}}-\frac{b\log\left(1+\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{16c^{8/3}}$$

output
```
3/20*b*x^(5/2)/c+1/4*x^4*(a+b*arctanh(c*x^(3/2)))-1/4*b*arctanh(c^(1/3)*x^
(1/2))/c^(8/3)+1/16*b*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))/c^(8/3)-1/16*b*ln(1+
c^(2/3)*x+c^(1/3)*x^(1/2))/c^(8/3)-1/8*b*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*
3^(1/2))*3^(1/2)/c^(8/3)+1/8*b*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3
^(1/2)/c^(8/3)
```

3.213.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.17

$$\int x^3 \left(a + b\text{arctanh}\left(cx^{3/2}\right)\right) dx = \frac{3bx^{5/2}}{20c} + \frac{ax^4}{4} + \frac{\sqrt{3}b\arctan\left(\frac{-1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}} + \frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{8c^{8/3}} + \frac{1}{4}bx^4\text{arctanh}\left(cx^{3/2}\right) + \frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}\right)}{8c^{8/3}} - \frac{b\log\left(1+\sqrt[3]{c}\sqrt{x}\right)}{8c^{8/3}} + \frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{16c^{8/3}} - \frac{b\log\left(1+\sqrt[3]{c}\sqrt{x}\right.}{16c^{8/3}}$$

input
```
Integrate[x^3*(a + b*ArcTanh[c*x^(3/2)]),x]
```

output
```
(3*b*x^(5/2))/(20*c) + (a*x^4)/4 + (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*Sqrt[
x])/Sqrt[3]])/(8*c^(8/3)) + (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt
[3]])/(8*c^(8/3)) + (b*x^4*ArcTanh[c*x^(3/2)])/4 + (b*Log[1 - c^(1/3)*Sqrt
[x]])/(8*c^(8/3)) - (b*Log[1 + c^(1/3)*Sqrt[x]])/(8*c^(8/3)) + (b*Log[1 -
c^(1/3)*Sqrt[x] + c^(2/3)*x])/(16*c^(8/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] +
c^(2/3)*x])/(16*c^(8/3))
```

3.213.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6452, 843, 851, 825, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3 \left(a + b\text{arctanh}\left(cx^{3/2}\right)\right) dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{4}x^4\left(a + b\text{arctanh}\left(cx^{3/2}\right)\right) - \frac{3}{8}bc\int \frac{x^{9/2}}{1-c^2x^3}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{4}x^4\left(a + b\text{arctanh}\left(cx^{3/2}\right)\right) - \frac{3}{8}bc\left(\frac{\int \frac{x^{3/2}}{1-c^2x^3}dx}{c^2} - \frac{2x^{5/2}}{5c^2}\right)$$

$$\Big\downarrow \text{851}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{8}bc\left(\frac{2\int \frac{x^2}{1-c^2x^3}d\sqrt{x}}{c^2}-\frac{2x^{5/2}}{5c^2}\right)$$

$$\Big\downarrow \text{825}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{8}bc\left(\frac{2\left(\frac{\int \frac{1}{1-c^{2/3}x}d\sqrt{x}}{3c^{4/3}}+\frac{\int -\frac{\sqrt[3]{c}\sqrt{x}+1}{2\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}d\sqrt{x}}{3c^{4/3}}+\frac{\int -\frac{1-\sqrt[3]{c}\sqrt{x}}{2\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}d\sqrt{x}}{3c^{4/3}}\right)}{c^2}-\frac{2x^{5/2}}{5c^2}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{8}bc\left(\frac{2\left(\frac{\int \frac{1}{1-c^{2/3}x}d\sqrt{x}}{3c^{4/3}}-\frac{\int \frac{\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}-\frac{\int \frac{1-\sqrt[3]{c}\sqrt{x}}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}\right)}{c^2}-\frac{2x^{5/2}}{5c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{8}bc\left(\frac{2\left(-\frac{\int \frac{\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}-\frac{\int \frac{1-\sqrt[3]{c}\sqrt{x}}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)}{3c^{5/3}}\right)}{c^2}-\frac{2x^{5/2}}{5c^2}\right)$$

$$\Big\downarrow \text{1142}$$

3.213. $\int x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{8}bc\left(\frac{2\left(-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\int-\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}\sqrt{x}+1\right)}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}\sqrt{\ldots}\right)}{3c^{5/3}}\ldots\right)}{c^2}\ldots\right)$$

$\downarrow$ 25

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{8}bc\left(\frac{2\left(-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\int\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}\sqrt{x}+1\right)}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\ldots\right)}{3c^{5/3}}\ldots\right)}{c^2}\ldots\right)$$

$\downarrow$ 27

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{8}bc\left(\frac{2\left(-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}-\frac{\frac{3}{2}\int\frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{1}{2}\int\frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}\ldots\right)}{3c^{5/3}}\ldots\right)}{c^2}\ldots\right)$$

$\downarrow$ 1082

3.213. $\int x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{8}bc\left(\frac{2\left(-\frac{\frac{3\int\frac{1}{-x-3}d\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{\sqrt[3]{c}}-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}-\frac{-\frac{1}{2}\int\frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{3\int\frac{1}{-x-3}d\left(2\sqrt[3]{c}\sqrt{x}+1\right)}{\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}\sqrt{\right.}{3c^{5/3}}\right.}{c^2}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{8}bc\left(\frac{2\left(-\frac{-\frac{1}{2}\int\frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{1}{2}\int\frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}\sqrt{\right.}{3c^{5/3}}\right.}{c^2}\right.$$

$$\Big\downarrow \text{1103}$$

$$\frac{1}{4}x^4\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{8}bc\left(\frac{2\left(-\frac{\frac{\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{2\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)}{3c^{5/3}}\right)}{c^2}-\right.$$

input
```
Int[x^3*(a + b*ArcTanh[c*x^(3/2)]),x]
```

3.213. $\int x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx$

output
```
(x^4*(a + b*ArcTanh[c*x^(3/2)]))/4 - (3*b*c*((-2*x^(5/2))/(5*c^2) + (2*(Ar
cTanh[c^(1/3)*Sqrt[x]]/(3*c^(5/3)) - (-((Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*Sqr
t[x])/Sqrt[3]])/c^(1/3)) + Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x]/(2*c^(1/3)
))/(6*c^(4/3)) - ((Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/c^(1/3
) - Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x]/(2*c^(1/3)))/(6*c^(4/3))))/c^2))/
8
```

3.213.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.213.4 Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a\,x^4}{4}+\frac{b\,x^4\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{4}+\frac{3b\,x^{\frac{5}{2}}}{20c}+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{}}{\left(\frac{1}{c}\right.}\right.}{3}\right.}{8c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}$
default	$\frac{a\,x^4}{4}+\frac{b\,x^4\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{4}+\frac{3b\,x^{\frac{5}{2}}}{20c}+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{}}{\left(\frac{1}{c}\right.}\right.}{3}\right.}{8c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}$
parts	$\frac{a\,x^4}{4}+\frac{b\,x^4\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{4}+\frac{3b\,x^{\frac{5}{2}}}{20c}+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{8c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{16c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{}}{\left(\frac{1}{c}\right.}\right.}{3}\right.}{8c^3\left(\frac{1}{c}\right)^{\frac{1}{3}}}$

input
```
int(x^3*(a+b*arctanh(c*x^(3/2))),x,method=_RETURNVERBOSE)
```

output
```
1/4*a*x^4+1/4*b*x^4*arctanh(c*x^(3/2))+3/20*b*x^(5/2)/c+1/8*b/c^3/(1/c)^(1
/3)*ln(x^(1/2)-(1/c)^(1/3))-1/16*b/c^3/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2
)+(1/c)^(2/3))+1/8*b/c^3*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(
1/3)*x^(1/2)+1))-1/8*b/c^3/(1/c)^(1/3)*ln(x^(1/2)+(1/c)^(1/3))+1/16*b/c^3/
(1/c)^(1/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/8*b/c^3*3^(1/2)/(1/c)^
(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

3.213.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 2039, normalized size of antiderivative = 10.73

$$\int x^3\left(a+b\,\mathrm{arctanh}\left(cx^{3/2}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")
```

output
```
1/160*(40*a*c*x^4 + 24*b*x^(5/2) - 10*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^
8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c*log(-1/4*((1/2)^(1/3)*(b^3 - (
c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^5 + ((1/2)^(1
/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^5
 - b^2*c^5 + b^2*sqrt(x)) - 20*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8
+ 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)*c*log((4*(-1/1024*b^3 + 1/102
4*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqrt(3) + 1) - b)^2*c^5 + 2
*(4*(-1/1024*b^3 + 1/1024*(c^8 - 1)*b^3/c^8 + 1/1024*b^3/c^8)^(1/3)*(I*sqr
t(3) + 1) - b)*b*c^5 + b^2*c^5 + b^2*sqrt(x)) + 5*(((1/2)^(1/3)*(b^3 - (c^
8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - 6*b*c - 2*sqrt(
-3/4*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1
) + 2*b)^2 + 3*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*s
qrt(3) + 1) + 2*b)*b - 3*b^2)*c)*log(1/4*((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3
/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c^5 - ((1/2)^(1/3)*(b^3 - (
c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b*c^5 + b^2*c^5 +
 2*b^2*sqrt(x) + 1/2*(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/
3)*(I*sqrt(3) + 1) + 2*b)*c^5 - 2*b*c^5)*sqrt(-3/4*((1/2)^(1/3)*(b^3 - (c^
8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 + 3*((1/2)^(1/3)*
(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b - 3*b^2
)) + 5*(((1/2)^(1/3)*(b^3 - (c^8 - 1)*b^3/c^8 + b^3/c^8)^(1/3)*(I*sqrt(...
```

3.213.6 Sympy [F(-1)]

Timed out.

$$\int x^3 \left(a + b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx = \text{Timed out}$$

input
```
integrate(x**3*(a+b*atanh(c*x**(3/2))),x)
```

output
```
Timed out
```

3.213.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{1}{4}\,ax^4+\frac{1}{80}\left(20\,x^4\,\text{artanh}\left(cx^{\frac{3}{2}}\right)+c\left(\frac{12\,x^{\frac{5}{2}}}{c^2}+\frac{10\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}+\frac{10\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\,c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3\,c^{\frac{1}{3}}}\right)}{c^{\frac{11}{3}}}-\right.\right.$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

output
```
1/4*a*x^4 + 1/80*(20*x^4*arctanh(c*x^(3/2)) + c*(12*x^(5/2)/c^2 + 10*sqrt(
3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))/c^(11/3) + 10
*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/3))/c^(11/3
) - 5*log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1)/c^(11/3) + 5*log(c^(2/3)*x - c^
(1/3)*sqrt(x) + 1)/c^(11/3) - 10*log((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(11/
3) + 10*log((c^(1/3)*sqrt(x) - 1)/c^(1/3))/c^(11/3)))*b
```

3.213.8 Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.19

$$\int x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{1}{4}\,ax^4+\frac{1}{320}\left(40\,x^4\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)+c\left(\frac{48\,x^{\frac{5}{2}}}{c^2}-\frac{10\,\sqrt{3}\left(-i\,\sqrt{3}-1\right)^2|c|^{\frac{4}{3}}\arctan\left(\frac{\sqrt{3}\left(2\,\sqrt{x}+\left(-\frac{1}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{1}{c}\right)^{\frac{1}{3}}}\right)}{c^5}+\frac{5\left(-i\,\sqrt{3}\right.}{}\right.\right.$$

input
```
integrate(x^3*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

output
```
1/4*a*x^4 + 1/320*(40*x^4*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1)) + c*(48*x^
(5/2)/c^2 - 10*sqrt(3)*(-I*sqrt(3) - 1)^2*abs(c)^(4/3)*arctan(1/3*sqrt(3)*
(2*sqrt(x) + (-1/c)^(1/3))/(-1/c)^(1/3))/c^5 + 5*(-I*sqrt(3) - 1)^2*abs(c)
^(4/3)*log(x + sqrt(x)*(-1/c)^(1/3) + (-1/c)^(2/3))/c^5 - 40*(-1/c)^(2/3)*
log(abs(sqrt(x) - (-1/c)^(1/3)))/c^3 + 40*sqrt(3)*abs(c)^(4/3)*arctan(1/3*
sqrt(3)*c^(1/3)*(2*sqrt(x) + 1/c^(1/3)))/c^5 - 20*abs(c)^(4/3)*log(x + sqr
t(x)/c^(1/3) + 1/c^(2/3))/c^5 + 40*log(abs(sqrt(x) - 1/c^(1/3)))/c^(11/3))
)*b
```

3.213.9 Mupad [B] (verification not implemented)

Time = 16.76 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.22

$$\int x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{a\,x^4}{4}+\frac{3\,b\,x^{5/2}}{20\,c}+\frac{b\,\ln\left(\frac{c^{1/3}\,\sqrt{x}-1}{c^{1/3}\,\sqrt{x}+1}\right)}{8\,c^{8/3}}+\frac{\ln\left(1-c\,x^{3/2}\right)\,\left(\frac{b\,x^4}{4}-\frac{b\,c^2\,x^7}{4}\right)}{2\,c^2\,x^3-2}+\frac{b\,x^4\,\ln\left(c\,x^{3/2}+1\right)}{8}+\frac{b\,\ln\left(\frac{\sqrt{3}+c^{2/3}\,x\,1\text{i}-c^{1/3}\,\sqrt{x}\,4\text{i}-\sqrt{3}\,c^{2/3}\,x+1\text{i}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1\text{i}}\right)\,\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1\text{i}}{2}}}{8\,c^{8/3}}+\frac{\sqrt{2}\,b\,\ln\left(\frac{\sqrt{3}\,c^{2/3}\,x+c^{2/3}\,x\,1\text{i}+c^{1/3}\,\sqrt{x}\,4\text{i}-\sqrt{3}+1\text{i}}{2\,c^{2/3}\,x+1-\sqrt{3}\,1\text{i}}\right)\,\sqrt{1+\sqrt{3}\,1\text{i}}\,1\text{i}}{16\,c^{8/3}}$$

input
```
int(x^3*(a + b*atanh(c*x^(3/2))),x)
```

output
```
(a*x^4)/4 + (3*b*x^(5/2))/(20*c) + (b*log((c^(1/3)*x^(1/2) - 1)/(c^(1/3)*x
^(1/2) + 1)))/(8*c^(8/3)) + (log(1 - c*x^(3/2))*((b*x^4)/4 - (b*c^2*x^7)/4
))/(2*c^2*x^3 - 2) + (b*x^4*log(c*x^(3/2) + 1))/8 + (b*log((3^(1/2) + c^(2
/3)*x*1i - c^(1/3)*x^(1/2)*4i - 3^(1/2)*c^(2/3)*x + 1i)/(3^(1/2)*1i + 2*c^
(2/3)*x + 1))*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(8*c^(8/3)) + (2^(1/2)*b*log((
c^(2/3)*x*1i - 3^(1/2) + c^(1/3)*x^(1/2)*4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c
^(2/3)*x - 3^(1/2)*1i + 1))*(3^(1/2)*1i + 1)^(1/2)*1i)/(16*c^(8/3))
```

3.214 $\int x^2(a+b\text{arctanh}(cx^{3/2}))\,dx$

3.214.1 Optimal result

Integrand size = 16, antiderivative size = 49

$$\int x^2(a+b\text{arctanh}(cx^{3/2}))\,dx=\frac{bx^{3/2}}{3c}-\frac{b\text{arctanh}(cx^{3/2})}{3c^2}+\frac{1}{3}x^3(a+b\text{arctanh}(cx^{3/2}))$$

output
```
1/3*b*x^(3/2)/c-1/3*b*arctanh(c*x^(3/2))/c^2+1/3*x^3*(a+b*arctanh(c*x^(3/2
)))
```

3.214.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int x^2(a+b\text{arctanh}(cx^{3/2}))\,dx=\frac{bx^{3/2}}{3c}+\frac{ax^3}{3}+\frac{1}{3}bx^3\text{arctanh}(cx^{3/2})+\frac{b\log(1-cx^{3/2})}{6c^2}-\frac{b\log(1+cx^{3/2})}{6c^2}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x^(3/2)]),x]
```

output
```
(b*x^(3/2))/(3*c) + (a*x^3)/3 + (b*x^3*ArcTanh[c*x^(3/2)])/3 + (b*Log[1 -
c*x^(3/2)])/(6*c^2) - (b*Log[1 + c*x^(3/2)])/(6*c^2)
```

3.214.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {6452, 843, 851, 807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\text{arctanh}\left(cx^{3/2}\right)\right) \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{1}{2}bc\int \frac{x^{7/2}}{1-c^2x^3}dx$$

$$\Big\downarrow \text{843}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{1}{2}bc\left(\frac{\int \frac{\sqrt{x}}{1-c^2x^3}dx}{c^2}-\frac{2x^{3/2}}{3c^2}\right)$$

$$\Big\downarrow \text{851}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{1}{2}bc\left(\frac{2\int \frac{x}{1-c^2x^3}d\sqrt{x}}{c^2}-\frac{2x^{3/2}}{3c^2}\right)$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{1}{2}bc\left(\frac{2\int \frac{1}{1-c^2x}dx^{3/2}}{3c^2}-\frac{2x^{3/2}}{3c^2}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{3}x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{1}{2}bc\left(\frac{2\text{arctanh}\left(cx^{3/2}\right)}{3c^3}-\frac{2x^{3/2}}{3c^2}\right)$$

input
```
Int[x^2*(a + b*ArcTanh[c*x^(3/2)]),x]
```

output
```
(x^3*(a + b*ArcTanh[c*x^(3/2)]))/3 - (b*c*((-2*x^(3/2))/(3*c^2) + (2*ArcTa
nh[c*x^(3/2)])/(3*c^3)))/2
```

3.214.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.214.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.12

method	result	size
parts	$\frac{a x^{3}}{3}+\frac{2 b \left(\frac{c^{2} x^{3} \operatorname{arctanh}\left(c \,x^{\frac{3}{2}}\right)}{2}+\frac{c \,x^{\frac{3}{2}}}{2}+\frac{\ln\left(c \,x^{\frac{3}{2}}-1\right)}{4}-\frac{\ln\left(c \,x^{\frac{3}{2}}+1\right)}{4}\right)}{3 c^{2}}$	55
derivativedivides	$\frac{\frac{a \,c^{2} x^{3}}{3}+\frac{2 b \left(\frac{c^{2} x^{3} \operatorname{arctanh}\left(c \,x^{\frac{3}{2}}\right)}{2}+\frac{c \,x^{\frac{3}{2}}}{2}+\frac{\ln\left(c \,x^{\frac{3}{2}}-1\right)}{4}-\frac{\ln\left(c \,x^{\frac{3}{2}}+1\right)}{4}\right)}{3}}{c^{2}}$	59
default	$\frac{\frac{a \,c^{2} x^{3}}{3}+\frac{2 b \left(\frac{c^{2} x^{3} \operatorname{arctanh}\left(c \,x^{\frac{3}{2}}\right)}{2}+\frac{c \,x^{\frac{3}{2}}}{2}+\frac{\ln\left(c \,x^{\frac{3}{2}}-1\right)}{4}-\frac{\ln\left(c \,x^{\frac{3}{2}}+1\right)}{4}\right)}{3}}{c^{2}}$	59

input
```
int(x^2*(a+b*arctanh(c*x^(3/2))),x,method=_RETURNVERBOSE)
```

output
```
1/3*a*x^3+2/3*b/c^2*(1/2*c^2*x^3*arctanh(c*x^(3/2))+1/2*c*x^(3/2)+1/4*ln(c
*x^(3/2)-1)-1/4*ln(c*x^(3/2)+1))
```

3.214.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{2\,ac^2x^3+2\,bcx^{\frac{3}{2}}+\left(bc^2x^3-b\right)\log\left(-\frac{c^2x^3+2\,cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)}{6\,c^2}$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")
```

output
```
1/6*(2*a*c^2*x^3 + 2*b*c*x^(3/2) + (b*c^2*x^3 - b)*log(-(c^2*x^3 + 2*c*x^(
3/2) + 1)/(c^2*x^3 - 1)))/c^2
```

3.214.6 Sympy [F(-1)]

Timed out.

$$\int x^2\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x**2*(a+b*atanh(c*x**(3/2))),x)
```

output
```
Timed out
```

3.214.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int x^2\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{1}{3}ax^3+\frac{1}{6}\left(2x^3\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+c\left(\frac{2x^{\frac{3}{2}}}{c^2}-\frac{\log\left(cx^{\frac{3}{2}}+1\right)}{c^3}+\frac{\log\left(cx^{\frac{3}{2}}-1\right)}{c^3}\right)\right)b$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(2*x^3*arctanh(c*x^(3/2)) + c*(2*x^(3/2)/c^2 - log(c*x^(3/
2) + 1)/c^3 + log(c*x^(3/2) - 1)/c^3))*b
```

3.214.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(37) = 74.

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.98

$$\int x^2\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{1}{3}ax^3+\frac{2}{3}bc\left(\frac{1}{c^3\left(\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}-1\right)}+\frac{\left(cx^{\frac{3}{2}}+1\right)\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{\left(cx^{\frac{3}{2}}-1\right)c^3\left(\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}-1\right)^2}\right)$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

output
```
1/3*a*x^3 + 2/3*b*c*(1/(c^3*((c*x^(3/2) + 1)/(c*x^(3/2) - 1) - 1)) + (c*x^
(3/2) + 1)*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1))/((c*x^(3/2) - 1)*c^3*((c*
x^(3/2) + 1)/(c*x^(3/2) - 1) - 1)^2))
```

3.214.9 Mupad [B] (verification not implemented)

Time = 4.87 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.24

$$\int x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{a\,x^3}{3}+\frac{b\,x^{3/2}}{3\,c}+\frac{b\,\ln\left(\frac{c\,x^{3/2}-1}{c\,x^{3/2}+1}\right)}{6\,c^2}+\frac{b\,x^3\,\ln\left(c\,x^{3/2}+1\right)}{6}+\frac{b\,x^3\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,c^2\,x^3-2\right)}-\frac{b\,c^2\,x^6\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,c^2\,x^3-2\right)}$$

input
```
int(x^2*(a + b*atanh(c*x^(3/2))),x)
```

output
```
(a*x^3)/3 + (b*x^(3/2))/(3*c) + (b*log((c*x^(3/2) - 1)/(c*x^(3/2) + 1)))/(
6*c^2) + (b*x^3*log(c*x^(3/2) + 1))/6 + (b*x^3*log(1 - c*x^(3/2)))/(3*(2*c
^2*x^3 - 2)) - (b*c^2*x^6*log(1 - c*x^(3/2)))/(3*(2*c^2*x^3 - 2))
```

3.215 $\int x\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx$

3.215.1 Optimal result

Integrand size = 14, antiderivative size = 190

$$\int x\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{3b\sqrt{x}}{2c}+\frac{\sqrt{3}b\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{b\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)}{2c^{4/3}}+\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{8c^{4/3}}-\frac{b\log\left(1+\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{8c^{4/3}}$$

output
```
1/2*x^2*(a+b*arctanh(c*x^(3/2)))-1/2*b*arctanh(c^(1/3)*x^(1/2))/c^(4/3)+1/
8*b*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))/c^(4/3)-1/8*b*ln(1+c^(2/3)*x+c^(1/3)*x
^(1/2))/c^(4/3)+1/4*b*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^
(4/3)-1/4*b*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(4/3)+3/2*
b*x^(1/2)/c
```

3.215.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.17

$$\int x\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{3b\sqrt{x}}{2c}+\frac{ax^2}{2}$$
$$-\frac{\sqrt{3}b\arctan\left(\frac{-1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}-\frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{4c^{4/3}}$$
$$+\frac{1}{2}bx^2\text{arctanh}\left(cx^{3/2}\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}\right)}{4c^{4/3}}-\frac{b\log\left(1+\sqrt[3]{c}\sqrt{x}\right)}{4c^{4/3}}+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{8c^{4/3}}-\frac{b\log\left(1+\sqrt[3]{c}\sqrt{\right.}}{8c^{4/3}}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x^(3/2)]),x]
```

output
```
(3*b*Sqrt[x])/(2*c) + (a*x^2)/2 - (Sqrt[3]*b*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x
])/Sqrt[3]])/(4*c^(4/3)) - (Sqrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[
3]])/(4*c^(4/3)) + (b*x^2*ArcTanh[c*x^(3/2)])/2 + (b*Log[1 - c^(1/3)*Sqrt[
x]])/(4*c^(4/3)) - (b*Log[1 + c^(1/3)*Sqrt[x]])/(4*c^(4/3)) + (b*Log[1 - c
^(1/3)*Sqrt[x] + c^(2/3)*x])/(8*c^(4/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^
(2/3)*x])/(8*c^(4/3))
```

3.215.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {6452, 843, 851, 754, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx$$
$$\Big\downarrow \text{6452}$$
$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\int\frac{x^{5/2}}{1-c^2x^3}dx$$
$$\Big\downarrow \text{843}$$
$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\left(\frac{\int\frac{1}{\sqrt{x}(1-c^2x^3)}dx}{c^2}-\frac{2\sqrt{x}}{c^2}\right)$$

$$\Big\downarrow \; 851$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\left(\frac{2\int \frac{1}{1-c^2x^3}d\sqrt{x}}{c^2}-\frac{2\sqrt{x}}{c^2}\right)$$

$$\Big\downarrow \; 754$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\left(\frac{2\left(\frac{1}{3}\int \frac{1}{1-c^{2/3}x}d\sqrt{x}+\frac{1}{3}\int \frac{2-\sqrt[3]{c}\sqrt{x}}{2\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}d\sqrt{x}+\frac{1}{3}\int \frac{\sqrt[3]{c}\sqrt{x}+2}{2\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}d\sqrt{x}\right)}{c^2}-\frac{2\sqrt{x}}{c^2}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\left(\frac{2\left(\frac{1}{3}\int \frac{1}{1-c^{2/3}x}d\sqrt{x}+\frac{1}{6}\int \frac{2-\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{1}{6}\int \frac{\sqrt[3]{c}\sqrt{x}+2}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}\right)}{c^2}-\frac{2\sqrt{x}}{c^2}\right)$$

$$\Big\downarrow \; 219$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\left(\frac{2\left(\frac{1}{6}\int \frac{2-\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{1}{6}\int \frac{\sqrt[3]{c}\sqrt{x}+2}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)}{3\sqrt[3]{c}}\right)}{c^2}-\frac{2\sqrt{x}}{c^2}\right)$$

$$\Big\downarrow \; 1142$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\left(\frac{2\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\int -\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}\sqrt{x}+1\right)}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}\right.\right.}{c^2}\right.$$

$$\Big\downarrow \; 25$$

3.215. $\int x\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\left(\frac{2\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\int\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\int\frac{\sqrt[3]{c}\left(2\sqrt[3]{c}\sqrt{x}+1\right)}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}\right)}{c^2}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\left(\frac{2\left(\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int\frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{6}\left(\frac{3}{2}\int\frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int\frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\right.}{c^2}\right.$$

$$\Big\downarrow \text{1082}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\left(\frac{2\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{3\int\frac{1}{-x-3}d\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{3\int\frac{1}{-x-3}d\left(2\sqrt[3]{c}\sqrt{x}+1\right)}{\sqrt[3]{c}}\right)}{c^2}\right.$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-\frac{3}{4}bc\left(\frac{2\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}\right)}{c^2}\right.$$

$$\Big\downarrow \text{1103}$$

3.215. $\int x\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx$

$$\frac{1}{2}x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)-$$
$$\frac{3}{4}bc\left(\frac{2\left(\frac{1}{6}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}+\frac{\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{2\sqrt[3]{c}}\right)+\text{ar}\ldots}{c^2}\right.$$

input
```
Int[x*(a + b*ArcTanh[c*x^(3/2)]),x]
```

output
```
(x^2*(a + b*ArcTanh[c*x^(3/2)]))/2 - (3*b*c*((-2*Sqrt[x])/c^2 + (2*(ArcTan
h[c^(1/3)*Sqrt[x]]/(3*c^(1/3)) + (-((Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*Sqrt[x]
)/Sqrt[3]])/c^(1/3)) - Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x]/(2*c^(1/3)))/6
 + ((Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/c^(1/3) + Log[1 + c^
(1/3)*Sqrt[x] + c^(2/3)*x]/(2*c^(1/3)))/6))/c^2))/4
```

3.215.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 843
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Simp[
a*c^n*((m - n + 1)/(b*(m + n*p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^p, x]
, x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*
p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.215. $\int x\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx$

3.215.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a x^2}{2}+\frac{\operatorname{arctanh}\left(c x^{\frac{3}{2}}\right) b x^2}{2}+\frac{3b\sqrt{x}}{2c}+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{}}{\left(\frac{1}{c}\right)}\right.}{3}\right.}{4c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}$
default	$\frac{a x^2}{2}+\frac{\operatorname{arctanh}\left(c x^{\frac{3}{2}}\right) b x^2}{2}+\frac{3b\sqrt{x}}{2c}+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{}}{\left(\frac{1}{c}\right)}\right.}{3}\right.}{4c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}$
parts	$\frac{a x^2}{2}+\frac{\operatorname{arctanh}\left(c x^{\frac{3}{2}}\right) b x^2}{2}+\frac{3b\sqrt{x}}{2c}+\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{}}{\left(\frac{1}{c}\right)}\right.}{3}\right.}{4c^2\left(\frac{1}{c}\right)^{\frac{2}{3}}}$

input
```
int(x*(a+b*arctanh(c*x^(3/2))),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x^2+1/2*arctanh(c*x^(3/2))*b*x^2+3/2*b*x^(1/2)/c+1/4*b/c^2/(1/c)^(2/
3)*ln(x^(1/2)-(1/c)^(1/3))-1/8*b/c^2/(1/c)^(2/3)*ln(x+(1/c)^(1/3)*x^(1/2)+
(1/c)^(2/3))-1/4*b/c^2/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/
3)*x^(1/2)+1))-1/4*b/c^2/(1/c)^(2/3)*ln(x^(1/2)+(1/c)^(1/3))+1/8*b/c^2/(1/
c)^(2/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))-1/4*b/c^2/(1/c)^(2/3)*3^(1/
2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

3.215.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 1488, normalized size of antiderivative = 7.83

$$\int x\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="fricas")
```

output
```
1/16*(8*a*c*x^2 - 2*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)
*(I*sqrt(3) + 1) + 2*b)*c*log(1/2*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 +
b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - b*c + b*sqrt(x)) - 4*(2*(-1/128*
b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*sqrt(3) + 1) - b)*
c*log((2*(-1/128*b^3 + 1/128*(c^4 - 1)*b^3/c^4 + 1/128*b^3/c^4)^(1/3)*(I*s
qrt(3) + 1) - b)*c + b*c + b*sqrt(x)) + (((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3
/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - 6*b*c - 2*sqrt(-3/4*((1/2
)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2
 + 3*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1
) + 2*b)*b - 3*b^2)*c)*log(-1/2*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^
3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c + b*c + sqrt(-3/4*((1/2)^(1/3)*(b^3
- (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 + 3*((1/2)^(
1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b -
3*b^2)*c + 2*b*sqrt(x)) + (((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4
)^(1/3)*(I*sqrt(3) + 1) + 2*b)*c - 6*b*c + 2*sqrt(-3/4*((1/2)^(1/3)*(b^3 -
 (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 + 3*((1/2)^(1
/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b - 3
*b^2)*c)*log(-1/2*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^3/c^4 + b^3/c^4)^(1/3)*(
I*sqrt(3) + 1) + 2*b)*c + b*c - sqrt(-3/4*((1/2)^(1/3)*(b^3 - (c^4 - 1)*b^
3/c^4 + b^3/c^4)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 + 3*((1/2)^(1/3)*(b^3 -...
```

3.215.6 Sympy [F(-1)]

Timed out.

$$\int x\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(x*(a+b*atanh(c*x**(3/2))),x)
```

output
```
Timed out
```

3.215.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.91

$$\int x\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{1}{2}ax^2$$
$$+\frac{1}{8}\left(4x^2\,\text{artanh}\left(cx^{\frac{3}{2}}\right)-c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{7}{3}}}+\frac{\log\left(c^{\frac{2}{3}}x+\cdots\right.}{c^{\cdots}}\right.\right.$$

input
```
integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/8*(4*x^2*arctanh(c*x^(3/2)) - c*(2*sqrt(3)*arctan(1/3*sqrt(3
)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))/c^(7/3) + 2*sqrt(3)*arctan(1/3*sq
rt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/3))/c^(7/3) + log(c^(2/3)*x + c^(
1/3)*sqrt(x) + 1)/c^(7/3) - log(c^(2/3)*x - c^(1/3)*sqrt(x) + 1)/c^(7/3) +
 2*log((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(7/3) - 2*log((c^(1/3)*sqrt(x) - 1
)/c^(1/3))/c^(7/3) - 12*sqrt(x)/c^2))*b
```

3.215.8 Giac [F]

$$\int x\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx=\int\left(b\,\text{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^(3/2))),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^(3/2)) + a)*x, x)
```

3.215.9 Mupad [B] (verification not implemented)

Time = 15.20 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.30

$$\int x\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx=\frac{a\,x^2}{2}+\frac{3\,b\,\sqrt{x}}{2\,c}+\frac{b\,\ln\left(\frac{c^{1/3}\,\sqrt{x}-1}{c^{1/3}\,\sqrt{x}+1}\right)}{4\,c^{4/3}}+\frac{\ln\left(1-c\,x^{3/2}\right)\,\left(\frac{b\,x^2}{2}-\frac{b\,c^2\,x^5}{2}\right)}{2\,c^2\,x^3-2}+\frac{b\,x^2\,\ln\left(c\,x^{3/2}+1\right)}{4}+\frac{b\,\ln\left(\frac{\sqrt{3}\,c^{2/3}\,x+c^{2/3}\,x\,1\mathrm{i}-c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{3}+1\mathrm{i}}{2\,c^{2/3}\,x+1-\sqrt{3}\,1\mathrm{i}}\right)\,\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}{4\,c^{4/3}}+\frac{\sqrt{2}\,b\,\ln\left(\frac{2\,\sqrt{2}-c^{1/3}\,\sqrt{x}\,\left(1+\sqrt{3}\,1\mathrm{i}\right)^{5/2}\,1\mathrm{i}-\sqrt{2}\,c^{2/3}\,x+\sqrt{2}\,\sqrt{3}\,c^{2/3}\,x\,1\mathrm{i}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1\mathrm{i}}\right)\,\sqrt{1+\sqrt{3}\,1\mathrm{i}}\,1\mathrm{i}}{8\,c^{4/3}}$$

input
```
int(x*(a + b*atanh(c*x^(3/2))),x)
```

output
```
(a*x^2)/2 + (3*b*x^(1/2))/(2*c) + (b*log((c^(1/3)*x^(1/2) - 1)/(c^(1/3)*x^
(1/2) + 1)))/(4*c^(4/3)) + (log(1 - c*x^(3/2))*((b*x^2)/2 - (b*c^2*x^5)/2)
)/(2*c^2*x^3 - 2) + (b*x^2*log(c*x^(3/2) + 1))/4 + (b*log((c^(2/3)*x*1i -
3^(1/2) - c^(1/3)*x^(1/2)*4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^(2/3)*x - 3^(1
/2)*1i + 1))*((3^(1/2)*1i)/2 - 1/2)^(1/2))/(4*c^(4/3)) + (2^(1/2)*b*log((2
*2^(1/2) - c^(1/3)*x^(1/2)*(3^(1/2)*1i + 1)^(5/2)*1i - 2^(1/2)*c^(2/3)*x +
 2^(1/2)*3^(1/2)*c^(2/3)*x*1i)/(3^(1/2)*1i + 2*c^(2/3)*x + 1))*(3^(1/2)*1i
 + 1)^(1/2)*1i)/(8*c^(4/3))
```

3.216 $\int \left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx$

3.216.1 Optimal result

Integrand size = 12, antiderivative size = 170

$$\int \left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx = ax-\frac{\sqrt{3}b\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}b\arctan\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}-\frac{b\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}}+bx\text{arctanh}\left(cx^{3/2}\right)+\frac{b\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{4c^{2/3}}-\frac{b\log\left(1+\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)}{4c^{2/3}}$$

output
```
a*x+b*x*arctanh(c*x^(3/2))-b*arctanh(c^(1/3)*x^(1/2))/c^(2/3)+1/4*b*ln(1+c
^(2/3)*x-c^(1/3)*x^(1/2))/c^(2/3)-1/4*b*ln(1+c^(2/3)*x+c^(1/3)*x^(1/2))/c^
(2/3)-1/2*b*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(2/3)+1/2*
b*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)/c^(2/3)
```

3.216.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

$$\int \left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx = ax+bx\text{arctanh}\left(cx^{3/2}\right)-\frac{b\left(\sqrt{3}\left(\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-\arctan\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)\right)+2\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)+\text{arctanh}\left(\frac{\sqrt[3]{c}\sqrt{x}}{1+c^{2/3}x}\right)\right)}{2c^{2/3}}$$

input
```
Integrate[a + b*ArcTanh[c*x^(3/2)],x]
```

output
```
a*x + b*x*ArcTanh[c*x^(3/2)] - (b*(Sqrt[3]*(ArcTan[(1 - 2*c^(1/3)*Sqrt[x])
/Sqrt[3]] - ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]]) + 2*ArcTanh[c^(1/3)*S
qrt[x]] + ArcTanh[(c^(1/3)*Sqrt[x])/(1 + c^(2/3)*x)]))/(2*c^(2/3))
```

3.216.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ax-\frac{\sqrt{3}b\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{2c^{2/3}}+\frac{\sqrt{3}b\arctan\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{2c^{2/3}}-\frac{b\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}}+bx\text{arctanh}\left(cx^{3/2}\right)+\frac{b\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{2/3}}-\frac{b\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{4c^{2/3}}$$

input
```
Int[a + b*ArcTanh[c*x^(3/2)],x]
```

output
```
a*x - (Sqrt[3]*b*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(2*c^(2/3)) + (S
qrt[3]*b*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/(2*c^(2/3)) - (b*ArcTanh
[c^(1/3)*Sqrt[x]])/c^(2/3) + b*x*ArcTanh[c*x^(3/2)] + (b*Log[1 - c^(1/3)*S
qrt[x] + c^(2/3)*x])/(4*c^(2/3)) - (b*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x]
)/(4*c^(2/3))
```

3.216.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.216.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.05

method	result
derivativedivides	$ax + bx\,\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right) + \frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}} - \frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}} + \frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$
default	$ax + bx\,\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right) + \frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}} - \frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}} + \frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$
parts	$ax + bx\,\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right) + \frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}} - \frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4c\left(\frac{1}{c}\right)^{\frac{1}{3}}} + \frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2c\left(\frac{1}{c}\right)^{\frac{1}{3}}}$

input
```
int(a+b*arctanh(c*x^(3/2)),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*x*arctanh(c*x^(3/2))+1/2*b/c/(1/c)^(1/3)*ln(x^(1/2)-(1/c)^(1/3))-1/4
*b/c/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*b*3^(1/2)/c/(1/
c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))-1/2*b/c/(1/c)^(1/3)
*ln(x^(1/2)+(1/c)^(1/3))+1/4*b/c/(1/c)^(1/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c
)^(2/3))+1/2*b*3^(1/2)/c/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(
1/2)-1))
```

3.216.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.92 (sec) , antiderivative size = 1914, normalized size of antiderivative = 11.26

$$\int \left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="fricas")
```

output
```
a*x + 1/8*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3
) + 1) - 4*b - 2*sqrt(-3/4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2
)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 + 3*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2
 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b - 3*b^2))*log(1/4*((1/2)^(1/3)*
(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2*c - ((1
/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)
*b*c + b^2*c + 2*b^2*sqrt(x) + 1/2*sqrt(-3/4*((1/2)^(1/3)*(b^3 - (c^2 - 1)
*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 + 3*((1/2)^(1/3)*(b^3 -
 (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b - 3*b^2)*(((1
/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)
*c - 2*b*c)) + 1/8*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*
(I*sqrt(3) + 1) - 4*b + 2*sqrt(-3/4*((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2
+ b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 + 3*((1/2)^(1/3)*(b^3 - (c^2 - 1
)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b - 3*b^2))*log(1/4*((1/
2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^
2*c - ((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) +
1) + 2*b)*b*c + b^2*c + 2*b^2*sqrt(x) - 1/2*sqrt(-3/4*((1/2)^(1/3)*(b^3 -
(c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)^2 + 3*((1/2)^(1/
3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3) + 1) + 2*b)*b - 3*
b^2)*(((1/2)^(1/3)*(b^3 - (c^2 - 1)*b^3/c^2 + b^3/c^2)^(1/3)*(I*sqrt(3)...
```

3.216.6 Sympy [F(-1)]

Timed out.

$$\int \left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)\,dx=\text{Timed out}$$

input
```
integrate(a+b*atanh(c*x**(3/2)),x)
```

output
```
Timed out
```

3.216.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.93

$$\int (a + b\operatorname{arctanh}(cx^{3/2}))\,dx = \frac{1}{4}\left(c\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}} + \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{5}{3}}} - \frac{\log\left(c^{\frac{2}{3}}x\right.}{}\right.\right.$$
$$+ ax$$

input
```
integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="maxima")
```

output
```
1/4*(c*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3)
)/c^(5/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(
1/3))/c^(5/3) - log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1)/c^(5/3) + log(c^(2/3)
*x - c^(1/3)*sqrt(x) + 1)/c^(5/3) - 2*log((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c
^(5/3) + 2*log((c^(1/3)*sqrt(x) - 1)/c^(1/3))/c^(5/3)) + 4*x*arctanh(c*x^(
3/2)))*b + a*x
```

3.216.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.09

$$\int (a + b\operatorname{arctanh}(cx^{3/2}))\,dx = \frac{1}{4}\left(c\left(\frac{2\sqrt{3}|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{c^2} + \frac{2\sqrt{3}|c|^{\frac{1}{3}}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}\right.\right.}{c^2}\right.\right.$$
$$+ ax$$

input
```
integrate(a+b*arctanh(c*x^(3/2)),x, algorithm="giac")
```

output
```
1/4*(c*(2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1
/3))*abs(c)^(1/3))/c^2 + 2*sqrt(3)*abs(c)^(1/3)*arctan(1/3*sqrt(3)*(2*sqrt
(x) - 1/abs(c)^(1/3))*abs(c)^(1/3))/c^2 - abs(c)^(1/3)*log(x + sqrt(x)/abs
(c)^(1/3) + 1/abs(c)^(2/3))/c^2 + abs(c)^(1/3)*log(x - sqrt(x)/abs(c)^(1/3
) + 1/abs(c)^(2/3))/c^2 - 2*abs(c)^(1/3)*log(sqrt(x) + 1/abs(c)^(1/3))/c^2
 + 2*abs(c)^(1/3)*log(abs(sqrt(x) - 1/abs(c)^(1/3)))/c^2) + 2*x*log(-(c*x^
(3/2) + 1)/(c*x^(3/2) - 1)))*b + a*x
```

3.216.9 Mupad [B] (verification not implemented)

Time = 8.38 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

$$\int \left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\,dx = a\,x + b\,x\,\text{atanh}\left(c\,x^{3/2}\right) - \frac{b\,\text{atanh}\left(c^{1/3}\,\sqrt{x}\right)}{c^{2/3}} + \frac{b\,\text{atanh}\left(\frac{486\,c^8\,\sqrt{x}}{-243\,c^{23/3}+\sqrt{3}\,c^{23/3}\,243\text{i}}\right)\,\left(1+\sqrt{3}\,1\text{i}\right)}{2\,c^{2/3}} + \frac{b\,\text{atanh}\left(\frac{486\,c^8\,\sqrt{x}}{243\,c^{23/3}+\sqrt{3}\,c^{23/3}\,243\text{i}}\right)\,\left(-1+\sqrt{3}\,1\text{i}\right)}{2\,c^{2/3}}$$

input
```
int(a + b*atanh(c*x^(3/2)),x)
```

output
```
a*x + b*x*atanh(c*x^(3/2)) - (b*atanh(c^(1/3)*x^(1/2)))/c^(2/3) + (b*atanh
((486*c^8*x^(1/2))/(3^(1/2)*c^(23/3)*243i - 243*c^(23/3)))*(3^(1/2)*1i + 1
))/(2*c^(2/3)) + (b*atanh((486*c^8*x^(1/2))/(3^(1/2)*c^(23/3)*243i + 243*c
^(23/3)))*(3^(1/2)*1i - 1))/(2*c^(2/3))
```

3.217 $\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}\,dx$

3.217.1 Optimal result

Integrand size = 16, antiderivative size = 34

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}\,dx = a\log(x) - \frac{1}{3}b\,\text{PolyLog}\left(2,-cx^{3/2}\right) + \frac{1}{3}b\,\text{PolyLog}\left(2,cx^{3/2}\right)$$

output
```
a*ln(x)-1/3*b*polylog(2,-c*x^(3/2))+1/3*b*polylog(2,c*x^(3/2))
```

3.217.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}\,dx = a\log(x) + \frac{1}{3}b\left(-\text{PolyLog}\left(2,-cx^{3/2}\right) + \text{PolyLog}\left(2,cx^{3/2}\right)\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^(3/2)])/x,x]
```

output
```
a*Log[x] + (b*(-PolyLog[2, -(c*x^(3/2))] + PolyLog[2, c*x^(3/2)]))/3
```

3.217.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6450, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}\,dx$$

$$\Big\downarrow \ 6450$$

$$\frac{2}{3}\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^{3/2}}dx^{3/2}$$

$$\Big\downarrow \ 6446$$

$$\frac{2}{3}\left(a\log\left(x^{3/2}\right)-\frac{1}{2}b\,\text{PolyLog}\left(2,-cx^{3/2}\right)+\frac{1}{2}b\,\text{PolyLog}\left(2,cx^{3/2}\right)\right)$$

input
```
Int[(a + b*ArcTanh[c*x^(3/2)])/x,x]
```

output
```
(2*(a*Log[x^(3/2)] - (b*PolyLog[2, -(c*x^(3/2))])/2 + (b*PolyLog[2, c*x^(3
/2)])/2))/3
```

3.217.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.217.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(26) = 52.

Time = 0.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68

method	result	size
parts	$a \ln\left(x\right)+b \left(\frac{2 \ln\left(c \,x^{\frac{3}{2}}\right) \operatorname{arctanh}\left(c \,x^{\frac{3}{2}}\right)}{3}-\frac{\operatorname{dilog}\left(c \,x^{\frac{3}{2}}+1\right)}{3}-\frac{\ln\left(c \,x^{\frac{3}{2}}\right) \ln\left(c \,x^{\frac{3}{2}}+1\right)}{3}-\frac{\operatorname{dilog}\left(c \,x^{\frac{3}{2}}\right)}{3}\right)$	57
derivativedivides	$\frac{2 a \ln\left(c \,x^{\frac{3}{2}}\right)}{3}+\frac{2 b \left(\ln\left(c \,x^{\frac{3}{2}}\right) \operatorname{arctanh}\left(c \,x^{\frac{3}{2}}\right)-\frac{\operatorname{dilog}\left(c \,x^{\frac{3}{2}}+1\right)}{2}-\frac{\ln\left(c \,x^{\frac{3}{2}}\right) \ln\left(c \,x^{\frac{3}{2}}+1\right)}{2}-\frac{\operatorname{dilog}\left(c \,x^{\frac{3}{2}}\right)}{2}\right)}{3}$	62
default	$\frac{2 a \ln\left(c \,x^{\frac{3}{2}}\right)}{3}+\frac{2 b \left(\ln\left(c \,x^{\frac{3}{2}}\right) \operatorname{arctanh}\left(c \,x^{\frac{3}{2}}\right)-\frac{\operatorname{dilog}\left(c \,x^{\frac{3}{2}}+1\right)}{2}-\frac{\ln\left(c \,x^{\frac{3}{2}}\right) \ln\left(c \,x^{\frac{3}{2}}+1\right)}{2}-\frac{\operatorname{dilog}\left(c \,x^{\frac{3}{2}}\right)}{2}\right)}{3}$	62

input
```
int((a+b*arctanh(c*x^(3/2)))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+b*(2/3*ln(c*x^(3/2))*arctanh(c*x^(3/2))-1/3*dilog(c*x^(3/2)+1)-1/3
*ln(c*x^(3/2))*ln(c*x^(3/2)+1)-1/3*dilog(c*x^(3/2)))
```

3.217.5 Fricas [F]

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}\,dx=\int \frac{b\,\text{artanh}\left(cx^{\frac{3}{2}}\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x^(3/2)) + a)/x, x)
```

3.217.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arctanh}(cx^{3/2})}{x}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**(3/2)))/x,x)
```

output
```
Timed out
```

3.217.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(24) = 48.

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.82

$$\int \frac{a+b\operatorname{arctanh}(cx^{3/2})}{x}\,dx = -\frac{1}{3}\left(\log\left(cx^{\frac{3}{2}}\right)\log\left(-cx^{\frac{3}{2}}+1\right)+\operatorname{Li}_2\left(-cx^{\frac{3}{2}}+1\right)\right)b + \frac{1}{3}\left(\log\left(cx^{\frac{3}{2}}+1\right)\log\left(-cx^{\frac{3}{2}}\right)+\operatorname{Li}_2\left(cx^{\frac{3}{2}}+1\right)\right)b + a\log(x)$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="maxima")
```

output
```
-1/3*(log(c*x^(3/2))*log(-c*x^(3/2) + 1) + dilog(-c*x^(3/2) + 1))*b + 1/3*
(log(c*x^(3/2) + 1)*log(-c*x^(3/2)) + dilog(c*x^(3/2) + 1))*b + a*log(x)
```

3.217.8 Giac [F]

$$\int \frac{a+b\operatorname{arctanh}(cx^{3/2})}{x}\,dx = \int \frac{b\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^(3/2)) + a)/x, x)
```

3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^{3/2})}{x}\, dx = \int \frac{a + b\,\text{atanh}\left(c\,x^{3/2}\right)}{x}\, dx$$

input
```
int((a + b*atanh(c*x^(3/2)))/x,x)
```

output
```
int((a + b*atanh(c*x^(3/2)))/x, x)
```

3.218 $\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^2}\,dx$

3.218.1 Optimal result

Integrand size = 16, antiderivative size = 172

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^2}\,dx = -\frac{1}{2}\sqrt{3}bc^{2/3}\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)$$
$$+\frac{1}{2}\sqrt{3}bc^{2/3}\arctan\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)+bc^{2/3}\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}-\frac{1}{4}bc^{2/3}\log\left(1-\sqrt[3]{c}\sqrt{x}+c^2\right.$$

output
```
(-a-b*arctanh(c*x^(3/2)))/x+b*c^(2/3)*arctanh(c^(1/3)*x^(1/2))-1/4*b*c^(2/
3)*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))+1/4*b*c^(2/3)*ln(1+c^(2/3)*x+c^(1/3)*x^
(1/2))-1/2*b*c^(2/3)*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)+1/2
*b*c^(2/3)*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)
```

3.218.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.19

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^2}\,dx = -\frac{a}{x}+\frac{1}{2}\sqrt{3}bc^{2/3}\arctan\left(\frac{-1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)$$
$$+\frac{1}{2}\sqrt{3}bc^{2/3}\arctan\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-\frac{b\text{arctanh}\left(cx^{3/2}\right)}{x}-\frac{1}{2}bc^{2/3}\log\left(1-\sqrt[3]{c}\sqrt{x}\right)+\frac{1}{2}bc^{2/3}\log\left(1+\sqrt[3]{c}\sqrt{x}\right)-\frac{1}{4}bc^2$$

input
```
Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^2,x]
```

output
```
-(a/x) + (Sqrt[3]*b*c^(2/3)*ArcTan[(-1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 +
(Sqrt[3]*b*c^(2/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/2 - (b*ArcTanh
[c*x^(3/2)])/x - (b*c^(2/3)*Log[1 - c^(1/3)*Sqrt[x]])/2 + (b*c^(2/3)*Log[1
 + c^(1/3)*Sqrt[x]])/2 - (b*c^(2/3)*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/
4 + (b*c^(2/3)*Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x])/4
```

3.218.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {6452, 851, 754, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^2} \, dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{3}{2}bc\int \frac{1}{\sqrt{x}\left(1-c^2x^3\right)}dx-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

$$\Big\downarrow \text{851}$$

$$3bc\int \frac{1}{1-c^2x^3}d\sqrt{x}-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

$$\Big\downarrow \text{754}$$

$$3bc\left(\frac{1}{3}\int \frac{1}{1-c^{2/3}x}d\sqrt{x}+\frac{1}{3}\int \frac{2-\sqrt[3]{c}\sqrt{x}}{2\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}d\sqrt{x}+\frac{1}{3}\int \frac{\sqrt[3]{c}\sqrt{x}+2}{2\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}d\sqrt{x}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

$$\Big\downarrow \text{27}$$

$$3bc\left(\frac{1}{3}\int \frac{1}{1-c^{2/3}x}d\sqrt{x}+\frac{1}{6}\int \frac{2-\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{1}{6}\int \frac{\sqrt[3]{c}\sqrt{x}+2}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

$$\Big\downarrow \text{219}$$

$$3bc\left(\frac{1}{6}\int \frac{2-\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{1}{6}\int \frac{\sqrt[3]{c}\sqrt{x}+2}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)}{3\sqrt[3]{c}}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

$$\Big\downarrow \; 1142$$

$$3bc\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\int -\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{\vphantom{c}}\right.}{c^{2/3}x+\sqrt[3]{\vphantom{c}}}}{2}\right.\right.$$
$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

$$\Big\downarrow \; 25$$

$$3bc\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\int \frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}\right.}{c^{2/3}x+\sqrt[3]{c}}}{2\sqrt[3]{\vphantom{c}}}\right.\right.$$
$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

$$\Big\downarrow \; 27$$

$$3bc\left(\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}\right)+\frac{1}{6}\left(\frac{3}{2}\int \frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{1}{2}\int \frac{}{c}\right.\right.$$
$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

$$\Big\downarrow \; 1082$$

$$3bc\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{3\int \frac{1}{-x-3}d\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{3\int \frac{1}{-x-3}c}{}\right.\right.$$
$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

$$\Big\downarrow \; 217$$

3.218. $\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^2}\,dx$

$$3bc\left(\frac{1}{6}\left(\frac{1}{2}\int \frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{1}{2}\int \frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\sqrt{3}\arctan\right.\right.$$
$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

$\downarrow$ 1103

$$3bc\left(\frac{1}{6}\left(-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{2\sqrt[3]{c}}\right)+\frac{1}{6}\left(\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}+\frac{\log\left(c^{2/3}x+\sqrt[3]{c}\right.}{2\sqrt[3]{c}}\right.\right.$$
$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x}$$

input
```
Int[(a + b*ArcTanh[c*x^(3/2)])/x^2,x]
```

output
```
-((a + b*ArcTanh[c*x^(3/2)])/x) + 3*b*c*(ArcTanh[c^(1/3)*Sqrt[x]]/(3*c^(1/
3)) + (-((Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/c^(1/3)) - Log[
1 - c^(1/3)*Sqrt[x] + c^(2/3)*x]/(2*c^(1/3)))/6 + ((Sqrt[3]*ArcTan[(1 + 2*
c^(1/3)*Sqrt[x])/Sqrt[3]])/c^(1/3) + Log[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x]/
(2*c^(1/3)))/6)
```

3.218.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 754
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[-a
/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r - s*Cos[(2*k*
Pi)/n]*x)/(r^2 - 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x] + Int[(r + s*Cos[(2
*k*Pi)/n]*x)/(r^2 + 2*r*s*Cos[(2*k*Pi)/n]*x + s^2*x^2), x]; 2*(r^2/(a*n))
 Int[1/(r^2 - s^2*x^2), x] + 2*(r/(a*n))   Sum[u, {k, 1, (n - 2)/4}], x]] /
; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && NegQ[a/b]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.218. $\int \frac{a+b\text{arctanh}(cx^{3/2})}{x^2}\,dx$

3.218.4 Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.97

method	result
derivativedivides	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{x}-\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}+$
default	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{x}-\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}+$
parts	$-\frac{a}{x}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{x}-\frac{b\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{2}{3}}}+\frac{b\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sqrt{x}}{\left(\frac{1}{c}\right)^{\frac{1}{3}}}+1\right)}{3}\right)}{2\left(\frac{1}{c}\right)^{\frac{2}{3}}}+$

input
```
int((a+b*arctanh(c*x^(3/2)))/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a/x-b/x*arctanh(c*x^(3/2))-1/2*b/(1/c)^(2/3)*ln(x^(1/2)-(1/c)^(1/3))+1/4*
b/(1/c)^(2/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+1/2*b/(1/c)^(2/3)*3^(1
/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)+1))+1/2*b/(1/c)^(2/3)*ln(x^(
1/2)+(1/c)^(1/3))-1/4*b/(1/c)^(2/3)*ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))+
1/2*b/(1/c)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

3.218.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.36

$$\int \frac{a+b\operatorname{arctanh}(cx^{3/2})}{x^2}\,dx =$$

$$-\frac{2\sqrt{3}(-c^2)^{\frac{1}{3}}\,bx\arctan\left(\frac{2\sqrt{3}(-c^2)^{\frac{2}{3}}\sqrt{x}+\sqrt{3}c}{3c}\right)-2\sqrt{3}b(c^2)^{\frac{1}{3}}x\arctan\left(\frac{2\sqrt{3}(c^2)^{\frac{2}{3}}\sqrt{x}-\sqrt{3}c}{3c}\right)+(-c^2)^{\frac{1}{3}}\,bx\log\left(c^2x\right.}{}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="fricas")
```

output
```
-1/4*(2*sqrt(3)*(-c^2)^(1/3)*b*x*arctan(1/3*(2*sqrt(3)*(-c^2)^(2/3)*sqrt(x
) + sqrt(3)*c)/c) - 2*sqrt(3)*b*(c^2)^(1/3)*x*arctan(1/3*(2*sqrt(3)*(c^2)^
(2/3)*sqrt(x) - sqrt(3)*c)/c) + (-c^2)^(1/3)*b*x*log(c^2*x - (-c^2)^(1/3)*
c*sqrt(x) + (-c^2)^(2/3)) + b*(c^2)^(1/3)*x*log(c^2*x - (c^2)^(1/3)*c*sqrt
(x) + (c^2)^(2/3)) - 2*(-c^2)^(1/3)*b*x*log(c*sqrt(x) + (-c^2)^(1/3)) - 2*
b*(c^2)^(1/3)*x*log(c*sqrt(x) + (c^2)^(1/3)) + 2*b*log(-(c^2*x^3 + 2*c*x^(
3/2) + 1)/(c^2*x^3 - 1)) + 4*a)/x
```

3.218.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arctanh}\left(cx^{3/2}\right)}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**(3/2)))/x**2,x)
```

output
```
Timed out
```

3.218.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

$$\int \frac{a + b\operatorname{arctanh}\left(cx^{3/2}\right)}{x^2}\,dx = \frac{1}{4}\left(\left(\left(\frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}} + \frac{2\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)}{c^{\frac{1}{3}}} + \frac{\log\left(c\right.}{}\right.\right.\right.$$
$$-\frac{a}{x}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="maxima")
```

output
```
1/4*((2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/c^(1/3))/
c^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/3))/c^(1/
3))/c^(1/3) + log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1)/c^(1/3) - log(c^(2/3)*x
 - c^(1/3)*sqrt(x) + 1)/c^(1/3) + 2*log((c^(1/3)*sqrt(x) + 1)/c^(1/3))/c^(
1/3) - 2*log((c^(1/3)*sqrt(x) - 1)/c^(1/3))/c^(1/3))*c - 4*arctanh(c*x^(3/
2))/x)*b - a/x
```

3.218.8 Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^2}\,dx = \frac{1}{4}\left(\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}+\frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{|c|^{\frac{1}{3}}}+\frac{2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x}-\frac{1}{|c|^{\frac{1}{3}}}\right.\right.}{|c|^{\frac{1}{3}}}\right.$$
$$-\frac{b\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{2x}-\frac{a}{x}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x^2,x, algorithm="giac")
```

output
```
1/4*(2*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/3))*abs(c)^(1/3
))/abs(c)^(1/3) + 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) - 1/abs(c)^(1/3)
)*abs(c)^(1/3))/abs(c)^(1/3) + log(x + sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/
3))/abs(c)^(1/3) - log(x - sqrt(x)/abs(c)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(
1/3) + 2*log(sqrt(x) + 1/abs(c)^(1/3))/abs(c)^(1/3) - 2*log(abs(sqrt(x) -
1/abs(c)^(1/3)))/abs(c)^(1/3))*b*c - 1/2*b*log(-(c*x^(3/2) + 1)/(c*x^(3/2)
 - 1))/x - a/x
```

3.218.9 Mupad [B] (verification not implemented)

Time = 11.40 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.28

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^2}\,dx = \frac{b\,c^{2/3}\,\ln\left(\frac{c^{1/3}\,\sqrt{x}+1}{c^{1/3}\,\sqrt{x}-1}\right)}{2}-\frac{a}{x}+\frac{\ln\left(1-c\,x^{3/2}\right)\,\left(b\,x-b\,c^2\,x^4\right)}{2\,x^2-2\,c^2\,x^5}$$
$$-\frac{b\,\ln\left(c\,x^{3/2}+1\right)}{2\,x}+\frac{b\,c^{2/3}\,\ln\left(\frac{\sqrt{3}+c^{2/3}\,x\,1\mathrm{i}-c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{3}\,c^{2/3}\,x+1\mathrm{i}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1\mathrm{i}}\right)\,\sqrt{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{2}$$
$$+\frac{b\,c^{2/3}\,\ln\left(\frac{\sqrt{3}\,c^{2/3}\,x+c^{2/3}\,x\,1\mathrm{i}+c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{3}+1\mathrm{i}}{2\,c^{2/3}\,x+1-\sqrt{3}\,1\mathrm{i}}\right)\,\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}{2}$$

input
```
int((a + b*atanh(c*x^(3/2)))/x^2,x)
```

output

```
(b*c^(2/3)*log((c^(1/3)*x^(1/2) + 1)/(c^(1/3)*x^(1/2) - 1)))/2 - a/x + (lo
g(1 - c*x^(3/2))*(b*x - b*c^2*x^4))/(2*x^2 - 2*c^2*x^5) - (b*log(c*x^(3/2)
 + 1))/(2*x) + (b*c^(2/3)*log((3^(1/2) + c^(2/3)*x*1i - c^(1/3)*x^(1/2)*4i
 - 3^(1/2)*c^(2/3)*x + 1i)/(3^(1/2)*1i + 2*c^(2/3)*x + 1))*((3^(1/2)*1i)/2
 + 1/2)^(1/2)*1i)/2 + (b*c^(2/3)*log((c^(2/3)*x*1i - 3^(1/2) + c^(1/3)*x^(
1/2)*4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^(2/3)*x - 3^(1/2)*1i + 1))*((3^(1/2
)*1i)/2 - 1/2)^(1/2))/2
```

3.218. $\int \frac{a+b\operatorname{arctanh}(cx^{3/2})}{x^2}\,dx$

3.219 $\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^3} dx$

3.219.1 Optimal result

Integrand size = 16, antiderivative size = 188

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^3} dx = -\frac{3bc}{2\sqrt{x}}+\frac{1}{4}\sqrt{3}bc^{4/3}\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right) -\frac{1}{4}\sqrt{3}bc^{4/3}\arctan\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)+\frac{1}{2}bc^{4/3}\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{8}bc^{4/3}\log\left(1-\sqrt[3]{c}\sqrt{x}+\right.$$

output
```
1/2*(-a-b*arctanh(c*x^(3/2)))/x^2+1/2*b*c^(4/3)*arctanh(c^(1/3)*x^(1/2))-1
/8*b*c^(4/3)*ln(1+c^(2/3)*x-c^(1/3)*x^(1/2))+1/8*b*c^(4/3)*ln(1+c^(2/3)*x+
c^(1/3)*x^(1/2))+1/4*b*c^(4/3)*arctan(1/3*(1-2*c^(1/3)*x^(1/2))*3^(1/2))*3
^(1/2)-1/4*b*c^(4/3)*arctan(1/3*(1+2*c^(1/3)*x^(1/2))*3^(1/2))*3^(1/2)-3/2
*b*c/x^(1/2)
```

3.219.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.17

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^3} dx = -\frac{a}{2x^2}-\frac{3bc}{2\sqrt{x}}-\frac{1}{4}\sqrt{3}bc^{4/3}\arctan\left(\frac{-1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right) -\frac{1}{4}\sqrt{3}bc^{4/3}\arctan\left(\frac{1+2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)-\frac{b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}-\frac{1}{4}bc^{4/3}\log\left(1-\sqrt[3]{c}\sqrt{x}\right)+\frac{1}{4}bc^{4/3}\log\left(1+\sqrt[3]{c}\sqrt{x}\right)-\frac{1}{8}bc$$

input
```
Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^3,x]
```

output
```
-1/2*a/x^2 - (3*b*c)/(2*Sqrt[x]) - (Sqrt[3]*b*c^(4/3)*ArcTan[(-1 + 2*c^(1/
3)*Sqrt[x])/Sqrt[3]])/4 - (Sqrt[3]*b*c^(4/3)*ArcTan[(1 + 2*c^(1/3)*Sqrt[x]
)/Sqrt[3]])/4 - (b*ArcTanh[c*x^(3/2)])/(2*x^2) - (b*c^(4/3)*Log[1 - c^(1/3
)*Sqrt[x]])/4 + (b*c^(4/3)*Log[1 + c^(1/3)*Sqrt[x]])/4 - (b*c^(4/3)*Log[1
- c^(1/3)*Sqrt[x] + c^(2/3)*x])/8 + (b*c^(4/3)*Log[1 + c^(1/3)*Sqrt[x] + c
^(2/3)*x])/8
```

3.219.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6452, 847, 851, 825, 27, 219, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^3}dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{3}{4}bc\int \frac{1}{x^{3/2}\left(1-c^2x^3\right)}dx-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$$\Big\downarrow \text{847}$$

$$\frac{3}{4}bc\left(c^2\int \frac{x^{3/2}}{1-c^2x^3}dx-\frac{2}{\sqrt{x}}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$$\Big\downarrow \text{851}$$

$$\frac{3}{4}bc\left(2c^2\int \frac{x^2}{1-c^2x^3}d\sqrt{x}-\frac{2}{\sqrt{x}}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$$\Big\downarrow \text{825}$$

$$\frac{3}{4}bc\left(2c^2\left(\frac{\int \frac{1}{1-c^{2/3}x}d\sqrt{x}}{3c^{4/3}}+\frac{\int -\frac{\sqrt[3]{c}\sqrt{x}+1}{2\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}d\sqrt{x}}{3c^{4/3}}+\frac{\int -\frac{1-\sqrt[3]{c}\sqrt{x}}{2\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}d\sqrt{x}}{3c^{4/3}}\right)-\frac{2}{\sqrt{x}}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$$\Big\downarrow \ 27$$

$$\frac{3}{4}bc\left(2c^2\left(\frac{\int \frac{1}{1-c^{2/3}x}d\sqrt{x}}{3c^{4/3}}-\frac{\int \frac{\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}-\frac{\int \frac{1-\sqrt[3]{c}\sqrt{x}}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}\right)-\frac{2}{\sqrt{x}}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$$\Big\downarrow \ 219$$

$$\frac{3}{4}bc\left(2c^2\left(-\frac{\int \frac{\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}-\frac{\int \frac{1-\sqrt[3]{c}\sqrt{x}}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}+\frac{\text{arctanh}\left(\sqrt[3]{c}\sqrt{x}\right)}{3c^{5/3}}\right)-\frac{2}{\sqrt{x}}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$$\Big\downarrow \ 1142$$

$$\frac{3}{4}bc\left(2c^2\left(-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}+\frac{\int -\frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}\sqrt{x}+1\right)}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}}{6c^{4/3}}+ \text{ar}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$$\Big\downarrow \ 25$$

$$\frac{3}{4}bc\left(2c^2\left(-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\int \frac{\sqrt[3]{c}\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\int \frac{\sqrt[3]{c}\left(2\sqrt[3]{c}\sqrt{x}+1\right)}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{2\sqrt[3]{c}}}{6c^{4/3}}+ \text{arc}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$$\Big\downarrow \ 27$$

3.219. $\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^3}\,dx$

$$\frac{3}{4}bc\left(2c^2\left(-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{1}{2}\int \frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}-\frac{\frac{3}{2}\int \frac{1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{1}{2}\int \frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$\big\downarrow$ 1082

$$\frac{3}{4}bc\left(2c^2\left(-\frac{\frac{3\int \frac{1}{-x-3}d\left(1-2\sqrt[3]{c}\sqrt{x}\right)}{\sqrt[3]{c}}-\frac{1}{2}\int \frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}-\frac{-\frac{1}{2}\int \frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{3\int \frac{1}{-x-3}d\left(2\sqrt[3]{c}\sqrt{x}+1\right)}{\sqrt[3]{c}}}{6c^{4/3}}+\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$\big\downarrow$ 217

$$\frac{3}{4}bc\left(2c^2\left(-\frac{-\frac{1}{2}\int \frac{1-2\sqrt[3]{c}\sqrt{x}}{c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{1}{2}\int \frac{2\sqrt[3]{c}\sqrt{x}+1}{c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1}d\sqrt{x}}{6c^{4/3}}+\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

$\big\downarrow$ 1103

$$\frac{3}{4}bc\left(2c^2\left(-\frac{\frac{\log\left(c^{2/3}x-\sqrt[3]{c}\sqrt{x}+1\right)}{2\sqrt[3]{c}}-\frac{\sqrt{3}\arctan\left(\frac{1-2\sqrt[3]{c}\sqrt{x}}{\sqrt{3}}\right)}{\sqrt[3]{c}}}{6c^{4/3}}-\frac{\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{c}\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt[3]{c}}-\frac{\log\left(c^{2/3}x+\sqrt[3]{c}\sqrt{x}+1\right)}{2\sqrt[3]{c}}}{6c^{4/3}}+\frac{\text{arctanh}\left(\cdots\right.}{3c^{5/3}}\right.\right.$$

$$\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{2x^2}$$

input
```
Int[(a + b*ArcTanh[c*x^(3/2)])/x^3,x]
```

3.219. $\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^3}\,dx$

output
```
-1/2*(a + b*ArcTanh[c*x^(3/2)])/x^2 + (3*b*c*(-2/Sqrt[x] + 2*c^2*(ArcTanh[
c^(1/3)*Sqrt[x]]/(3*c^(5/3)) - (-((Sqrt[3]*ArcTan[(1 - 2*c^(1/3)*Sqrt[x])/
Sqrt[3]])/c^(1/3)) + Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x]/(2*c^(1/3)))/(6*
c^(4/3)) - ((Sqrt[3]*ArcTan[(1 + 2*c^(1/3)*Sqrt[x])/Sqrt[3]])/c^(1/3) - Lo
g[1 + c^(1/3)*Sqrt[x] + c^(2/3)*x]/(2*c^(1/3)))/(6*c^(4/3)))))/4
```

3.219.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 825
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator
[Rt[-a/b, n]], s = Denominator[Rt[-a/b, n]], k, u}, Simp[u = Int[(r*Cos[2*k
*m*(Pi/n)] - s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[2*k*(Pi/n)]*x +
s^2*x^2), x] + Int[(r*Cos[2*k*m*(Pi/n)] + s*Cos[2*k*(m + 1)*(Pi/n)]*x)/(r^2
 + 2*r*s*Cos[2*k*(Pi/n)]*x + s^2*x^2), x]; 2*(r^(m + 2)/(a*n*s^m))   Int[1/
(r^2 - s^2*x^2), x] + 2*(r^(m + 1)/(a*n*s^m))   Sum[u, {k, 1, (n - 2)/4}],
x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1
] && NegQ[a/b]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

3.219. $\int \frac{a+b\operatorname{arctanh}(cx^{3/2})}{x^3}\,dx$

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.219.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.96

method	result
derivativedivides	$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{2x^2}-\frac{3bc}{2\sqrt{x}}-\frac{bc\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2}{(}\right.}{}\right.}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}$
default	$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{2x^2}-\frac{3bc}{2\sqrt{x}}-\frac{bc\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2}{(}\right.}{}\right.}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}$
parts	$-\frac{a}{2x^2}-\frac{b\,\mathrm{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{2x^2}-\frac{3bc}{2\sqrt{x}}-\frac{bc\ln\left(\sqrt{x}-\left(\frac{1}{c}\right)^{\frac{1}{3}}\right)}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}+\frac{bc\ln\left(x+\left(\frac{1}{c}\right)^{\frac{1}{3}}\sqrt{x}+\left(\frac{1}{c}\right)^{\frac{2}{3}}\right)}{8\left(\frac{1}{c}\right)^{\frac{1}{3}}}-\frac{bc\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2}{(}\right.}{}\right.}{4\left(\frac{1}{c}\right)^{\frac{1}{3}}}$

input
```
int((a+b*arctanh(c*x^(3/2)))/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/x^2-1/2*b/x^2*arctanh(c*x^(3/2))-3/2*b*c/x^(1/2)-1/4*b*c/(1/c)^(1/3
)*ln(x^(1/2)-(1/c)^(1/3))+1/8*b*c/(1/c)^(1/3)*ln(x+(1/c)^(1/3)*x^(1/2)+(1/
c)^(2/3))-1/4*b*c*3^(1/2)/(1/c)^(1/3)*arctan(1/3*3^(1/2)*(2/(1/c)^(1/3)*x^
(1/2)+1))+1/4*b*c/(1/c)^(1/3)*ln(x^(1/2)+(1/c)^(1/3))-1/8*b*c/(1/c)^(1/3)*
ln(x-(1/c)^(1/3)*x^(1/2)+(1/c)^(2/3))-1/4*b*c*3^(1/2)/(1/c)^(1/3)*arctan(1
/3*3^(1/2)*(2/(1/c)^(1/3)*x^(1/2)-1))
```

3.219.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.14

$$\int \frac{a+b\,\mathrm{arctanh}\left(cx^{3/2}\right)}{x^3}\,dx =$$

$$-\frac{2\sqrt{3}b(-c)^{\frac{1}{3}}cx^2\arctan\left(\frac{2}{3}\sqrt{3}(-c)^{\frac{1}{3}}\sqrt{x}-\frac{1}{3}\sqrt{3}\right)+2\sqrt{3}bc^{\frac{4}{3}}x^2\arctan\left(\frac{2}{3}\sqrt{3}c^{\frac{1}{3}}\sqrt{x}-\frac{1}{3}\sqrt{3}\right)+b(-c)^{\frac{1}{3}}cx^2\,\mathrm{l}}{}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(3)*b*(-c)^(1/3)*c*x^2*arctan(2/3*sqrt(3)*(-c)^(1/3)*sqrt(x) -
 1/3*sqrt(3)) + 2*sqrt(3)*b*c^(4/3)*x^2*arctan(2/3*sqrt(3)*c^(1/3)*sqrt(x)
 - 1/3*sqrt(3)) + b*(-c)^(1/3)*c*x^2*log(c*x + (-c)^(2/3)*sqrt(x) - (-c)^(
1/3)) + b*c^(4/3)*x^2*log(c*x - c^(2/3)*sqrt(x) + c^(1/3)) - 2*b*(-c)^(1/3
)*c*x^2*log(c*sqrt(x) - (-c)^(2/3)) - 2*b*c^(4/3)*x^2*log(c*sqrt(x) + c^(2
/3)) + 12*b*c*x^(3/2) + 2*b*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)
) + 4*a)/x^2
```

3.219.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arctanh}\left(cx^{3/2}\right)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**(3/2)))/x**3,x)
```

output
```
Timed out
```

3.219.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.89

$$\int \frac{a + b\operatorname{arctanh}\left(cx^{3/2}\right)}{x^3}\,dx =$$
$$-\frac{1}{8}\left(\left(2\sqrt{3}c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}+c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)+2\sqrt{3}c^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2c^{\frac{2}{3}}\sqrt{x}-c^{\frac{1}{3}}\right)}{3c^{\frac{1}{3}}}\right)-c^{\frac{1}{3}}\log\left(c^{\frac{2}{3}}x+c^{\frac{1}{3}}\sqrt{\right.\right.\right.$$
$$-\frac{a}{2x^2}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="maxima")
```

output
```
-1/8*((2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) + c^(1/3))/
c^(1/3)) + 2*sqrt(3)*c^(1/3)*arctan(1/3*sqrt(3)*(2*c^(2/3)*sqrt(x) - c^(1/
3))/c^(1/3)) - c^(1/3)*log(c^(2/3)*x + c^(1/3)*sqrt(x) + 1) + c^(1/3)*log(
c^(2/3)*x - c^(1/3)*sqrt(x) + 1) - 2*c^(1/3)*log((c^(1/3)*sqrt(x) + 1)/c^(
1/3)) + 2*c^(1/3)*log((c^(1/3)*sqrt(x) - 1)/c^(1/3)) + 12/sqrt(x))*c + 4*a
rctanh(c*x^(3/2))/x^2)*b - 1/2*a/x^2
```

3.219.8 Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.06

$$\int \frac{a + b\text{arctanh}(cx^{3/2})}{x^3}\,dx = -\frac{\sqrt{3}bc^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} + \frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{4\,|c|^{\frac{5}{3}}} - \frac{\sqrt{3}bc^3\arctan\left(\frac{1}{3}\sqrt{3}\left(2\sqrt{x} - \frac{1}{|c|^{\frac{1}{3}}}\right)|c|^{\frac{1}{3}}\right)}{4\,|c|^{\frac{5}{3}}} + \frac{bc^3\log\left(x + \frac{\sqrt{x}}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{8\,|c|^{\frac{5}{3}}} - \frac{bc^3\log\left(x - \frac{\sqrt{x}}{|c|^{\frac{1}{3}}} + \frac{1}{|c|^{\frac{2}{3}}}\right)}{8\,|c|^{\frac{5}{3}}} + \frac{bc^3\log\left(\sqrt{x} + \frac{1}{|c|^{\frac{1}{3}}}\right)}{4\,|c|^{\frac{5}{3}}} - \frac{bc^3\log\left(\left|\sqrt{x} - \frac{1}{|c|^{\frac{1}{3}}}\right|\right)}{4\,|c|^{\frac{5}{3}}} - \frac{b\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{4\,x^2} - \frac{3\,bcx^{\frac{3}{2}} + a}{2\,x^2}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x^3,x, algorithm="giac")
```

output
```
-1/4*sqrt(3)*b*c^3*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1/abs(c)^(1/3))*abs(c)^
(1/3))/abs(c)^(5/3) - 1/4*sqrt(3)*b*c^3*arctan(1/3*sqrt(3)*(2*sqrt(x) - 1/
abs(c)^(1/3))*abs(c)^(1/3))/abs(c)^(5/3) + 1/8*b*c^3*log(x + sqrt(x)/abs(c
)^(1/3) + 1/abs(c)^(2/3))/abs(c)^(5/3) - 1/8*b*c^3*log(x - sqrt(x)/abs(c)^
(1/3) + 1/abs(c)^(2/3))/abs(c)^(5/3) + 1/4*b*c^3*log(sqrt(x) + 1/abs(c)^(1
/3))/abs(c)^(5/3) - 1/4*b*c^3*log(abs(sqrt(x) - 1/abs(c)^(1/3)))/abs(c)^(5
/3) - 1/4*b*log(-(c*x^(3/2) + 1)/(c*x^(3/2) - 1))/x^2 - 1/2*(3*b*c*x^(3/2)
 + a)/x^2
```

3.219.9 Mupad [B] (verification not implemented)

Time = 10.36 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.21

$$\int \frac{a+b\text{arctanh}(cx^{3/2})}{x^3}\,dx = \frac{b\,c^{4/3}\,\ln\left(\frac{c^{1/3}\,\sqrt{x}+1}{c^{1/3}\,\sqrt{x}-1}\right)}{4} - \frac{a}{2\,x^2} + \frac{\ln\left(1-c\,x^{3/2}\right)\,\left(\frac{b\,x}{2}-\frac{b\,c^2\,x^4}{2}\right)}{2\,x^3-2\,c^2\,x^6} - \frac{3\,b\,c}{2\,\sqrt{x}} - \frac{b\,\ln\left(c\,x^{3/2}+1\right)}{4\,x^2} + \frac{b\,c^{4/3}\,\ln\left(\frac{\sqrt{3}+c^{2/3}\,x\,1\mathrm{i}+c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{3}\,c^{2/3}\,x+1\mathrm{i}}{2\,c^{2/3}\,x+1+\sqrt{3}\,1\mathrm{i}}\right)\,\sqrt{-\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}}{4} + \frac{b\,c^{4/3}\,\ln\left(\frac{\sqrt{3}\,c^{2/3}\,x+c^{2/3}\,x\,1\mathrm{i}-c^{1/3}\,\sqrt{x}\,4\mathrm{i}-\sqrt{3}+1\mathrm{i}}{2\,c^{2/3}\,x+1-\sqrt{3}\,1\mathrm{i}}\right)\,\sqrt{\frac{1}{2}+\frac{\sqrt{3}\,1\mathrm{i}}{2}}\,1\mathrm{i}}{4}$$

input
```
int((a + b*atanh(c*x^(3/2)))/x^3,x)
```

output
```
(b*c^(4/3)*log((c^(1/3)*x^(1/2) + 1)/(c^(1/3)*x^(1/2) - 1)))/4 - a/(2*x^2)
 + (log(1 - c*x^(3/2))*((b*x)/2 - (b*c^2*x^4)/2))/(2*x^3 - 2*c^2*x^6) - (3
*b*c)/(2*x^(1/2)) - (b*log(c*x^(3/2) + 1))/(4*x^2) + (b*c^(4/3)*log((3^(1/
2) + c^(2/3)*x*1i + c^(1/3)*x^(1/2)*4i - 3^(1/2)*c^(2/3)*x + 1i)/(3^(1/2)*
1i + 2*c^(2/3)*x + 1))*((3^(1/2)*1i)/2 - 1/2)^(1/2))/4 + (b*c^(4/3)*log((c
^(2/3)*x*1i - 3^(1/2) - c^(1/3)*x^(1/2)*4i + 3^(1/2)*c^(2/3)*x + 1i)/(2*c^
(2/3)*x - 3^(1/2)*1i + 1))*((3^(1/2)*1i)/2 + 1/2)^(1/2)*1i)/4
```

3.220 $\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^4} dx$

3.220.1 Optimal result

Integrand size = 16, antiderivative size = 47

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^4} dx = -\frac{bc}{3x^{3/2}} + \frac{1}{3}bc^2\text{arctanh}\left(cx^{3/2}\right) - \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{3x^3}$$

output
```
-1/3*b*c/x^(3/2)+1/3*b*c^2*arctanh(c*x^(3/2))+1/3*(-a-b*arctanh(c*x^(3/2))
)/x^3
```

3.220.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 140 vs. 2(47) = 94.

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.98

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^4} dx = -\frac{a}{3x^3} - \frac{bc}{3x^{3/2}} - \frac{b\text{arctanh}\left(cx^{3/2}\right)}{3x^3} - \frac{1}{6}bc^2\log\left(1-\sqrt[3]{c}\sqrt{x}\right) + \frac{1}{6}bc^2\log\left(1+\sqrt[3]{c}\sqrt{x}\right) + \frac{1}{6}bc^2\log\left(1-\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right) - \frac{1}{6}bc^2\log\left(1+\sqrt[3]{c}\sqrt{x}+c^{2/3}x\right)$$

input
```
Integrate[(a + b*ArcTanh[c*x^(3/2)])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*c)/(3*x^(3/2)) - (b*ArcTanh[c*x^(3/2)])/(3*x^3) - (b*c^2*L
og[1 - c^(1/3)*Sqrt[x]])/6 + (b*c^2*Log[1 + c^(1/3)*Sqrt[x]])/6 + (b*c^2*L
og[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x])/6 - (b*c^2*Log[1 + c^(1/3)*Sqrt[x] +
c^(2/3)*x])/6
```

3.220.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6452, 847, 851, 807, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^4}dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{2}bc\int \frac{1}{x^{5/2}\left(1-c^2x^3\right)}dx-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{3x^3}$$

$$\Big\downarrow \text{847}$$

$$\frac{1}{2}bc\left(c^2\int \frac{\sqrt{x}}{1-c^2x^3}dx-\frac{2}{3x^{3/2}}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{3x^3}$$

$$\Big\downarrow \text{851}$$

$$\frac{1}{2}bc\left(2c^2\int \frac{x}{1-c^2x^3}d\sqrt{x}-\frac{2}{3x^{3/2}}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{3x^3}$$

$$\Big\downarrow \text{807}$$

$$\frac{1}{2}bc\left(\frac{2}{3}c^2\int \frac{1}{1-c^2x}dx^{3/2}-\frac{2}{3x^{3/2}}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{3x^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}bc\left(\frac{2}{3}c\text{arctanh}\left(cx^{3/2}\right)-\frac{2}{3x^{3/2}}\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{3x^3}$$

input
```
Int[(a + b*ArcTanh[c*x^(3/2)])/x^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c*x^(3/2)])/x^3 + (b*c*(-2/(3*x^(3/2)) + (2*c*ArcTanh[
c*x^(3/2)])/3))/2
```

3.220.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.220.4 Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$-\frac{a}{3x^3}-\frac{b\,\text{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3x^3}-\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}-1\right)}{6}-\frac{bc}{3x^{\frac{3}{2}}}+\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}+1\right)}{6}$	55
default	$-\frac{a}{3x^3}-\frac{b\,\text{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3x^3}-\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}-1\right)}{6}-\frac{bc}{3x^{\frac{3}{2}}}+\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}+1\right)}{6}$	55
parts	$-\frac{a}{3x^3}-\frac{b\,\text{arctanh}\left(c\,x^{\frac{3}{2}}\right)}{3x^3}-\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}-1\right)}{6}-\frac{bc}{3x^{\frac{3}{2}}}+\frac{b\,c^2\ln\left(c\,x^{\frac{3}{2}}+1\right)}{6}$	55

input
```
int((a+b*arctanh(c*x^(3/2)))/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a/x^3-1/3*b/x^3*arctanh(c*x^(3/2))-1/6*b*c^2*ln(c*x^(3/2)-1)-1/3*b*c/
x^(3/2)+1/6*b*c^2*ln(c*x^(3/2)+1)
```

3.220.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^4}\,dx=-\frac{2\,bcx^{\frac{3}{2}}-\left(bc^2x^3-b\right)\log\left(-\frac{c^2x^3+2\,cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)+2\,a}{6\,x^3}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="fricas")
```

output
```
-1/6*(2*b*c*x^(3/2) - (b*c^2*x^3 - b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^
2*x^3 - 1)) + 2*a)/x^3
```

3.220.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^4}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**(3/2)))/x**4,x)
```

output
```
Timed out
```

3.220.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{a+b\text{arctanh}(cx^{3/2})}{x^4}\,dx = \frac{1}{6}\left(\left(c\log\left(cx^{\frac{3}{2}}+1\right)-c\log\left(cx^{\frac{3}{2}}-1\right)-\frac{2}{x^{\frac{3}{2}}}\right)c-\frac{2\ \text{artanh}\left(cx^{\frac{3}{2}}\right)}{x^3}\right)b-\frac{a}{3\,x^3}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="maxima")
```

output
```
1/6*((c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*c - 2*arcta
nh(c*x^(3/2))/x^3)*b - 1/3*a/x^3
```

3.220.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{a+b\text{arctanh}(cx^{3/2})}{x^4}\,dx = \frac{1}{6}\,bc^2\log\left(cx^{\frac{3}{2}}+1\right)-\frac{1}{6}\,bc^2\log\left(cx^{\frac{3}{2}}-1\right)-\frac{b\log\left(-\frac{cx^{\frac{3}{2}}+1}{cx^{\frac{3}{2}}-1}\right)}{6\,x^3}-\frac{bcx^{\frac{3}{2}}+a}{3\,x^3}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))/x^4,x, algorithm="giac")
```

output
```
1/6*b*c^2*log(c*x^(3/2) + 1) - 1/6*b*c^2*log(c*x^(3/2) - 1) - 1/6*b*log(-(
c*x^(3/2) + 1)/(c*x^(3/2) - 1))/x^3 - 1/3*(b*c*x^(3/2) + a)/x^3
```

3.220.9 Mupad [B] (verification not implemented)

Time = 3.83 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.43

$$\int \frac{a+b\text{arctanh}(cx^{3/2})}{x^4}\,dx = \frac{b\,c^2\,\ln\left(\frac{c\,x^{3/2}+1}{c\,x^{3/2}-1}\right)}{6}-\frac{a}{3\,x^3}-\frac{b\,c}{3\,x^{3/2}}-\frac{b\,\ln\left(c\,x^{3/2}+1\right)}{6\,x^3}+\frac{b\,x\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,x^4-2\,c^2\,x^7\right)}-\frac{b\,c^2\,x^4\,\ln\left(1-c\,x^{3/2}\right)}{3\,\left(2\,x^4-2\,c^2\,x^7\right)}$$

input
```
int((a + b*atanh(c*x^(3/2)))/x^4,x)
```

output
```
(b*c^2*log((c*x^(3/2) + 1)/(c*x^(3/2) - 1)))/6 - a/(3*x^3) - (b*c)/(3*x^(3
/2)) - (b*log(c*x^(3/2) + 1))/(6*x^3) + (b*x*log(1 - c*x^(3/2)))/(3*(2*x^4
 - 2*c^2*x^7)) - (b*c^2*x^4*log(1 - c*x^(3/2)))/(3*(2*x^4 - 2*c^2*x^7))
```

3.220. $\int \frac{a+b\text{arctanh}(cx^{3/2})}{x^4} dx$

3.221 $\int x^2 \left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2 \, dx$

3.221.1 Optimal result

Integrand size = 18, antiderivative size = 101

$$\int x^2 \left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2 \, dx = \frac{2abx^{3/2}}{3c}+\frac{2b^2x^{3/2}\text{arctanh}\left(cx^{3/2}\right)}{3c} - \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{3c^2}+\frac{1}{3}x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2+\frac{b^2\log\left(1-c^2x^3\right)}{3c^2}$$

output
```
2/3*a*b*x^(3/2)/c+2/3*b^2*x^(3/2)*arctanh(c*x^(3/2))/c-1/3*(a+b*arctanh(c*
x^(3/2)))^2/c^2+1/3*x^3*(a+b*arctanh(c*x^(3/2)))^2+1/3*b^2*ln(-c^2*x^3+1)/
c^2
```

3.221.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.21

$$\int x^2 (a +b\text{arctanh}\left(cx^{3/2}\right))^2 \, dx = \frac{2abcx^{3/2}+a^2c^2x^3+2bcx^{3/2}\left(b+acx^{3/2}\right)\text{arctanh}\left(cx^{3/2}\right)+b^2\left(-1+c^2x^3\right)\text{arctanh}(}{3c^2}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x^(3/2)])^2,x]
```

output
```
(2*a*b*c*x^(3/2) + a^2*c^2*x^3 + 2*b*c*x^(3/2)*(b + a*c*x^(3/2))*ArcTanh[c
*x^(3/2)] + b^2*(-1 + c^2*x^3)*ArcTanh[c*x^(3/2)]^2 + b*(a + b)*Log[1 - c*
x^(3/2)] - a*b*Log[1 + c*x^(3/2)] + b^2*Log[1 + c*x^(3/2)])/(3*c^2)
```

3.221.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6454, 6452, 6542, 2009, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2 dx$$

$$\Big\downarrow \text{ 6454}$$

$$\frac{2}{3}\int x^{3/2}\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2dx^{3/2}$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{2}{3}\left(\frac{1}{2}x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2-bc\int \frac{x^3\left(a+b\text{arctanh}(cx^{3/2})\right)}{1-c^2x^3}dx^{3/2}\right)$$

$$\Big\downarrow \text{ 6542}$$

$$\frac{2}{3}\left(\frac{1}{2}x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2-bc\left(\frac{\int \frac{a+b\text{arctanh}(cx^{3/2})}{1-c^2x^3}dx^{3/2}}{c^2}-\frac{\int \left(a+b\text{arctanh}(cx^{3/2})\right)dx^{3/2}}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}\left(\frac{1}{2}x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2-bc\left(\frac{\int \frac{a+b\text{arctanh}(cx^{3/2})}{1-c^2x^3}dx^{3/2}}{c^2}-\frac{ax^{3/2}+bx^{3/2}\text{arctanh}(cx^{3/2})+\frac{b\log(1-c^2x^3)}{2c}}{c^2}\right)\right)$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{2}{3}\left(\frac{1}{2}x^3\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2-bc\left(\frac{\left(a+b\text{arctanh}(cx^{3/2})\right)^2}{2bc^3}-\frac{ax^{3/2}+bx^{3/2}\text{arctanh}(cx^{3/2})+\frac{b\log(1-c^2x^3)}{2c}}{c^2}\right)\right)$$

input
```
Int[x^2*(a + b*ArcTanh[c*x^(3/2)])^2,x]
```

output
```
(2*((x^3*(a + b*ArcTanh[c*x^(3/2)])^2)/2 - b*c*((a + b*ArcTanh[c*x^(3/2)])
^2/(2*b*c^3) - (a*x^(3/2) + b*x^(3/2)*ArcTanh[c*x^(3/2)] + (b*Log[1 - c^2*
x^3])/(2*c))/c^2)))/3
```

3.221.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6542
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[f^2/e   Int[(f*x)^(m - 2)*(a + b*ArcTanh[c*
x])^p, x], x] - Simp[d*(f^2/e)   Int[(f*x)^(m - 2)*((a + b*ArcTanh[c*x])^p/
(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && GtQ[m,
 1]
```

3.221.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(81) = 162$.

Time = 1.83 (sec) , antiderivative size = 220, normalized size of antiderivative = 2.18

method	result
parts	$\frac{a^2x^3}{3}+\frac{2b^2\left(\frac{c^2x^3\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)^2}{2}+cx^{\frac{3}{2}}\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)+\frac{\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)\ln\left(cx^{\frac{3}{2}}-1\right)}{2}-\frac{\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)\ln\left(cx^{\frac{3}{2}}+1\right)}{2}-\frac{\ln \cdots}{\cdots}\right.}{\cdots}$
derivativedivides	$\frac{\frac{a^2c^2x^3}{3}+\frac{2b^2\left(\frac{c^2x^3\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)^2}{2}+cx^{\frac{3}{2}}\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)+\frac{\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)\ln\left(cx^{\frac{3}{2}}-1\right)}{2}-\frac{\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)\ln\left(cx^{\frac{3}{2}}+1\right)}{2}-\frac{\ln\left(c x \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{a^2c^2x^3}{3}+\frac{2b^2\left(\frac{c^2x^3\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)^2}{2}+cx^{\frac{3}{2}}\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)+\frac{\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)\ln\left(cx^{\frac{3}{2}}-1\right)}{2}-\frac{\operatorname{arctanh}\left(cx^{\frac{3}{2}}\right)\ln\left(cx^{\frac{3}{2}}+1\right)}{2}-\frac{\ln\left(c x \cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$

input
```
int(x^2*(a+b*arctanh(c*x^(3/2)))^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*x^3+2/3*b^2/c^2*(1/2*c^2*x^3*arctanh(c*x^(3/2))^2+c*x^(3/2)*arctan
h(c*x^(3/2))+1/2*arctanh(c*x^(3/2))*ln(c*x^(3/2)-1)-1/2*arctanh(c*x^(3/2))
*ln(c*x^(3/2)+1)-1/4*ln(c*x^(3/2)-1)*ln(1/2*c*x^(3/2)+1/2)+1/8*ln(c*x^(3/2
)-1)^2+1/2*ln(c*x^(3/2)-1)+1/2*ln(c*x^(3/2)+1)+1/8*ln(c*x^(3/2)+1)^2-1/4*(
ln(c*x^(3/2)+1)-ln(1/2*c*x^(3/2)+1/2))*ln(-1/2*c*x^(3/2)+1/2))+4/3*a*b/c^2
*(1/2*c^2*x^3*arctanh(c*x^(3/2))+1/2*c*x^(3/2)+1/4*ln(c*x^(3/2)-1)-1/4*ln(
c*x^(3/2)+1))
```

3.221.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(81) = 162$.

Time = 0.28 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.77

$$\int x^2\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2\,dx=\frac{4a^2c^2x^3+8abcx^{\frac{3}{2}}+\left(b^2c^2x^3-b^2\right)\log\left(-\frac{c^2x^3+2cx^{\frac{3}{2}}+1}{c^2x^3-1}\right)^2+4\left(abc^2-ab+b^2\right)\log\left(c\cdots\right.}{\cdots}$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="fricas")
```

output
```
1/12*(4*a^2*c^2*x^3 + 8*a*b*c*x^(3/2) + (b^2*c^2*x^3 - b^2)*log(-(c^2*x^3
+ 2*c*x^(3/2) + 1)/(c^2*x^3 - 1))^2 + 4*(a*b*c^2 - a*b + b^2)*log(c*x^(3/2
) + 1) - 4*(a*b*c^2 - a*b - b^2)*log(c*x^(3/2) - 1) + 4*(a*b*c^2*x^3 + b^2
*c*x^(3/2) - a*b*c^2)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/c^2
```

3.221.6 Sympy [F(-1)]

Timed out.

$$\int x^2 (a + b\text{arctanh}(cx^{3/2}))^2 \, dx = \text{Timed out}$$

input
```
integrate(x**2*(a+b*atanh(c*x**(3/2)))**2,x)
```

output
```
Timed out
```

3.221.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. 2(81) = 162.

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.84

$$\int x^2 (a + b\text{arctanh}(cx^{3/2}))^2 \, dx = \frac{1}{3} b^2 x^3 \text{artanh}\left(cx^{\frac{3}{2}}\right)^2 + \frac{1}{3} a^2 x^3$$
$$+ \frac{1}{3} \left(2 x^3 \text{artanh}\left(cx^{\frac{3}{2}}\right) + c\left(\frac{2 x^{\frac{3}{2}}}{c^2} - \frac{\log\left(cx^{\frac{3}{2}} + 1\right)}{c^3} + \frac{\log\left(cx^{\frac{3}{2}} - 1\right)}{c^3}\right)\right) ab$$
$$+ \frac{1}{12} \left(4 c\left(\frac{2 x^{\frac{3}{2}}}{c^2} - \frac{\log\left(cx^{\frac{3}{2}} + 1\right)}{c^3} + \frac{\log\left(cx^{\frac{3}{2}} - 1\right)}{c^3}\right) \text{artanh}\left(cx^{\frac{3}{2}}\right) - \frac{2\left(\log\left(cx^{\frac{3}{2}} - 1\right) - 2\right) \log\left(cx^{\frac{3}{2}} + 1\right)}{\quad}\right.$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="maxima")
```

output
```
1/3*b^2*x^3*arctanh(c*x^(3/2))^2 + 1/3*a^2*x^3 + 1/3*(2*x^3*arctanh(c*x^(3
/2)) + c*(2*x^(3/2)/c^2 - log(c*x^(3/2) + 1)/c^3 + log(c*x^(3/2) - 1)/c^3)
)*a*b + 1/12*(4*c*(2*x^(3/2)/c^2 - log(c*x^(3/2) + 1)/c^3 + log(c*x^(3/2)
- 1)/c^3)*arctanh(c*x^(3/2)) - (2*(log(c*x^(3/2) - 1) - 2)*log(c*x^(3/2) +
 1) - log(c*x^(3/2) + 1)^2 - log(c*x^(3/2) - 1)^2 - 4*log(c*x^(3/2) - 1))/
c^2)*b^2
```

3.221.8 Giac [F]

$$\int x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2\,dx=\int\left(b\,\text{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)^2x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^(3/2)))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^(3/2)) + a)^2*x^2, x)
```

3.221.9 Mupad [B] (verification not implemented)

Time = 4.06 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.04

$$\int x^2\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2\,dx=\frac{c\left(\frac{2\,b^2\,x^{3/2}\,\text{atanh}\left(c\,x^{3/2}\right)}{3}+\frac{2\,a\,b\,x^{3/2}}{3}\right)-\frac{b^2\,\text{atanh}\left(c\,x^{3/2}\right)^2}{3}+\frac{b^2\,\ln\left(c^2\,x^3-1\right)}{3}-\frac{2\,a\,b\,\text{atanh}\left(c\,x^{3/2}\right)}{3}}{c^2}+\frac{a^2\,x^3}{3}+\frac{b^2\,x^3\,\text{atanh}\left(c\,x^{3/2}\right)^2}{3}+\frac{2\,a\,b\,x^3\,\text{atanh}\left(c\,x^{3/2}\right)}{3}$$

input
```
int(x^2*(a + b*atanh(c*x^(3/2)))^2,x)
```

output
```
(c*((2*b^2*x^(3/2)*atanh(c*x^(3/2)))/3 + (2*a*b*x^(3/2))/3) - (b^2*atanh(c
*x^(3/2))^2)/3 + (b^2*log(c^2*x^3 - 1))/3 - (2*a*b*atanh(c*x^(3/2)))/3)/c^
2 + (a^2*x^3)/3 + (b^2*x^3*atanh(c*x^(3/2))^2)/3 + (2*a*b*x^3*atanh(c*x^(3
/2)))/3
```

3.222 $\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x}\,dx$

3.222.1 Optimal result

Integrand size = 18, antiderivative size = 156

$$\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x}\,dx=\frac{4}{3}(a+b\text{arctanh}\left(cx^{3/2}\right))^2\text{arctanh}\left(1-\frac{2}{1-cx^{3/2}}\right)-\frac{2}{3}b(a+b\text{arctanh}\left(cx^{3/2}\right))\text{PolyLog}\left(2,1-\frac{2}{1-cx^{3/2}}\right)+\frac{2}{3}b(a+$$

output
```
-4/3*(a+b*arctanh(c*x^(3/2)))^2*arctanh(-1+2/(1-c*x^(3/2)))-2/3*b*(a+b*arc
tanh(c*x^(3/2)))*polylog(2,1-2/(1-c*x^(3/2)))+2/3*b*(a+b*arctanh(c*x^(3/2)
))*polylog(2,-1+2/(1-c*x^(3/2)))+1/3*b^2*polylog(3,1-2/(1-c*x^(3/2)))-1/3*
b^2*polylog(3,-1+2/(1-c*x^(3/2)))
```

3.222.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.33

$$\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x}\,dx=a^2\log(x)+\frac{2}{3}ab\left(-\text{PolyLog}\left(2,-cx^{3/2}\right)+\text{PolyLog}\left(2,cx^{3/2}\right)\right)+\frac{2}{3}b^2\left(\frac{i\pi^3}{24}-\frac{2}{3}\text{arctanh}\left(cx^{3/2}\right)^3-\text{arctanh}\left(cx^{3/2}\right)^2\log\left(1+\right.\right.$$

input
```
Integrate[(a + b*ArcTanh[c*x^(3/2)])^2/x,x]
```

output
```
a^2*Log[x] + (2*a*b*(-PolyLog[2, -(c*x^(3/2))] + PolyLog[2, c*x^(3/2)]))/3
 + (2*b^2*((I/24)*Pi^3 - (2*ArcTanh[c*x^(3/2)]^3)/3 - ArcTanh[c*x^(3/2)]^2
*Log[1 + E^(-2*ArcTanh[c*x^(3/2)])] + ArcTanh[c*x^(3/2)]^2*Log[1 - E^(2*Ar
cTanh[c*x^(3/2)])] + ArcTanh[c*x^(3/2)]*PolyLog[2, -E^(-2*ArcTanh[c*x^(3/2
)])] + ArcTanh[c*x^(3/2)]*PolyLog[2, E^(2*ArcTanh[c*x^(3/2)])] + PolyLog[3
, -E^(-2*ArcTanh[c*x^(3/2)])]/2 - PolyLog[3, E^(2*ArcTanh[c*x^(3/2)])]/2))
/3
```

3.222.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {6450, 6448, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x}dx$$

$$\Big\downarrow \text{6450}$$

$$\frac{2}{3}\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x^{3/2}}dx^{3/2}$$

$$\Big\downarrow \text{6448}$$

$$\frac{2}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2-4bc\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\text{arctanh}\left(1-\frac{2}{1-cx^{3/2}}\right)}{1-c^2x^3}dx^{3/}$$

$$\Big\downarrow \text{6614}$$

$$\frac{2}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2-4bc\left(\frac{1}{2}\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)\log\left(2-\frac{2}{1-cx^{3/2}}\right)}{1-c^2x^3}dx^{3/}$$

$$\Big\downarrow \text{6620}$$

$$\frac{2}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)}{2c}\right.\right.\right.$$

$\downarrow$ 7164

$$\frac{2}{3}\left(2\text{arctanh}\left(1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2-4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^{3/2}}\right)\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)}{2c}\right.\right.\right.$$

input
```
Int[(a + b*ArcTanh[c*x^(3/2)])^2/x,x]
```

output
```
(2*(2*(a + b*ArcTanh[c*x^(3/2)])^2*ArcTanh[1 - 2/(1 - c*x^(3/2))] - 4*b*c*
((((a + b*ArcTanh[c*x^(3/2)])*PolyLog[2, 1 - 2/(1 - c*x^(3/2))])/(2*c) - (
b*PolyLog[3, 1 - 2/(1 - c*x^(3/2))])/(4*c))/2 + (-1/2*((a + b*ArcTanh[c*x^
(3/2)])*PolyLog[2, -1 + 2/(1 - c*x^(3/2))])/c + (b*PolyLog[3, -1 + 2/(1 -
c*x^(3/2))])/(4*c))/2)))/3
```

3.222.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.222.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 31.97 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.53

method	result
parts	$a^2\ln(x)+b^2\left(\frac{2\ln\left(c\,x^{\frac{3}{2}}\right)\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)^2}{3}-\frac{2\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)\operatorname{polylog}\left(2,-\frac{\left(c\,x^{\frac{3}{2}}+1\right)^2}{-x^3c^2+1}\right)}{3}+\frac{\operatorname{polylog}\left(3,-\frac{\left(c\right.}{-}\right.}{3}\right.$
derivativedivides	$\frac{2a^2\ln\left(c\,x^{\frac{3}{2}}\right)}{3}+\frac{2b^2\left(\ln\left(c\,x^{\frac{3}{2}}\right)\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)^2-\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)\operatorname{polylog}\left(2,-\frac{\left(c\,x^{\frac{3}{2}}+1\right)^2}{-x^3c^2+1}\right)+\frac{\operatorname{polylog}\left(3,-\frac{\left(c\,x^{\frac{3}{2}}+1\right)^2}{-x^3c^2+1}\right.}{2}\right.}{}$
default	$\frac{2a^2\ln\left(c\,x^{\frac{3}{2}}\right)}{3}+\frac{2b^2\left(\ln\left(c\,x^{\frac{3}{2}}\right)\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)^2-\operatorname{arctanh}\left(c\,x^{\frac{3}{2}}\right)\operatorname{polylog}\left(2,-\frac{\left(c\,x^{\frac{3}{2}}+1\right)^2}{-x^3c^2+1}\right)+\frac{\operatorname{polylog}\left(3,-\frac{\left(c\,x^{\frac{3}{2}}+1\right)^2}{-x^3c^2+1}\right.}{2}\right.}{}$

input
```
int((a+b*arctanh(c*x^(3/2)))^2/x,x,method=_RETURNVERBOSE)
```

3.222. $\int \frac{\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2}{x}\,dx$

output
```
a^2*ln(x)+b^2*(2/3*ln(c*x^(3/2))*arctanh(c*x^(3/2))^2-2/3*arctanh(c*x^(3/2
))*polylog(2,-(c*x^(3/2)+1)^2/(-c^2*x^3+1))+1/3*polylog(3,-(c*x^(3/2)+1)^2
/(-c^2*x^3+1))-2/3*arctanh(c*x^(3/2))^2*ln((c*x^(3/2)+1)^2/(-c^2*x^3+1)-1)
+2/3*arctanh(c*x^(3/2))^2*ln(1+(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+4/3*arcta
nh(c*x^(3/2))*polylog(2,-(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))-4/3*polylog(3,-
(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+2/3*arctanh(c*x^(3/2))^2*ln(1-(c*x^(3/2)
+1)/(-c^2*x^3+1)^(1/2))+4/3*arctanh(c*x^(3/2))*polylog(2,(c*x^(3/2)+1)/(-c
^2*x^3+1)^(1/2))-4/3*polylog(3,(c*x^(3/2)+1)/(-c^2*x^3+1)^(1/2))+1/3*I*Pi*
csgn(I*(-(c*x^(3/2)+1)^2/(c^2*x^3-1)-1)/(1-(c*x^(3/2)+1)^2/(c^2*x^3-1)))*(
csgn(I*(-(c*x^(3/2)+1)^2/(c^2*x^3-1)-1))*csgn(I/(1-(c*x^(3/2)+1)^2/(c^2*x^
3-1)))-csgn(I*(-(c*x^(3/2)+1)^2/(c^2*x^3-1)-1))*csgn(I*(-(c*x^(3/2)+1)^2/(
c^2*x^3-1)-1)/(1-(c*x^(3/2)+1)^2/(c^2*x^3-1)))-csgn(I*(-(c*x^(3/2)+1)^2/(c
^2*x^3-1)-1)/(1-(c*x^(3/2)+1)^2/(c^2*x^3-1)))*csgn(I/(1-(c*x^(3/2)+1)^2/(c
^2*x^3-1)))+csgn(I*(-(c*x^(3/2)+1)^2/(c^2*x^3-1)-1)/(1-(c*x^(3/2)+1)^2/(c^
2*x^3-1)))^2)*arctanh(c*x^(3/2))^2)+2*a*b*(2/3*ln(c*x^(3/2))*arctanh(c*x^(
3/2))-1/3*dilog(c*x^(3/2)+1)-1/3*ln(c*x^(3/2))*ln(c*x^(3/2)+1)-1/3*dilog(c
*x^(3/2)))
```

3.222.5 Fricas [F]

$$\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x}\,dx=\int \frac{\left(b\,\text{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^(3/2))^2 + 2*a*b*arctanh(c*x^(3/2)) + a^2)/x, x)
```

3.222.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x}\,dx=\text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**(3/2)))**2/x,x)
```

output
```
Timed out
```

3.222.7 Maxima [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2}{x}\,dx=\int \frac{\left(b\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="maxima")
```

output
```
1/4*b^2*integrate(log(c*x^(3/2) + 1)^2/x, x) - 1/2*b^2*integrate(log(c*x^(
3/2) + 1)*log(-c*x^(3/2) + 1)/x, x) + 1/4*b^2*integrate(log(-c*x^(3/2) + 1
)^2/x, x) + a*b*integrate(log(c*x^(3/2) + 1)/x, x) - a*b*integrate(log(-c*
x^(3/2) + 1)/x, x) + a^2*log(x)
```

3.222.8 Giac [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2}{x}\,dx=\int \frac{\left(b\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^(3/2)) + a)^2/x, x)
```

3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2}{x}\,dx=\int \frac{\left(a+b\operatorname{atanh}\left(c\,x^{3/2}\right)\right)^2}{x}\,dx$$

input
```
int((a + b*atanh(c*x^(3/2)))^2/x,x)
```

output
```
int((a + b*atanh(c*x^(3/2)))^2/x, x)
```

3.223 $\int \frac{\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx$

3.223.1 Optimal result

Integrand size = 18, antiderivative size = 96

$$\int \frac{\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx = -\frac{2bc\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)}{3x^{3/2}} + \frac{1}{3}c^2\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2 - \frac{\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2}{3x^3} + b^2c^2\log(x) - \frac{1}{3}b^2c^2\log\left(1-c^2x^3\right)$$

output
```
-2/3*b*c*(a+b*arctanh(c*x^(3/2)))/x^(3/2)+1/3*c^2*(a+b*arctanh(c*x^(3/2)))
^2-1/3*(a+b*arctanh(c*x^(3/2)))^2/x^3+b^2*c^2*ln(x)-1/3*b^2*c^2*ln(-c^2*x^
3+1)
```

3.223.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 210 vs. 2(96) = 192.

Time = 0.10 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.19

$$\int \frac{\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx = -\frac{a^2+2abcx^{3/2}+2b\left(a+bcx^{3/2}\right)\operatorname{arctanh}\left(cx^{3/2}\right)-b^2\left(-1+c^2x^3\right)\operatorname{arctanh}\left(cx^{3/2}\right)^2+abc^2x^3\log\left(1-\sqrt[3]{c}\sqrt{x}\right)}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x^(3/2)])^2/x^4,x]
```

output
```
-1/3*(a^2 + 2*a*b*c*x^(3/2) + 2*b*(a + b*c*x^(3/2))*ArcTanh[c*x^(3/2)] - b
^2*(-1 + c^2*x^3)*ArcTanh[c*x^(3/2)]^2 + a*b*c^2*x^3*Log[1 - c^(1/3)*Sqrt[
x]] - a*b*c^2*x^3*Log[1 + c^(1/3)*Sqrt[x]] - 3*b^2*c^2*x^3*Log[x] - a*b*c^
2*x^3*Log[1 - c^(1/3)*Sqrt[x] + c^(2/3)*x] + a*b*c^2*x^3*Log[1 + c^(1/3)*S
qrt[x] + c^(2/3)*x] + b^2*c^2*x^3*Log[1 - c^2*x^3])/x^3
```

3.223.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6454, 6452, 6544, 6452, 243, 47, 14, 16, 6510}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x^4}dx$$

$$\Big\downarrow \text{6454}$$

$$\frac{2}{3}\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x^{9/2}}dx^{3/2}$$

$$\Big\downarrow \text{6452}$$

$$\frac{2}{3}\left(bc\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^3\left(1-c^2x^3\right)}dx^{3/2}-\frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{2x^3}\right)$$

$$\Big\downarrow \text{6544}$$

$$\frac{2}{3}\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{1-c^2x^3}dx^{3/2}+\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^3}dx^{3/2}\right)-\frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{2x^3}\right)$$

$$\Big\downarrow \text{6452}$$

$$\frac{2}{3}\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{1-c^2x^3}dx^{3/2}+bc\int \frac{1}{x^{3/2}\left(1-c^2x^3\right)}dx^{3/2}-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^{3/2}}\right)-\frac{(a+b\text{arctanh}(cx}{2x^3}$$

$$\Big\downarrow \text{243}$$

$$\frac{2}{3}\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{1-c^2x^3}dx^{3/2}+\frac{1}{2}bc\int \frac{1}{x^{3/2}\left(1-c^2x^3\right)}dx^3-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^{3/2}}\right)-\frac{\left(a+b\text{arctanh}\left(cx\right.\right.}{2x^3}\right.$$

$$\Big\downarrow \text{ 47}$$

$$\frac{2}{3}\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{1-c^2x^3}dx^{3/2}+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^3}dx^3+\int \frac{1}{x^{3/2}}dx^3\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^{3/2}}\right)-\frac{(a+}{}\right.$$

$$\Big\downarrow \text{ 14}$$

$$\frac{2}{3}\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{1-c^2x^3}dx^{3/2}+\frac{1}{2}bc\left(c^2\int \frac{1}{1-c^2x^3}dx^3+\log\left(x^3\right)\right)-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^{3/2}}\right)-\frac{(a+ba}{}\right.$$

$$\Big\downarrow \text{ 16}$$

$$\frac{2}{3}\left(bc\left(c^2\int \frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{1-c^2x^3}dx^{3/2}-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^{3/2}}+\frac{1}{2}bc\left(\log\left(x^3\right)-\log\left(1-c^2x^3\right)\right)\right)-\frac{(a+b\text{arctan}}{2x}\right.$$

$$\Big\downarrow \text{ 6510}$$

$$\frac{2}{3}\left(bc\left(\frac{c\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{2b}-\frac{a+b\text{arctanh}\left(cx^{3/2}\right)}{x^{3/2}}+\frac{1}{2}bc\left(\log\left(x^3\right)-\log\left(1-c^2x^3\right)\right)\right)-\frac{\left(a+b\text{arctanh}\left(cx^{3}\right.\right.}{2x^3}\right.$$

input
```
Int[(a + b*ArcTanh[c*x^(3/2)])^2/x^4,x]
```

output
```
(2*(-1/2*(a + b*ArcTanh[c*x^(3/2)])^2/x^3 + b*c*(-((a + b*ArcTanh[c*x^(3/2
)])/x^(3/2)) + (c*(a + b*ArcTanh[c*x^(3/2)])^2)/(2*b) + (b*c*(Log[x^3] - L
og[1 - c^2*x^3]))/2)))/3
```

3.223.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

3.223. $\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx$

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6454
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :>
 Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*ArcTanh[c*x])^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 1] && IntegerQ[Simpl
ify[(m + 1)/n]]
```

rule 6510
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Simp[(a + b*ArcTanh[c*x])^(p + 1)/(b*c*d*(p + 1)), x] /; FreeQ[{a, b
, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && NeQ[p, -1]
```

rule 6544
```
Int[(((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.)*((f_.)*(x_))^(m_))/((d_) + (
e_.)*(x_)^2), x_Symbol] :> Simp[1/d   Int[(f*x)^m*(a + b*ArcTanh[c*x])^p, x
], x] - Simp[e/(d*f^2)   Int[(f*x)^(m + 2)*((a + b*ArcTanh[c*x])^p/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1]
```

3.223.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.27 (sec) , antiderivative size = 3062, normalized size of antiderivative = 31.90

method	result	size
parts	Expression too large to display	3062
derivativedivides	Expression too large to display	3063
default	Expression too large to display	3063

input
```
int((a+b*arctanh(c*x^(3/2)))^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*a^2/x^3+b^2*(-1/3/x^3*arctanh(c*x^(3/2))^2+2*c*(-1/6*arctanh(c*x^(3/2
))*c*ln(c*x^(3/2)-1)-1/3/x^(3/2)*arctanh(c*x^(3/2))+1/6*arctanh(c*x^(3/2))
*c*ln(c*x^(3/2)+1)-1/2*c*(c*(Sum(1/6*(ln(x^(1/2)-_alpha)*ln(c*x^(3/2)-1)-3
*c*(1/6/_alpha^2/c*ln(x^(1/2)-_alpha)^2-1/3*_alpha*ln(x^(1/2)-_alpha)*(2*l
n((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x^(1/2)+_alpha)/RootOf(_Z^2
+3*_Z*_alpha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index
=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)+6*ln((RootOf(_Z^2+3*_Z*_al
pha+3*_alpha^2,index=2)-x^(1/2)+_alpha)/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2
,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*_alpha+3*ln((RootOf
(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x^(1/2)+_alpha)/RootOf(_Z^2+3*_Z*_al
pha+3*_alpha^2,index=2))*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*_alph
a+9*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)-x^(1/2)+_alpha)/RootOf
(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2))*_alpha^2+2*RootOf(_Z^2+3*_Z*_alpha+
3*_alpha^2,index=1)*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=2)*ln((RootOf
(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x^(1/2)+_alpha)/RootOf(_Z^2+3*_Z*_al
pha+3*_alpha^2,index=1))+3*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)*ln(
(RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x^(1/2)+_alpha)/RootOf(_Z^2+3
*_Z*_alpha+3*_alpha^2,index=1))*_alpha+6*RootOf(_Z^2+3*_Z*_alpha+3*_alpha^
2,index=2)*ln((RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1)-x^(1/2)+_alpha)
/RootOf(_Z^2+3*_Z*_alpha+3*_alpha^2,index=1))*_alpha+9*ln((RootOf(_Z^2+...
```

3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(80) = 160$.

Time = 0.27 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.80

$$\int \frac{\left(a + b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx = \frac{24\,b^2c^2x^3\log\left(\sqrt{x}\right) + 4\,(ab - b^2)c^2x^3\log\left(cx^{\frac{3}{2}} + 1\right) - 4\,(ab + b^2)c^2x^3\log\left(c\right.}{}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="fricas")
```

output
```
1/12*(24*b^2*c^2*x^3*log(sqrt(x)) + 4*(a*b - b^2)*c^2*x^3*log(c*x^(3/2) +
1) - 4*(a*b + b^2)*c^2*x^3*log(c*x^(3/2) - 1) - 8*a*b*c*x^(3/2) + (b^2*c^2
*x^3 - b^2)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1))^2 - 4*a^2 - 4*
(b^2*c*x^(3/2) + a*b)*log(-(c^2*x^3 + 2*c*x^(3/2) + 1)/(c^2*x^3 - 1)))/x^3
```

3.223.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((a+b*atanh(c*x**(3/2)))**2/x**4,x)
```

output
```
Timed out
```

3.223.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(80) = 160$.

Time = 0.19 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.82

$$\int \frac{\left(a+b\text{arctanh}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx = \frac{1}{3}\left(\left(c\log\left(cx^{\frac{3}{2}}+1\right)-c\log\left(cx^{\frac{3}{2}}-1\right)-\frac{2}{x^{\frac{3}{2}}}\right)c-\frac{2\,\text{artanh}\left(cx^{\frac{3}{2}}\right)}{x^3}\right)ab + \frac{1}{12}\Big(\Big(2\left(\log\left(cx^{\frac{3}{2}}-1\right)-2\right)\log\left(cx^{\frac{3}{2}}+1\right)-\log\left(cx^{\frac{3}{2}}+1\right)^2-\log\left(cx^{\frac{3}{2}}-1\right)^2-4\log\left(cx^{\frac{3}{2}}-1\right)+12\,\text{l} - \frac{b^2\,\text{artanh}\left(cx^{\frac{3}{2}}\right)^2}{3\,x^3}-\frac{a^2}{3\,x^3}$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="maxima")
```

output
```
1/3*((c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/2))*c - 2*arcta
nh(c*x^(3/2))/x^3)*a*b + 1/12*((2*(log(c*x^(3/2) - 1) - 2)*log(c*x^(3/2) +
 1) - log(c*x^(3/2) + 1)^2 - log(c*x^(3/2) - 1)^2 - 4*log(c*x^(3/2) - 1) +
 12*log(x))*c^2 + 4*(c*log(c*x^(3/2) + 1) - c*log(c*x^(3/2) - 1) - 2/x^(3/
2))*c*arctanh(c*x^(3/2)))*b^2 - 1/3*b^2*arctanh(c*x^(3/2))^2/x^3 - 1/3*a^2
/x^3
```

3.223.8 Giac [F]

$$\int \frac{\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx=\int \frac{\left(b\operatorname{artanh}\left(cx^{\frac{3}{2}}\right)+a\right)^2}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x^(3/2)))^2/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^(3/2)) + a)^2/x^4, x)
```

3.223.9 Mupad [B] (verification not implemented)

Time = 4.56 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.93

$$\begin{aligned}\int \frac{\left(a+b\operatorname{arctanh}\left(cx^{3/2}\right)\right)^2}{x^4}\,dx&=\frac{2\,b^2\,c^2\,\ln\left(x^{3/2}\right)}{3}-\frac{a^2}{3\,x^3}-\frac{b^2\,c^2\,\ln\left(c\,x^{3/2}-1\right)}{3}\\&-\frac{b^2\,c^2\,\ln\left(c\,x^{3/2}+1\right)}{3}+\frac{b^2\,c^2\ln\left(c\,x^{3/2}+1\right)^2}{12}+\frac{b^2\,c^2\ln\left(1-c\,x^{3/2}\right)^2}{12}\\&-\frac{b^2\ln\left(c\,x^{3/2}+1\right)^2}{12\,x^3}-\frac{b^2\ln\left(1-c\,x^{3/2}\right)^2}{12\,x^3}-\frac{a\,b\,c^2\,\ln\left(c\,x^{3/2}-1\right)}{3}\\&+\frac{a\,b\,c^2\,\ln\left(c\,x^{3/2}+1\right)}{3}-\frac{2\,a\,b\,c}{3\,x^{3/2}}-\frac{a\,b\,\ln\left(c\,x^{3/2}+1\right)}{3\,x^3}+\frac{a\,b\,\ln\left(1-c\,x^{3/2}\right)}{3\,x^3}\\&-\frac{b^2\,c^2\,\ln\left(c\,x^{3/2}+1\right)\,\ln\left(1-c\,x^{3/2}\right)}{6}-\frac{b^2\,c\,\ln\left(c\,x^{3/2}+1\right)}{3\,x^{3/2}}\\&+\frac{b^2\,c\,\ln\left(1-c\,x^{3/2}\right)}{3\,x^{3/2}}+\frac{b^2\,\ln\left(c\,x^{3/2}+1\right)\,\ln\left(1-c\,x^{3/2}\right)}{6\,x^3}\end{aligned}$$

input
```
int((a + b*atanh(c*x^(3/2)))^2/x^4,x)
```

output
```
(2*b^2*c^2*log(x^(3/2)))/3 - a^2/(3*x^3) - (b^2*c^2*log(c*x^(3/2) - 1))/3
- (b^2*c^2*log(c*x^(3/2) + 1))/3 + (b^2*c^2*log(c*x^(3/2) + 1)^2)/12 + (b^
2*c^2*log(1 - c*x^(3/2))^2)/12 - (b^2*log(c*x^(3/2) + 1)^2)/(12*x^3) - (b^
2*log(1 - c*x^(3/2))^2)/(12*x^3) - (a*b*c^2*log(c*x^(3/2) - 1))/3 + (a*b*c
^2*log(c*x^(3/2) + 1))/3 - (2*a*b*c)/(3*x^(3/2)) - (a*b*log(c*x^(3/2) + 1)
)/(3*x^3) + (a*b*log(1 - c*x^(3/2)))/(3*x^3) - (b^2*c^2*log(c*x^(3/2) + 1)
*log(1 - c*x^(3/2)))/6 - (b^2*c*log(c*x^(3/2) + 1))/(3*x^(3/2)) + (b^2*c*l
og(1 - c*x^(3/2)))/(3*x^(3/2)) + (b^2*log(c*x^(3/2) + 1)*log(1 - c*x^(3/2)
))/(6*x^3)
```

3.224 $\int x^2(a+b\operatorname{arctanh}(cx^n))\,dx$

3.224.1 Optimal result

Integrand size = 14, antiderivative size = 66

$$\int x^2(a+b\operatorname{arctanh}(cx^n))\,dx=\frac{1}{3}x^3(a+b\operatorname{arctanh}(cx^n))-\frac{bcnx^{3+n}\,\text{Hypergeometric2F1}\left(1,\frac{3+n}{2n},\frac{3(1+n)}{2n},c^2x^{2n}\right)}{3(3+n)}$$

output
```
1/3*x^3*(a+b*arctanh(c*x^n))-1/3*b*c*n*x^(3+n)*hypergeom([1, 1/2*(3+n)/n],
[3/2*(1+n)/n],c^2*x^(2*n))/(3+n)
```

3.224.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int x^2(a+b\operatorname{arctanh}(cx^n))\,dx=\frac{ax^3}{3}+\frac{1}{3}bx^3\operatorname{arctanh}(cx^n)-\frac{bcnx^{3+n}\,\text{Hypergeometric2F1}\left(1,\frac{3+n}{2n},1+\frac{3+n}{2n},c^2x^{2n}\right)}{3(3+n)}$$

input
```
Integrate[x^2*(a + b*ArcTanh[c*x^n]),x]
```

output
```
(a*x^3)/3 + (b*x^3*ArcTanh[c*x^n])/3 - (b*c*n*x^(3 + n)*Hypergeometric2F1[
1, (3 + n)/(2*n), 1 + (3 + n)/(2*n), c^2*x^(2*n)])/(3*(3 + n))
```

3.224.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6452, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+b\text{arctanh}(cx^n))\,dx$$

$$\Big\downarrow \text{6452}$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx^n))-\frac{1}{3}bcn\int\frac{x^{n+2}}{1-c^2x^{2n}}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{1}{3}x^3(a+b\text{arctanh}(cx^n))-\frac{bcnx^{n+3}\,\text{Hypergeometric2F1}\left(1,\frac{n+3}{2n},\frac{3(n+1)}{2n},c^2x^{2n}\right)}{3(n+3)}$$

input
```
Int[x^2*(a + b*ArcTanh[c*x^n]),x]
```

output
```
(x^3*(a + b*ArcTanh[c*x^n]))/3 - (b*c*n*x^(3 + n)*Hypergeometric2F1[1, (3
+ n)/(2*n), (3*(1 + n))/(2*n), c^2*x^(2*n)])/(3*(3 + n))
```

3.224.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.224.4 Maple [F]

$$\int x^2(a + b \operatorname{arctanh}(c\,x^n))\,dx$$

input
```
int(x^2*(a+b*arctanh(c*x^n)),x)
```

output
```
int(x^2*(a+b*arctanh(c*x^n)),x)
```

3.224.5 Fricas [F]

$$\int x^2(a + b\operatorname{arctanh}(cx^n))\;dx = \int (b\operatorname{artanh}(cx^n) + a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

output
```
integral(b*x^2*arctanh(c*x^n) + a*x^2, x)
```

3.224.6 Sympy [F]

$$\int x^2(a + b\operatorname{arctanh}(cx^n))\;dx = \int x^2(a + b\operatorname{atanh}(cx^n))\;dx$$

input
```
integrate(x**2*(a+b*atanh(c*x**n)),x)
```

output
```
Integral(x**2*(a + b*atanh(c*x**n)), x)
```

3.224.7 Maxima [F]

$$\int x^2(a+b\text{arctanh}(cx^n))\,dx=\int (b\,\text{artanh}\,(cx^n)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

output
```
1/3*a*x^3 + 1/6*(x^3*log(c*x^n + 1) - x^3*log(-c*x^n + 1) + 3*n*integrate(
1/3*x^2/(c*x^n + 1), x) + 3*n*integrate(1/3*x^2/(c*x^n - 1), x))*b
```

3.224.8 Giac [F]

$$\int x^2(a+b\text{arctanh}(cx^n))\,dx=\int (b\,\text{artanh}\,(cx^n)+a)x^2\,dx$$

input
```
integrate(x^2*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)*x^2, x)
```

3.224.9 Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\text{arctanh}(cx^n))\,dx=\int x^2\,(a+b\,\text{atanh}(c\,x^n))\,dx$$

input
```
int(x^2*(a + b*atanh(c*x^n)),x)
```

output
```
int(x^2*(a + b*atanh(c*x^n)), x)
```

3.225 $\int x(a + b\text{arctanh}(cx^n))\,dx$

3.225.1 Optimal result

Integrand size = 12, antiderivative size = 67

$$\int x(a + b\text{arctanh}(cx^n))\,dx = \frac{1}{2}x^2(a + b\text{arctanh}(cx^n)) - \frac{bcnx^{2+n}\,\text{Hypergeometric2F1}\left(1, \frac{2+n}{2n}, \frac{1}{2}\left(3 + \frac{2}{n}\right), c^2x^{2n}\right)}{2(2+n)}$$

output
```
1/2*x^2*(a+b*arctanh(c*x^n))-1/2*b*c*n*x^(2+n)*hypergeom([1, 1/2*(2+n)/n],
[3/2+1/n],c^2*x^(2*n))/(2+n)
```

3.225.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.09

$$\int x(a + b\text{arctanh}(cx^n))\,dx = \frac{ax^2}{2} + \frac{1}{2}bx^2\text{arctanh}(cx^n) - \frac{bcnx^{2+n}\,\text{Hypergeometric2F1}\left(1, \frac{2+n}{2n}, 1 + \frac{2+n}{2n}, c^2x^{2n}\right)}{2(2+n)}$$

input
```
Integrate[x*(a + b*ArcTanh[c*x^n]),x]
```

output
```
(a*x^2)/2 + (b*x^2*ArcTanh[c*x^n])/2 - (b*c*n*x^(2 + n)*Hypergeometric2F1[
1, (2 + n)/(2*n), 1 + (2 + n)/(2*n), c^2*x^(2*n)])/(2*(2 + n))
```

3.225.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {6452, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+b\text{arctanh}(cx^n))\,dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx^n))-\frac{1}{2}bcn\int\frac{x^{n+1}}{1-c^2x^{2n}}dx$$

$$\Big\downarrow \text{ 888}$$

$$\frac{1}{2}x^2(a+b\text{arctanh}(cx^n))-\frac{bcnx^{n+2}\,\text{Hypergeometric2F1}\left(1,\frac{n+2}{2n},\frac{1}{2}\left(3+\frac{2}{n}\right),c^2x^{2n}\right)}{2(n+2)}$$

input
```
Int[x*(a + b*ArcTanh[c*x^n]),x]
```

output
```
(x^2*(a + b*ArcTanh[c*x^n]))/2 - (b*c*n*x^(2 + n)*Hypergeometric2F1[1, (2
+ n)/(2*n), (3 + 2/n)/2, c^2*x^(2*n)])/(2*(2 + n))
```

3.225.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.225.4 Maple [F]

$$\int x(a+b\operatorname{arctanh}(c\,x^n))\,dx$$

input
```
int(x*(a+b*arctanh(c*x^n)),x)
```

output
```
int(x*(a+b*arctanh(c*x^n)),x)
```

3.225.5 Fricas [F]

$$\int x(a+b\mathrm{arctanh}(cx^n))\,dx=\int (b\operatorname{artanh}(cx^n)+a)x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

output
```
integral(b*x*arctanh(c*x^n) + a*x, x)
```

3.225.6 Sympy [F]

$$\int x(a+b\mathrm{arctanh}(cx^n))\,dx=\int x(a+b\operatorname{atanh}(cx^n))\,dx$$

input
```
integrate(x*(a+b*atanh(c*x**n)),x)
```

output
```
Integral(x*(a + b*atanh(c*x**n)), x)
```

3.225.7 Maxima [F]

$$\int x(a + b\text{arctanh}(cx^n))\,dx = \int (b\,\text{artanh}\,(cx^n) + a)x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*a*x^2 + 1/4*(x^2*log(c*x^n + 1) - x^2*log(-c*x^n + 1) + 2*n*integrate(
1/2*x/(c*x^n + 1), x) + 2*n*integrate(1/2*x/(c*x^n - 1), x))*b
```

3.225.8 Giac [F]

$$\int x(a + b\text{arctanh}(cx^n))\,dx = \int (b\,\text{artanh}\,(cx^n) + a)x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)*x, x)
```

3.225.9 Mupad [F(-1)]

Timed out.

$$\int x(a + b\text{arctanh}(cx^n))\,dx = \int x\,(a + b\,\text{atanh}(c\,x^n))\,dx$$

input
```
int(x*(a + b*atanh(c*x^n)),x)
```

output
```
int(x*(a + b*atanh(c*x^n)), x)
```

3.226 $\int (a + b\text{arctanh}(cx^n)) \, dx$

3.226.1 Optimal result

Integrand size = 10, antiderivative size = 58

$$\int (a + b\text{arctanh}(cx^n)) \, dx = ax + bx\text{arctanh}(cx^n) - \frac{bcnx^{1+n} \text{Hypergeometric2F1}\left(1, \frac{1+n}{2n}, \frac{1}{2}\left(3 + \frac{1}{n}\right), c^2x^{2n}\right)}{1+n}$$

output
```
a*x+b*x*arctanh(c*x^n)-b*c*n*x^(1+n)*hypergeom([1, 1/2*(1+n)/n],[3/2+1/2/n
],c^2*x^(2*n))/(1+n)
```

3.226.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00

$$\int (a + b\text{arctanh}(cx^n)) \, dx = ax + bx\text{arctanh}(cx^n) - \frac{bcnx^{1+n} \text{Hypergeometric2F1}\left(1, \frac{1+n}{2n}, \frac{1}{2}\left(3 + \frac{1}{n}\right), c^2x^{2n}\right)}{1+n}$$

input
```
Integrate[a + b*ArcTanh[c*x^n],x]
```

output
```
a*x + b*x*ArcTanh[c*x^n] - (b*c*n*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(
2*n), (3 + n^(-1))/2, c^2*x^(2*n)])/(1 + n)
```

3.226.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arctanh}(cx^n))\,dx$$

$$\Big\downarrow \ 2009$$

$$ax+bx\text{arctanh}(cx^n)-\frac{bcnx^{n+1}\text{Hypergeometric2F1}\left(1,\frac{n+1}{2n},\frac{1}{2}\left(3+\frac{1}{n}\right),c^2x^{2n}\right)}{n+1}$$

input
```
Int[a + b*ArcTanh[c*x^n],x]
```

output
```
a*x + b*x*ArcTanh[c*x^n] - (b*c*n*x^(1 + n)*Hypergeometric2F1[1, (1 + n)/(
2*n), (3 + n^(-1))/2, c^2*x^(2*n)])/(1 + n)
```

3.226.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.226.4 Maple [F]

$$\int (a+b\ \text{arctanh}\,(c\,x^n))\,dx$$

input
```
int(a+b*arctanh(c*x^n),x)
```

output
```
int(a+b*arctanh(c*x^n),x)
```

3.226.5 Fricas [F]

$$\int (a + b\text{arctanh}(cx^n)) \, dx = \int b \, \text{artanh}\,(cx^n) + a \, dx$$

input
```
integrate(a+b*arctanh(c*x^n),x, algorithm="fricas")
```

output
```
integral(b*arctanh(c*x^n) + a, x)
```

3.226.6 Sympy [F]

$$\int (a + b\text{arctanh}(cx^n)) \, dx = \int (a + b \, \text{atanh}\,(cx^n)) \, dx$$

input
```
integrate(a+b*atanh(c*x**n),x)
```

output
```
Integral(a + b*atanh(c*x**n), x)
```

3.226.7 Maxima [F]

$$\int (a + b\text{arctanh}(cx^n)) \, dx = \int b \, \text{artanh}\,(cx^n) + a \, dx$$

input
```
integrate(a+b*arctanh(c*x^n),x, algorithm="maxima")
```

output
```
1/2*(n*integrate(1/(c*x^n + 1), x) + n*integrate(1/(c*x^n - 1), x) + x*log
(c*x^n + 1) - x*log(-c*x^n + 1))*b + a*x
```

3.226.8 Giac [F]

$$\int (a + b\text{arctanh}(cx^n))\, dx = \int b\, \text{artanh}\,(cx^n) + a\, dx$$

input
```
integrate(a+b*arctanh(c*x^n),x, algorithm="giac")
```

output
```
integrate(b*arctanh(c*x^n) + a, x)
```

3.226.9 Mupad [F(-1)]

Timed out.

$$\int (a + b\text{arctanh}(cx^n))\, dx = \int a + b\, \text{atanh}(c\, x^n)\, dx$$

input
```
int(a + b*atanh(c*x^n),x)
```

output
```
int(a + b*atanh(c*x^n), x)
```

3.227 $\int \frac{a+b\text{arctanh}(cx^n)}{x}\,dx$

3.227.1 Optimal result

Integrand size = 14, antiderivative size = 36

$$\int \frac{a+b\text{arctanh}(cx^n)}{x}\,dx = a\log(x) - \frac{b\,\text{PolyLog}(2,-cx^n)}{2n} + \frac{b\,\text{PolyLog}(2,cx^n)}{2n}$$

output
```
a*ln(x)-1/2*b*polylog(2,-c*x^n)/n+1/2*b*polylog(2,c*x^n)/n
```

3.227.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{a+b\text{arctanh}(cx^n)}{x}\,dx = \frac{bcx^n\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},1;\frac{3}{2},\frac{3}{2};c^2x^{2n}\right)}{n} + a\log(x)$$

input
```
Integrate[(a + b*ArcTanh[c*x^n])/x,x]
```

output
```
(b*c*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, c^2*x^(2*n)])/n + a*
Log[x]
```

3.227.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6450, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^n)}{x}\,dx$$

$$\Big\downarrow \text{ 6450}$$

$$\frac{\int x^{-n}(a+b\text{arctanh}(cx^n))\,dx^n}{n}$$

$$\Big\downarrow \text{ 6446}$$

$$\frac{a\log(x^n)-\frac{1}{2}b\,\text{PolyLog}(2,-cx^n)+\frac{1}{2}b\,\text{PolyLog}(2,cx^n)}{n}$$

input
```
Int[(a + b*ArcTanh[c*x^n])/x,x]
```

output
```
(a*Log[x^n] - (b*PolyLog[2, -(c*x^n)])/2 + (b*PolyLog[2, c*x^n])/2)/n
```

3.227.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.227.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
risch	$a\ln(x)+\frac{b\,\mathrm{dilog}(1-c\,x^{n})}{2n}-\frac{b\,\mathrm{dilog}(c\,x^{n}+1)}{2n}$	35
parts	$a\ln(x)+\frac{b\left(\ln(c\,x^{n})\operatorname{arctanh}(c\,x^{n})-\frac{\mathrm{dilog}(c\,x^{n}+1)}{2}-\frac{\ln(c\,x^{n})\ln(c\,x^{n}+1)}{2}-\frac{\mathrm{dilog}(c\,x^{n})}{2}\right)}{n}$	59
derivativedivides	$\frac{a\ln(c\,x^{n})+b\left(\ln(c\,x^{n})\operatorname{arctanh}(c\,x^{n})-\frac{\mathrm{dilog}(c\,x^{n}+1)}{2}-\frac{\ln(c\,x^{n})\ln(c\,x^{n}+1)}{2}-\frac{\mathrm{dilog}(c\,x^{n})}{2}\right)}{n}$	64
default	$\frac{a\ln(c\,x^{n})+b\left(\ln(c\,x^{n})\operatorname{arctanh}(c\,x^{n})-\frac{\mathrm{dilog}(c\,x^{n}+1)}{2}-\frac{\ln(c\,x^{n})\ln(c\,x^{n}+1)}{2}-\frac{\mathrm{dilog}(c\,x^{n})}{2}\right)}{n}$	64

input
```
int((a+b*arctanh(c*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
a*ln(x)+1/2*b/n*dilog(1-c*x^n)-1/2/n*b*dilog(c*x^n+1)
```

3.227.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(30) = 60.

Time = 0.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.92

$$\int \frac{a+b\operatorname{arctanh}(cx^{n})}{x}\,dx =$$

$$-\frac{bn\log\left(c\cosh\left(n\log\left(x\right)\right)+c\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-bn\log\left(-c\cosh\left(n\log\left(x\right)\right)-c\sinh\left(n\log\left(x\right)\right)\right.}{}$$

input
```
integrate((a+b*arctanh(c*x^n))/x,x, algorithm="fricas")
```

output
```
-1/2*(b*n*log(c*cosh(n*log(x)) + c*sinh(n*log(x)) + 1)*log(x) - b*n*log(-c
*cosh(n*log(x)) - c*sinh(n*log(x)) + 1)*log(x) - b*n*log(x)*log(-(c*cosh(n
*log(x)) + c*sinh(n*log(x)) + 1)/(c*cosh(n*log(x)) + c*sinh(n*log(x)) - 1)
) - 2*a*n*log(x) - b*dilog(c*cosh(n*log(x)) + c*sinh(n*log(x))) + b*dilog(
-c*cosh(n*log(x)) - c*sinh(n*log(x))))/n
```

3.227.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx^n)}{x}\,dx=\int \frac{a+b\,\text{atanh}\,(cx^n)}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x**n))/x,x)
```

output
```
Integral((a + b*atanh(c*x**n))/x, x)
```

3.227.7 Maxima [F]

$$\int \frac{a+b\text{arctanh}(cx^n)}{x}\,dx=\int \frac{b\,\text{artanh}\,(cx^n)+a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x,x, algorithm="maxima")
```

output
```
1/2*(n*integrate(log(x)/(c*x*x^n + x), x) + n*integrate(log(x)/(c*x*x^n -
x), x) + log(c*x^n + 1)*log(x) - log(-c*x^n + 1)*log(x))*b + a*log(x)
```

3.227.8 Giac [F]

$$\int \frac{a+b\text{arctanh}(cx^n)}{x}\,dx=\int \frac{b\,\text{artanh}\,(cx^n)+a}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)/x, x)
```

3.227.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\operatorname{arctanh}(cx^n)}{x}\,dx=\int \frac{a+b\,\mathrm{atanh}(c\,x^n)}{x}\,dx$$

input
```
int((a + b*atanh(c*x^n))/x,x)
```

output
```
int((a + b*atanh(c*x^n))/x, x)
```

3.228 $\int \frac{a+b\text{arctanh}(cx^n)}{x^2}\,dx$

3.228.1 Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^2}\,dx = -\frac{a+b\text{arctanh}(cx^n)}{x} - \frac{bcnx^{-1+n}\,\text{Hypergeometric2F1}\left(1,-\frac{1-n}{2n},\frac{1}{2}\left(3-\frac{1}{n}\right),c^2x^{2n}\right)}{1-n}$$

output
```
(-a-b*arctanh(c*x^n))/x-b*c*n*x^(-1+n)*hypergeom([1, 1/2*(-1+n)/n],[3/2-1/
2/n],c^2*x^(2*n))/(1-n)
```

3.228.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.99

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^2}\,dx = -\frac{a}{x} - \frac{b\text{arctanh}(cx^n)}{x} + \frac{bcnx^{-1+n}\,\text{Hypergeometric2F1}\left(1,\frac{-1+n}{2n},1+\frac{-1+n}{2n},c^2x^{2n}\right)}{-1+n}$$

input
```
Integrate[(a + b*ArcTanh[c*x^n])/x^2,x]
```

output
```
-(a/x) - (b*ArcTanh[c*x^n])/x + (b*c*n*x^(-1 + n)*Hypergeometric2F1[1, (-1
 + n)/(2*n), 1 + (-1 + n)/(2*n), c^2*x^(2*n)])/(-1 + n)
```

3.228.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6452, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^2}\,dx$$

$$\Big\downarrow \text{6452}$$

$$bcn\int \frac{x^{n-2}}{1-c^2x^{2n}}dx-\frac{a+b\text{arctanh}(cx^n)}{x}$$

$$\Big\downarrow \text{888}$$

$$-\frac{a+b\text{arctanh}(cx^n)}{x}-\frac{bcnx^{n-1}\,\text{Hypergeometric2F1}\left(1,-\frac{1-n}{2n},\frac{1}{2}\left(3-\frac{1}{n}\right),c^2x^{2n}\right)}{1-n}$$

input
```
Int[(a + b*ArcTanh[c*x^n])/x^2,x]
```

output
```
-((a + b*ArcTanh[c*x^n])/x) - (b*c*n*x^(-1 + n)*Hypergeometric2F1[1, -1/2*
(1 - n)/n, (3 - n^(-1))/2, c^2*x^(2*n)])/(1 - n)
```

3.228.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.228.4 Maple [F]

$$\int \frac{a+b \operatorname{arctanh}(c\, x^{n})}{x^{2}}dx$$

input
```
int((a+b*arctanh(c*x^n))/x^2,x)
```

output
```
int((a+b*arctanh(c*x^n))/x^2,x)
```

3.228.5 Fricas [F]

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^2}\,dx = \int \frac{b\,\text{artanh}\,(cx^n)+a}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x^n) + a)/x^2, x)
```

3.228.6 Sympy [F]

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^2}\,dx = \int \frac{a+b\,\text{atanh}\,(cx^n)}{x^2}\,dx$$

input
```
integrate((a+b*atanh(c*x**n))/x**2,x)
```

output
```
Integral((a + b*atanh(c*x**n))/x**2, x)
```

3.228.7 Maxima [F]

$$\int \frac{a + b\operatorname{arctanh}(cx^n)}{x^2}\,dx = \int \frac{b\operatorname{artanh}(cx^n) + a}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="maxima")
```

output
```
-1/2*(n*integrate(1/(c*x^2*x^n + x^2), x) + n*integrate(1/(c*x^2*x^n - x^2
), x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x)*b - a/x
```

3.228.8 Giac [F]

$$\int \frac{a + b\operatorname{arctanh}(cx^n)}{x^2}\,dx = \int \frac{b\operatorname{artanh}(cx^n) + a}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)/x^2, x)
```

3.228.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\operatorname{arctanh}(cx^n)}{x^2}\,dx = \int \frac{a + b\operatorname{atanh}(c\,x^n)}{x^2}\,dx$$

input
```
int((a + b*atanh(c*x^n))/x^2,x)
```

output
```
int((a + b*atanh(c*x^n))/x^2, x)
```

3.229 $\int \frac{a+b\,\text{arctanh}(cx^n)}{x^3}\,dx$

3.229.1 Optimal result

Integrand size = 14, antiderivative size = 70

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^3}\,dx = -\frac{a+b\text{arctanh}(cx^n)}{2x^2} - \frac{bcnx^{-2+n}\,\text{Hypergeometric2F1}\left(1,\frac{1}{2}\left(1-\frac{2}{n}\right),\frac{1}{2}\left(3-\frac{2}{n}\right),c^2x^{2n}\right)}{2(2-n)}$$

output
```
1/2*(-a-b*arctanh(c*x^n))/x^2-1/2*b*c*n*x^(-2+n)*hypergeom([1, 1/2-1/n],[3
/2-1/n],c^2*x^(2*n))/(2-n)
```

3.229.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.04

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^3}\,dx = -\frac{a}{2x^2} - \frac{b\text{arctanh}(cx^n)}{2x^2} + \frac{bcnx^{-2+n}\,\text{Hypergeometric2F1}\left(1,\frac{-2+n}{2n},1+\frac{-2+n}{2n},c^2x^{2n}\right)}{2(-2+n)}$$

input
```
Integrate[(a + b*ArcTanh[c*x^n])/x^3,x]
```

output
```
-1/2*a/x^2 - (b*ArcTanh[c*x^n])/(2*x^2) + (b*c*n*x^(-2 + n)*Hypergeometric
2F1[1, (-2 + n)/(2*n), 1 + (-2 + n)/(2*n), c^2*x^(2*n)])/(2*(-2 + n))
```

3.229.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6452, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^3}dx$$

$$\Big\downarrow \text{ 6452}$$

$$\frac{1}{2}bcn\int \frac{x^{n-3}}{1-c^2x^{2n}}dx-\frac{a+b\text{arctanh}(cx^n)}{2x^2}$$

$$\Big\downarrow \text{ 888}$$

$$-\frac{a+b\text{arctanh}(cx^n)}{2x^2}-\frac{bcnx^{n-2}\text{Hypergeometric2F1}\left(1,-\frac{2-n}{2n},\frac{1}{2}\left(3-\frac{2}{n}\right),c^2x^{2n}\right)}{2(2-n)}$$

input
```
Int[(a + b*ArcTanh[c*x^n])/x^3,x]
```

output
```
-1/2*(a + b*ArcTanh[c*x^n])/x^2 - (b*c*n*x^(-2 + n)*Hypergeometric2F1[1, -
1/2*(2 - n)/n, (3 - 2/n)/2, c^2*x^(2*n)])/(2*(2 - n))
```

3.229.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.229.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arctanh}\left(c\,x^{n}\right)}{x^{3}}dx$$

input
```
int((a+b*arctanh(c*x^n))/x^3,x)
```

output
```
int((a+b*arctanh(c*x^n))/x^3,x)
```

3.229.5 Fricas [F]

$$\int \frac{a+b\mathrm{arctanh}(cx^n)}{x^3}\,dx = \int \frac{b\,\mathrm{artanh}\left(cx^{n}\right)+a}{x^{3}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x^n) + a)/x^3, x)
```

3.229.6 Sympy [F]

$$\int \frac{a+b\mathrm{arctanh}(cx^n)}{x^3}\,dx = \int \frac{a+b\,\mathrm{atanh}\left(cx^{n}\right)}{x^{3}}\,dx$$

input
```
integrate((a+b*atanh(c*x**n))/x**3,x)
```

output
```
Integral((a + b*atanh(c*x**n))/x**3, x)
```

3.229.7 Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx^n)}{x^3}\,dx = \int \frac{b\,\text{artanh}\,(cx^n) + a}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="maxima")
```

output
```
-1/4*(2*n*integrate(1/2/(c*x^3*x^n + x^3), x) + 2*n*integrate(1/2/(c*x^3*x
^n - x^3), x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x^2)*b - 1/2*a/x^2
```

3.229.8 Giac [F]

$$\int \frac{a + b\text{arctanh}(cx^n)}{x^3}\,dx = \int \frac{b\,\text{artanh}\,(cx^n) + a}{x^3}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)/x^3, x)
```

3.229.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^n)}{x^3}\,dx = \int \frac{a + b\,\text{atanh}(c\,x^n)}{x^3}\,dx$$

input
```
int((a + b*atanh(c*x^n))/x^3,x)
```

output
```
int((a + b*atanh(c*x^n))/x^3, x)
```

3.230 $\int \frac{a+b\text{arctanh}(cx^n)}{x^4}\,dx$

3.230.1 Optimal result

Integrand size = 14, antiderivative size = 72

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^4}\,dx = -\frac{a+b\text{arctanh}(cx^n)}{3x^3} - \frac{bcnx^{-3+n}\,\text{Hypergeometric2F1}\left(1,-\frac{3-n}{2n},-\frac{3(1-n)}{2n},c^2x^{2n}\right)}{3(3-n)}$$

output
```
1/3*(-a-b*arctanh(c*x^n))/x^3-1/3*b*c*n*x^(-3+n)*hypergeom([1, 1/2*(-3+n)/
n],[-3/2*(1-n)/n],c^2*x^(2*n))/(3-n)
```

3.230.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.01

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^4}\,dx = -\frac{a}{3x^3} - \frac{b\text{arctanh}(cx^n)}{3x^3} + \frac{bcnx^{-3+n}\,\text{Hypergeometric2F1}\left(1,\frac{-3+n}{2n},1+\frac{-3+n}{2n},c^2x^{2n}\right)}{3(-3+n)}$$

input
```
Integrate[(a + b*ArcTanh[c*x^n])/x^4,x]
```

output
```
-1/3*a/x^3 - (b*ArcTanh[c*x^n])/(3*x^3) + (b*c*n*x^(-3 + n)*Hypergeometric
2F1[1, (-3 + n)/(2*n), 1 + (-3 + n)/(2*n), c^2*x^(2*n)])/(3*(-3 + n))
```

3.230.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {6452, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\text{arctanh}(cx^n)}{x^4}dx$$
$$\Big\downarrow \text{ 6452}$$
$$\frac{1}{3}bcn\int \frac{x^{n-4}}{1-c^2x^{2n}}dx-\frac{a+b\text{arctanh}(cx^n)}{3x^3}$$
$$\Big\downarrow \text{ 888}$$
$$-\frac{a+b\text{arctanh}(cx^n)}{3x^3}-\frac{bcnx^{n-3}\text{Hypergeometric2F1}\left(1,-\frac{3-n}{2n},-\frac{3(1-n)}{2n},c^2x^{2n}\right)}{3(3-n)}$$

input
```
Int[(a + b*ArcTanh[c*x^n])/x^4,x]
```

output
```
-1/3*(a + b*ArcTanh[c*x^n])/x^3 - (b*c*n*x^(-3 + n)*Hypergeometric2F1[1, -
1/2*(3 - n)/n, (-3*(1 - n))/(2*n), c^2*x^(2*n)])/(3*(3 - n))
```

3.230.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

3.230.4 Maple [F]

$$\int \frac{a+b\,\mathrm{arctanh}\,(c\,x^{n})}{x^{4}}dx$$

input
```
int((a+b*arctanh(c*x^n))/x^4,x)
```

output
```
int((a+b*arctanh(c*x^n))/x^4,x)
```

3.230.5 Fricas [F]

$$\int \frac{a+b\mathrm{arctanh}(cx^{n})}{x^{4}}\,dx = \int \frac{b\,\mathrm{artanh}\,(cx^{n})+a}{x^{4}}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="fricas")
```

output
```
integral((b*arctanh(c*x^n) + a)/x^4, x)
```

3.230.6 Sympy [F]

$$\int \frac{a+b\mathrm{arctanh}(cx^{n})}{x^{4}}\,dx = \int \frac{a+b\,\mathrm{atanh}\,(cx^{n})}{x^{4}}\,dx$$

input
```
integrate((a+b*atanh(c*x**n))/x**4,x)
```

output
```
Integral((a + b*atanh(c*x**n))/x**4, x)
```

3.230.7 Maxima [F]

$$\int \frac{a + b\text{arctanh}(cx^n)}{x^4}\,dx = \int \frac{b\,\text{artanh}\,(cx^n) + a}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="maxima")
```

output
```
-1/6*(3*n*integrate(1/3/(c*x^4*x^n + x^4), x) + 3*n*integrate(1/3/(c*x^4*x
^n - x^4), x) + (log(c*x^n + 1) - log(-c*x^n + 1))/x^3)*b - 1/3*a/x^3
```

3.230.8 Giac [F]

$$\int \frac{a + b\text{arctanh}(cx^n)}{x^4}\,dx = \int \frac{b\,\text{artanh}\,(cx^n) + a}{x^4}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))/x^4,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)/x^4, x)
```

3.230.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a + b\text{arctanh}(cx^n)}{x^4}\,dx = \int \frac{a + b\,\text{atanh}(c\,x^n)}{x^4}\,dx$$

input
```
int((a + b*atanh(c*x^n))/x^4,x)
```

output
```
int((a + b*atanh(c*x^n))/x^4, x)
```

3.231 $\int x(a+b\text{arctanh}(cx^n))^2\,dx$

3.231.1 Optimal result

Integrand size = 14, antiderivative size = 14

$$\int x(a+b\text{arctanh}(cx^n))^2\,dx = \text{Int}\left(x(a+b\text{arctanh}(cx^n))^2,x\right)$$

output
```
Unintegrable(x*(a+b*arctanh(c*x^n))^2,x)
```

3.231.2 Mathematica [N/A]

Not integrable

Time = 12.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int x(a+b\text{arctanh}(cx^n))^2\,dx = \int x(a+b\text{arctanh}(cx^n))^2\,dx$$

input
```
Integrate[x*(a + b*ArcTanh[c*x^n])^2,x]
```

output
```
Integrate[x*(a + b*ArcTanh[c*x^n])^2, x]
```

3.231.3 Rubi [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a + b\text{arctanh}(cx^n))^2 \, dx$$

$$\Big\downarrow \text{6468}$$

$$\int x(a + b\text{arctanh}(cx^n))^2 \, dx$$

input
```
Int[x*(a + b*ArcTanh[c*x^n])^2,x]
```

output
```
$Aborted
```

3.231.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.231.4 Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(a + b \operatorname{arctanh}(c x^n))^2 dx$$

input
```
int(x*(a+b*arctanh(c*x^n))^2,x)
```

output
```
int(x*(a+b*arctanh(c*x^n))^2,x)
```

3.231.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.29

$$\int x(a+b\text{arctanh}(cx^n))^2\,dx=\int (b\,\text{artanh}\,(cx^n)+a)^2x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*x*arctanh(c*x^n)^2 + 2*a*b*x*arctanh(c*x^n) + a^2*x, x)
```

3.231.6 Sympy [N/A]

Not integrable

Time = 28.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int x(a+b\text{arctanh}(cx^n))^2\,dx=\int x(a+b\,\text{atanh}\,(cx^n))^2\,dx$$

input
```
integrate(x*(a+b*atanh(c*x**n))**2,x)
```

output
```
Integral(x*(a + b*atanh(c*x**n))**2, x)
```

3.231.7 Maxima [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 11.07

$$\int x(a+b\text{arctanh}(cx^n))^2\,dx=\int (b\,\text{artanh}\,(cx^n)+a)^2x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*x^2*log(-c*x^n + 1)^2 + 1/2*a^2*x^2 - integrate(-1/4*((b^2*c*x*x^n
 - b^2*x)*log(c*x^n + 1)^2 + 4*(a*b*c*x*x^n - a*b*x)*log(c*x^n + 1) + (4*a
*b*x - (b^2*c*n + 4*a*b*c)*x*x^n - 2*(b^2*c*x*x^n - b^2*x)*log(c*x^n + 1))
*log(-c*x^n + 1))/(c*x^n - 1), x)
```

3.231.8 Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int x(a+b\text{arctanh}(cx^n))^2\,dx=\int (b\,\text{artanh}\left(cx^n\right)+a)^2x\,dx$$

input
```
integrate(x*(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)^2*x, x)
```

3.231.9 Mupad [N/A]

Not integrable

Time = 3.40 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int x(a+b\text{arctanh}(cx^n))^2\,dx=\int x\left(a+b\,\text{atanh}(c\,x^n)\right)^2\,dx$$

input
```
int(x*(a + b*atanh(c*x^n))^2,x)
```

output
```
int(x*(a + b*atanh(c*x^n))^2, x)
```

3.232 $\int (a+b\,\mathrm{arctanh}(cx^n))^2\,dx$

3.232.1 Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (a+b\,\mathrm{arctanh}(cx^n))^2\,dx = \mathrm{Int}\left((a+b\,\mathrm{arctanh}(cx^n))^2,x\right)$$

output
```
Unintegrable((a+b*arctanh(c*x^n))^2,x)
```

3.232.2 Mathematica [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a+b\,\mathrm{arctanh}(cx^n))^2\,dx = \int (a+b\,\mathrm{arctanh}(cx^n))^2\,dx$$

input
```
Integrate[(a + b*ArcTanh[c*x^n])^2,x]
```

output
```
Integrate[(a + b*ArcTanh[c*x^n])^2, x]
```

3.232.3 Rubi [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {6444}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\text{arctanh}(cx^n))^2\,dx$$

$$\Big\downarrow \text{ 6444}$$

$$\int (a+b\text{arctanh}(cx^n))^2dx$$

input
```
Int[(a + b*ArcTanh[c*x^n])^2,x]
```

output
```
$Aborted
```

3.232.3.1 Defintions of rubi rules used

rule 6444
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Unintegrab
le[(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b, c, n, p}, x]
```

3.232.4 Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+b\,\text{arctanh}\,(c\,x^{n}))^{2}dx$$

input
```
int((a+b*arctanh(c*x^n))^2,x)
```

output
```
int((a+b*arctanh(c*x^n))^2,x)
```

3.232.5 Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int (a+b\text{arctanh}(cx^n))^2\,dx=\int (b\,\text{artanh}\,(cx^n)+a)^2\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral(b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2, x)
```

3.232.6 Sympy [N/A]

Not integrable

Time = 17.64 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (a+b\text{arctanh}(cx^n))^2\,dx=\int (a+b\,\text{atanh}\,(cx^n))^2\,dx$$

input
```
integrate((a+b*atanh(c*x**n))**2,x)
```

output
```
Integral((a + b*atanh(c*x**n))**2, x)
```

3.232.7 Maxima [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 143, normalized size of antiderivative = 11.92

$$\int (a+b\text{arctanh}(cx^n))^2\,dx=\int (b\,\text{artanh}\,(cx^n)+a)^2\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*x*log(-c*x^n + 1)^2 + a^2*x - integrate(-1/4*((b^2*c*x^n - b^2)*lo
g(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b - (b^2*c*n
+ 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*x^n
 - 1), x)
```

3.232.8 Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a + b\text{arctanh}(cx^n))^2\, dx = \int (b\,\text{artanh}\,(cx^n) + a)^2\, dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)^2, x)
```

3.232.9 Mupad [N/A]

Not integrable

Time = 3.54 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (a + b\text{arctanh}(cx^n))^2\, dx = \int (a + b\,\text{atanh}(c\,x^n))^2\, dx$$

input
```
int((a + b*atanh(c*x^n))^2,x)
```

output
```
int((a + b*atanh(c*x^n))^2, x)
```

3.233 $\int \frac{\left(a+b\,\mathrm{arctanh}(cx^n)\right)^2}{x}\,dx$

3.233.1 Optimal result

Integrand size = 16, antiderivative size = 148

$$\begin{aligned}\int \frac{(a+b\,\mathrm{arctanh}(cx^n))^2}{x}\,dx = {} & \frac{2(a+b\,\mathrm{arctanh}(cx^n))^2\,\mathrm{arctanh}\left(1-\frac{2}{1-cx^n}\right)}{n} \\ & - \frac{b(a+b\,\mathrm{arctanh}(cx^n))\,\mathrm{PolyLog}\left(2,1-\frac{2}{1-cx^n}\right)}{n} \\ & + \frac{b(a+b\,\mathrm{arctanh}(cx^n))\,\mathrm{PolyLog}\left(2,-1+\frac{2}{1-cx^n}\right)}{n} \\ & + \frac{b^2\,\mathrm{PolyLog}\left(3,1-\frac{2}{1-cx^n}\right)}{2n} - \frac{b^2\,\mathrm{PolyLog}\left(3,-1+\frac{2}{1-cx^n}\right)}{2n}\end{aligned}$$

output
```
-2*(a+b*arctanh(c*x^n))^2*arctanh(-1+2/(1-c*x^n))/n-b*(a+b*arctanh(c*x^n))
*polylog(2,1-2/(1-c*x^n))/n+b*(a+b*arctanh(c*x^n))*polylog(2,-1+2/(1-c*x^n
))/n+1/2*b^2*polylog(3,1-2/(1-c*x^n))/n-1/2*b^2*polylog(3,-1+2/(1-c*x^n))/
n
```

3.233.2 Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.22

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x}\,dx = a^2\log(x) + \frac{ab(-\text{PolyLog}(2,-cx^n)+\text{PolyLog}(2,cx^n))}{n}$$
$$+ \frac{b^2\left(\frac{i\pi^3}{24} - \frac{2}{3}\text{arctanh}(cx^n)^3 - \text{arctanh}(cx^n)^2\log\left(1+e^{-2\text{arctanh}(cx^n)}\right) + \text{arctanh}(cx^n)^2\log\left(1-e^{2\text{arctanh}(cx^n)}\right)\right.}{}$$

input
```
Integrate[(a + b*ArcTanh[c*x^n])^2/x,x]
```

output
```
a^2*Log[x] + (a*b*(-PolyLog[2, -(c*x^n)] + PolyLog[2, c*x^n]))/n + (b^2*((
I/24)*Pi^3 - (2*ArcTanh[c*x^n]^3)/3 - ArcTanh[c*x^n]^2*Log[1 + E^(-2*ArcTa
nh[c*x^n])] + ArcTanh[c*x^n]^2*Log[1 - E^(2*ArcTanh[c*x^n])] + ArcTanh[c*x
^n]*PolyLog[2, -E^(-2*ArcTanh[c*x^n])] + ArcTanh[c*x^n]*PolyLog[2, E^(2*Ar
cTanh[c*x^n])] + PolyLog[3, -E^(-2*ArcTanh[c*x^n])]/2 - PolyLog[3, E^(2*Ar
cTanh[c*x^n])]/2))/n
```

3.233.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {6450, 6448, 6614, 6620, 7164}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x}\,dx$$

$$\Big\downarrow \; 6450$$

$$\frac{\int x^{-n}(a+b\text{arctanh}(cx^n))^2\,dx^n}{n}$$

$$\Big\downarrow \; 6448$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1-cx^n}\right)(a+b\text{arctanh}(cx^n))^2 - 4bc\int \frac{(a+b\text{arctanh}(cx^n))\text{arctanh}\left(1-\frac{2}{1-cx^n}\right)}{1-c^2x^{2n}}\,dx^n}{n}$$

$$\Big\downarrow \; 6614$$

$$\frac{2\text{arctanh}\left(1-\frac{2}{1-cx^n}\right)(a+b\text{arctanh}(cx^n))^2-4bc\left(\frac{1}{2}\int\frac{(a+b\text{arctanh}(cx^n))\log\left(2-\frac{2}{1-cx^n}\right)}{1-c^2x^{2n}}dx^n-\frac{1}{2}\int\frac{(a+b\text{arctanh}(cx^n))}{1-c^2x^{2n}}\right.}{n}$$

$\big\downarrow$ 6620

$$\frac{2\text{arctanh}\left(1-\frac{2}{1-cx^n}\right)(a+b\text{arctanh}(cx^n))^2-4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^n}\right)(a+b\text{arctanh}(cx^n))}{2c}-\frac{1}{2}b\int\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^n}\right)}{1-c^2x^{2n}}\right.\right.}{n}$$

$\big\downarrow$ 7164

$$\frac{2\text{arctanh}\left(1-\frac{2}{1-cx^n}\right)(a+b\text{arctanh}(cx^n))^2-4bc\left(\frac{1}{2}\left(\frac{\text{PolyLog}\left(2,1-\frac{2}{1-cx^n}\right)(a+b\text{arctanh}(cx^n))}{2c}-\frac{b\,\text{PolyLog}\left(3,1-\frac{2}{1-cx^n}\right)}{4c}\right)\right.}{n}$$

input
```
Int[(a + b*ArcTanh[c*x^n])^2/x,x]
```

output
```
(2*(a + b*ArcTanh[c*x^n])^2*ArcTanh[1 - 2/(1 - c*x^n)] - 4*b*c*((((a + b*A
rcTanh[c*x^n])*PolyLog[2, 1 - 2/(1 - c*x^n)])/(2*c) - (b*PolyLog[3, 1 - 2/
(1 - c*x^n)])/(4*c))/2 + (-1/2*((a + b*ArcTanh[c*x^n])*PolyLog[2, -1 + 2/(
1 - c*x^n)])/c + (b*PolyLog[3, -1 + 2/(1 - c*x^n)])/(4*c))/2))/n
```

3.233.3.1 Defintions of rubi rules used

rule 6448
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_)/(x_), x_Symbol] :> Simp[2*(a +
 b*ArcTanh[c*x])^p*ArcTanh[1 - 2/(1 - c*x)], x] - Simp[2*b*c*p   Int[(a + b
*ArcTanh[c*x])^(p - 1)*(ArcTanh[1 - 2/(1 - c*x)]/(1 - c^2*x^2)), x], x] /;
FreeQ[{a, b, c}, x] && IGtQ[p, 1]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.233. $\int \frac{(a+b\text{arctanh}(cx^n))^2}{x}\,dx$

rule 6614
```
Int[(ArcTanh[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(
x_)^2), x_Symbol] :> Simp[1/2   Int[Log[1 + u]*((a + b*ArcTanh[c*x])^p/(d +
 e*x^2)), x], x] - Simp[1/2   Int[Log[1 - u]*((a + b*ArcTanh[c*x])^p/(d + e
*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d + e,
 0] && EqQ[u^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 6620
```
Int[(Log[u_]*((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))^(p_.))/((d_) + (e_.)*(x_)^
2), x_Symbol] :> Simp[(-(a + b*ArcTanh[c*x])^p)*(PolyLog[2, 1 - u]/(2*c*d))
, x] + Simp[b*(p/2)   Int[(a + b*ArcTanh[c*x])^(p - 1)*(PolyLog[2, 1 - u]/(
d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[p, 0] && EqQ[c^2*d
 + e, 0] && EqQ[(1 - u)^2 - (1 - 2/(1 - c*x))^2, 0]
```

rule 7164
```
Int[(u_)*PolyLog[n_, v_], x_Symbol] :> With[{w = DerivativeDivides[v, u*v,
x]}, Simp[w*PolyLog[n + 1, v], x] /;  !FalseQ[w]] /; FreeQ[n, x]
```

3.233.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 23.72 (sec) , antiderivative size = 750, normalized size of antiderivative = 5.07

method	result
parts	$a^2\ln(x)+\frac{b^2\left(\ln(cx^n)\operatorname{arctanh}(cx^n)^2-\operatorname{arctanh}(cx^n)\operatorname{polylog}\left(2,-\frac{(cx^n+1)^2}{-c^2x^{2n}+1}\right)+\frac{\operatorname{polylog}\left(3,-\frac{(cx^n+1)^2}{-c^2x^{2n}+1}\right)}{2}-\operatorname{arctanh}\right.}{}$
derivativedivides	$\frac{a^2\ln(cx^n)+b^2\left(\ln(cx^n)\operatorname{arctanh}(cx^n)^2-\operatorname{arctanh}(cx^n)\operatorname{polylog}\left(2,-\frac{(cx^n+1)^2}{-c^2x^{2n}+1}\right)+\frac{\operatorname{polylog}\left(3,-\frac{(cx^n+1)^2}{-c^2x^{2n}+1}\right)}{2}-\operatorname{arctanh}(c\right.}{}$
default	$\frac{a^2\ln(cx^n)+b^2\left(\ln(cx^n)\operatorname{arctanh}(cx^n)^2-\operatorname{arctanh}(cx^n)\operatorname{polylog}\left(2,-\frac{(cx^n+1)^2}{-c^2x^{2n}+1}\right)+\frac{\operatorname{polylog}\left(3,-\frac{(cx^n+1)^2}{-c^2x^{2n}+1}\right)}{2}-\operatorname{arctanh}(c\right.}{}$

input
```
int((a+b*arctanh(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

output
```
a^2*ln(x)+b^2/n*(ln(c*x^n)*arctanh(c*x^n)^2-arctanh(c*x^n)*polylog(2,-(c*x
^n+1)^2/(-c^2*(x^n)^2+1))+1/2*polylog(3,-(c*x^n+1)^2/(-c^2*(x^n)^2+1))-arc
tanh(c*x^n)^2*ln((c*x^n+1)^2/(-c^2*(x^n)^2+1)-1)+arctanh(c*x^n)^2*ln(1+(c*
x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+2*arctanh(c*x^n)*polylog(2,-(c*x^n+1)/(-c^2
*(x^n)^2+1)^(1/2))-2*polylog(3,-(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+arctanh(
c*x^n)^2*ln(1-(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))+2*arctanh(c*x^n)*polylog(2
,(c*x^n+1)/(-c^2*(x^n)^2+1)^(1/2))-2*polylog(3,(c*x^n+1)/(-c^2*(x^n)^2+1)^
(1/2))+1/2*I*Pi*csgn(I*(-(c*x^n+1)^2/(c^2*(x^n)^2-1)-1)/(1-(c*x^n+1)^2/(c^
2*(x^n)^2-1)))*(csgn(I*(-(c*x^n+1)^2/(c^2*(x^n)^2-1)-1))*csgn(I/(1-(c*x^n+
1)^2/(c^2*(x^n)^2-1)))-csgn(I*(-(c*x^n+1)^2/(c^2*(x^n)^2-1)-1))*csgn(I*(-(
c*x^n+1)^2/(c^2*(x^n)^2-1)-1)/(1-(c*x^n+1)^2/(c^2*(x^n)^2-1)))-csgn(I*(-(c
*x^n+1)^2/(c^2*(x^n)^2-1)-1)/(1-(c*x^n+1)^2/(c^2*(x^n)^2-1)))*csgn(I/(1-(c
*x^n+1)^2/(c^2*(x^n)^2-1)))+csgn(I*(-(c*x^n+1)^2/(c^2*(x^n)^2-1)-1)/(1-(c*
x^n+1)^2/(c^2*(x^n)^2-1)))^2)*arctanh(c*x^n)^2)+2*a*b/n*(ln(c*x^n)*arctanh
(c*x^n)-1/2*dilog(c*x^n+1)-1/2*ln(c*x^n)*ln(c*x^n+1)-1/2*dilog(c*x^n))
```

3.233.5 Fricas [F]

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x}\,dx = \int \frac{(b\,\text{artanh}\,(cx^n)+a)^2}{x}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x, x)
```

3.233.6 Sympy [F]

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x}\,dx = \int \frac{(a+b\,\text{atanh}\,(cx^n))^2}{x}\,dx$$

input
```
integrate((a+b*atanh(c*x**n))**2/x,x)
```

output
```
Integral((a + b*atanh(c*x**n))**2/x, x)
```

3.233.7 Maxima [F]

$$\int \frac{(a + b\text{arctanh}(cx^n))^2}{x} dx = \int \frac{(b\,\text{artanh}\,(cx^n) + a)^2}{x} dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
1/4*b^2*log(-c*x^n + 1)^2*log(x) + a^2*log(x) - integrate(-1/4*((b^2*c*x^n
 - b^2)*log(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b -
 (b^2*c*n*log(x) + 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c
*x^n + 1))/(c*x*x^n - x), x)
```

3.233.8 Giac [F]

$$\int \frac{(a + b\text{arctanh}(cx^n))^2}{x} dx = \int \frac{(b\,\text{artanh}\,(cx^n) + a)^2}{x} dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2/x,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)^2/x, x)
```

3.233.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a + b\text{arctanh}(cx^n))^2}{x} dx = \int \frac{(a + b\,\text{atanh}(c\,x^n))^2}{x} dx$$

input
```
int((a + b*atanh(c*x^n))^2/x,x)
```

output
```
int((a + b*atanh(c*x^n))^2/x, x)
```

3.234 $\int \frac{\left(a+b\text{arctanh}(cx^n)\right)^2}{x^2}\,dx$

3.234.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^2}\,dx=\text{Int}\left(\frac{(a+b\text{arctanh}(cx^n))^2}{x^2},x\right)$$

output
```
Unintegrable((a+b*arctanh(c*x^n))^2/x^2,x)
```

3.234.2 Mathematica [N/A]

Not integrable

Time = 12.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^2}\,dx=\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^2}\,dx$$

input
```
Integrate[(a + b*ArcTanh[c*x^n])^2/x^2,x]
```

output
```
Integrate[(a + b*ArcTanh[c*x^n])^2/x^2, x]
```

3.234.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^2} \, dx$$

$$\Big\downarrow \; 6468$$

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^2}dx$$

input
```
Int[(a + b*ArcTanh[c*x^n])^2/x^2,x]
```

output
```
$Aborted
```

3.234.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.234.4 Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b \,\operatorname{arctanh}\left(c \,x^{n}\right)\right)^{2}}{x^{2}}dx$$

input
```
int((a+b*arctanh(c*x^n))^2/x^2,x)
```

output
```
int((a+b*arctanh(c*x^n))^2/x^2,x)
```

3.234.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx^n)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x^2, x)
```

3.234.6 Sympy [N/A]

Not integrable

Time = 19.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^2}\,dx = \int \frac{(a+b\,\text{atanh}\,(cx^n))^2}{x^2}\,dx$$

input
```
integrate((a+b*atanh(c*x**n))**2/x**2,x)
```

output
```
Integral((a + b*atanh(c*x**n))**2/x**2, x)
```

3.234.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 154, normalized size of antiderivative = 9.62

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^2}\,dx = \int \frac{(b\,\text{artanh}\,(cx^n)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*log(-c*x^n + 1)^2/x - a^2/x - integrate(-1/4*((b^2*c*x^n - b^2)*l
og(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + 2*(2*a*b + (b^2*c*n
 - 2*a*b*c)*x^n - (b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1))/(c*x^
2*x^n - x^2), x)
```

3.234.8 Giac [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^2}\,dx=\int \frac{(b\,\text{artanh}\,(cx^n)+a)^2}{x^2}\,dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2/x^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)^2/x^2, x)
```

3.234.9 Mupad [N/A]

Not integrable

Time = 3.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^2}\,dx=\int \frac{(a+b\,\text{atanh}(c\,x^n))^2}{x^2}\,dx$$

input
```
int((a + b*atanh(c*x^n))^2/x^2,x)
```

output
```
int((a + b*atanh(c*x^n))^2/x^2, x)
```

3.235 $\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^3}\,dx$

3.235.1 Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^3}\,dx = \text{Int}\left(\frac{(a+b\text{arctanh}(cx^n))^2}{x^3},x\right)$$

output
```
Unintegrable((a+b*arctanh(c*x^n))^2/x^3,x)
```

3.235.2 Mathematica [N/A]

Not integrable

Time = 17.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^3}\,dx = \int \frac{(a+b\text{arctanh}(cx^n))^2}{x^3}\,dx$$

input
```
Integrate[(a + b*ArcTanh[c*x^n])^2/x^3,x]
```

output
```
Integrate[(a + b*ArcTanh[c*x^n])^2/x^3, x]
```

3.235.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^3} dx$$
$$\Big\downarrow \text{ 6468}$$
$$\int \frac{(a+b\text{arctanh}(cx^n))^2}{x^3}dx$$

input
```
Int[(a + b*ArcTanh[c*x^n])^2/x^3,x]
```

output
```
$Aborted
```

3.235.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.235.4 Maple [N/A] (verified)

Not integrable

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\left(a+b\,\operatorname{arctanh}\left(c\,x^{n}\right)\right)^{2}}{x^{3}}dx$$

input
```
int((a+b*arctanh(c*x^n))^2/x^3,x)
```

output
```
int((a+b*arctanh(c*x^n))^2/x^3,x)
```

3.235.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.00

$$\int \frac{(a + b\text{arctanh}(cx^n))^2}{x^3} \, dx = \int \frac{(b \operatorname{artanh}(cx^n) + a)^2}{x^3} \, dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="fricas")
```

output
```
integral((b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2)/x^3, x)
```

3.235.6 Sympy [N/A]

Not integrable

Time = 46.57 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(a + b\text{arctanh}(cx^n))^2}{x^3} \, dx = \int \frac{(a + b \operatorname{atanh}(cx^n))^2}{x^3} \, dx$$

input
```
integrate((a+b*atanh(c*x**n))**2/x**3,x)
```

output
```
Integral((a + b*atanh(c*x**n))**2/x**3, x)
```

3.235.7 Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 9.56

$$\int \frac{(a + b\text{arctanh}(cx^n))^2}{x^3} \, dx = \int \frac{(b \operatorname{artanh}(cx^n) + a)^2}{x^3} \, dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="maxima")
```

output
```
-1/8*b^2*log(-c*x^n + 1)^2/x^2 - 1/2*a^2/x^2 - integrate(-1/4*((b^2*c*x^n
- b^2)*log(c*x^n + 1)^2 + 4*(a*b*c*x^n - a*b)*log(c*x^n + 1) + (4*a*b + (b
^2*c*n - 4*a*b*c)*x^n - 2*(b^2*c*x^n - b^2)*log(c*x^n + 1))*log(-c*x^n + 1
))/(c*x^3*x^n - x^3), x)
```

3.235.8 Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a + b\text{arctanh}(cx^n))^2}{x^3} \, dx = \int \frac{(b\,\text{artanh}\,(cx^n) + a)^2}{x^3} \, dx$$

input
```
integrate((a+b*arctanh(c*x^n))^2/x^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)^2/x^3, x)
```

3.235.9 Mupad [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a + b\text{arctanh}(cx^n))^2}{x^3} \, dx = \int \frac{(a + b\,\text{atanh}(c\,x^n))^2}{x^3} \, dx$$

input
```
int((a + b*atanh(c*x^n))^2/x^3,x)
```

output
```
int((a + b*atanh(c*x^n))^2/x^3, x)
```

3.236 $\int \frac{\operatorname{arctanh}(ax^n)}{x} \, dx$

3.236.1 Optimal result

Integrand size = 10, antiderivative size = 30

$$\int \frac{\operatorname{arctanh}(ax^n)}{x} \, dx = -\frac{\operatorname{PolyLog}(2, -ax^n)}{2n} + \frac{\operatorname{PolyLog}(2, ax^n)}{2n}$$

output
```
-1/2*polylog(2,-a*x^n)/n+1/2*polylog(2,a*x^n)/n
```

3.236.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.10

$$\int \frac{\operatorname{arctanh}(ax^n)}{x} \, dx = \frac{ax^n \, {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}; a^2x^{2n}\right)}{n}$$

input
```
Integrate[ArcTanh[a*x^n]/x,x]
```

output
```
(a*x^n*HypergeometricPFQ[{1/2, 1/2, 1}, {3/2, 3/2}, a^2*x^(2*n)])/n
```

3.236.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {6450, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{arctanh}(ax^n)}{x} \, dx$$

$$\Big\downarrow \text{ 6450}$$

$$\frac{\int x^{-n} \operatorname{arctanh}(ax^n) \, dx^n}{n}$$

$$\Big\downarrow \text{ 6446}$$

$$\frac{\frac{\operatorname{PolyLog}(2,ax^n)}{2} - \frac{1}{2}\operatorname{PolyLog}(2,-ax^n)}{n}$$

input
```
Int[ArcTanh[a*x^n]/x,x]
```

output
```
(-1/2*PolyLog[2, -(a*x^n)] + PolyLog[2, a*x^n]/2)/n
```

3.236.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.236.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{\operatorname{dilog}(1-a\,x^{n})}{2n}-\frac{\operatorname{dilog}(a\,x^{n}+1)}{2n}$	29
derivativedivides	$\frac{\ln(a\,x^{n})\operatorname{arctanh}(a\,x^{n})-\frac{\operatorname{dilog}(a\,x^{n})}{2}-\frac{\operatorname{dilog}(a\,x^{n}+1)}{2}-\frac{\ln(a\,x^{n})\ln(a\,x^{n}+1)}{2}}{n}$	53
default	$\frac{\ln(a\,x^{n})\operatorname{arctanh}(a\,x^{n})-\frac{\operatorname{dilog}(a\,x^{n})}{2}-\frac{\operatorname{dilog}(a\,x^{n}+1)}{2}-\frac{\ln(a\,x^{n})\ln(a\,x^{n}+1)}{2}}{n}$	53
meijerg	$-\frac{i\left(\frac{2ia\,x^{n}\operatorname{polylog}\left(2,\sqrt{a^{2}x^{2n}}\right)}{\sqrt{a^{2}x^{2n}}}-\frac{2ia\,x^{n}\operatorname{polylog}\left(2,-\sqrt{a^{2}x^{2n}}\right)}{\sqrt{a^{2}x^{2n}}}\right)}{4n}$	72

input
```
int(arctanh(a*x^n)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2/n*dilog(1-a*x^n)-1/2/n*dilog(a*x^n+1)
```

3.236.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(24) = 48$.

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 4.30

$$\int \frac{\operatorname{arctanh}(ax^n)}{x}\,dx =$$

$$-\frac{n\log\left(a\cosh\left(n\log\left(x\right)\right)+a\sinh\left(n\log\left(x\right)\right)+1\right)\log\left(x\right)-n\log\left(-a\cosh\left(n\log\left(x\right)\right)-a\sinh\left(n\log\left(x\right)\right)\right.}{}$$

input
```
integrate(arctanh(a*x^n)/x,x, algorithm="fricas")
```

output
```
-1/2*(n*log(a*cosh(n*log(x)) + a*sinh(n*log(x)) + 1)*log(x) - n*log(-a*cos
h(n*log(x)) - a*sinh(n*log(x)) + 1)*log(x) - n*log(x)*log(-(a*cosh(n*log(x
)) + a*sinh(n*log(x)) + 1)/(a*cosh(n*log(x)) + a*sinh(n*log(x)) - 1)) - di
log(a*cosh(n*log(x)) + a*sinh(n*log(x))) + dilog(-a*cosh(n*log(x)) - a*sin
h(n*log(x))))/n
```

3.236.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax^n)}{x}\,dx = \int \frac{\operatorname{atanh}(ax^n)}{x}\,dx$$

input
```
integrate(atanh(a*x**n)/x,x)
```

output
```
Integral(atanh(a*x**n)/x, x)
```

3.236.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(24) = 48.

Time = 0.25 (sec) , antiderivative size = 147, normalized size of antiderivative = 4.90

$$\int \frac{\operatorname{arctanh}(ax^n)}{x}\,dx = -\frac{1}{2}an\left(\frac{\log\left(\frac{ax^n+1}{a}\right)}{an} - \frac{\log\left(\frac{ax^n-1}{a}\right)}{an}\right)\log(x) + \frac{1}{2}an\left(\frac{\log(ax^n+1)\log(x) - \log(ax^n-1)\log(x)}{an} - \frac{n\log(ax^n+1)\log(x) + \operatorname{Li}_2(-ax^n)}{an^2} + \frac{n\log(-ax\right. + \operatorname{artanh}(ax^n)\log(x)$$

input
```
integrate(arctanh(a*x^n)/x,x, algorithm="maxima")
```

output
```
-1/2*a*n*(log((a*x^n + 1)/a)/(a*n) - log((a*x^n - 1)/a)/(a*n))*log(x) + 1/
2*a*n*((log(a*x^n + 1)*log(x) - log(a*x^n - 1)*log(x))/(a*n) - (n*log(a*x^
n + 1)*log(x) + dilog(-a*x^n))/(a*n^2) + (n*log(-a*x^n + 1)*log(x) + dilog
(a*x^n))/(a*n^2)) + arctanh(a*x^n)*log(x)
```

3.236.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(ax^n)}{x}\,dx = \int \frac{\operatorname{artanh}(ax^n)}{x}\,dx$$

input
```
integrate(arctanh(a*x^n)/x,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x^n)/x, x)
```

3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax^n)}{x}\,dx = \int \frac{\operatorname{atanh}(a\,x^n)}{x}\,dx$$

input
```
int(atanh(a*x^n)/x,x)
```

output
```
int(atanh(a*x^n)/x, x)
```

3.237 $\int \frac{\operatorname{arctanh}(ax^5)}{x} \, dx$

3.237.1 Optimal result

Integrand size = 10, antiderivative size = 24

$$\int \frac{\operatorname{arctanh}(ax^5)}{x} \, dx = -\frac{1}{10}\operatorname{PolyLog}\left(2,-ax^5\right)+\frac{\operatorname{PolyLog}\left(2,ax^5\right)}{10}$$

output
```
-1/10*polylog(2,-a*x^5)+1/10*polylog(2,a*x^5)
```

3.237.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{arctanh}(ax^5)}{x} \, dx = \frac{1}{10}\left(-\operatorname{PolyLog}\left(2,-ax^5\right)+\operatorname{PolyLog}\left(2,ax^5\right)\right)$$

input
```
Integrate[ArcTanh[a*x^5]/x,x]
```

output
```
(-PolyLog[2, -(a*x^5)] + PolyLog[2, a*x^5])/10
```

3.237.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6450, 6446}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{arctanh}(ax^5)}{x} \, dx$$

$$\Big\downarrow \text{6450}$$

$$\frac{1}{5} \int \frac{\text{arctanh}(ax^5)}{x^5} dx^5$$

$$\Big\downarrow \text{6446}$$

$$\frac{1}{5} \left(\frac{\text{PolyLog}(2, ax^5)}{2} - \frac{1}{2} \text{PolyLog}(2, -ax^5) \right)$$

input
```
Int[ArcTanh[a*x^5]/x,x]
```

output
```
(-1/2*PolyLog[2, -(a*x^5)] + PolyLog[2, a*x^5]/2)/5
```

3.237.3.1 Defintions of rubi rules used

rule 6446
```
Int[((a_.) + ArcTanh[(c_.)*(x_)]*(b_.))/(x_), x_Symbol] :> Simp[a*Log[x], x
] + (-Simp[(b/2)*PolyLog[2, (-c)*x], x] + Simp[(b/2)*PolyLog[2, c*x], x]) /
; FreeQ[{a, b, c}, x]
```

rule 6450
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[
1/n   Subst[Int[(a + b*ArcTanh[c*x])^p/x, x], x, x^n], x] /; FreeQ[{a, b, c
, n}, x] && IGtQ[p, 0]
```

3.237.4 Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

method	result
meijerg	$-\frac{i\left(\frac{2 i a x^{5} \operatorname{polylog}\left(2,\sqrt{a^{2} x^{10}}\right)}{\sqrt{a^{2} x^{10}}}-\frac{2 i a x^{5} \operatorname{polylog}\left(2,-\sqrt{a^{2} x^{10}}\right)}{\sqrt{a^{2} x^{10}}}\right)}{20}$
default	$\ln\left(x\right) \operatorname{arctanh}\left(a \,x^{5}\right)-5 a \left(-\frac{\sum_{\_R1 =\operatorname{RootOf}\left(a \_Z^{5}-1\right)}\left(\ln\left(x\right) \ln\left(\frac{\_R1 -x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1 -x}{\_R1}\right)\right)}{10 a}+\right.$
parts	$\ln\left(x\right) \operatorname{arctanh}\left(a \,x^{5}\right)-5 a \left(-\frac{\sum_{\_R1 =\operatorname{RootOf}\left(a \_Z^{5}-1\right)}\left(\ln\left(x\right) \ln\left(\frac{\_R1 -x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1 -x}{\_R1}\right)\right)}{10 a}+\right.$
risch	$\frac{\ln\left(x\right) \ln\left(a \,x^{5}+1\right)}{2}-\frac{\left(\sum_{\_R1 =\operatorname{RootOf}\left(a \_Z^{5}+1\right)}\left(\ln\left(x\right) \ln\left(\frac{\_R1 -x}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1 -x}{\_R1}\right)\right)\right)}{2}-\frac{\ln\left(x\right) \ln\left(-a \,x^{5}+1\right)}{2}+$

input
```
int(arctanh(a*x^5)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/20*I*(2*I*a*x^5/(a^2*x^10)^(1/2)*polylog(2,(a^2*x^10)^(1/2))-2*I*a*x^5/
(a^2*x^10)^(1/2)*polylog(2,-(a^2*x^10)^(1/2)))
```

3.237.5 Fricas [F]

$$\int \frac{\operatorname{arctanh}(ax^5)}{x}\,dx = \int \frac{\operatorname{artanh}\left(ax^5\right)}{x}\,dx$$

input
```
integrate(arctanh(a*x^5)/x,x, algorithm="fricas")
```

output
```
integral(arctanh(a*x^5)/x, x)
```

3.237.6 Sympy [F]

$$\int \frac{\operatorname{arctanh}(ax^5)}{x}\,dx = \int \frac{\operatorname{atanh}(ax^5)}{x}\,dx$$

input
```
integrate(atanh(a*x**5)/x,x)
```

output
```
Integral(atanh(a*x**5)/x, x)
```

3.237.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(18) = 36.

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.33

$$\int \frac{\operatorname{arctanh}(ax^5)}{x}\,dx = -\frac{1}{2}a\left(\frac{\log(ax^5+1)}{a} - \frac{\log(ax^5-1)}{a}\right)\log(x) - \frac{1}{10}a\left(\frac{\log(ax^5-1)\log(ax^5) + \operatorname{Li}_2(-ax^5+1)}{a} - \frac{\log(ax^5+1)\log(-ax^5) + \operatorname{Li}_2(ax^5+1)}{a}\right) + \operatorname{artanh}(ax^5)\log(x)$$

input
```
integrate(arctanh(a*x^5)/x,x, algorithm="maxima")
```

output
```
-1/2*a*(log(a*x^5 + 1)/a - log(a*x^5 - 1)/a)*log(x) - 1/10*a*((log(a*x^5 -
 1)*log(a*x^5) + dilog(-a*x^5 + 1))/a - (log(a*x^5 + 1)*log(-a*x^5) + dilo
g(a*x^5 + 1))/a) + arctanh(a*x^5)*log(x)
```

3.237.8 Giac [F]

$$\int \frac{\operatorname{arctanh}(ax^5)}{x}\,dx = \int \frac{\operatorname{artanh}(ax^5)}{x}\,dx$$

input
```
integrate(arctanh(a*x^5)/x,x, algorithm="giac")
```

output
```
integrate(arctanh(a*x^5)/x, x)
```

3.237.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{arctanh}(ax^5)}{x}\,dx = \int \frac{\operatorname{atanh}(a\,x^5)}{x}\,dx$$

input
```
int(atanh(a*x^5)/x,x)
```

output
```
int(atanh(a*x^5)/x, x)
```

3.238 $\int \operatorname{arctanh}\left(\frac{1}{x}\right) dx$

3.238.1 Optimal result

Integrand size = 4, antiderivative size = 19

$$\int \operatorname{arctanh}\left(\frac{1}{x}\right) dx = x\operatorname{arctanh}\left(\frac{1}{x}\right) + \frac{1}{2}\log\left(1-x^2\right)$$

output
```
x*arctanh(1/x)+1/2*ln(-x^2+1)
```

3.238.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \operatorname{arctanh}\left(\frac{1}{x}\right) dx = x\operatorname{arctanh}\left(\frac{1}{x}\right) + \frac{1}{2}\log\left(-1+x^2\right)$$

input
```
Integrate[ArcTanh[x^(-1)],x]
```

output
```
x*ArcTanh[x^(-1)] + Log[-1 + x^2]/2
```

3.238.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {6436, 795, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{arctanh}\left(\frac{1}{x}\right) dx$$

$$\Big\downarrow \ 6436$$

$$\int \frac{1}{\left(1-\frac{1}{x^2}\right) x}dx + x\operatorname{arctanh}\left(\frac{1}{x}\right)$$

$$\Big\downarrow \ 795$$

$$\int \frac{x}{x^2-1}dx + x\operatorname{arctanh}\left(\frac{1}{x}\right)$$

$$\Big\downarrow \ 240$$

$$x\operatorname{arctanh}\left(\frac{1}{x}\right) + \frac{1}{2}\log\left(1-x^2\right)$$

input
```
Int[ArcTanh[x^(-1)],x]
```

output
```
x*ArcTanh[x^(-1)] + Log[1 - x^2]/2
```

3.238.3.1 Defintions of rubi rules used

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 795
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[x^(m + n*p)*
(b + a/x^n)^p, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && NegQ[n]
```

rule 6436
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a
 + b*ArcTanh[c*x^n])^p, x] - Simp[b*c*n*p   Int[x^n*((a + b*ArcTanh[c*x^n])
^(p - 1)/(1 - c^2*x^(2*n))), x], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[p, 0]
 && (EqQ[n, 1] || EqQ[p, 1])
```

3.238.4 Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

method	result
parallelrisch	$x \operatorname{arctanh}\left(\frac{1}{x}\right)+\ln\left(x-1\right)+\operatorname{arctanh}\left(\frac{1}{x}\right)$
parts	$x \operatorname{arctanh}\left(\frac{1}{x}\right)+\frac{\ln(x-1)}{2}+\frac{\ln(1+x)}{2}$
derivativedivides	$x \operatorname{arctanh}\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}+1\right)}{2}+\frac{\ln\left(\frac{1}{x}-1\right)}{2}-\ln\left(\frac{1}{x}\right)$
default	$x \operatorname{arctanh}\left(\frac{1}{x}\right)+\frac{\ln\left(\frac{1}{x}+1\right)}{2}+\frac{\ln\left(\frac{1}{x}-1\right)}{2}-\ln\left(\frac{1}{x}\right)$
meijerg	$\ln\left(x\right)-\frac{i\pi}{2}-\frac{\ln\left(1-\sqrt{\frac{1}{x^2}}\right)-\ln\left(1+\sqrt{\frac{1}{x^2}}\right)}{2\sqrt{\frac{1}{x^2}}}+\frac{\ln\left(-\frac{1}{x^2}+1\right)}{2}$
risch	$-\frac{x\ln(x-1)}{2}+\frac{x\ln(1+x)}{2}-\frac{i\pi\ \operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}(i(1+x))\operatorname{csgn}\left(\frac{i(1+x)}{x}\right)x}{4}+\frac{i\pi\ \operatorname{csgn}\left(\frac{i}{x}\right)\operatorname{csgn}(i(x-1))\operatorname{csgn}\left(\frac{i(x-1)}{x}\right)x}{4}$

input
```
int(arctanh(1/x),x,method=_RETURNVERBOSE)
```

output
```
x*arctanh(1/x)+ln(x-1)+arctanh(1/x)
```

3.238.5 Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \operatorname{arctanh}\left(\frac{1}{x}\right)\,dx=\frac{1}{2}\,x\log\left(\frac{x+1}{x-1}\right)+\frac{1}{2}\,\log\left(x^2-1\right)$$

input
```
integrate(arctanh(1/x),x, algorithm="fricas")
```

output
```
1/2*x*log((x + 1)/(x - 1)) + 1/2*log(x^2 - 1)
```

3.238.6 Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \operatorname{arctanh}\left(\frac{1}{x}\right) dx = x \operatorname{atanh}\left(\frac{1}{x}\right) + \log(x+1) - \operatorname{atanh}\left(\frac{1}{x}\right)$$

input
```
integrate(atanh(1/x),x)
```

output
```
x*atanh(1/x) + log(x + 1) - atanh(1/x)
```

3.238.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \operatorname{arctanh}\left(\frac{1}{x}\right) dx = x \operatorname{artanh}\left(\frac{1}{x}\right) + \frac{1}{2}\log\left(x^2-1\right)$$

input
```
integrate(arctanh(1/x),x, algorithm="maxima")
```

output
```
x*arctanh(1/x) + 1/2*log(x^2 - 1)
```

3.238.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(17) = 34.

Time = 0.26 (sec) , antiderivative size = 101, normalized size of antiderivative = 5.32

$$\int \operatorname{arctanh}\left(\frac{1}{x}\right) dx = \frac{\log\left(-\frac{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}+1}{\frac{\frac{x+1}{x-1}-1}{\frac{x+1}{x-1}+1}-1}\right)}{\frac{x+1}{x-1}-1} + \log\left(\frac{|x+1|}{|x-1|}\right) - \log\left(\left|\frac{x+1}{x-1}-1\right|\right)$$

input
```
integrate(arctanh(1/x),x, algorithm="giac")
```

output
```
log(-(((x + 1)/(x - 1) - 1)/((x + 1)/(x - 1) + 1) + 1)/(((x + 1)/(x - 1) -
 1)/((x + 1)/(x - 1) + 1) - 1))/((x + 1)/(x - 1) - 1) + log(abs(x + 1)/abs
(x - 1)) - log(abs((x + 1)/(x - 1) - 1))
```

3.238.9 Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.79

$$\int \operatorname{arctanh}\left(\frac{1}{x}\right) dx = \frac{\ln(x^2-1)}{2} + x \operatorname{atanh}\left(\frac{1}{x}\right)$$

input
```
int(atanh(1/x),x)
```

output
```
log(x^2 - 1)/2 + x*atanh(1/x)
```

3.239 $\int (dx)^m (a+b\text{arctanh}(cx^n))^3 \, dx$

3.239.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^3 \, dx = \text{Int}\left((dx)^m (a+b\text{arctanh}(cx^n))^3 , x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

3.239.2 Mathematica [N/A]

Not integrable

Time = 6.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^3 \, dx = \int (dx)^m (a+b\text{arctanh}(cx^n))^3 \, dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^3,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^3, x]
```

3.239.3 Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m (a + b\text{arctanh}(cx^n))^3 \, dx$$

$$\Big\downarrow \; 6468$$

$$\int (dx)^m (a + b\text{arctanh}(cx^n))^3 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x^n])^3,x]
```

output
```
$Aborted
```

3.239.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.239.4 Maple [N/A] (verified)

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m (a + b \,\text{arctanh}\,(c\,x^n))^3 dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x^n))^3,x)
```

3.239.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 65, normalized size of antiderivative = 3.61

$$\int (dx)^m (a + b\text{arctanh}(cx^n))^3 \, dx = \int (b \operatorname{artanh}(cx^n) + a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="fricas")
```

output
```
integral((d*x)^m*b^3*arctanh(c*x^n)^3 + 3*(d*x)^m*a*b^2*arctanh(c*x^n)^2 +
 3*(d*x)^m*a^2*b*arctanh(c*x^n) + (d*x)^m*a^3, x)
```

3.239.6 Sympy [F(-1)]

Timed out.

$$\int (dx)^m (a + b\text{arctanh}(cx^n))^3 \, dx = \text{Timed out}$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x**n))**3,x)
```

output
```
Timed out
```

3.239.7 Maxima [N/A]

Not integrable

Time = 1.66 (sec) , antiderivative size = 468, normalized size of antiderivative = 26.00

$$\int (dx)^m (a + b\text{arctanh}(cx^n))^3 \, dx = \int (b \operatorname{artanh}(cx^n) + a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="maxima")
```

output
```
-1/8*b^3*d^m*x*x^m*log(-c*x^n + 1)^3/(m + 1) + (d*x)^(m + 1)*a^3/(d*(m + 1
)) + integrate(1/8*((b^3*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) - b^3*d^m*(
m + 1)*x^m)*log(c*x^n + 1)^3 + 6*(a*b^2*c*d^m*(m + 1)*e^(m*log(x) + n*log(
x)) - a*b^2*d^m*(m + 1)*x^m)*log(c*x^n + 1)^2 - 3*(2*a*b^2*d^m*(m + 1)*x^m
 - (2*a*b^2*c*d^m*(m + 1) + b^3*c*d^m*n)*e^(m*log(x) + n*log(x)) - (b^3*c*
d^m*(m + 1)*e^(m*log(x) + n*log(x)) - b^3*d^m*(m + 1)*x^m)*log(c*x^n + 1))
*log(-c*x^n + 1)^2 + 12*(a^2*b*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) - a^2
*b*d^m*(m + 1)*x^m)*log(c*x^n + 1) - 3*(4*a^2*b*c*d^m*(m + 1)*e^(m*log(x)
+ n*log(x)) - 4*a^2*b*d^m*(m + 1)*x^m + (b^3*c*d^m*(m + 1)*e^(m*log(x) + n
*log(x)) - b^3*d^m*(m + 1)*x^m)*log(c*x^n + 1)^2 + 4*(a*b^2*c*d^m*(m + 1)*
e^(m*log(x) + n*log(x)) - a*b^2*d^m*(m + 1)*x^m)*log(c*x^n + 1))*log(-c*x^
n + 1))/(c*(m + 1)*x^n - m - 1), x)
```

3.239.8 Giac [N/A]

Not integrable

Time = 0.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^3 \, dx = \int (b\,\text{artanh}\,(cx^n) + a)^3 (dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^n))^3,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)^3*(d*x)^m, x)
```

3.239.9 Mupad [N/A]

Not integrable

Time = 3.44 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^3 \, dx = \int (d\,x)^m\,(a+b\,\text{atanh}(c\,x^n))^3 \, dx$$

input
```
int((d*x)^m*(a + b*atanh(c*x^n))^3,x)
```

output
```
int((d*x)^m*(a + b*atanh(c*x^n))^3, x)
```

3.240 $\int (dx)^m (a+b\,\mathrm{arctanh}(cx^n))^2\,dx$

3.240.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^m (a+b\,\mathrm{arctanh}(cx^n))^2\,dx = \mathrm{Int}\left((dx)^m (a+b\,\mathrm{arctanh}(cx^n))^2, x\right)$$

output
```
Unintegrable((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

3.240.2 Mathematica [N/A]

Not integrable

Time = 14.32 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m (a+b\,\mathrm{arctanh}(cx^n))^2\,dx = \int (dx)^m (a+b\,\mathrm{arctanh}(cx^n))^2\,dx$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^2,x]
```

output
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n])^2, x]
```

3.240.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^2 \, dx$$

$$\big\downarrow \text{ 6468}$$

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^2 dx$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x^n])^2,x]
```

output
```
$Aborted
```

3.240.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.240.4 Maple [N/A] (verified)

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^m \left(a+b\,\text{arctanh}\left(c\,x^n\right)\right)^2 dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x^n))^2,x)
```

3.240.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^2\, dx = \int (b\,\text{artanh}\,(cx^n)+a)^2 (dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m*b^2*arctanh(c*x^n)^2 + 2*(d*x)^m*a*b*arctanh(c*x^n) + (d*
x)^m*a^2, x)
```

3.240.6 Sympy [N/A]

Not integrable

Time = 177.90 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^2\, dx = \int (dx)^m (a+b\,\text{atanh}\,(cx^n))^2\, dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x**n))**2,x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c*x**n))**2, x)
```

3.240.7 Maxima [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 265, normalized size of antiderivative = 14.72

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^2\, dx = \int (b\,\text{artanh}\,(cx^n)+a)^2 (dx)^m\, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/4*b^2*d^m*x*x^m*log(-c*x^n + 1)^2/(m + 1) + (d*x)^(m + 1)*a^2/(d*(m + 1)
) - integrate(-1/4*((b^2*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)) - b^2*d^m*(
m + 1)*x^m)*log(c*x^n + 1)^2 + 4*(a*b*c*d^m*(m + 1)*e^(m*log(x) + n*log(x)
) - a*b*d^m*(m + 1)*x^m)*log(c*x^n + 1) + 2*(2*a*b*d^m*(m + 1)*x^m - (2*a*
b*c*d^m*(m + 1) + b^2*c*d^m*n)*e^(m*log(x) + n*log(x)) - (b^2*c*d^m*(m + 1
)*e^(m*log(x) + n*log(x)) - b^2*d^m*(m + 1)*x^m)*log(c*x^n + 1))*log(-c*x^
n + 1))/(c*(m + 1)*x^n - m - 1), x)
```

3.240.8 Giac [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^2\,dx=\int (b\,\text{artanh}\,(cx^n)+a)^2(dx)^m\,dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)^2*(d*x)^m, x)
```

3.240.9 Mupad [N/A]

Not integrable

Time = 3.64 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^m (a+b\text{arctanh}(cx^n))^2\,dx=\int (d\,x)^m\,(a+b\,\text{atanh}(c\,x^n))^2\,dx$$

input
```
int((d*x)^m*(a + b*atanh(c*x^n))^2,x)
```

output
```
int((d*x)^m*(a + b*atanh(c*x^n))^2, x)
```

3.241 $\int (dx)^m (a + b\text{arctanh}(cx^n))\,dx$

3.241.1 Optimal result

Integrand size = 16, antiderivative size = 84

$$\int (dx)^m (a + b\text{arctanh}(cx^n))\,dx = \frac{x(dx)^m (a + b\text{arctanh}(cx^n))}{1+m} - \frac{bcnx^{1+n}(dx)^m \text{Hypergeometric2F1}\left(1, \frac{1+m+n}{2n}, \frac{1+m+3n}{2n}, c^2x^{2n}\right)}{(1+m)(1+m+n)}$$

output
```
x*(d*x)^m*(a+b*arctanh(c*x^n))/(1+m)-b*c*n*x^(1+n)*(d*x)^m*hypergeom([1, 1
/2*(1+m+n)/n],[1/2*(1+m+3*n)/n],c^2*x^(2*n))/(1+m)/(1+m+n)
```

3.241.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.92

$$\int (dx)^m (a + b\text{arctanh}(cx^n))\,dx = \frac{x(dx)^m \left((1+m+n)(a + b\text{arctanh}(cx^n)) - bcnx^n \text{Hypergeometric2F1}\left(1, \frac{1+m+n}{2n}, \frac{1+m+3n}{2n}, c^2x^{2n}\right)\right)}{(1+m)(1+m+n)}$$

input
```
Integrate[(d*x)^m*(a + b*ArcTanh[c*x^n]),x]
```

output
```
(x*(d*x)^m*((1 + m + n)*(a + b*ArcTanh[c*x^n]) - b*c*n*x^n*Hypergeometric2
F1[1, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), c^2*x^(2*n)]))/((1 + m)*(1 +
 m + n))
```

3.241.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {6466, 6452, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^m (a+b\text{arctanh}(cx^n)) \, dx$$

$$\Big\downarrow \text{ 6466}$$

$$x^{-m}(dx)^m\int x^m(a+b\text{arctanh}(cx^n))dx$$

$$\Big\downarrow \text{ 6452}$$

$$x^{-m}(dx)^m\left(\frac{x^{m+1}(a+b\text{arctanh}(cx^n))}{m+1}-\frac{bcn\int \frac{x^{m+n}}{1-c^2x^{2n}}dx}{m+1}\right)$$

$$\Big\downarrow \text{ 888}$$

$$x^{-m}(dx)^m\left(\frac{x^{m+1}(a+b\text{arctanh}(cx^n))}{m+1}-\frac{bcnx^{m+n+1}\text{Hypergeometric2F1}\left(1,\frac{m+n+1}{2n},\frac{m+3n+1}{2n},c^2x^{2n}\right)}{(m+1)(m+n+1)}\right)$$

input
```
Int[(d*x)^m*(a + b*ArcTanh[c*x^n]),x]
```

output
```
((d*x)^m*((x^(1 + m)*(a + b*ArcTanh[c*x^n]))/(1 + m) - (b*c*n*x^(1 + m + n
)*Hypergeometric2F1[1, (1 + m + n)/(2*n), (1 + m + 3*n)/(2*n), c^2*x^(2*n)
])/((1 + m)*(1 + m + n))))/x^m
```

3.241.3.1 Defintions of rubi rules used

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 6452
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :
> Simp[x^(m + 1)*((a + b*ArcTanh[c*x^n])^p/(m + 1)), x] - Simp[b*c*n*(p/(m
+ 1))   Int[x^(m + n)*((a + b*ArcTanh[c*x^n])^(p - 1)/(1 - c^2*x^(2*n))), x
], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0] && (EqQ[p, 1] || (EqQ[n, 1
] && IntegerQ[m])) && NeQ[m, -1]
```

rule 6466
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_)*(x_))^(m_), x_Sym
bol] :> Simp[d^IntPart[m]*((d*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a +
b*ArcTanh[c*x^n])^p, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IGtQ[p, 0] &
& (EqQ[p, 1] || RationalQ[m, n])
```

3.241.4 Maple [F]

$$\int (dx)^m (a + b \operatorname{arctanh}(c\, x^n))\, dx$$

input
```
int((d*x)^m*(a+b*arctanh(c*x^n)),x)
```

output
```
int((d*x)^m*(a+b*arctanh(c*x^n)),x)
```

3.241.5 Fricas [F]

$$\int (dx)^m (a + b\text{arctanh}(cx^n))\ dx = \int (b\,\text{artanh}\,(cx^n) + a)(dx)^m\ dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

output
```
integral((d*x)^m*b*arctanh(c*x^n) + (d*x)^m*a, x)
```

3.241.6 Sympy [F]

$$\int (dx)^m (a + b\text{arctanh}(cx^n)) \, dx = \int (dx)^m (a + b \operatorname{atanh}(cx^n)) \, dx$$

input
```
integrate((d*x)**m*(a+b*atanh(c*x**n)),x)
```

output
```
Integral((d*x)**m*(a + b*atanh(c*x**n)), x)
```

3.241.7 Maxima [F]

$$\int (dx)^m (a + b\text{arctanh}(cx^n)) \, dx = \int (b \operatorname{artanh}(cx^n) + a)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*(d^m*n*integrate(x^m/(c*(m + 1)*x^n + m + 1), x) + d^m*n*integrate(x^m
/(c*(m + 1)*x^n - m - 1), x) + (d^m*x*x^m*log(c*x^n + 1) - d^m*x*x^m*log(-
c*x^n + 1))/(m + 1))*b + (d*x)^(m + 1)*a/(d*(m + 1))
```

3.241.8 Giac [F]

$$\int (dx)^m (a + b\text{arctanh}(cx^n)) \, dx = \int (b \operatorname{artanh}(cx^n) + a)(dx)^m \, dx$$

input
```
integrate((d*x)^m*(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

output
```
integrate((b*arctanh(c*x^n) + a)*(d*x)^m, x)
```

3.241.9 Mupad [F(-1)]

Timed out.

$$\int (dx)^m (a + b\text{arctanh}(cx^n)) \, dx = \int (d\,x)^m \,(a + b\,\text{atanh}(c\,x^n)) \, dx$$

input
```
int((d*x)^m*(a + b*atanh(c*x^n)),x)
```

output
```
int((d*x)^m*(a + b*atanh(c*x^n)), x)
```

3.242 $\int \frac{(dx)^m}{a+b\mathbf{arctanh}(cx^n)}\,dx$

3.242.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{a+b\,\text{arctanh}\,(cx^n)}\,dx=\text{Int}\left(\frac{(dx)^m}{a+b\,\text{arctanh}\,(cx^n)},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctanh(c*x^n)),x)
```

3.242.2 Mathematica [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\,\text{arctanh}\,(cx^n)}\,dx=\int \frac{(dx)^m}{a+b\,\text{arctanh}\,(cx^n)}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n]),x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n]), x]
```

3.242.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(cx^n\right)} \, dx$$
$$\Big\downarrow \text{ 6468}$$
$$\int \frac{(dx)^m}{a+b\text{arctanh}\left(cx^n\right)}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTanh[c*x^n]),x]
```

output
```
$Aborted
```

3.242.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.242.4 Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{a+b\,\text{arctanh}\left(c\,x^n\right)}dx$$

input
```
int((d*x)^m/(a+b*arctanh(c*x^n)),x)
```

output
```
int((d*x)^m/(a+b*arctanh(c*x^n)),x)
```

3.242.5 Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\text{arctanh}\,(cx^n)}\,dx = \int \frac{(dx)^m}{b\,\text{artanh}\,(cx^n) + a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

3.242.6 Sympy [N/A]

Not integrable

Time = 34.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^m}{a + b\text{arctanh}\,(cx^n)}\,dx = \int \frac{(dx)^m}{a + b\,\text{atanh}\,(cx^n)}\,dx$$

input
```
integrate((d*x)**m/(a+b*atanh(c*x**n)),x)
```

output
```
Integral((d*x)**m/(a + b*atanh(c*x**n)), x)
```

3.242.7 Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a + b\text{arctanh}\,(cx^n)}\,dx = \int \frac{(dx)^m}{b\,\text{artanh}\,(cx^n) + a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="maxima")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

3.242.8 Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^n)}\,dx=\int \frac{(dx)^m}{b\,\text{artanh}\,(cx^n)+a}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^n)),x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x^n) + a), x)
```

3.242.9 Mupad [N/A]

Not integrable

Time = 3.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{a+b\text{arctanh}(cx^n)}\,dx=\int \frac{(d\,x)^m}{a+b\,\text{atanh}\,(c\,x^n)}\,dx$$

input
```
int((d*x)^m/(a + b*atanh(c*x^n)),x)
```

output
```
int((d*x)^m/(a + b*atanh(c*x^n)), x)
```

3.243 $\int \frac{(dx)^m}{\left(a+b\text{arctanh}(cx^n)\right)^2}\,dx$

3.243.1 Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^n))^2}\,dx = \text{Int}\left(\frac{(dx)^m}{(a+b\text{arctanh}(cx^n))^2},x\right)$$

output
```
Unintegrable((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

3.243.2 Mathematica [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^n))^2}\,dx = \int \frac{(dx)^m}{(a+b\text{arctanh}(cx^n))^2}\,dx$$

input
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n])^2,x]
```

output
```
Integrate[(d*x)^m/(a + b*ArcTanh[c*x^n])^2, x]
```

3.243.3 Rubi [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {6468}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^n))^2} dx$$

$$\Big\downarrow \text{6468}$$

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^n))^2}dx$$

input
```
Int[(d*x)^m/(a + b*ArcTanh[c*x^n])^2,x]
```

output
```
$Aborted
```

3.243.3.1 Defintions of rubi rules used

rule 6468
```
Int[((a_.) + ArcTanh[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_S
ymbol] :> Unintegrable[(d*x)^m*(a + b*ArcTanh[c*x^n])^p, x] /; FreeQ[{a, b,
 c, d, m, n, p}, x]
```

3.243.4 Maple [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^m}{(a+b\,\text{arctanh}\,(c\,x^n))^2}dx$$

input
```
int((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

output
```
int((d*x)^m/(a+b*arctanh(c*x^n))^2,x)
```

3.243.5 Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.00

$$\int \frac{(dx)^m}{(a + b\text{arctanh}\,(cx^n))^2}\,dx = \int \frac{(dx)^m}{(b\,\text{artanh}\,(cx^n) + a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="fricas")
```

output
```
integral((d*x)^m/(b^2*arctanh(c*x^n)^2 + 2*a*b*arctanh(c*x^n) + a^2), x)
```

3.243.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(dx)^m}{(a + b\text{arctanh}\,(cx^n))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x)**m/(a+b*atanh(c*x**n))**2,x)
```

output
```
Timed out
```

3.243.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 169, normalized size of antiderivative = 9.39

$$\int \frac{(dx)^m}{(a + b\text{arctanh}\,(cx^n))^2}\,dx = \int \frac{(dx)^m}{(b\,\text{artanh}\,(cx^n) + a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="maxima")
```

output
```
2*(c^2*d^m*x*e^(m*log(x) + 2*n*log(x)) - d^m*x*x^m)/(b^2*c*n*x^n*log(c*x^n
 + 1) - b^2*c*n*x^n*log(-c*x^n + 1) + 2*a*b*c*n*x^n) + integrate(-2*(c^2*d
^m*(m + n + 1)*e^(m*log(x) + 2*n*log(x)) - d^m*(m - n + 1)*x^m)/(b^2*c*n*x
^n*log(c*x^n + 1) - b^2*c*n*x^n*log(-c*x^n + 1) + 2*a*b*c*n*x^n), x)
```

3.243.8 Giac [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^n))^2}\,dx = \int \frac{(dx)^m}{(b\,\text{artanh}\,(cx^n)+a)^2}\,dx$$

input
```
integrate((d*x)^m/(a+b*arctanh(c*x^n))^2,x, algorithm="giac")
```

output
```
integrate((d*x)^m/(b*arctanh(c*x^n) + a)^2, x)
```

3.243.9 Mupad [N/A]

Not integrable

Time = 3.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^m}{(a+b\text{arctanh}(cx^n))^2}\,dx = \int \frac{(d\,x)^m}{(a+b\,\text{atanh}\,(c\,x^n))^2}\,dx$$

input
```
int((d*x)^m/(a + b*atanh(c*x^n))^2,x)
```

output
```
int((d*x)^m/(a + b*atanh(c*x^n))^2, x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):    #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
      return expnType(expn.args[0])    #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
    m1 = expnType(expn.args[0])
    m2 = expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
    return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
  elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
  elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
    return max(7,m1)
  elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).___name___ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```